Frequency Map Analysis for the HER with the Spin Rotator

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What is FMA?

- Created by Jaques Laskar in 1987 with the intent to study the movement of solar systems
- Very useful for quantifying the stability of particles around an accelerator
- Can be generalized to quasiperiodic systems
- Other accelerators (ALS – Advanced Light Source) as well as solar system studies have found good use of FMA
- Based on the robust and precise NAFF (Numerical Analysis of Fundamental Frequencies) algorithm

Laskar, J. (1999). Introduction to frequency map analysis. In *Hamiltonian systems with three or more degrees of freedom* (pp. 134-150). Springer, Dordrecht.

Accelerator: Frequency Map Principle

Taken from Nadolski's slides

Nadolski, L. S. (2010-2011). *Frequency Map Analysis.* Beam Dynamics Group, Synchrotron SOLEIL. https://npac2013.lal.in2p3.fr/2012-2013/Cours/AccConcept/fma2011_handout.pdf

- For each starting set (x_0, y_0), the particle trajectory is integrated over 2×1000 turns for computing the frequency and the diffusion coefficient.

$$D = \log_{10}\left(\sqrt{\left(\nu_x^{(2)} - \nu_x^{(1)}\right)^2 + \left(\nu_y^{(2)} - \nu_y^{(1)}\right)^2}\right) \qquad (28)$$

- This diffusion index is used **both** for the dynamics aperture and the frequency map
- **This diffusion index is an excellent criterium for the long term stability** (Dumas and Laskar, 1993).
- This construction scheme is repeated again and again for each point of a grid of initial conditions.

Diffusion Index

- Used to study the **stability of particle motion at different positions in x-y space,** and the **dynamic aperture**
- From the steps in the NAFF algorithm, we obtain two max frequencies:
- The diffusion index is the log10 of the averaged differences:

$$D = \log_{10}\left(\sqrt{\left(\nu_x^{(2)} - \nu_x^{(1)}\right)^2 + \left(\nu_y^{(2)} - \nu_y^{(1)}\right)^2}\right)$$

- **This is what generates the color map on the FMA plots.**
- A factor of 1/N may be included (N corresponding to the turns)
- In addition, Bmad allows the monitoring of diffusion in **tune space**

Nadolski, L. S. (2010-2011). *Frequency Map Analysis.* Beam Dynamics Group, Synchrotron SOLEIL.
https://npac2013.lal.in2p3.fr/2012-2013/Cours/AccConcept/fma2011_handout.pdf

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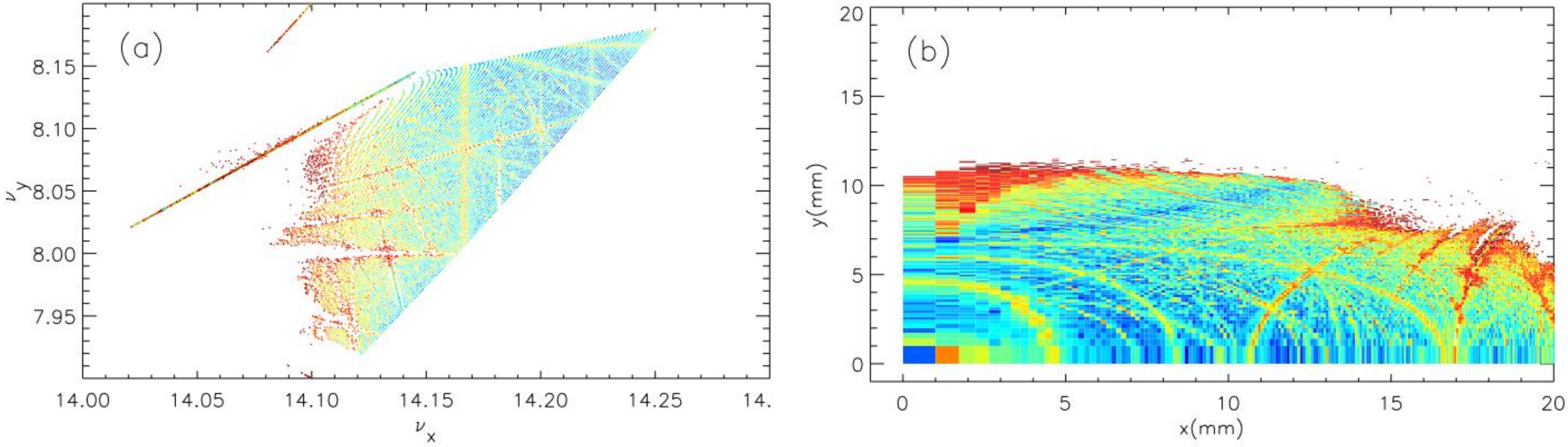
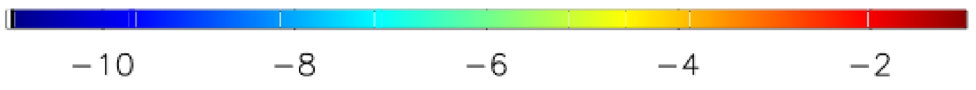
FMA Example (ALS)



The Diffusion Index (D), is represented by the color on the FMA plots.



