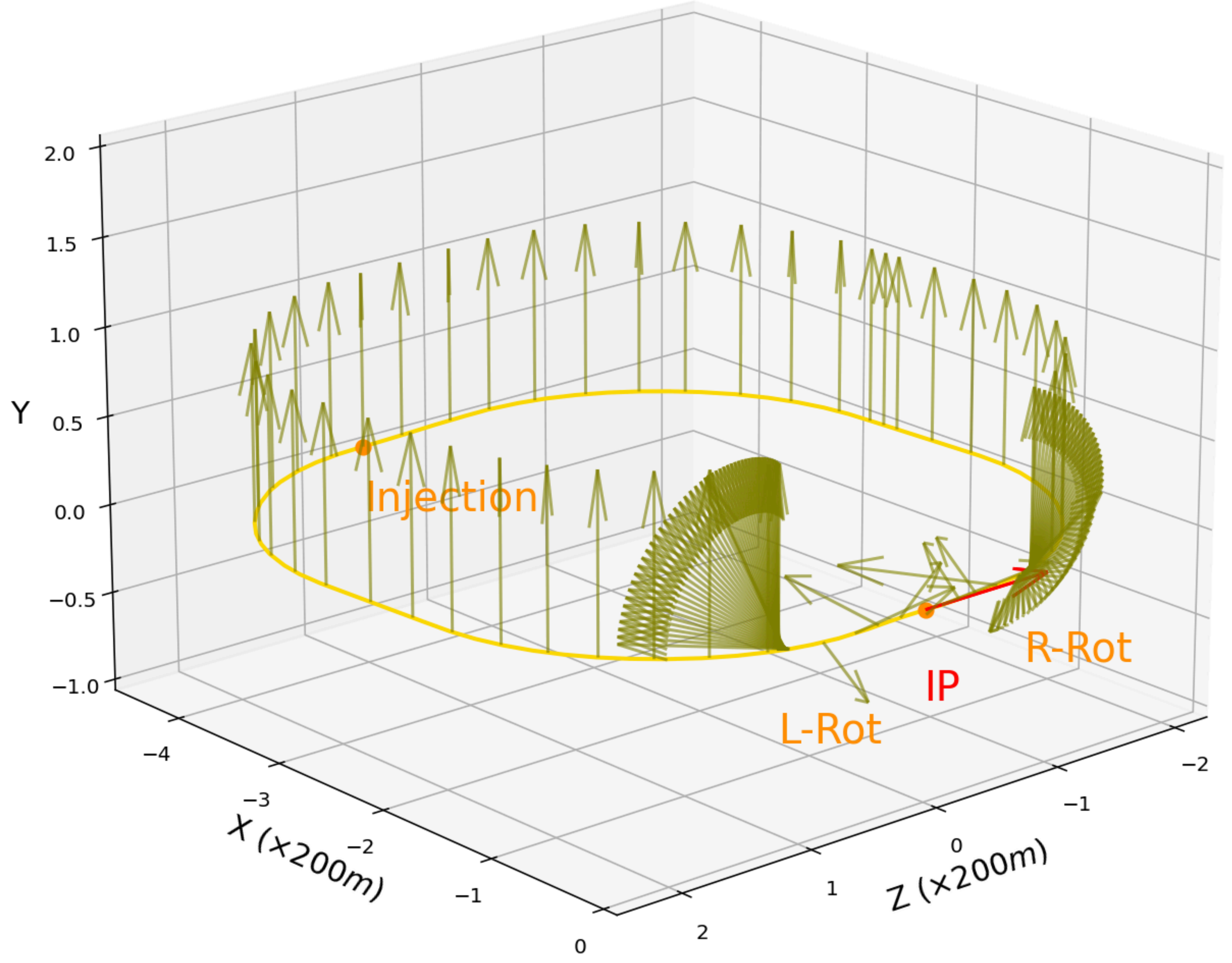
Spin Rotator Conceptual Design for SuperKEKB e-polarization Upgrade



Yuhao Peng

directed by Uli Wienands

2022.01.25

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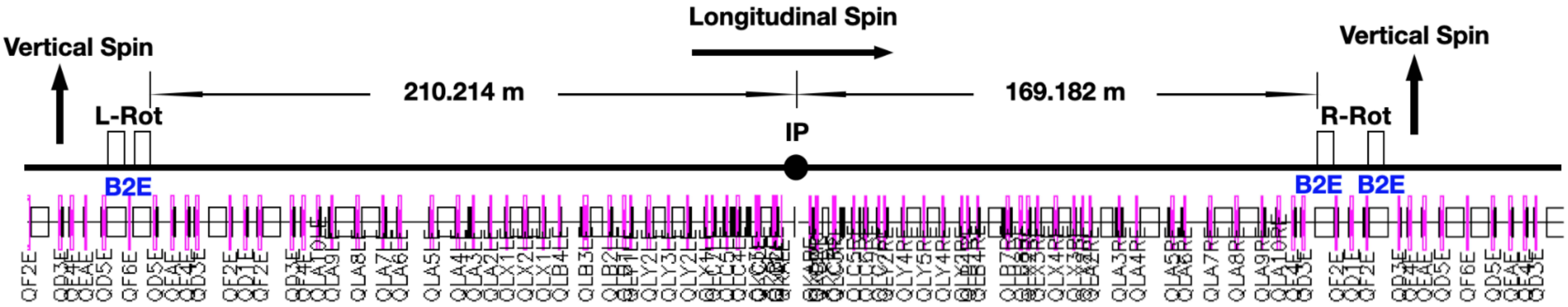
Overview of the Rot Design



- Replacing 4 B2E dipoles with rotator magnets (dipole-sol-quad)
- A pair of rotator (L/R-Rot) rotates the spin back and forth between the horizontal plane and the vertical direction

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Rot Design

- Rotator modelling with BMAD (proposed by Uli Wienands and David Sagan)

- Open-geometry optimization (Lattice segment)
  - Fit Solenoids to achieve the spin polarization
  - Fit skew-quads and ring quads to perform decoupling and optical rematch

- Closed-geometry optimization (full lattice)
  - Match the Tunes by tuning quads in the straight section
  - Match the Linear Chromaticity by tuning ring sextupoles

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Open-geometry Optimization

Replace B2E with Rotator lattice elements, and perform the optimization in the lattice segment containing the L-Rot and nearby elements, and repeat the same procedure for the R-Rot

- Fit the solenoids to match the spin
- Fit the skew-quads to perform decoupling
- Tune the ring quads near the rotator region to achieve the optical rematch

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Optimization Result

- Solenoid strength, below 5T

Rotator Manget	Toy Model	BMAD
B2EALSQ	-5.2672 T	-4.8431 T
B2EBLSQ	-2.2275 T	-2.5774 T

L-Rot

Rotator Manget	Toy Model	BMAD
B2EARSQ	-3.1649 T	-3.6084 T
B2EBRSQ	-4.5554 T	-3.9420 T

R-Rot

- Maximum skew-quad strength is ~20 T/m, which is below the technical limit we have assumed, 30 T/m

- Maximum Ring quad is ~ 14 T/m which is about the same as the strongest existing quad in the ring

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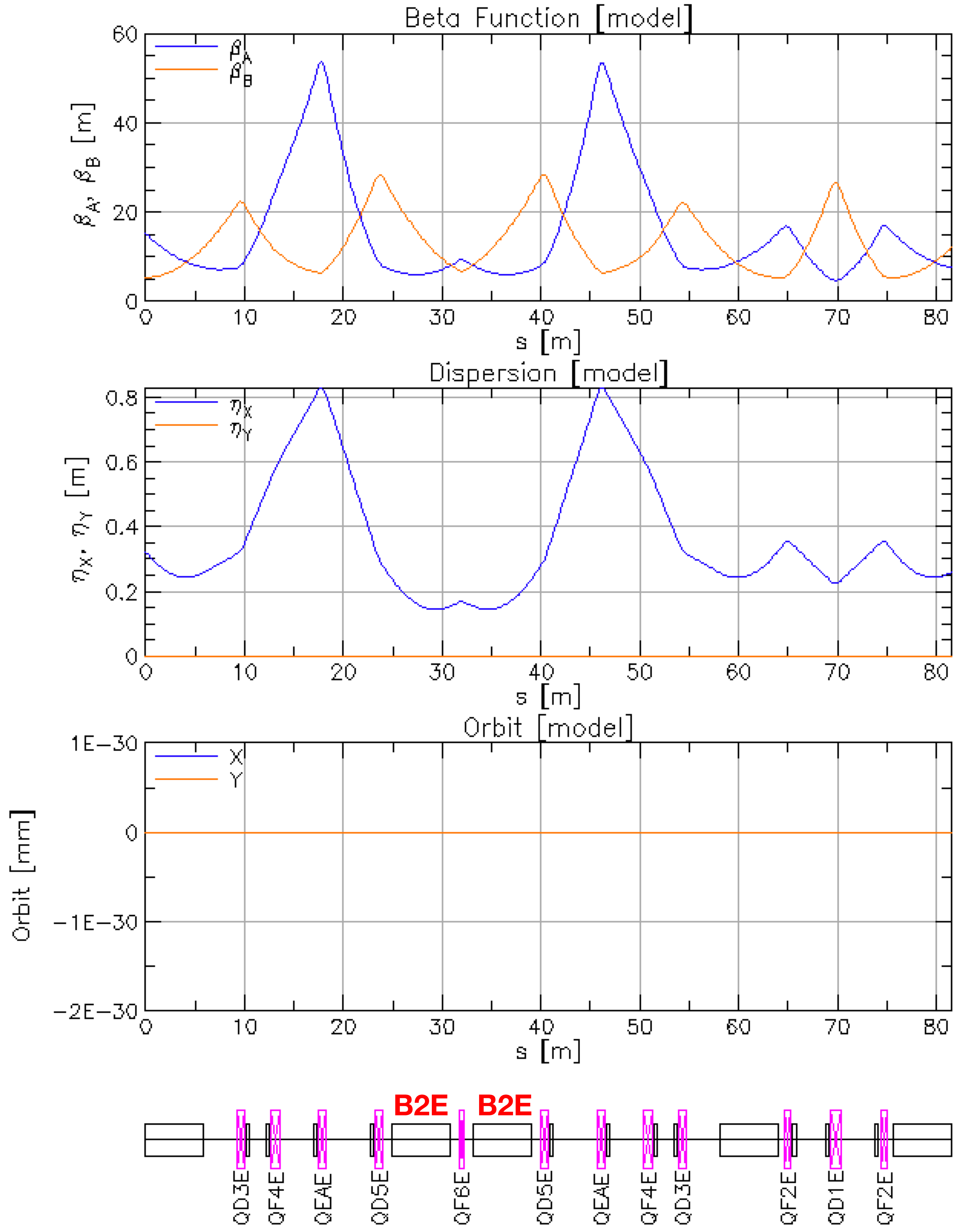
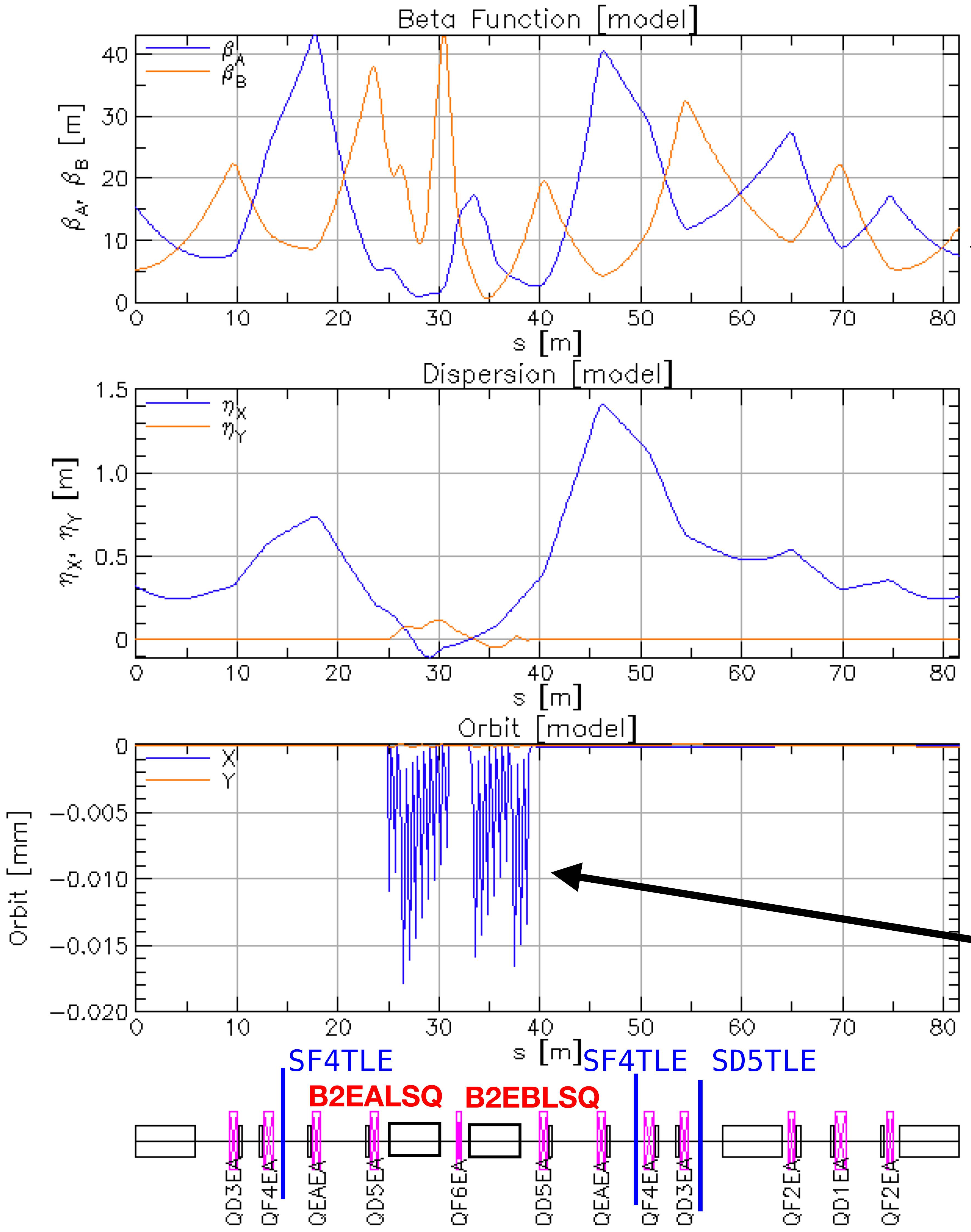
Comparison at L-Rot Region after Completing the Optical Rematch



