Towards a ppm measurement of the magnetic moment of the tau with polarized beams at SuperKEKB

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SuperKEKB e^- Polarization Upgrade

based on: A. Crivellin, MH, J. Michael Roney arXiv:2111.10378

Searching for physics beyond the SM with $(g-2)_\ell$

- **Muon** Bennett et al. 2006, Abi et al. 2021, Aoyama et al. 2020

$$a_\mu^{\text{exp}} = 116{,}592{,}061(41)\times 10^{-11} \quad vs. \quad a_\mu^{\text{SM}} = 116{,}591{,}810(43)\times 10^{-11}$$

$$\hookrightarrow a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 251(59)\times 10^{-11}[4.2\sigma]$$

- **Electron** Hanneke et al. 2008, Parker et al. 2018, Morel et al. 2020

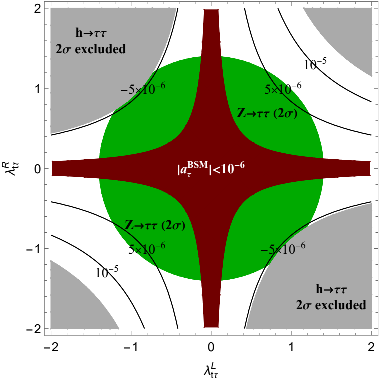
$$a_e^{\text{exp}} = 1{,}159{,}652{,}180.73(28)\times 10^{-12} \quad vs. \quad \begin{Bmatrix} a_e^{\text{SM}}[\text{Cs}] = 1{,}159{,}652{,}181.61(23)\times 10^{-12} \\ a_e^{\text{SM}}[\text{Rb}] = 1{,}159{,}652{,}180.25(9)\times 10^{-12} \end{Bmatrix}$$

$$\hookrightarrow a_e^{\text{exp}} - a_e^{\text{SM}} = \begin{Bmatrix} -0.88(36)\times 10^{-12}[-2.5\sigma] \\ 0.48(30)\times 10^{-12}[1.6\sigma] \end{Bmatrix}$$

- **Tau** Abdallah et al. 2004, Keshavarzi et al. 2020

$$a_\tau^{\text{exp}} = -0.018(17) \quad vs. \quad a_\tau^{\text{SM}} = 1{,}177.171(39)\times 10^{-6}$$

- **Scaling arguments**:
  - Minimal flavor violation:
    $a_\tau^{\text{BSM}} \simeq a_\mu^{\text{BSM}} \left(\frac{m_\tau}{m_\mu}\right)^2 \simeq 0.7 \times 10^{-6}$
  - Electroweak contribution: $a_\tau^{\text{EW}} \simeq 0.5 \times 10^{-6}$
- **Concrete models**:
  - S_1 leptoquark model promising due to **chiral enhancement** with $\frac{m_t}{m_\tau}$
    $\hookrightarrow$ can get $a_\tau^{\text{BSM}} \simeq$ (few) $\times 10^{-6}$ without violating $h \to \tau\tau$ and $Z \to \tau\tau$



The ultimate target has to be a measurement of a_τ at the level of 10^{-6}!

How can we get to 10^{-6}?

- Proposals to measure a_τ include:
  - Radiative τ decays Eidelman et al. 2016
  - Channeling in a bent crystal Fomin et al. 2018, Fu et al. 2019, ...
  - γp or heavy-ion reactions at LHC Koksal et al. 2017, Gutiérrez-Rodríguez et al. 2019, Beresford et al. 2019, Dyndal et al. 2020, ...
  - ...

  $\hookrightarrow$ none of these seem to reach much beyond the Schwinger term at 10^{-3}
- Exception: **$e^+e^- \to \tau^+\tau^-$ at Υ resonances** Bernabéu et al. 2007

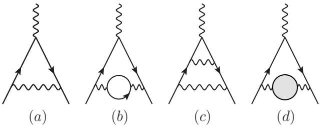
  $\hookrightarrow$ quotes projections at 10^{-6} level
- This talk: what would it take to actually make this idea work?
- Note: measure $F_2(s)$ at $s \simeq (10\,\text{GeV})^2$, but heavy new physics decouples

  $\hookrightarrow$ $a_\tau^{\text{BSM}} = F_2^{\text{exp}}(s) - F_2^{\text{SM}}(s)$ as long as $s \ll \Lambda_{\text{BSM}}^2$
- Bounds on light BSM become model dependent, but anyway better constrained in other processes