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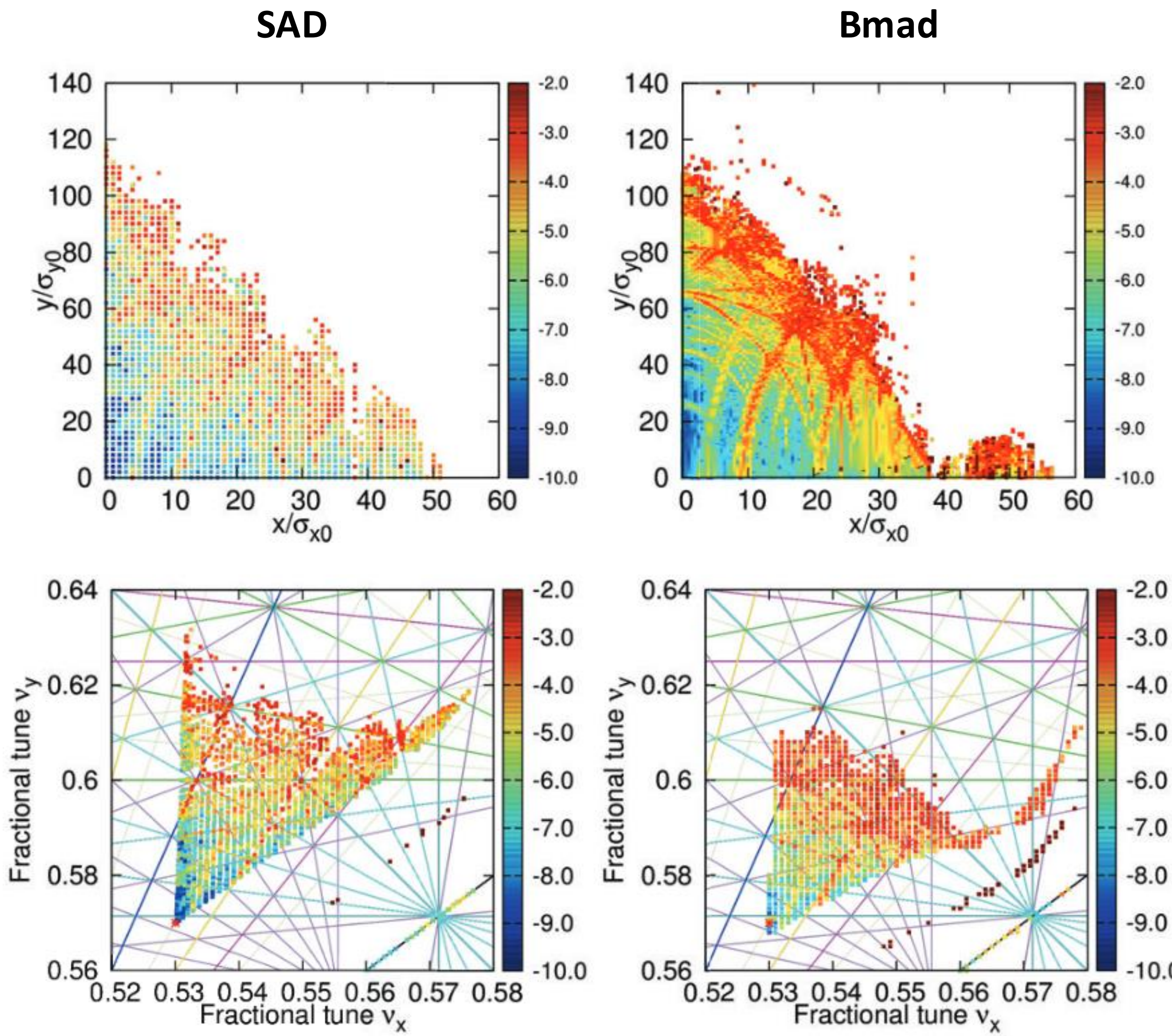
Laskar, J. (2003) *Frequency Map Analysis and Particle Accelerators.* [Paper]. Proceedings of 2003 Particle Accelerator Conference, Portland, Oregon. https://accelconf.web.cern.ch/p03/PAPERS/WOAB001.PDF

FMA Example (SuperKEKB LER):



From the paper validating the conversion of the lattice file from SAD to Bmad

Zhou, D., Biagini, M., Carmignani, N., Koiso, H., Liuzzo, S., Morita, A., Ohnishi, Y., Oide, K., Sagan, D., & Sugimoto, H. (2016). Lattice Translation Between Accelerator Simulation Codes for Superkekb. Proceedings of the 7th Int. Particle Accelerator Conf., IPAC2016, Korea. https://doi.org/10.18429/JACOW-IPAC2016-WEPOY040

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FMA with Bmad

- Generated each "grid point" on the FMA plot
- Specify start and endpoints in **x**, **y, e** as well as step size **dx, dy, de**
- Additionally, there are tools available to do momentum aperture studies