Original



L-Rot

Optical functions are matched to the original at the exit

Artificial effect of the BMAD rotator modelling

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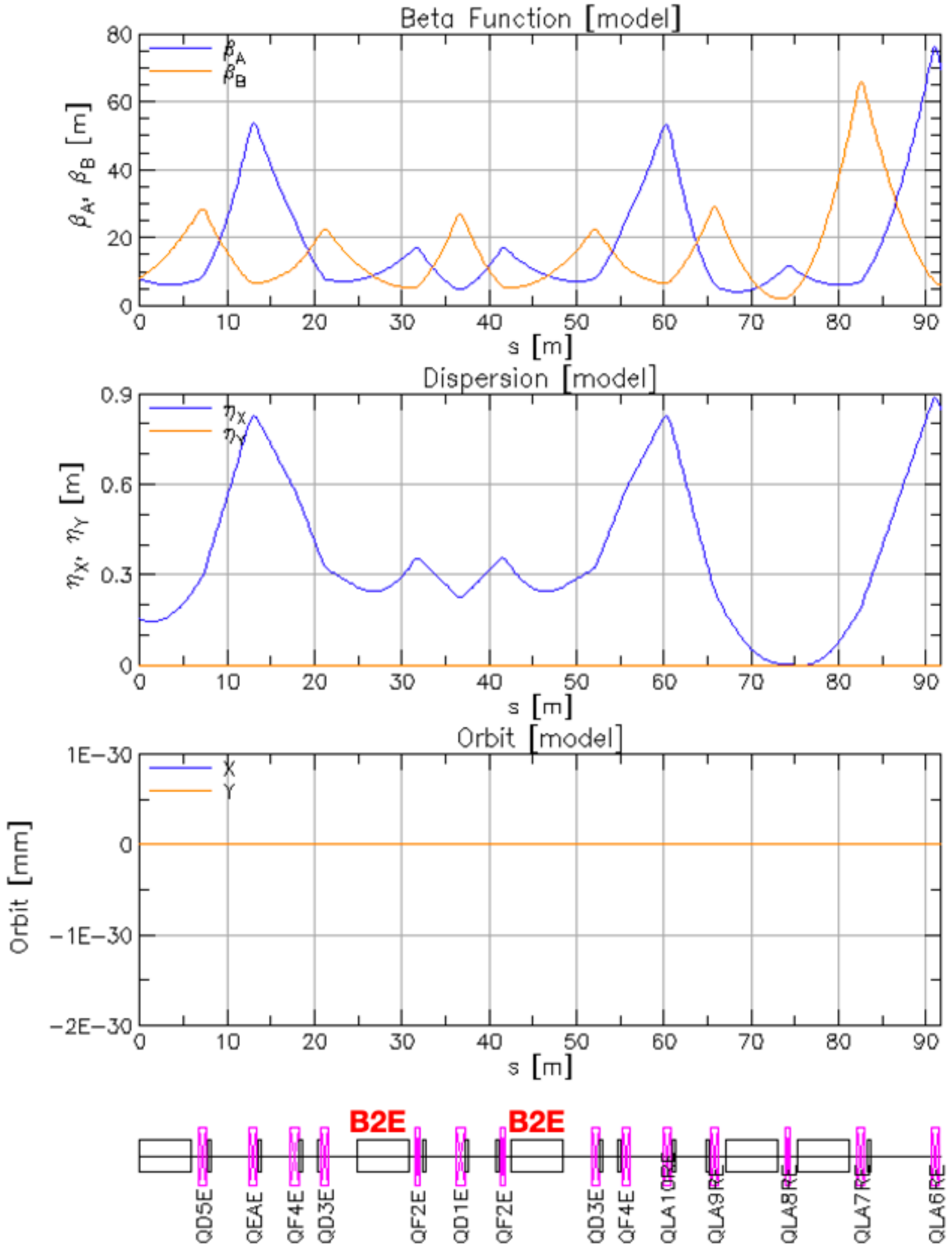
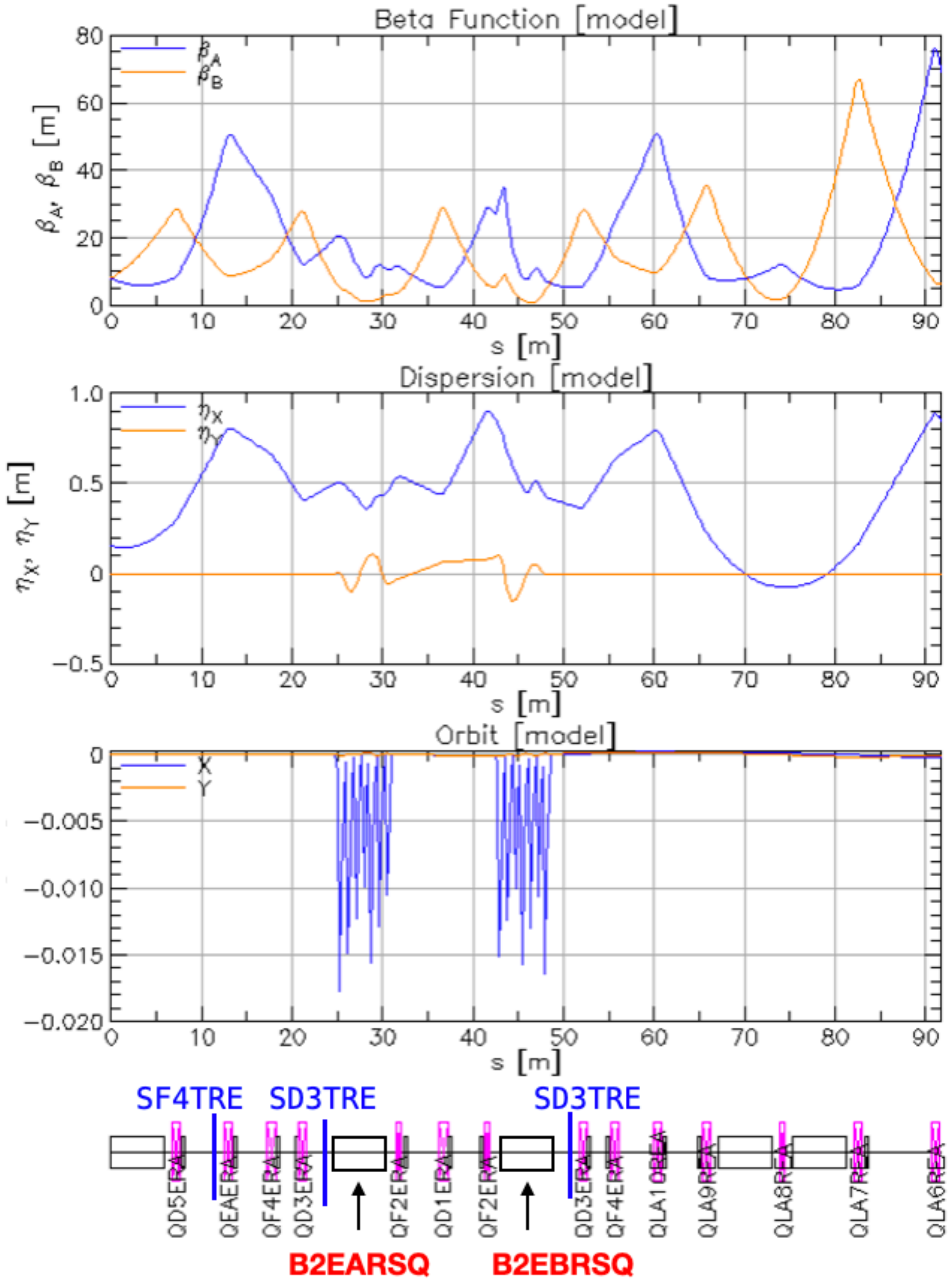
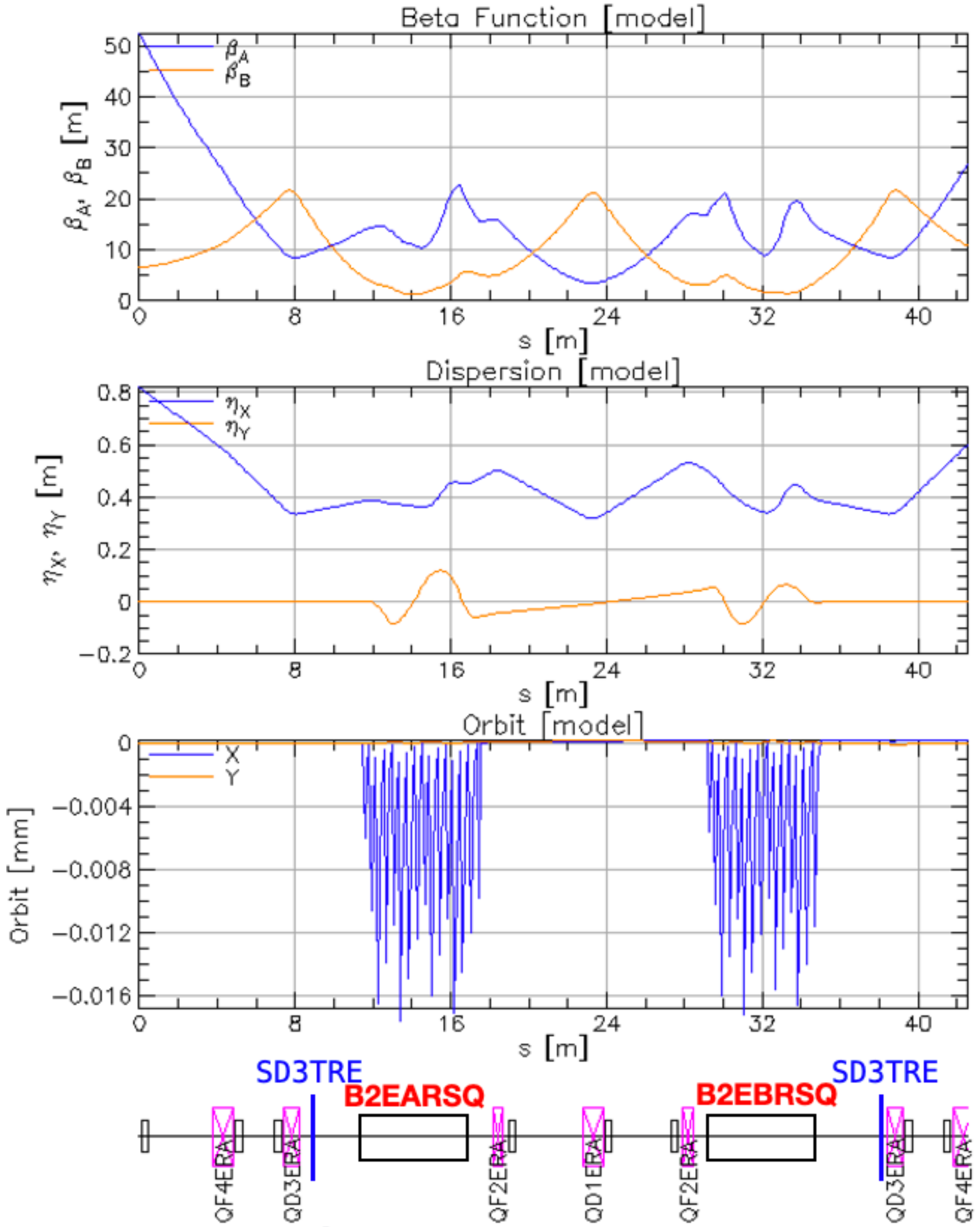
Comparison at R-Rot Region after completing the Optical Rematch



