The $\bar{B} \to D^* \ell \nu$ Rate

Starting with the Standard Model, the EW Lagrangian for $b \to c \tau \bar{\nu}$

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} V_{cb}\, \bar{c}_L \gamma^\mu W_\mu b_L + \frac{g}{\sqrt{2}} \bar{\tau}_L \gamma^\mu W_\mu \nu_L \qquad -(1)$$

this generates the 4-Fermi operator in the SM

$$\frac{4 G_F V_{cb}}{\sqrt{2}}\, \bar{c} \gamma^\mu P_L b\; \bar{\tau} \gamma_\mu P_L \nu \qquad P_L = \frac{1-\gamma^5}{2} \quad -(2)$$

$$G_F = \frac{g^2}{4\sqrt{2} M_W^2} = \frac{1}{\sqrt{2} v^2}$$

When considering NP, these currents can be generalized to

$$\frac{4 G_F V_{cb}}{\sqrt{2}}\, c_{XY}\, \bar{c}\, \Gamma_X\, b\; \bar{\tau}\, \Gamma_Y P_L \nu \qquad -(3)$$

c_{XY}: normalized coupling

$\frac{4 G_F V_{cb}}{\sqrt{2}}$: $\sim \frac{1}{\Lambda^2}$, $\Lambda \sim 870\,\text{GeV}$

For LH ν's: scalar, pseudoscalar, vector, axial vector, tensor

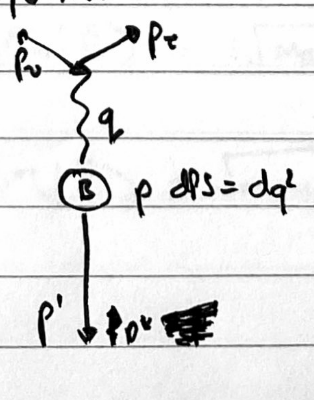
To write the amplitudes for $\bar{B} \to D^* \ell \nu$, need to "dress" the bare quark current, so that

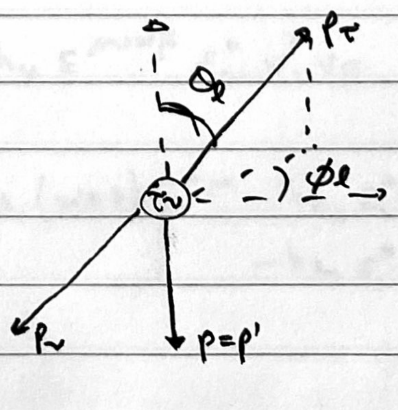
$$A^{\lambda s_\tau s_\nu} = \frac{4 G_F V_{cb}}{\sqrt{2}} \underbrace{\langle D^*_\lambda | \bar{c}\, \Gamma\, b | B \rangle}_{\text{Hadronic matrix element}} \underbrace{\bar{u}_\tau^{s_\tau}\, \Gamma' v_\nu^{s_\nu}}_{\text{lepton current}} \qquad -(4)$$

$\lambda s_\tau s_\nu$: q.n.'s

Γ': $\gamma^\mu P_L$ in SM

The kinematics of the decay is easiest to understand in two separate frames, the B and $q = p_\tau + p_\nu$ rest frames:

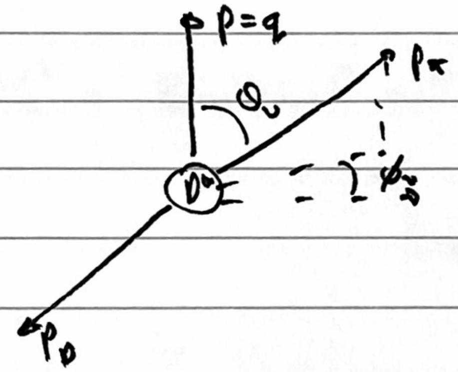




$dPS = d\cos\theta_\ell\, d\phi_\ell$

θ_ℓ, ϕ_ℓ helicity angles

If we want to include also the $D^* \to D\pi$ decay, the also in the D^* frame



$dPS = d\cos\theta_v \, d\phi_v$

In the SM, the hadronic matrix element contains a vector and axial vector current. These can be expressed, following angular momentum and parity conservation, in terms of form-factors (more on this later).