A look at the SM prediction for $F_2(s)$

	$s = 0$	$s = (10\,\text{GeV})^2$
1-loop QED	1161.41	$-265.90 + 246.48i$
e loop	10.92	$-2.43 + 2.95i$
μ loop	1.95	$-0.34 + 0.92i$
τ loop	0.08	$0.06 + 0.07i$
2-loop QED (mass independent, incl. τ loop)	-1.77	IR divergent
sum QED	1172.51	IR divergent
HVP	3.33	$-0.33 + 1.93i$
sum of the above	1175.84	
QED (incl. 3-loop) Eidelman et al. 2016	1173.24(2)	
HVP Keshavarzi et al. 2020	3.328(14)	
EW Eidelman et al. 2016	0.474(5)	
total Keshavarzi et al. 2020	1177.171(39)	

Need two-loop precision to reach 10^{-6}!



- All in units of 10^{-6}
- $a_\tau = F_2(0)$
- In $e^+e^- \to \tau^+\tau^-$: measure $F_2(s)$, $s \simeq (10\,\text{GeV})^2$
- Form factors: $\langle p'|j^\mu|p\rangle = e\bar{u}(p')\Big[\gamma^\mu F_1(s) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(s)\Big]u(p)$
- IR divergences to cancel with bremsstrahlung

First attempt: total cross section

- **Differential cross section** for $e^+e^- \to \tau^+\tau^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\left[(2-\beta^2\sin^2\theta)\left(|F_1|^2-\gamma^2|F_2|^2\right)+4\text{Re}\left(F_1F_2^*\right)+2(1+\gamma^2)|F_2|^2\right]$$

with scattering angle θ, $\beta = \sqrt{1-4m_\tau^2/s}$, $\gamma = \sqrt{s}/(2m_\tau)$

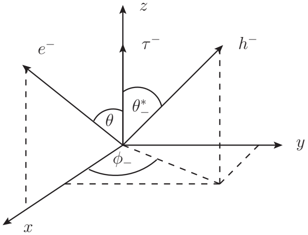
- Interference term $4\text{Re}\left(F_1F_2^*\right)$ sensitive to the sought two-loop effects
- Could be determined by fit to θ dependence
- But: need to measure total cross section at 10^{-6}

  $\hookrightarrow$ **can we use asymmetries instead**?
- Usual forward–backward asymmetry ($z = \cos\theta$)

$$\sigma_{\text{FB}} = 2\pi\left[\int_0^1 dz\frac{d\sigma}{d\Omega} - \int_{-1}^0 dz\frac{d\sigma}{d\Omega}\right]$$

alone does not help

Second attempt: normal asymmetry



- Idea: use **polarization information of the $\tau^\pm$**

  $\hookrightarrow$ semileptonic decays $\tau^\pm \to h^\pm \overset{(-)}{\nu}_\tau$, $h = \pi, \rho, \ldots$

  Bernabéu et al. 2007

- Polarization characterized by

$$\mathbf{n}^*_\pm = \mp\alpha_\pm \begin{pmatrix} \sin\theta^*_\pm \cos\phi_\pm \\ \sin\theta^*_\pm \sin\phi_\pm \\ \cos\theta^*_\pm \end{pmatrix} \qquad \alpha_\pm \equiv \frac{m_\tau^2 - 2m_{h^\pm}^2}{m_\tau^2 + 2m_{h^\pm}^2} = \begin{cases} 0.97 & h^\pm = \pi^\pm \\ 0.46 & h^\pm = \rho^\pm \end{cases}$$

  $\hookrightarrow$ angles in $\tau^\pm$ rest frame

- **Normal asymmetry**

$$A_N^\pm = \frac{\sigma_L^\pm - \sigma_R^\pm}{\sigma} \propto \operatorname{Im} F_2(s) \qquad \sigma_L^\pm = \int_\pi^{2\pi} d\phi_\pm \frac{d\sigma_{\text{FB}}}{d\phi_\pm} \quad \sigma_R^\pm = \int_0^\pi d\phi_\pm \frac{d\sigma_{\text{FB}}}{d\phi_\pm}$$

  $\hookrightarrow$ only get the imaginary part, **need electron polarization**

Third attempt: electron polarization

- **Transverse and longitudinal asymmetries** Bernabéu et al. 2007

$$A_T^{\pm} = \frac{\sigma_R^{\pm} - \sigma_L^{\pm}}{\sigma} \qquad A_L^{\pm} = \frac{\sigma_{\text{FB},\,R}^{\pm} - \sigma_{\text{FB},\,L}^{\pm}}{\sigma}$$