Original



R-Rot



Alternative R-Rot

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Closed-geometry Optimization

- Tune $Q \equiv \frac{\Delta\Psi}{2\pi} = \frac{1}{2\pi}\oint \frac{ds}{\beta(s)}$

- Chromaticity $\xi \equiv \frac{\Delta Q}{\Delta p/p} = -\frac{1}{4\pi}\oint k(s)\beta(s)ds$

They are overall ring (circular machine) parameters, can only be calculated when it's closed-geometry

Performing the optimization in the closed-geometry is challenging

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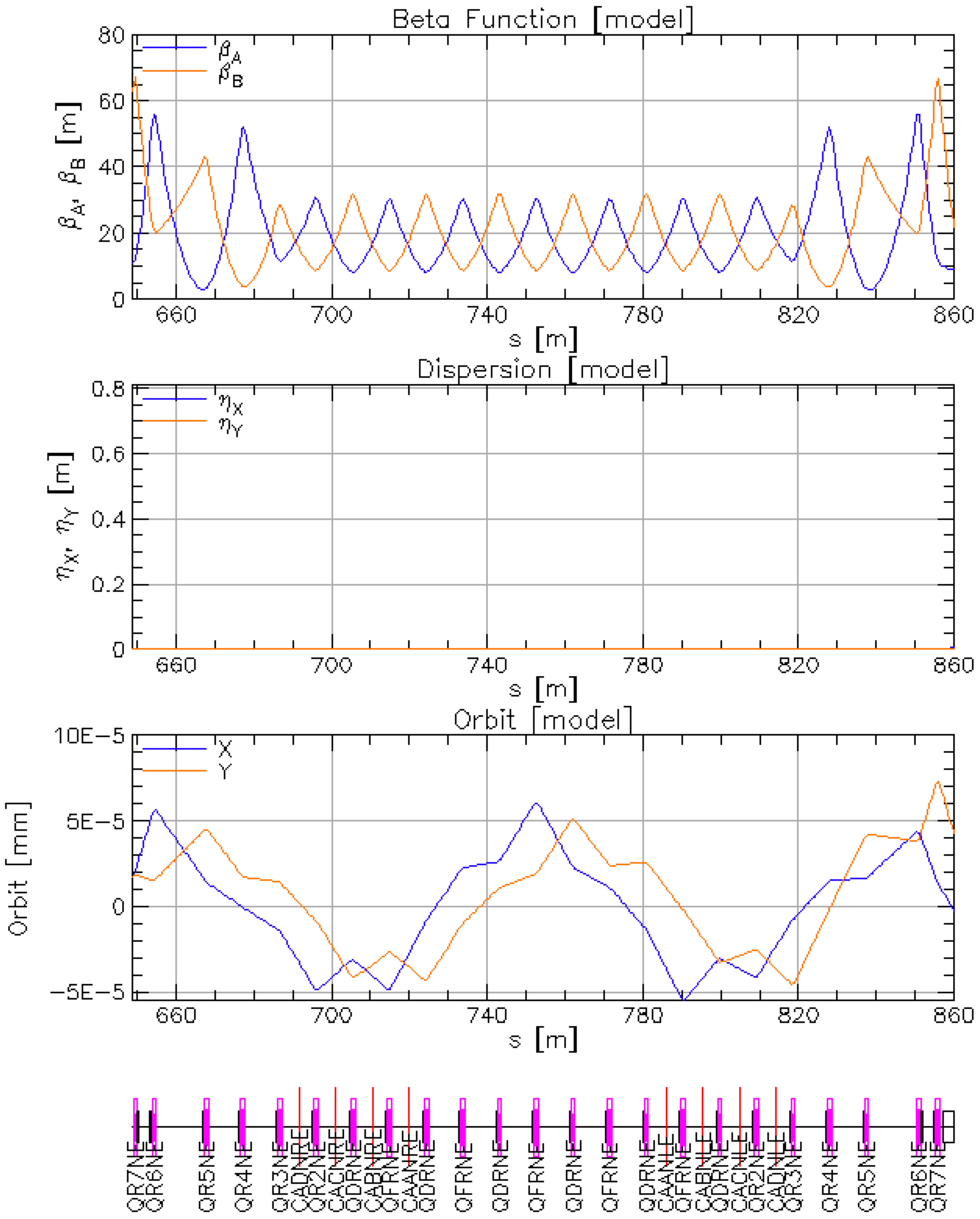
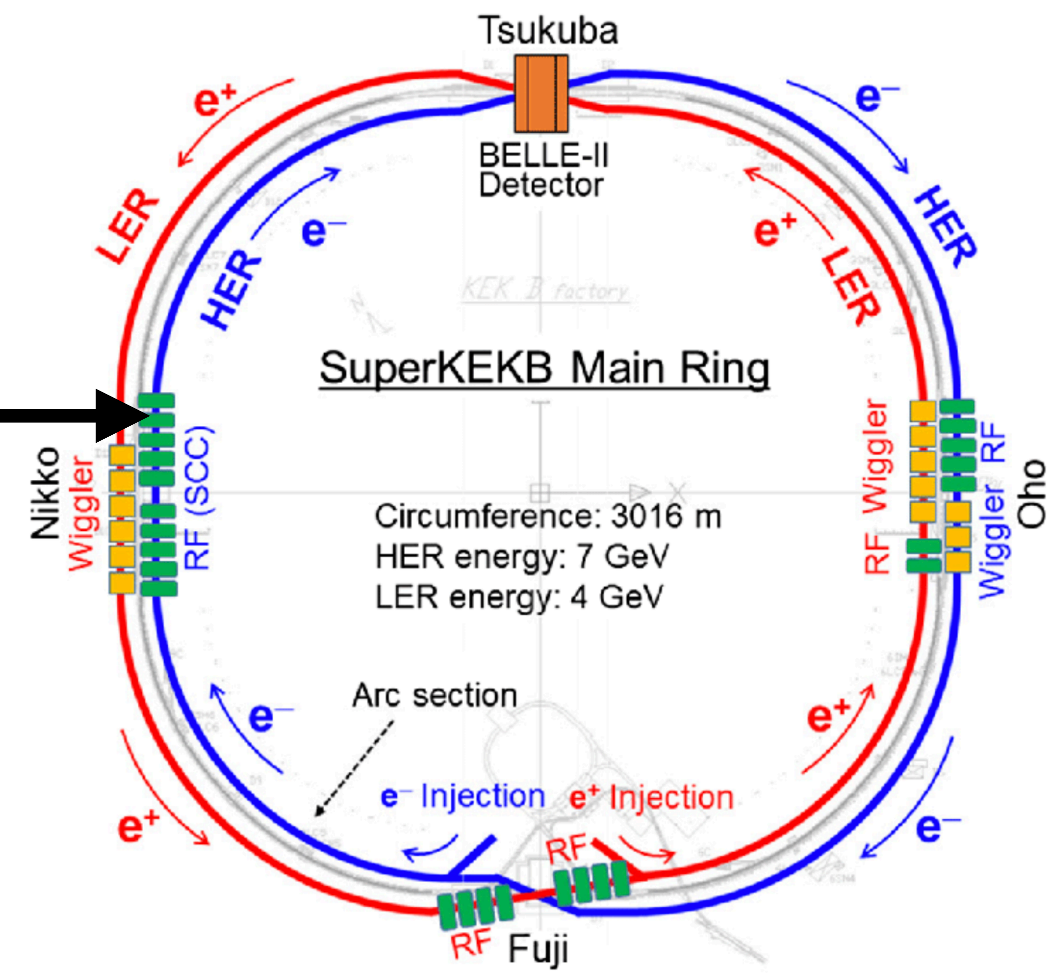
Tune Match