- In the below test, **dx, dy** are integer multiples of **σ** determined using: $\sigma_{x,y} = \sqrt{\varepsilon_{x,y} \times \beta^*_{x,y}}$
- HER lattice file, 1000 turns
- A typical output looks like:

x	y	e	NAFFx-1	NAFFy-1	NAFFe-1	NAFFx-2	NAFFy-2	NAFFe-2	Delta NAFFx	Delta NAFFy	Delta NAFFe
0.000000E+00	0.000000E+00	0.000000E+00	4.701500E-01	4.208500E-01	9.753200E-01	4.707800E-01	4.217400E-01	9.748700E-01	6.306400E-04	8.962100E-04	-4.528300E-04
8.162700E-05	0.000000E+00	0.000000E+00	4.706500E-01	4.173300E-01	9.746100E-01	4.706500E-01	4.173300E-01	9.746100E-01	-3.079800E-10	6.751200E-07	9.120800E-09
1.632500E-04	0.000000E+00	0.000000E+00	4.697300E-01	4.067600E-01	9.746100E-01	4.697300E-01	4.078900E-01	9.746100E-01	3.969500E-07	1.129000E-03	3.040100E-08
2.448800E-04	0.000000E+00	0.000000E+00	4.685400E-01	4.016100E-01	9.746100E-01	4.685400E-01	4.016100E-01	9.746100E-01	-3.695000E-10	3.553700E-06	-1.123900E-08
3.265100E-04	0.000000E+00	0.000000E+00	4.673900E-01	4.673900E-01	9.746100E-01	4.673900E-01	4.673900E-01	9.746100E-01	3.155800E-09	-1.663800E-07	1.304700E-08
4.081300E-04	0.000000E+00	0.000000E+00	4.666500E-01	1.996800E-03	1.999800E-03	4.666500E-01	2.002800E-03	2.000100E-03	-6.430200E-09	6.013000E-06	2.887000E-07
4.897600E-04	0.000000E+00	0.000000E+00	4.669300E-01	3.519500E-01	2.001700E-03	4.669300E-01	3.521500E-01	1.999700E-03	6.193600E-07	2.034000E-04	-1.965000E-06
0.000000E+00	1.186900E-07	0.000000E+00	2.000000E-03	4.212900E-01	9.746100E-01	2.000000E-03	4.212900E-01	9.746100E-01	-2.975600E-09	-2.278700E-12	5.517400E-06
8.162700E-05	1.186900E-07	0.000000E+00	4.706500E-01	4.173400E-01	9.746100E-01	4.706500E-01	4.173400E-01	9.746100E-01	-3.335300E-10	3.735200E-08	8.988100E-09

+ many more rows...

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Learning Bmad

- Yuhao is working on recreating Bmad's FMA approximately with FFTs
- Results seem to agree at a cursory glance

- We are also considering the HER with the rotator with different Qx, Qy in order to get a stable beam in the long-term tracking studies
- **Rot1** and **Rot2** designs are currently stable for 3 damping times with radiative fluctuations