$$(\varepsilon^{0123} = +1)$$

$$\langle D^*_\lambda(p') | \bar{c}\gamma^\mu b | \bar{B}(p) \rangle = i g \, \varepsilon^{\mu\nu\alpha\beta} \, \varepsilon^*_{\lambda\nu}(p') \, p'_\alpha \, p_\beta \qquad - (5a)$$

$$\langle D^*_\lambda(p') | \bar{c}\gamma^\mu\gamma^5 b | \bar{B}(p) \rangle = f \, \underbrace{\varepsilon^{*\mu}_\lambda(p')}_{\text{pol. vec. of } D^*} + a_+ \, \varepsilon^*_\lambda\cdot p \, (p+p')^\mu + a_- \, \varepsilon^*_\lambda \cdot p \, q^\mu \qquad - (5b)$$

Here $g, f, a_\pm$ are functions of q^2 (and masses). Note $q^2 = (p-p')^2$ has range $m_\ell^2 \le q^2 \le (M_B - M_{D^*})^2 \equiv q^2_-$. Defining velocities $v = p/M_B$, $v' = p'/M_{D^*}$, also convenient to express in terms of

$$w = v\cdot v' = \frac{M_B^2 + M_{D^*}^2 - q^2}{2 M_B M_{D^*}} \quad \left(= \frac{E_{D^*}}{M_B} \text{ in B frame, the } D^* \text{ boost} \right) \qquad - (6)$$

Another convenient FF basis

$$\langle D^*_\lambda | \bar{c}\gamma^\mu b | \bar{B} \rangle = i\sqrt{M_B M_{D^*}} \, h_V \, \varepsilon^{\mu\nu\alpha\beta} \, \varepsilon^*_{\lambda\nu} \, v'_\alpha \, v_\beta \qquad - (6a)$$

$$\langle D^*_\lambda | \bar{c}\gamma^\mu\gamma^5 b | \bar{B} \rangle = \sqrt{M_B M_{D^*}} \Big(h_{A_1}(w+1) \, \varepsilon^{*\mu}_\lambda - h_{A_2} \, \varepsilon^*_\lambda\cdot v \, v^\mu - h_{A_3} \, \varepsilon^*_\lambda \cdot v \, v'^\mu \Big) \qquad - (6b)$$

Using either (5) or (6), can compute the matrix element squared in the "usual" way

$$\sum_{\lambda, s_\ell, s_\nu} |A^{\lambda s_\ell s_\nu}|^2 = 8 G_F^2 |V_{cb}|^2 \, W^{\mu\nu} L_{\mu\nu} \qquad -(7)$$

$L_{\mu\nu} \to \mathrm{Tr}\left[\gamma^\mu P_L \not{p}_\nu \gamma^\nu P_L (\not{p}_\ell - m_\ell)\right]$

$W^{\mu\nu} \to \sum \langle B | V^\mu - A^\mu | D^* \rangle \langle D^* | V^\nu - A^\nu | B \rangle$

using completeness relation $\sum_\lambda \varepsilon^\lambda_\mu \varepsilon^{\lambda *}_\nu = -g_{\mu\nu} + \frac{p'_\mu p'_\nu}{M_{D^*}^2}$ $\quad -(8)$

$\sum_s u^s \bar{u}^s = \not{p} - M$ $\qquad$ etc

This is tractable, but a mess:

- Need to calculate all pieces in one frame (cf kinematics definitions)
- Completeness relations obscure the selection rules – the physics – that determine the structure of the result
- PS integral becomes nasty, too.

A better way: compute the amplitudes <u>directly</u>!