Fitting target: Q_x, Q_y

Constraints: Matching the Twiss parameters at the exit of the straight area ($\beta_{x,y}$, $\alpha_{x,y}$)

8 variables: QR*NE(6 different Quadruopole pairs), QDRNE, QFRNE

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It is challenging to match the Tune and optical functions at the same time using the LMDIF optimizer

For small quadrupole deviation Δk, the Tune shift is given by:

$$4\pi dQ = \Delta k \beta ds$$

The total Tune shift can be approximated by:

$$\Delta Q \sim \frac{1}{4\pi} \sum_i \beta_i \Delta k_i L_i$$

It can be seen that if the quadrupole deviation is sufficiently small, the Tune can be slightly shifted without changing the beta function too much

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Ladder Method

Taking steps to approach the original Tunes

Step	0th	1st	2nd	3rd	...	15th
Q_x	45.777566	45.761128	45.744690	45.728252	...	45.530994
Q_y	44.446774	44.389036	44.331299	44.273561	...	43.580709

In this approach, alphas are not matched due to the difficulty matching the beta and alpha simultaneously

the variation of alpha is small even though it is not involved in the optimization

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Quads adjusted to match the Tune

Quadrupole	Length (m)	k_1 (m^{-2}) original	k_1 (m^{-2}) Rot
QFRNE	1.080	0.122	0.099
QDRNE	1.080	-0.118	-0.085
QR7NE	0.826	-0.252	-0.249
QR6NE	1.015	0.196	0.202
QR5NE	1.080	-0.110	-0.091
QR4NE	1.080	0.144	0.127
QR3NE	1.080	-0.145	-0.071
QR2NE	1.080	0.110	0.067

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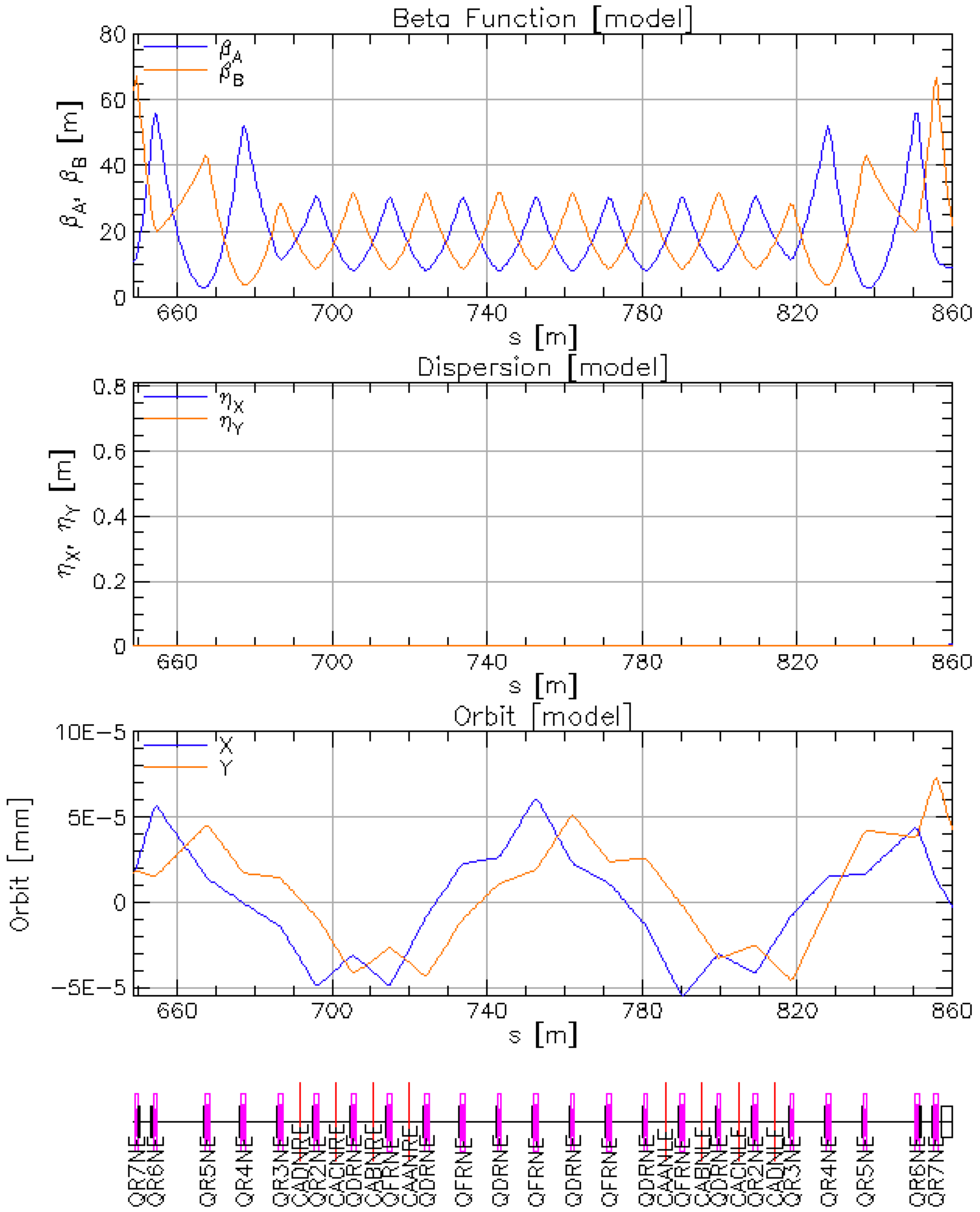
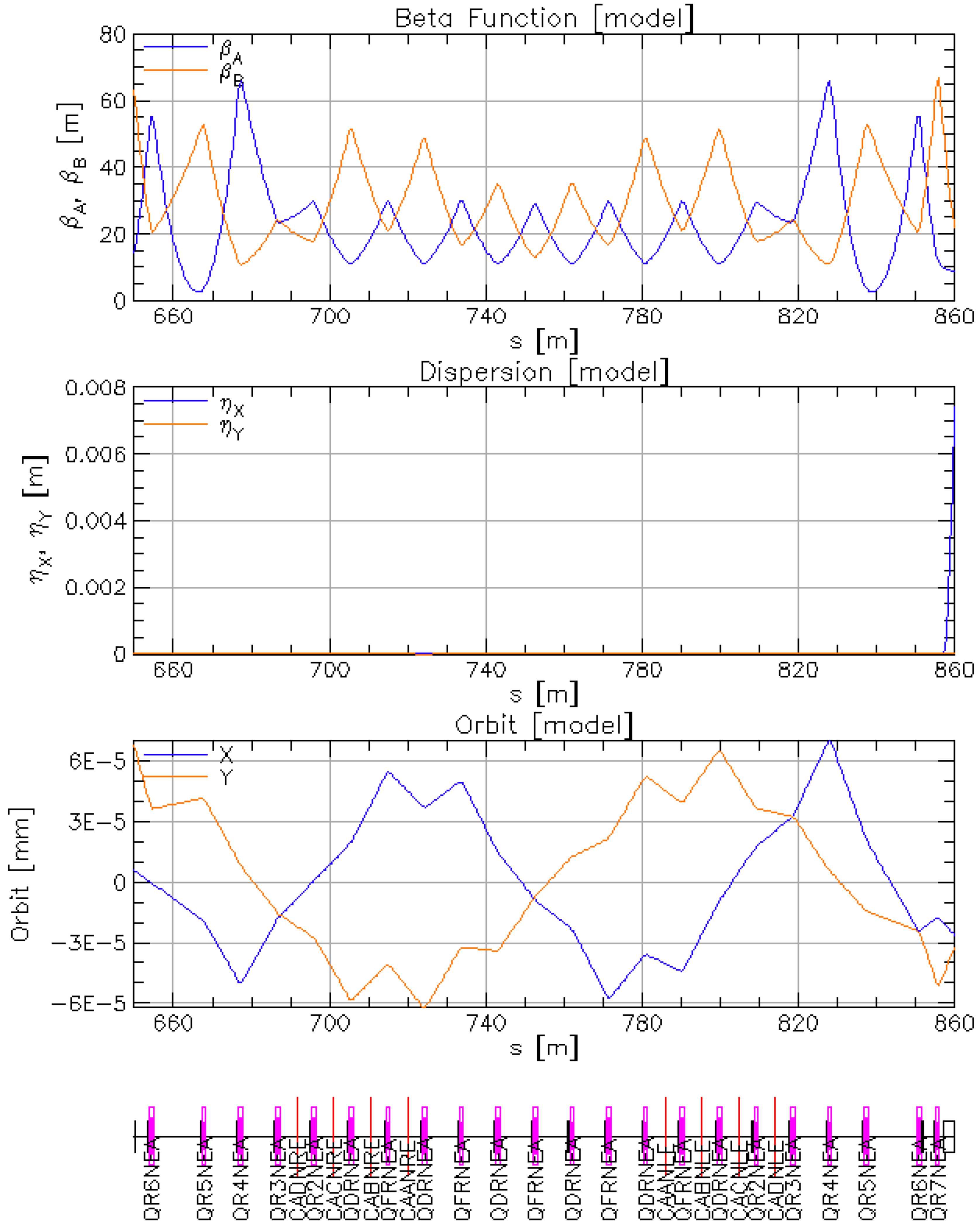
Comparison of the Straight section after the Tune match