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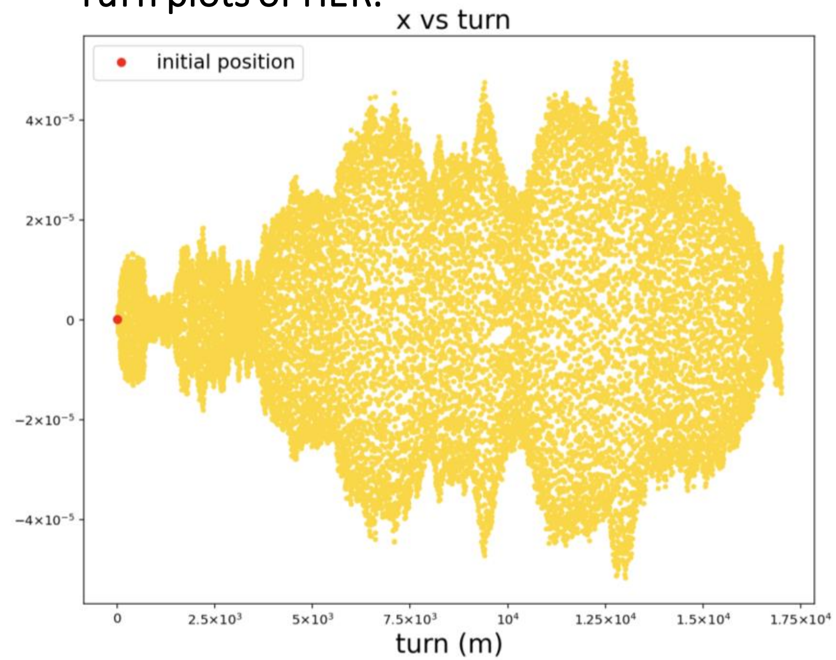
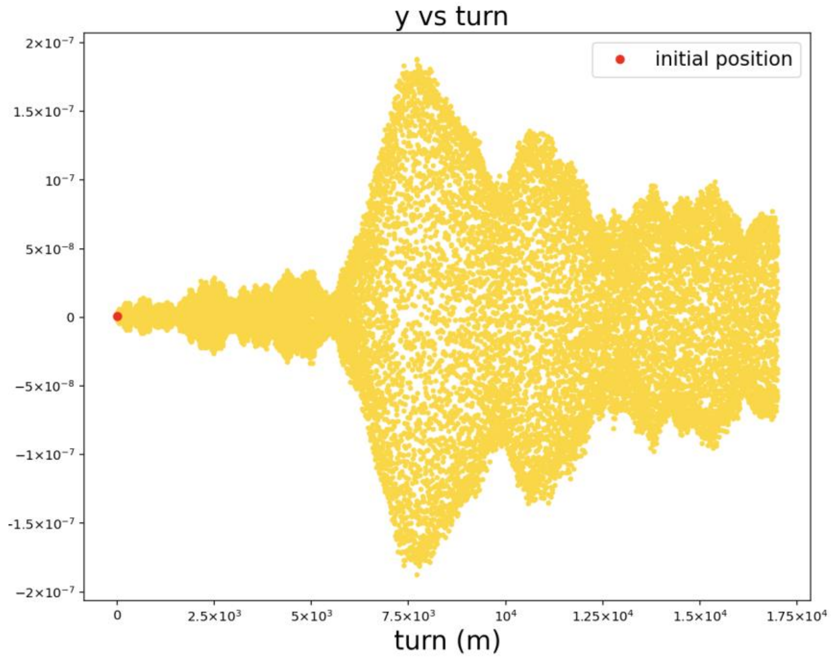
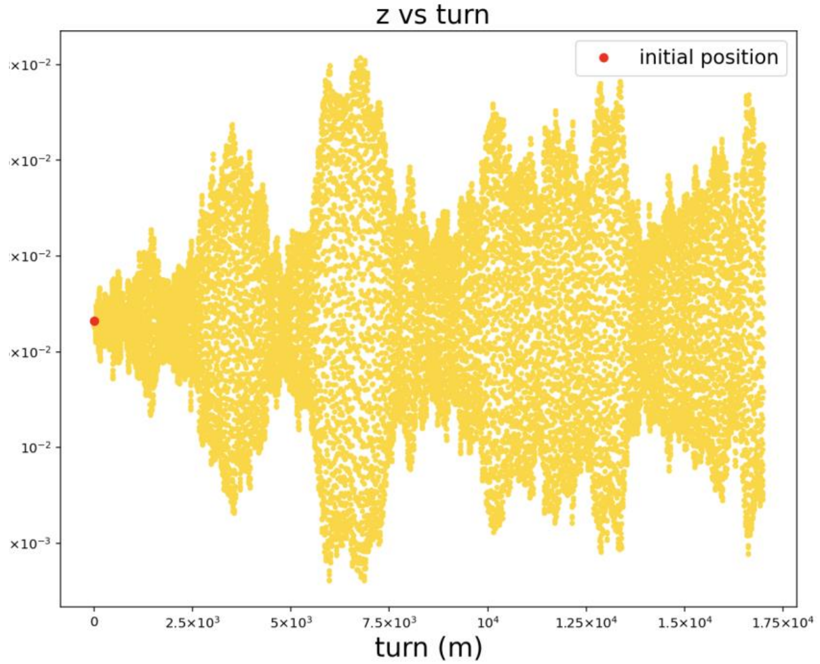
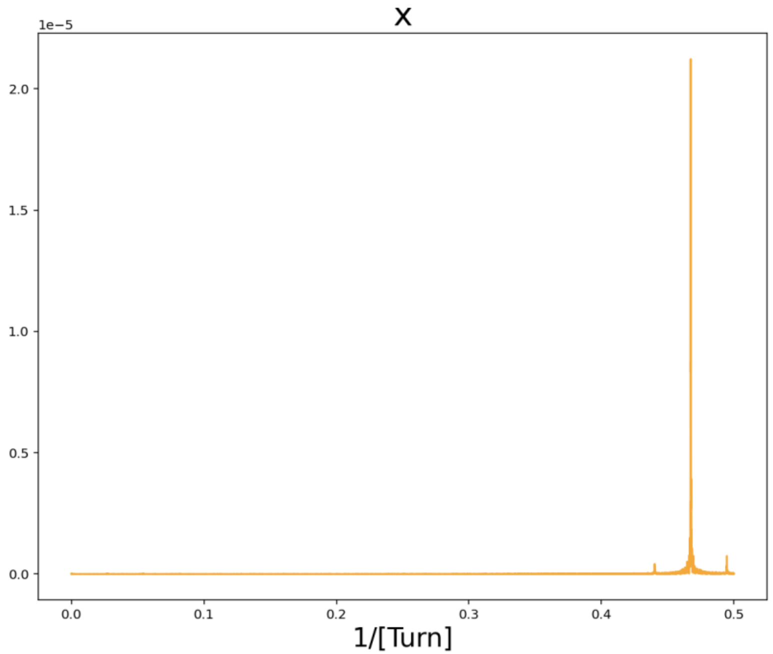
Turn plots of HER:

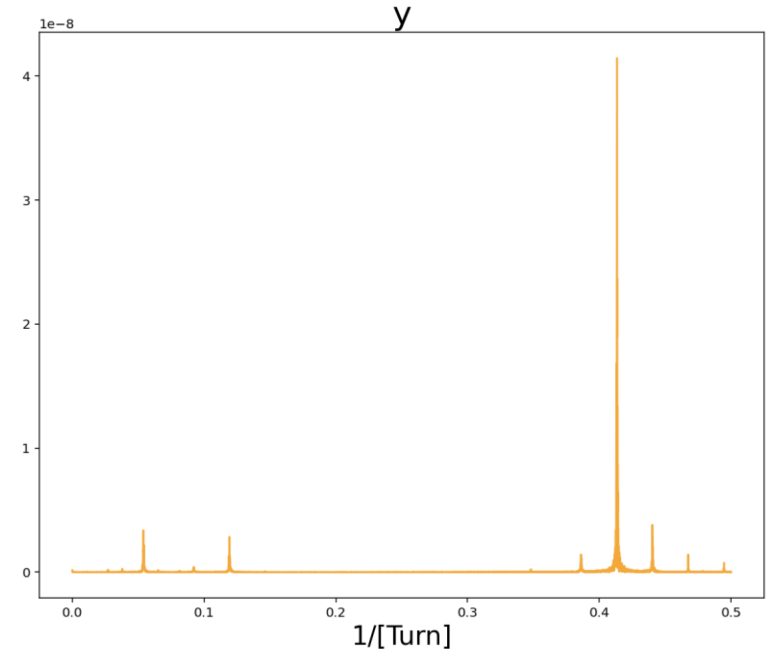


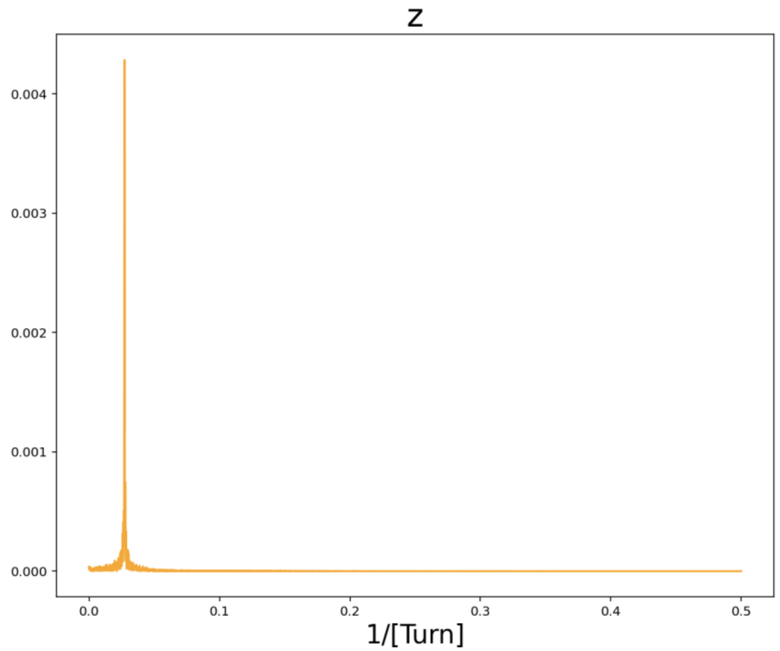




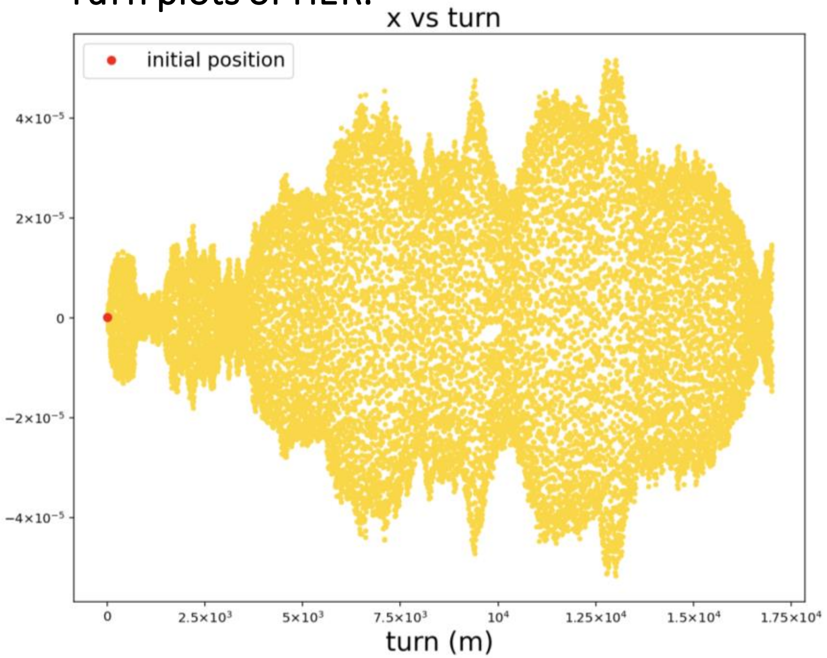
FFT frequency spectrum of HER:

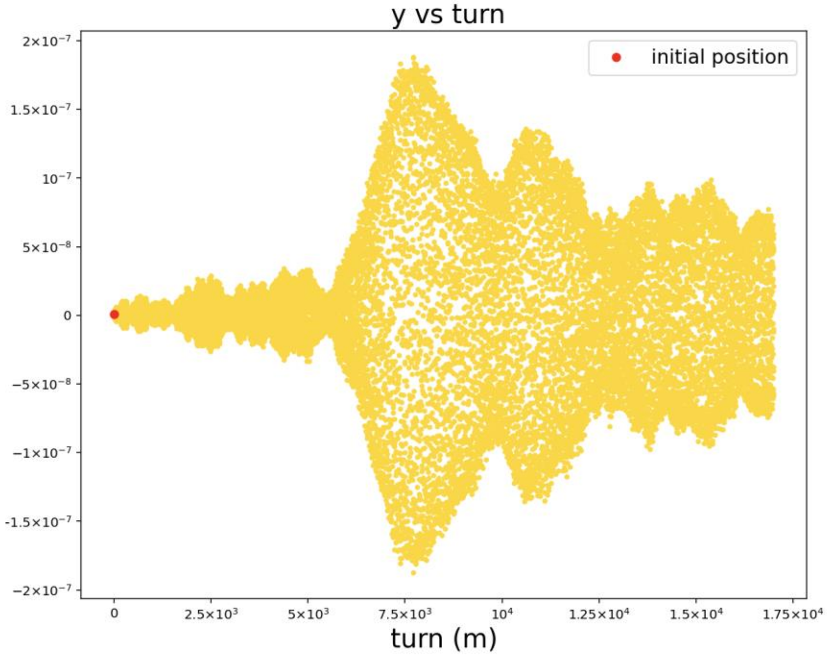


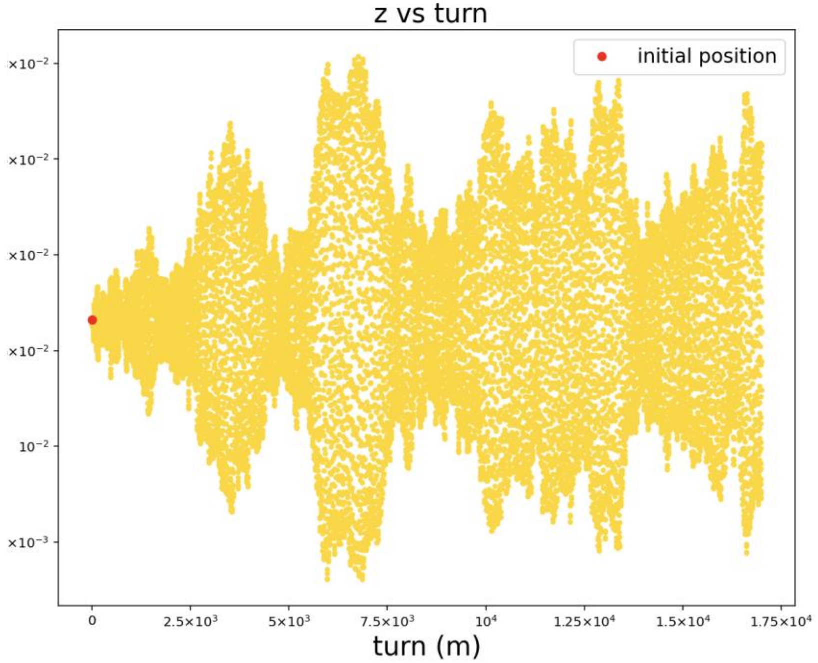




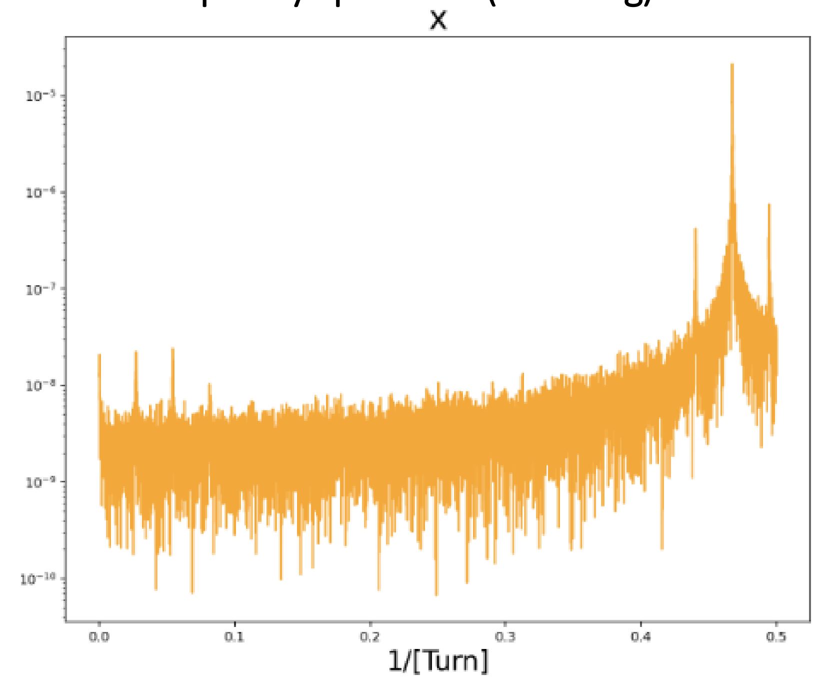
Turn plots of HER:

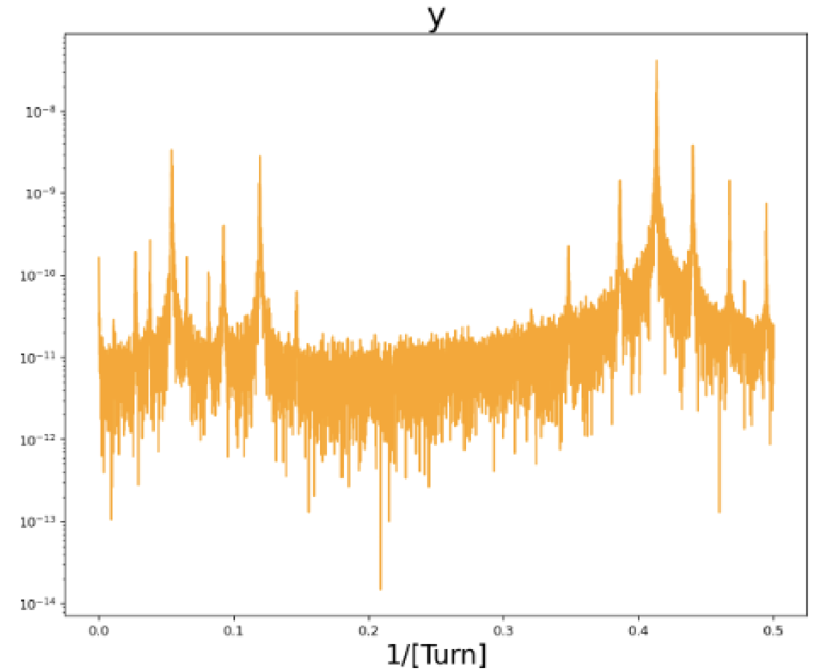


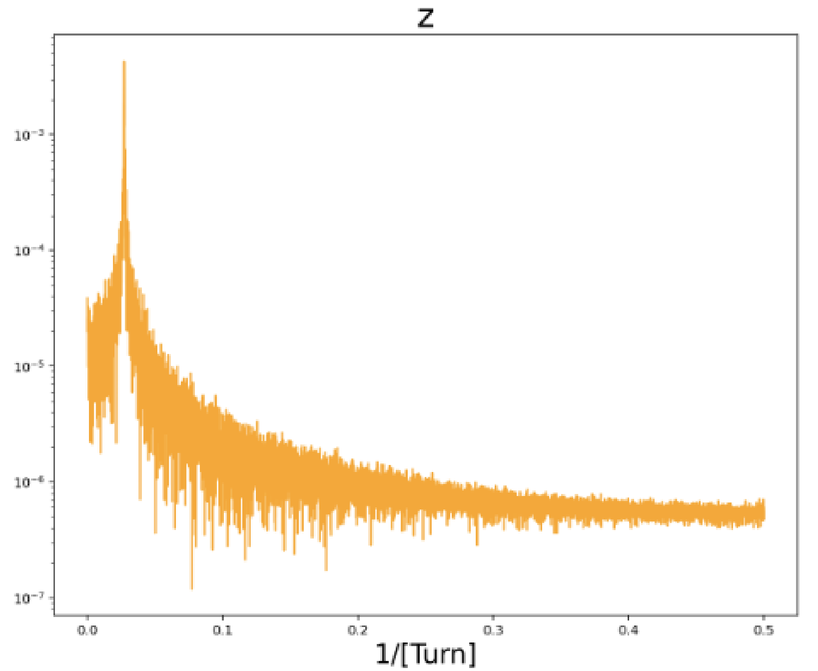




FFT frequency spectrum (semi-log):







First order NAFF frequencies (x) for the HER

HER	5σ_x	10σ_x	15σ_x
5σ_y	4.706500E-01	4.697300E-01	4.685400E-01
10σ_y	4.706500E-01	4.697300E-01	4.685400E-01
15σ_y	4.706500E-01	4.685400E-01	4.685400E-01

First order NAFF frequencies (x) for Rot1

Rot1	5σ_x	10σ_x	15σ_x
5σ_y	0.43160E+00	0.43053E+00	0.42898E+00
10σ_y	0.43159E+00	0.43052E+00	0.42898E+00
15σ_y	0.43157E+00	0.43051E+00	0.42897E+00

First order NAFF frequencies (y) for the HER

HER	5σ_x	10σ_x	15σ_x
5σ_y	4.173400E-01	4.076000E-01	3.957800E-01
10σ_y	4.173500E-01	4.076500E-01	3.958200E-01
15σ_y	4.173600E-01	4.076800E-01	3.958400E-01

First order NAFF frequencies (y) for Rot1

Rot1	5σ_x	10σ_x	15σ_x
5σ_y	0.39770E+00	0.38678E+00	0.37158E+00
10σ_y	0.39775E+00	0.38683E+00	0.37161E+00
15σ_y	0.39784E+00	0.38691E+00	0.37167E+00