To do this, go back to (4) and note (see (8))

$$g_{\mu\nu} = -\sum_{\lambda=\pm,0} \varepsilon^\lambda_\mu(q)\, \varepsilon^{\lambda *}_\nu(q) + \frac{q^\mu q^\nu}{q^2} \qquad -(9)$$

where $\varepsilon(q) \equiv \varepsilon_q$ is polarization vector associated with q. This completeness relation allows us to factorize the Lorentz contraction. NB

$$\underbrace{\varepsilon^\lambda_q \cdot \varepsilon^{\kappa *}_q = -\delta^{\lambda\kappa}}_{\text{orthonormal}}\ , \quad \underbrace{\varepsilon^\lambda_q \cdot q = 0}_{\text{transverse}}\ , \quad \lambda = \pm, 0 \qquad -(10)$$

Doing this, (4) becomes

$$A^{\lambda s_\tau s_\nu} = \frac{4G_F V_{cb}}{\sqrt{2}} \Bigg\{ -\sum_{\kappa} \overbrace{\langle D^*_\lambda | \bar{c}\, \not{\epsilon}_q^{\kappa *} P_L b | B \rangle}^{\text{Lorentz invariant}} \; \overbrace{\bar{u}_\tau^{s_\tau} \, \not{\epsilon}_q^{\kappa} P_L v_\nu^{s_\nu}}^{\text{Lorentz invariant}}$$

$$+ \frac{1}{q^2} \langle D^*_\lambda | \bar{c}\, \not{q} P_L b | B \rangle \; \underbrace{\bar{u}_\tau^{s_\tau} \, \not{q} P_L v_\nu^{s_\nu}}_{m_\tau \bar{u}_R^{s_\tau} v_L^{s_\nu}} \Bigg\} \quad \text{—(11)}$$

Can calculate pieces in different frames! Ie hadronic matrix element in B frame, ~~The~~ lepton current in $\tau\nu$ frame. Leads to tremendous simplifications! Sum and square at the end $\times$ PS Jacobian for the full differential rate.

Consider first the had matrix element. In the B rest frame, a sensible polarization basis can be directly chosen:

$$\left(r = m_{D^*}/m_B \right)$$

$$q = \left(m_B - w m_{D^*}, 0, 0, m_{D^*}\sqrt{w^2-1}\right) = m_B\big(\underbrace{1 - wr}_{E_q}, 0, 0, \underbrace{r\sqrt{w^2-1}}_{|p|}\big)$$

$$\epsilon_q^{\pm} = \pm\frac{1}{\sqrt{2}}(0, 1, \mp i, 0)$$

$$\epsilon_q^{0} = \frac{m_B}{\sqrt{q^2}}\left(r\sqrt{w^2-1}, 0, 0, 1-wr\right), \quad \frac{m_B}{\sqrt{q^2}} \to \frac{1}{\sqrt{1+r^2-2wr}}$$

$$\text{(B)} \quad p_B = (m_B, 0, 0, 0) \quad \text{—(12)}$$

$$p' = \left(w m_{D^*}, 0, 0, -m_{D^*}\sqrt{w^2-1}\right) \quad \leftarrow \text{see (6)}$$

$$= m_B\big(\underbrace{wr}_{E'}, 0, 0, \underbrace{-r\sqrt{w^2-1}}_{-|p|}\big)$$

$$\epsilon^{\pm} = \pm\frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

$$\epsilon^{0} = \frac{1}{r}\left(r\sqrt{w^2-1}, 0, 0, -rw\right)$$

Note $\epsilon_q^{\kappa} \cdot \epsilon^{\lambda} = -\delta^{\lambda\kappa}$ except for $\epsilon_q^0 \cdot \epsilon^0 = \cancel{r}(w - r)\dfrac{m_B}{\cancel{r}\sqrt{q^2}}$

Vector:

$$\langle D^*_\lambda | \bar{c} \not\epsilon^{\kappa *} b | \bar{B} \rangle = ig\, \epsilon^{ij30}\, m_B(-|p|)\, \epsilon_{q\,i}^{\kappa *} \epsilon_j^{\lambda *} \qquad i=1,2$$

$$= \begin{cases} -ig\, m_B|p| \left(\epsilon^{1230}\left(\frac{-i}{2}\right) + \epsilon^{2130}\left(\frac{+i}{2}\right)\right) & \lambda=\kappa=+ \\ -ig\, m_B|p| \left(\epsilon^{1230}\left(\frac{+i}{2}\right) + \epsilon^{2130}\left(\frac{-i}{2}\right)\right) & \lambda=\kappa=- \end{cases}$$

$$= \begin{cases} +m_B|p|g & \lambda\ \kappa:\ +\ + \\ -m_B|p|g & -\ - \quad \text{and rest } 0! \end{cases} \qquad (13)$$

$$\langle D^*_\lambda | \bar{c} \not q\, b | \bar{B} \rangle = 0 \text{ as } \epsilon^{\mu\nu\alpha\beta} q_\mu \epsilon^*_\nu p'_\alpha p_\beta = 0$$