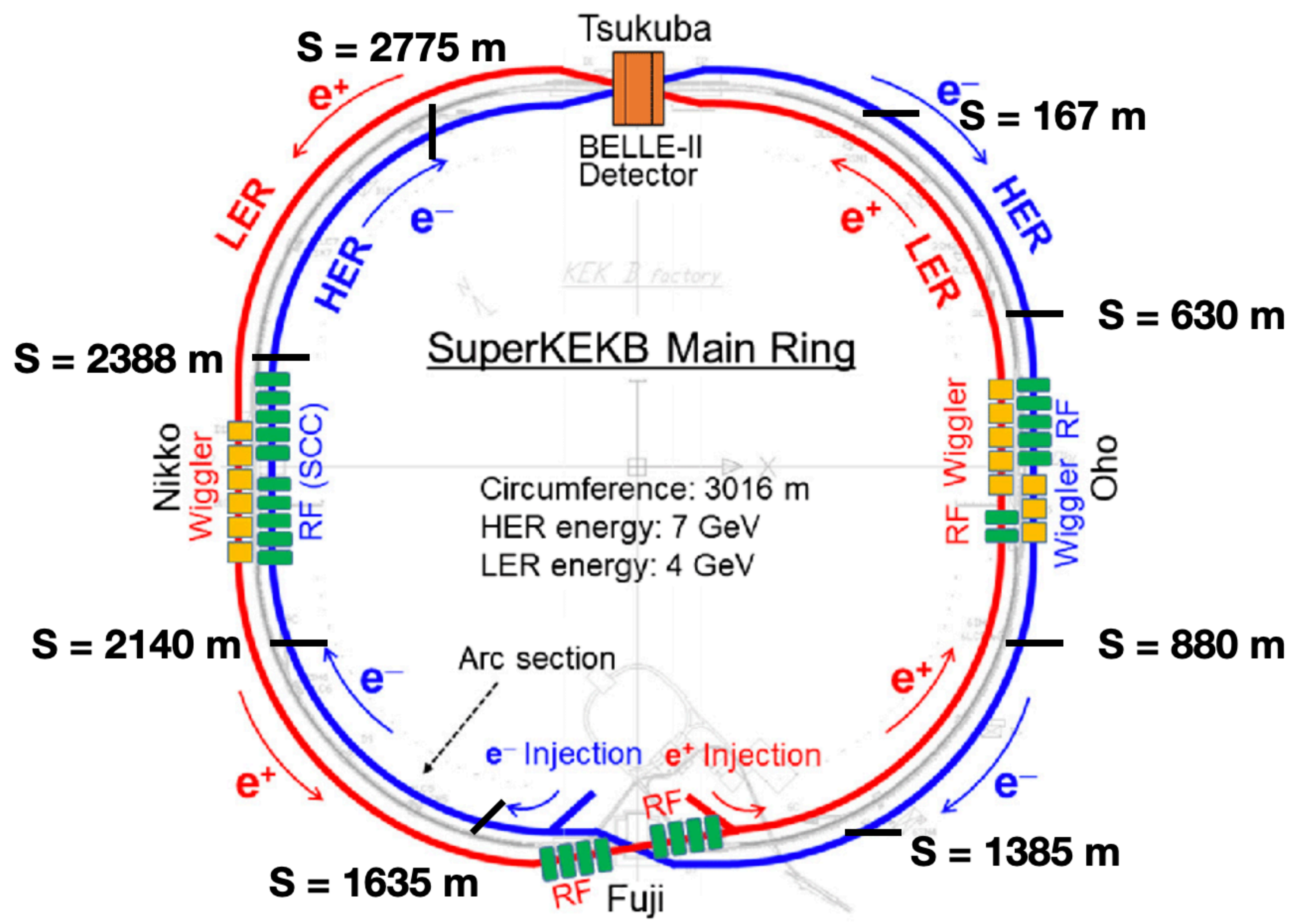
Original



Rot

Match the Linear Chromaticity

Adjust existing sextupoles at 4 arc sections shown in Figure below



SD5TLE, SF4TLE, and SD3TRE pairs (located in the Rot area) are turned off because the phase difference between these pairs is no longer π; can not cancel out their non-linear effects

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Theory

$$\xi_{\text{total}} = \frac{1}{4\pi} \oint \left[k(s) + m(s)\eta(s) \right] \beta(s) ds$$

Where k is the quadrupole strength, m is the sextupole strength, η is the dispersion function, β is the beta function

Varying the strength of sextupole does not change the beta function

the variation of chromaticity $\Delta\xi$ can be expressed as the linear combination of the variation of sextupole strength Δx (the sextupole strength $b_2 = \frac{k_2 L}{2}$, BMAD setting)

$$\begin{cases} \Delta\xi_x = \sum_i p_i \Delta x_i \\ \Delta\xi_y = \sum_j q_j \Delta x_j \end{cases} \quad \text{with} \quad \begin{cases} p_i = \frac{1}{2\pi} \beta_{x,i} \eta_{x,i} \\ q_j = -\frac{1}{2\pi} \beta_{y,j} \eta_{x,j} \end{cases}$$

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Assume N (N>2) sextupoles are used to correct the chromaticity, and denote:

$$\begin{cases} \overrightarrow{P} = (p_1, p_2, \ldots, p_N) \\ \overrightarrow{Q} = (q_1, q_2, \ldots, q_N) \\ \overrightarrow{x} = (\Delta x_1, \Delta x_2, \ldots, \Delta x_N) \end{cases} \quad \text{and} \quad \begin{cases} G_p = \overrightarrow{P} \cdot \overrightarrow{x} - \Delta\xi_x \\ G_q = \overrightarrow{Q} \cdot \overrightarrow{x} - \Delta\xi_y \end{cases}$$

Build function: $f(\overrightarrow{x}) = G_p^2 + G_q^2$

Use gradient descent to approach the global minimum (when $f(\overrightarrow{x}) \to 0$, the desired $\Delta\xi_x$ and $\Delta\xi_y$ are reached)

Gradient Descent

The gradient : $\nabla f(\overrightarrow{x}) = 2G_p\overrightarrow{P} + 2G_q\overrightarrow{Q}$

At jth iteration:

$$G_p^j = \overrightarrow{P} \cdot \overrightarrow{x}^j - \Delta\xi_x$$

$$G_q^j = \overrightarrow{Q} \cdot \overrightarrow{x}^j - \Delta\xi_y$$

$$\nabla f^j = 2G_p^j\overrightarrow{P} + 2G_q^j\overrightarrow{Q}$$

$$\overrightarrow{x}^{j+1} = \overrightarrow{x}^j - \varepsilon\nabla f^j$$

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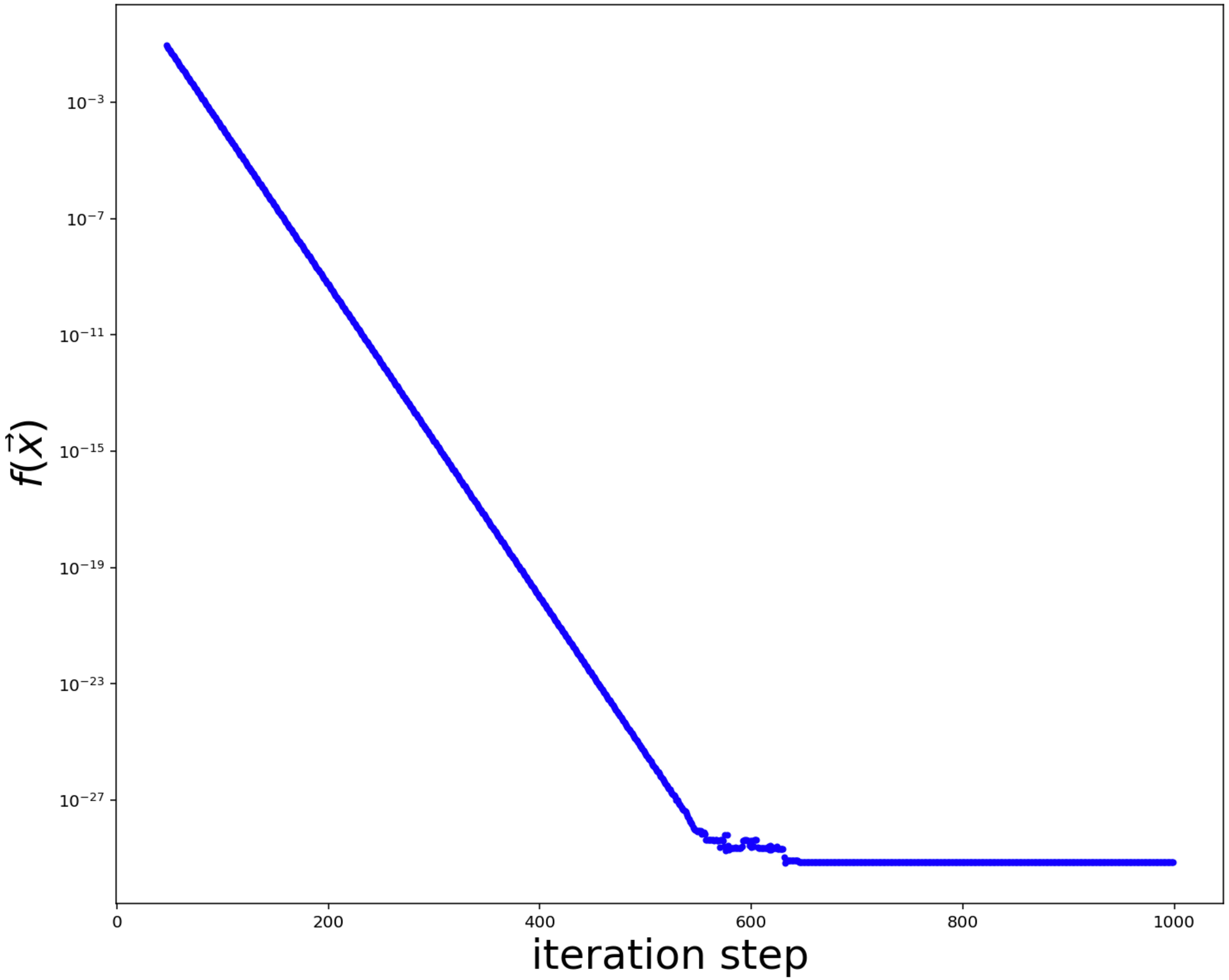
Where ε is the learning rate



Applying gradient descent algorithm to find a better starting point to run the BMAD optimization