- Constructed based on helicity difference

$$d\sigma^S_{\text{pol}} = \frac{1}{2}\Big(d\sigma^{S\lambda}\big|_{\lambda=1} - d\sigma^{S\lambda}\big|_{\lambda=-1}\Big)$$

and then integrating over angles

$$\sigma_R^{\pm} = \int_{-\pi/2}^{\pi/2} d\phi_{\pm} \frac{d\sigma^S_{\text{pol}}}{d\phi_{\pm}} \qquad \sigma_L^{\pm} = \int_{\pi/2}^{3\pi/2} d\phi_{\pm} \frac{d\sigma^S_{\text{pol}}}{d\phi_{\pm}} \qquad \sigma_{\text{FB},\,R}^{\pm} = \int_0^1 dz_{\pm}^* \frac{d\sigma^S_{\text{FB,pol}}}{dz_{\pm}^*} \qquad \sigma_{\text{FB},\,L}^{\pm} = \int_{-1}^0 dz_{\pm}^* \frac{d\sigma^S_{\text{FB,pol}}}{dz_{\pm}^*}$$

- Linear combination

$$A_T^{\pm} - \frac{\pi}{2\gamma} A_L^{\pm} = \mp\alpha_{\pm} \frac{\pi^2\alpha^2\beta^3\gamma}{4s\sigma}\Big[\text{Re}\,(F_2F_1^*) + |F_2|^2\Big]$$

isolates the interesting interference effect

How to make use of this?

Contributions to $\text{Re}\, F_2^{\text{eff}}(s)$	$s = 0$	$s = (10\,\text{GeV})^2$
1-loop QED	1161.41	−265.90
e loop	10.92	−2.43
μ loop	1.95	−0.34
2-loop QED (mass independent)	−0.42	−0.24
HVP	3.33	−0.33
EW	0.47	0.47
total	1177.66	−268.77

$$\text{Re}\, F_2^{\text{eff}}((10\,\text{GeV})^2) \simeq \mp \frac{0.73}{\alpha_\pm}\Big(A_T^\pm - 0.56 A_L^\pm\Big)$$

- **Strategy**:
  - Measure effective $F_2(s)$

$$\text{Re}\, F_2^{\text{eff}} = \mp \frac{8(3-\beta^2)}{3\pi\gamma\beta^2\alpha_\pm}\Big(A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm\Big)$$

  - Compare measurement to SM prediction for $\text{Re}\, F_2^{\text{eff}}$
  - Difference gives constraint on a_τ^{BSM}
  - A measurement of $A_T^\pm - \frac{\pi}{2\gamma}A_L^\pm$ at $\lesssim 1\%$ would already be competitive with current limits

How to make use of this?

- **Challenges**:
  - Cancellation in $A_T^{\pm} - \frac{\pi}{2\gamma} A_L^{\pm}$: $A_{T,L}^{\pm} = \mathcal{O}(1)$, difference $\mathcal{O}(\alpha)$
  - Two-loop calculation in SM see 2111.10378 for form factor and radiative corrections
  - Form factor only dominates for resonant $\tau^+\tau^-$ pairs

$$|H(M_\Upsilon)|^2 = \left(\frac{3}{\alpha}\text{Br}(\Upsilon \to e^+e^-)\right)^2 \simeq 100$$

  - However: continuum pairs dominate even at $\Upsilon(nS)$, $n = 1, 2, 3$, due to energy spread
  - Should consider $A_T^{\pm}$, $A_L^{\pm}$ also for nonresonant $\tau^+\tau^-$, but requires substantial investment in theory for SM prediction (box diagrams, ...)

Conclusions

- Realistic BSM test with a_τ requires 10^{-6} precision
- One possible strategy: $e^+e^- \to \tau^+\tau^-$, but absolute cross section measurement likely prohibitively difficult
- Alternative: **asymmetries using polarized electrons**
- Theoretically clean on $\Upsilon(nS)$, $n = 1, 2, 3$, but
  - Limited energy resolution
  - Limited statistics
- **Nonresonant data**
  - Massive increase in statistics
  - Could be used to constrain continuum $\tau^+\tau^-$ to isolate resonant pairs
  - Substantial investment in theory required for full two-loop prediction