Axial Vector

$$\langle D^*_\lambda | \bar{c} \not\epsilon^{\kappa *} \gamma^5 b | \bar{B} \rangle = f\, \epsilon^{\lambda *}\cdot\epsilon_q^{\kappa *} + 2a_+\, \epsilon^{\lambda *}\cdot p\ \epsilon_q^{\kappa *}\cdot p$$

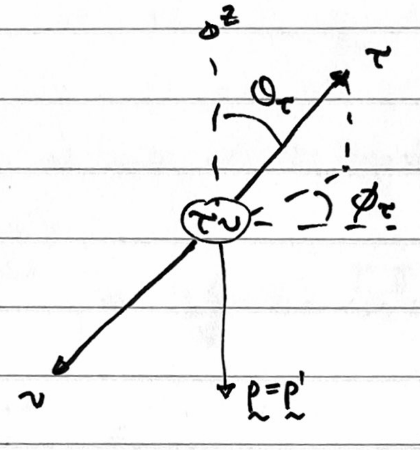
$$= \begin{cases} -f & \lambda\ \kappa:\ +\ + \\ f\frac{(w-r)m_B}{\sqrt{q^2}} + \frac{2m_B^3}{\sqrt{q^2}} a_+ (w^2-1)r & 0\ 0 \\ -f & -\ - \end{cases} \qquad (14)$$

$$\langle D^*_\lambda | \bar{c} \not q \gamma^5 b | \bar{B} \rangle = f\, \epsilon^{\lambda *}\cdot q + a_+\, \epsilon^{\lambda *}\cdot p(m_B^2 - m_{D^*}^2) + a_-\, \epsilon^{\lambda *}\cdot p\, q^2$$

$$= m_B\sqrt{w^2-1}\left[f + a_+(m_B^2 - m_{D^*}^2) + a_- q^2 \right] \quad \text{for } \lambda = 0 \qquad (15)$$

and rest are zero.

What about the lepton amplitude? Easiest in the $\tau\nu$ rest frame.



$$p_\tau = \left(\frac{q^2 + m_\tau^2}{2\sqrt{q^2}},\ \frac{q^2 - m_\tau^2}{2\sqrt{q^2}}\, \underset{\sim}{e}_\theta \right)$$

$$p_\nu = \frac{q^2 - m_\tau^2}{2\sqrt{q^2}} \left(1, -\underset{\sim}{e}_\theta \right) \qquad \text{—(16)}$$

$$\varepsilon_q^\pm = \frac{1}{\sqrt{2}} \left(0, \pm 1, -i, 0 \right) \qquad \text{(after boost)}$$

$$\varepsilon_q^0 = (0, 0, 0, -1)$$

Weyl spinors:

$$u_{\nu_L}^- = \left(\frac{q^2 - m_\tau^2}{2\sqrt{q^2}} \right)^{1/2} \begin{pmatrix} \sqrt{1 - \cos\theta} \\ \dfrac{-e^{-i\phi}\sin\theta}{\sqrt{1-\cos\theta}} \end{pmatrix}$$

$$\bar{u}_{\tau_L}^+ = \left(\frac{\sqrt{q^2}}{2} \right)^{1/2} \begin{pmatrix} \sqrt{1 + \cos\theta} & \dfrac{e^{i\phi}\sin\theta}{\sqrt{1+\cos\theta}} \end{pmatrix} \qquad \text{—(17)}$$

$$\bar{u}_{\tau_L}^- = \frac{-m_\tau}{(2\sqrt{q^2})^{1/2}} \begin{pmatrix} \sqrt{1 - \cos\theta} & \dfrac{-e^{i\phi}\sin\theta}{\sqrt{1-\cos\theta}} \end{pmatrix}$$