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Name	L (m)	b_2 (original)	b_2 (Rot) BMAD	b_2 (Rot) Python
SD3TLE	1.030	-3.577	-3.789	-3.789
SF6TLE	0.334	0.818	0.869	0.869
SD7TLE	1.030	-3.607	-3.819	-3.819
SF8TNE	0.334	1.751	1.554	1.553
SD7NRE	1.030	-4.582	-4.788	-4.789
SF6NRE	0.334	1.467	1.539	1.537
SD5NRE	1.030	-1.389	-1.573	-1.573
SF4NRE	0.334	2.092	2.175	2.175
SD3NRE	1.030	-1.443	-1.628	-1.628
SF2NRE	0.334	0.371	0.403	0.406
SF2NLE	0.334	0.077	0.109	0.100
SD3NLE	1.030	-3.070	-3.281	-3.282
SF4NLE	0.334	0.497	0.535	0.535
SD5NLE	1.030	-1.527	-1.714	-1.715
SF6NLE	0.334	0.660	0.705	0.705
SD7NLE	1.030	-1.537	-1.724	-1.724
SD7FRE	0.334	-5.461	-5.652	-5.652
SF6FRE	0.334	2.296	2.384	2.384
SD5FRE	1.030	-6.803	-6.954	-6.956
SF4FRE	0.334	0.691	0.737	0.737
SD3FRE	1.030	-1.903	-2.099	-2.100
SF2FRE	0.334	1.226	1.289	1.289
SF2FLE	0.334	0.856	0.897	0.908
SD3FLE	1.030	-1.359	-1.542	-1.542
SF4FLE	0.334	0.541	0.581	0.581
SD5FLE	1.030	-2.926	-3.136	-3.136
SF6FLE	0.334	2.260	2.353	2.347
SD7FLE	1.030	-6.909	-7.055	-7.057
SF8FOE	0.334	1.871	1.770	1.770
SD7ORE	1.030	-7.242	-7.375	-7.377
SF6ORE	0.334	0.217	0.245	0.245
SD5ORE	1.030	-2.833	-3.043	-3.043
SF4ORE	0.334	1.686	1.761	1.761
SD3ORE	1.030	-3.123	-3.335	-3.335
SF2ORE	0.334	0.362	0.397	0.397
SF2OLE	0.334	2.296	2.384	2.384
SD3OLE	1.030	-0.706	-0.868	-0.868
SF4OLE	0.334	0.585	0.628	0.628
SD5OLE	1.030	-2.483	-2.689	-2.690
SF6OLE	0.334	0.415	0.435	0.451
SD7OLE	1.030	-3.385	-3.598	-3.598
SF8OTE	0.334	0.353	0.216	0.206
SD7TRE	1.030	-1.730	-1.921	-1.921
SF6TRE	0.334	0.829	0.876	0.882
SD5TRE	1.030	-1.695	-1.885	-1.885

Sextupoles adjusted to correct the linear chromaticity of Rot Ring

The maximum shift of sextupole strength from the original ~ 0.21

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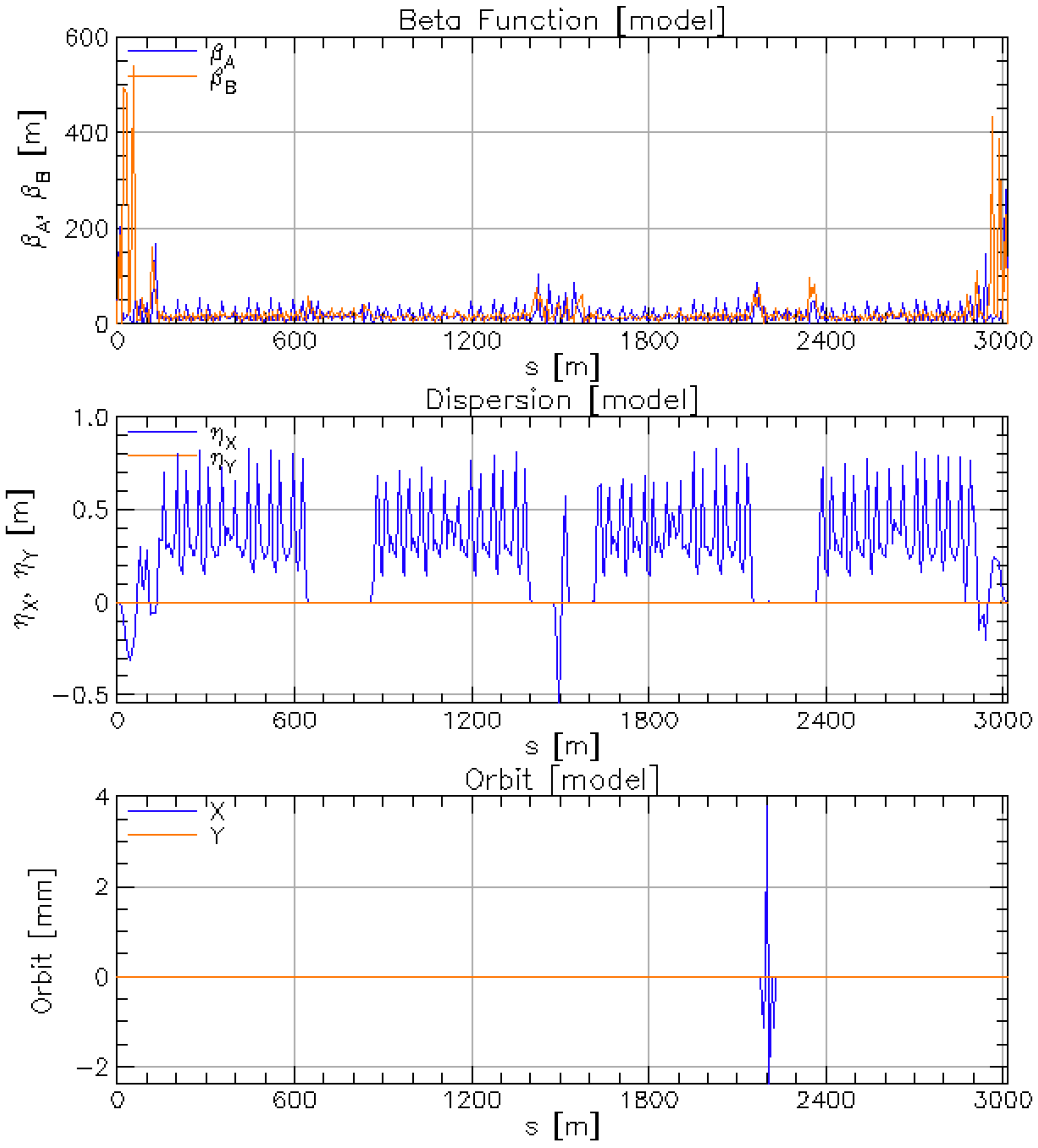
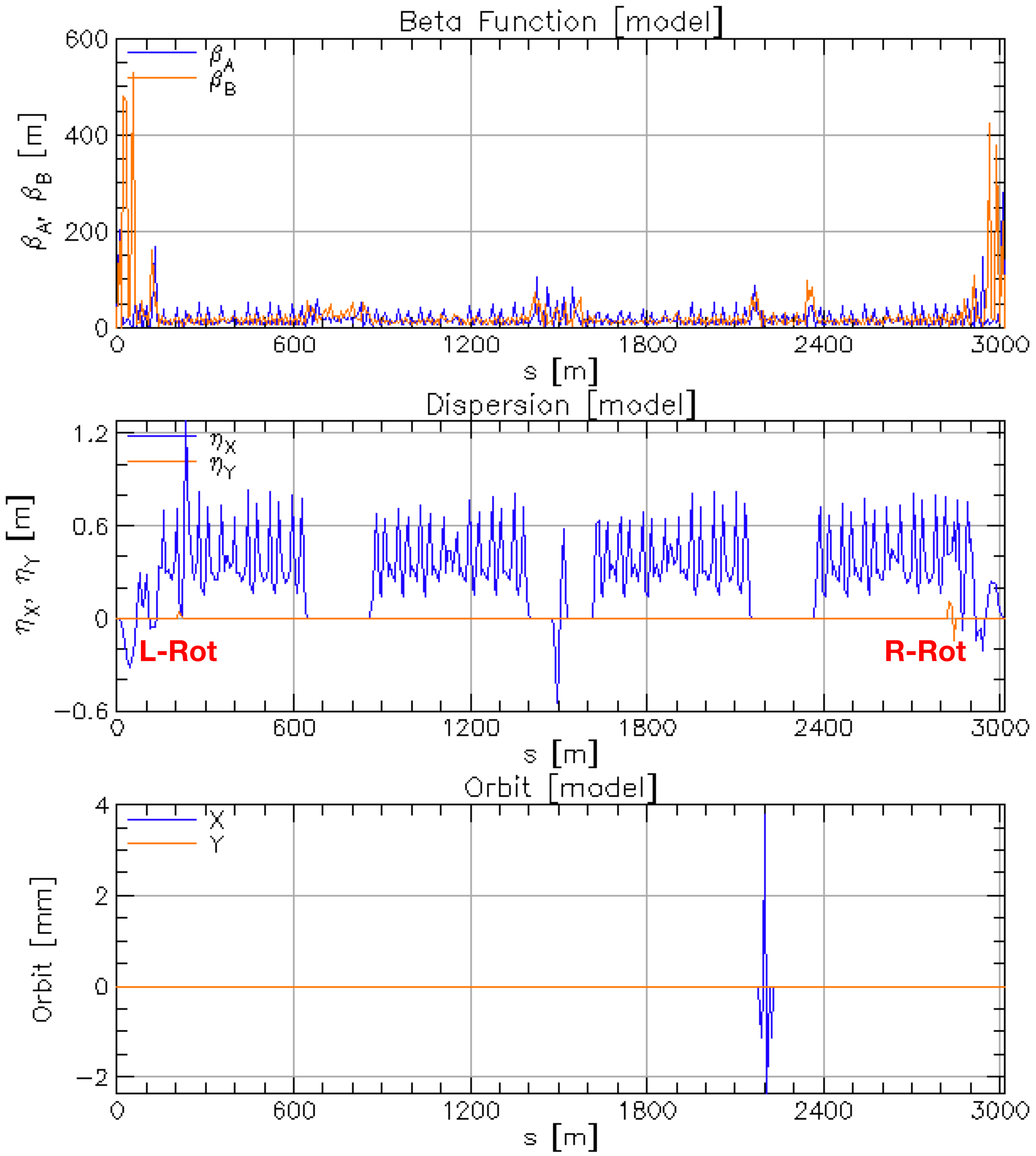
Full lattice Comparison with L/R-Rot installed in the ring



Original Ring



Rotator Ring

Ring Parameters Comparison after rematch the Tunes and Chromaticities

Original

	X Model	X Design	Y Model	Y Design	
Q	45.530994	45.530994	43.580709	43.580709	! Tune
Chrom	1.593508	1.591895	1.622865	1.621568	! dQ/(dE/E)
J_damp	1.000064	0.999662	1.000002	1.000002	! Damping Partition #
Emittance	4.44061E-09	4.44277E-09	5.65367E-13	5.65331E-13	! Meters

Rot

	X Model	X Design	Y Model	Y Design	
Q	45.530994	45.530994	43.580709	43.580709	! Tune
Chrom	1.593508	1.255194	1.622865	1.622979	! dQ/(dE/E)
J_damp	0.984216	0.983532	1.005266	1.005262	! Damping Partition #
Emittance	4.88967E-09	4.89624E-09	3.96631E-12	3.96983E-12	! Meters

Single Particle Spin Tracking Result of e^-

- The spin track result shows a longitudinal spin alignment >99.99% with the rotator installed in the High Energy Ring

Spin Component	Entrance of L-Rot	IP	Exit of R-Rot
X	-0.0000450734	0.0000066698	0.0000538792
Y	0.9999999959	0.0000926945	0.9999999959
Z	-0.0000788085	0.9999999957	-0.0000728110