Result is standard set of Wigner functions

$$\bar{u}_{\tau_L}^+ \sigma^\mu u_{\nu_L}^- \varepsilon^K_{q\mu} = -D^1_{1K}(\theta_\tau, -\phi_\tau)\, \sqrt{2}\, \sqrt{q^2 - m_\tau^2}$$

$$= -\sqrt{2}\sqrt{q^2 - m_\tau^2} \left\{ \begin{array}{ll} & K \\ e^{-i\phi}\cos^2(\theta/2) & + \\ \cancel{\sqrt{2}}\sin\theta/\sqrt{2} & 0 \\ e^{i\phi}\sin^2(\theta/2) & - \end{array} \right. \qquad \text{—(18)}$$

$$\bar{u}_{\tau_L}^- \sigma^\mu u_{\nu_L}^- \varepsilon^K_{q\mu} = -D^1_{0K}(\theta_\tau, -\phi_\tau)\, \frac{m_\tau}{\cancel{2}\sqrt{q^2}} \sqrt{q^2 - m_\tau^2}$$

$$= \frac{+m_\tau \sqrt{q^2 - m_\tau^2}}{\sqrt{q^2}} \left\{ \begin{array}{ll} & K \\ e^{-i\phi}\sin\theta & + \\ -\cos\theta & 0 \\ e^{i\phi}\sin\theta & - \end{array} \right. \qquad \text{—(19)}$$

and

$$\bar{u}_{\tau_L^-}\, \sigma^\mu\, u_{\bar{\nu}_L}\, q_\mu = -m_\tau \sqrt{q^2 - m_\tau^2} \qquad \left(D^0_{00} = 1 \right) \qquad -(20)$$

Now we have all the ingredients to put everything together in (11).

$$A^{\lambda s_\tau s_\nu}$$

$$A^{++-} = \frac{4 G_F}{\sqrt{2}} V_{cb} \sqrt{q^2 - m_\tau^2}\, \frac{1}{2} \left(f + m_B^2 r \sqrt{w^2-1}\, g \right) \left[+\sqrt{2}\, e^{-i\phi} \cos^2 \theta/_2 \right]$$

$$A^{+--} = \quad '' \quad \left[\frac{-m_\tau}{\sqrt{q^2}} e^{-i\phi} \sin\theta \right]$$

$$A^{0+-} = \frac{4 G_F}{\sqrt{2}} V_{cb} \sqrt{q^2 - m_\tau^2} \left[-\frac{1}{2} \left(f(w-r) \frac{m_B}{\sqrt{q^2}} + \frac{2 m_B^3}{\sqrt{q^2}} a_+ (w^2-1) r \right) \right] \sin\theta \qquad -(21)$$

$F_1/\sqrt{q^2}$

$$A^{0--} = \frac{4 G_F}{\sqrt{2}} V_{cb} \sqrt{q^2 - m_\tau^2} \left[-\frac{1}{2} \left(f(w-r) \frac{m_B}{\sqrt{q^2}} + \frac{2 m_B^3}{\sqrt{q^2}} a_+ (w^2-1) r \right) \frac{m_\tau}{\sqrt{q^2}} \cos\theta \right.$$

$$\left. + \sqrt{w^2-1}\, \frac{m_B m_\tau}{2 q^2} \left(f + a_+ m_B^2 (1-r^2) + a_- q^2 \right) \right]$$

$P_1 m_B \sqrt{r} (1+r)$

$$A^{-+-} = \frac{4 G_F}{\sqrt{2}} V_{cb} \sqrt{q^2 - m_\tau^2}\, \frac{1}{2} \left(f - m_B^2 r \sqrt{w^2-1}\, g \right) \left[+\sqrt{2}\, e^{i\phi} \sin^2 \theta/_2 \right]$$

$$A^{---} = \quad '' \quad \left[\frac{-m_\tau}{\sqrt{q^2}} e^{i\phi} \sin\theta \right]$$

All m_τ terms have $s_\tau = -$. To construct the rate, we have simply

$$d\Gamma = \frac{\overbrace{\eta_{EW}^2}^{1 + \alpha/\pi \log m_Z/m_B \simeq 1.0066}}{2 m_B} \sum_{\lambda s_\tau s_\nu} \left| A^{\lambda s_\tau s_\nu} \right|^2 \, dPS \qquad -(22)$$

$$dPS \to \frac{1}{256 \pi^4}\, r \sqrt{w^2-1}\, \underbrace{dq^2}_{2 m_B^2 r\, dw}\, d\Omega_\tau \left(\frac{q^2 - m_\tau^2}{q^2} \right)$$

Because we have the amplitudes it is a simple matter to add e.g. $D^* \to D\pi$ amplitudes (which are just $L=1$ spherical harmonics in D^* rest frame)

Note the FF basis maps

$$g = \frac{\overbrace{h_V}^{R_1 h_{A_1}}}{\cancel{2} M_B\sqrt{r}}\,, \qquad f = h_{A_1} M_B \sqrt{r}\,(w+1)$$

$$F_1 \equiv M_B f(w-r) + 2M_B^2\, a_+ (w^2-1)\, r \qquad -(23)$$

$$= M_B^2 \sqrt{r}(w+1)\, h_{A_1}\left[w - r - (w-1)R_2\right]$$

$$\text{with } R_2 \equiv \frac{h_{A_3} + r h_{A_2}}{h_{A_1}}$$

$$P_1 \equiv \frac{1}{M_B\sqrt{r}(1+r)}\left(f + a_+ M_B^2(1-r^2) + a_- q^2\right)$$

$$= \frac{h_{A_1}(w+1) + h_{A_2}(rw-1) + h_{A_3}(r-w)}{1+r} \qquad -(24)$$

$$= h_{A_1} R_0$$

so can express everything in terms of $\underbrace{h_{A_1},\ R_{1,2}}_{m_\tau \to 0}$ and R_0.

$r_\tau = \frac{m_\tau}{M_B}$

$\hat{q}^2 = q^2/M_B^2$

$= 1 + r^2 - 2rw$

$$\frac{d\Gamma}{dw\, d\cos\theta} = 2\Gamma_0 \sqrt{w^2-1}\, r^3 \left(\frac{\hat{q}^2 - r_\tau^2}{\hat{q}^2}\right)^2 \Bigg\{ \left(1 + \frac{r_\ell^2}{2\hat{q}^2}\right)\left[H_+ + 2\hat{q}^2 H_1\right]$$

$$+ \frac{3r_\tau^2}{2\hat{q}^2} H_0 \quad + \quad \cos\theta_\ell\, H_{+0}$$

$$+ \quad \frac{3\cos^2\theta_\ell - 1}{2}\left(\frac{\hat{q}^2 - r_\tau^2}{\hat{q}^2}\right)\left[\hat{q}^2 H_1 - H_+\right]\Bigg\}$$

$$H_1 = \frac{f^2}{r M_B^2} + g^2 r M_B^2 (w^2-1) \qquad -(25)$$

$$= h_{A_1}^2\left[(w+1)^2 + (w^2-1) R_1^2\right]$$

$$H_+ = \frac{F_1^2}{r M_B^4} = h_{A_1}^2 (w+1)^2 \left[w - r - (w-1) R_2\right]^2$$

$$H_0 = P_1^2 (1+r)^2 (w^2-1) = h_{A_1}^2 R_0^2 (1+r)^2 (w^2-1)$$

$$H_{+0} = 6\hat{q}^2 f g \sqrt{w^2-1} - \frac{3\,\cancel{m}_\tau^2}{\hat{q}^2}\left(H_+ H_0\right)^{1/2}$$

In the massless limit, everything depends only on

$$H_+ + 2\hat{q}^2 H_1 = h_{A_1}^2 (w+1)^2 \left\{ 2\hat{q}^2 \left[1 + \left(\frac{w-1}{w+1}\right) R_1^2 \right] + \left[w-r-(w-1)R_2 \right]^2 \right\}$$

$$= (w+1)^2 \left[(1-r)^2 + \frac{4w}{w+1} \hat{q}^2 \right] \mathcal{F}^2(w) \qquad \text{— (26)}$$

s.t. $\mathcal{F}(1) = h_{A_1}(1)$, $\mathcal{F}(w) \to h_{A_1}(w)$ in HQ limit $R_1 = R_2 = 1$