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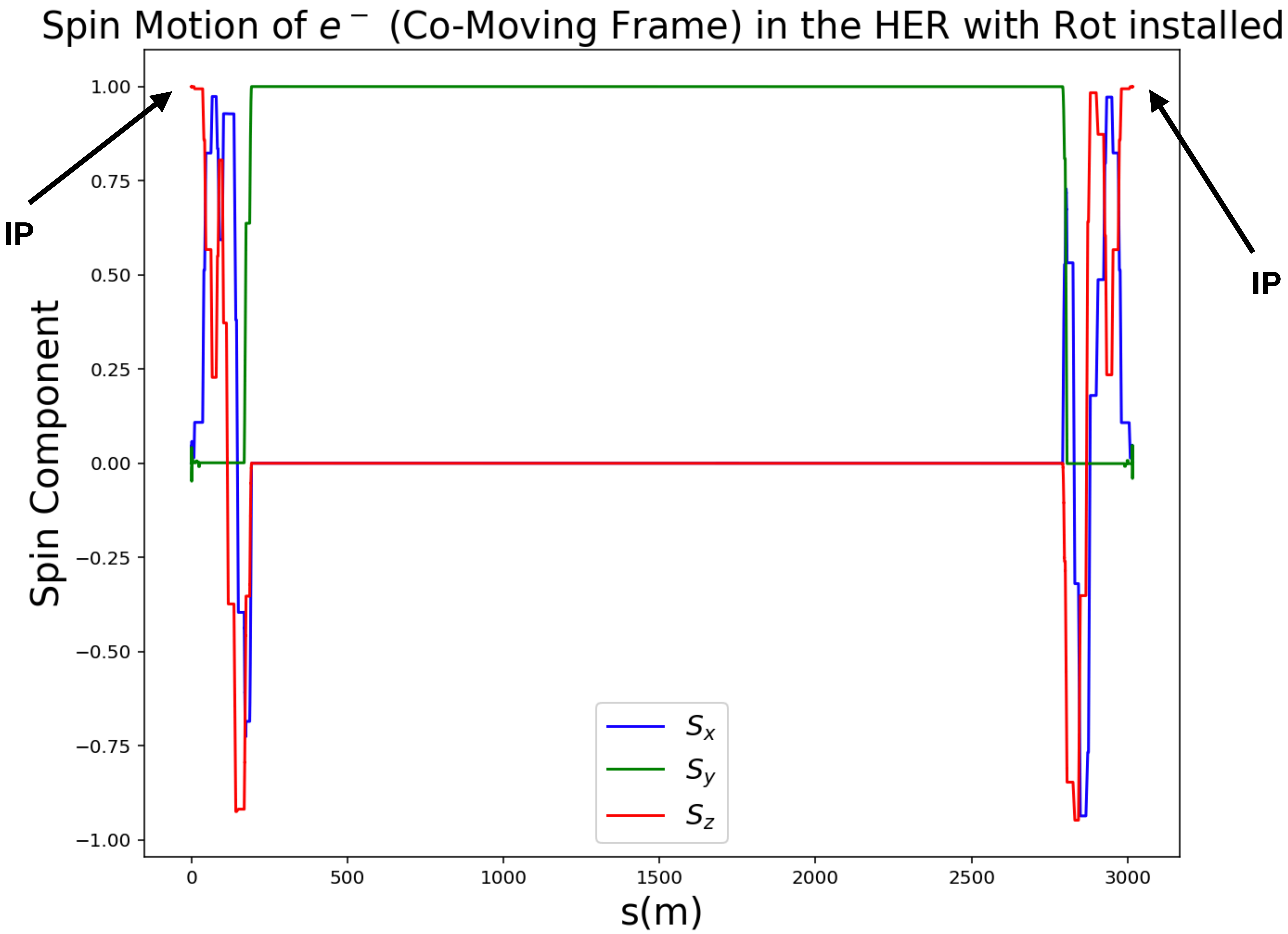

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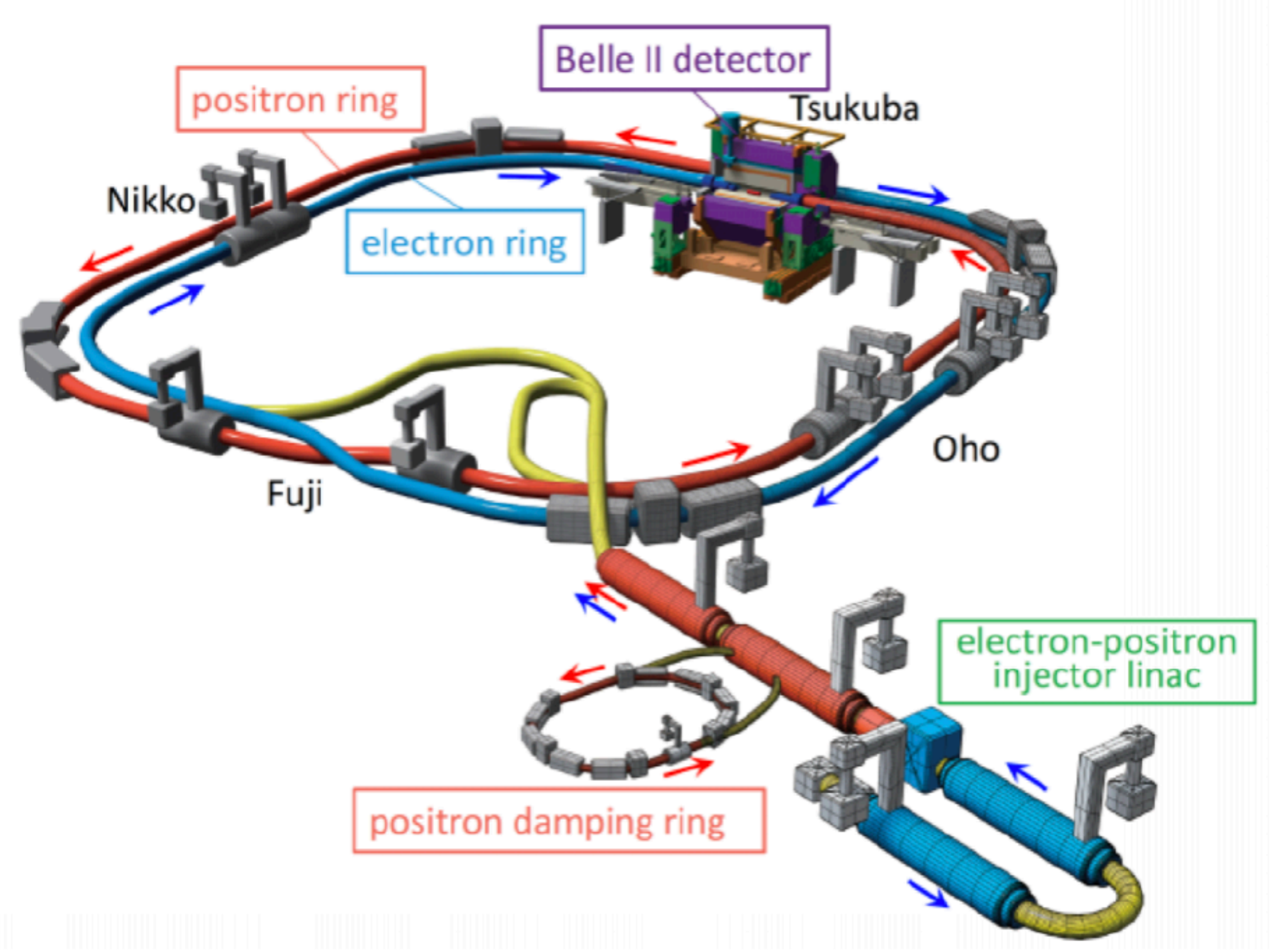
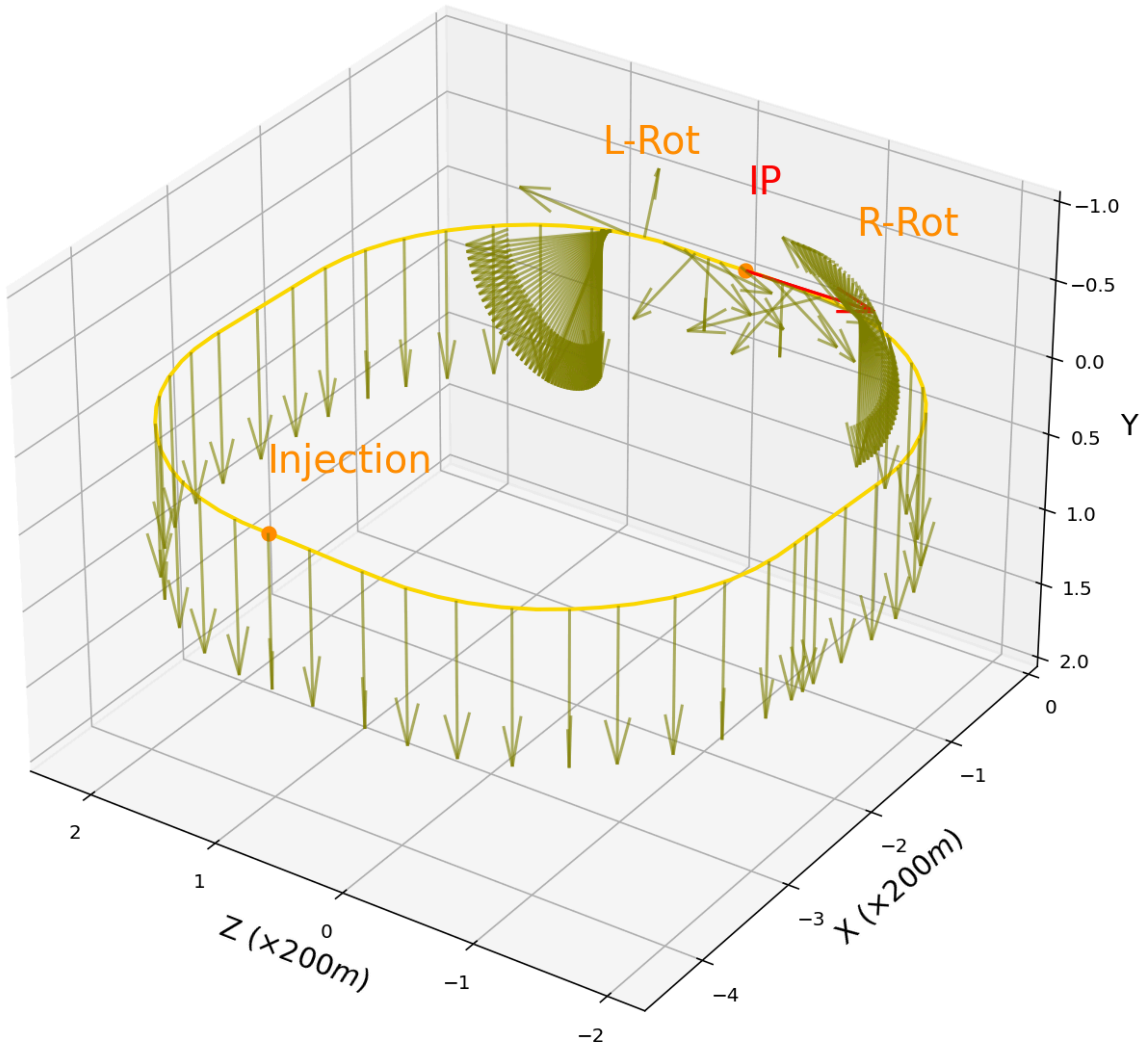
Spin Motion of the electron in the Rot Ring (KEK frame)





Summary

- We have a BMAD solution for the overlapping field spin rotator in the HER

- Transparency is achieved (original machine linear dynamics recovered except the rotator region)
  - Optical rematch
  - Ring Parmaters rematch (Tune and Chromaticity)

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Future Steps

Beam Tracking Studies (Long_Term_Tracking requiring computer cluster)

- investigate the dynamic aperture, adjust sextupoles to reach the maximum dynamic aperture
- determine the polarization lifetime and beam lifetime

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Appendix

- BMAD uses the normalized integrated multipole k_nL (equivalent to k_n in SAD) to specify magnetic multipole components

$$k_nL \equiv \frac{qB_nL}{p_0}$$

Where q is the charge of the reference particle, L is the element length, and p_0 is the reference momentum

- Another representation that BMAD uses divides the field into normal b_n and skew component a_n, the nth order multipole is given by:

$$\frac{qL}{p_0}(B_y + iB_x) = (b_n + ia_n)(x + iy)^n$$

- if there is no skew component $b_n = \dfrac{k_nL}{n!}$

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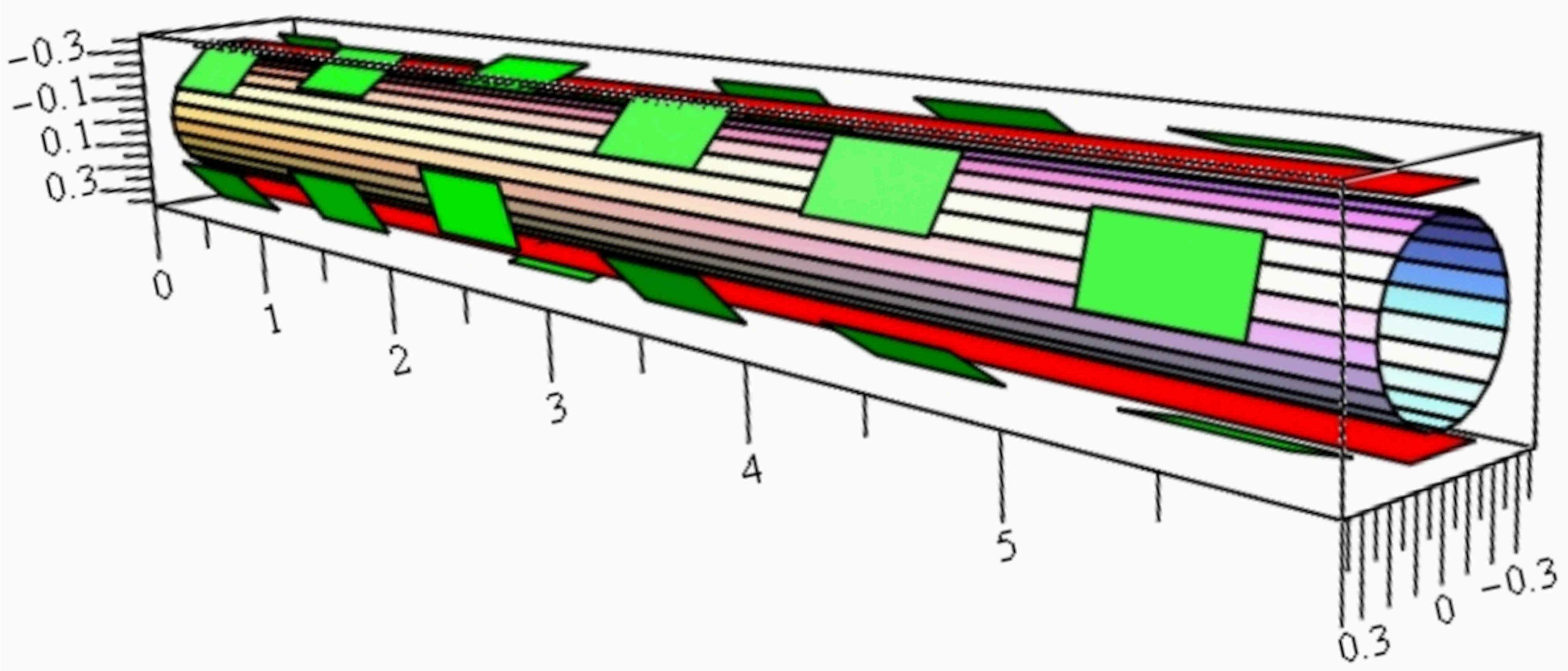
Rotator Magnet Structure

Follows Uli Wienands's (Argonne National Laboratory) idea and direction:

- replace some existing ring dipoles on both sides of the IP with the dipole-solenoid combined function magnets and keep the original dipole strength to preserve the machine geometry

- Install 6 skew-quadruple on top of each rotator section to compensate for the x-y plane coupling caused by solenoids

- Original machine can be recovered by turning off sol-quad field



Constraints of the Design

- **Transparency**: Need to maintain the original **beam dynamics,** make the spin rotator transparent to the ring as much as possible (the spin rotator is for the polarization purpose only)

- Physical constraints: All new magnets must be manufacturable and installable
  - Solenoid strength can not exceed **5** T
  - Skew-quad can not exceed **30** T/m ~ 3T at the coil

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Simulation Tool

- **BMAD** is an open-source software library (aka toolkit)created/maintained by David Sagan at Cornell University for simulating charged particles and X-rays. Étienne Forest's "Polymorphic Tracking Code" (**PTC**) is incorporated into it.

- **Tao** is a user-friendly interface to Bmad which gives general purpose simulation, based upon Bmad.

- **BMAD** via the **Tao** interface is a powerful and user-friendly tool used for viewing lattices, doing Twiss and orbit calculations, and performing nonlinear optimization on lattices

- Optimization Algorithm: **LMDIF** is to minimize the sum of the squares of nonlinear functions by a modification of the Levenberg-Marquardt algorithm

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L-Rot Magnets

Quads	L(m)	k_1L(Original)	k_1L(L-Rot)	B_1(Original) T/m	B_1(L-Rot) T/m
QD3E	0.826	-0.175	-0.177	-4.948	-5.012
QF4E	1.015	0.035	0.071	0.805	1.633
QEAE	0.826	0.183	0.175	5.178	4.961
QD5E	0.826	-0.179	-0.286	-5.074	-8.079
QF6E	0.557	0.163	0.342	6.855	14.366
QF2E	0.557	0.192	0.145	8.050	6.067
QD1E	1.015	-0.255	-0.203	-5.868	-4.682

Skew-Quad	L(m)	k_1L	B_1 (T/m)	Tilt (rad)
B2EALSQ1	0.9837	0.511	12.133	-0.426
B2EALSQ2	0.9837	0.510	12.130	1.053
B2EALSQ3	0.9837	-0.314	-7.457	-0.988
B2EALSQ4	0.9837	0.855	20.315	0.030
B2EALSQ5	0.9837	0.688	16.350	-0.630
B2EALSQ6	0.9837	0.814	19.340	1.383
B2EBLSQ1	0.9837	0.558	13.266	0.651
B2EBLSQ2	0.9837	-0.482	-11.444	0.992
B2EBLSQ3	0.9837	0.426	10.119	-1.494
B2EBLSQ4	0.9837	0.338	8.024	-0.931
B2EBLSQ5	0.9837	0.562	13.359	0.735
B2EBLSQ6	0.9837	-0.185	-4.404	0.868

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R-Rot Magnets

Quads	L(m)	k_1L(Original)	k_1L(R-Rot)	B_1(Original) T/m	B_1(R-Rot) T/m
QD5E	0.826	-0.179	-0.165	-5.074	-4.667
QEAE	0.826	0.183	0.154	5.178	4.362
QF4E	1.015	0.035	0.067	0.805	1.538
QD3E	0.826	-0.175	-0.251	-4.948	-7.088
QF2E	0.557	0.192	0.183	8.050	7.659
QD1E	1.015	-0.255	-0.274	-5.868	-6.311
QLA10RE	0.826	0.202	0.185	5.718	5.234
QLA9RE	0.826	-0.237	-0.226	-6.703	-6.385
QLA8RE	0.557	0.203	0.169	8.527	7.106
QLA7RE	0.826	-0.192	-0.195	-5.438	-5.522
QLA6RE	0.826	0.202	0.205	5.716	5.808

Skew-Quad	L(m)	k_1L	B_1 (T/m)	Tilt (rad)
B2EARSQ1	0.9837	0.435	10.341	-2.610
B2EARSQ2	0.9837	0.600	14.258	2.290
B2EARSQ3	0.9837	0.043	1.032	2.328
B2EARSQ4	0.9837	-0.566	-13.451	-0.180
B2EARSQ5	0.9837	0.600	14.258	-2.545
B2EARSQ6	0.9837	-0.591	-14.038	0.618
B2EBRSQ1	0.9837	0.495	11.769	-2.480
B2EBRSQ2	0.9837	0.532	12.648	2.238
B2EBRSQ3	0.9837	0.280	6.663	-0.960
B2EBRSQ4	0.9837	-0.565	-13.429	-0.197
B2EBRSQ5	0.9837	0.600	14.258	-2.846
B2EBRSQ6	0.9837	-0.383	-9.098	0.475

Alternative R-Rot Magnets

Quads	L(m)	k_1L(Original)	k_1L(R-Rot)	B_1(Original) T/m	B_1(R-Rot) T/m
QF4E	1.015	0.035	0.031	0.805	0.716
QD3E	0.826	-0.175	-0.256	-4.948	-7.230
QF2E	0.557	0.192	0.161	8.050	6.766
QD1E	1.015	-0.255	-0.273	-5.868	-6.285

Skew-Quad	L(m)	k_1L	B_1 (T/m)	Tilt (rad)
B2EARSQ1	0.9837	0.424	10.078	-2.522
B2EARSQ2	0.9837	0.600	14.258	2.266
B2EARSQ3	0.9837	0.086	2.054	2.264
B2EARSQ4	0.9837	-0.493	-11.714	-0.276
B2EARSQ5	0.9837	0.600	14.258	-2.592
B2EARSQ6	0.9837	-0.566	-13.451	0.654
B2EBRSQ1	0.9837	0.499	11.848	-2.457
B2EBRSQ2	0.9837	0.561	13.318	2.222
B2EBRSQ3	0.9837	0.278	6.599	-1.001
B2EBRSQ4	0.9837	-0.577	-13.710	-0.273
B2EBRSQ5	0.9837	0.600	14.258	-2.877
B2EBRSQ6	0.9837	-0.352	-8.355	0.597