Generated with BMAD

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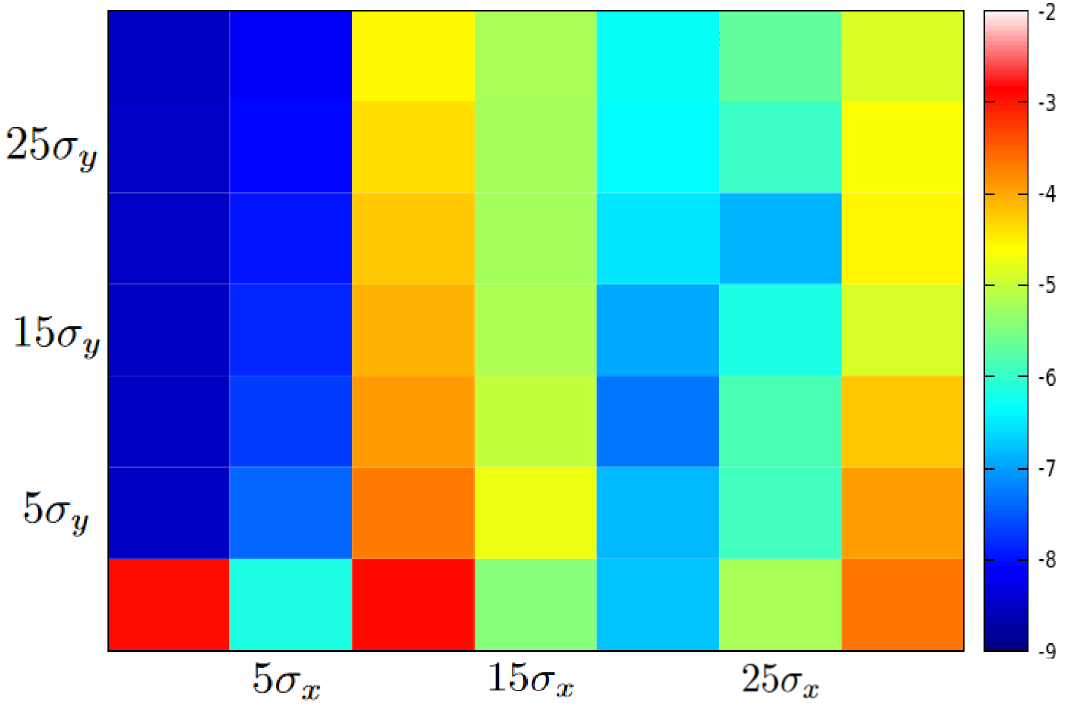
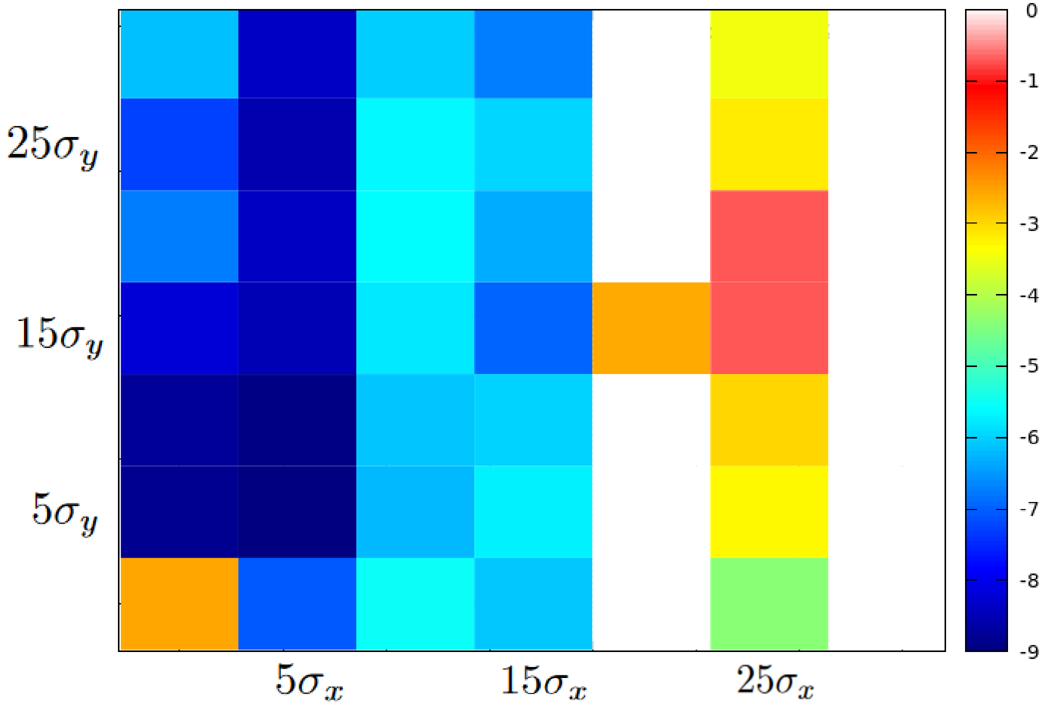
FMA Preliminary Plots

Coarse Frequency Map (HER):



Coarse Frequency Map (Rot1):



- I am getting a grasp on how to use Bmad's FMA calculation tools
- Not plotted with Bmad yet, just gnuplot (still some technical issues)

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Next Steps

- ➢Getting BMAD's FMA plotting working as intended
- ➢More detailed, higher resolution mapping
  - Most likely with grid computing on Compute Canada
  - More turns, smaller incrementation, larger scope...
  - We'd like to be able to compare them to the SAD FMA plots of the HER
- ➢These studies will guide which tunes will work for the combined HER + rotator ring
  - Need to discuss with experts about constraints to tune
- ➢Use BMAD and FMA to study the Momentum Aperture
  - The MA is sensitive to many beam parameters, could provide some new insight†
- ➢Further study the effects of radiation damping on the Spin Rotator Design

†Steier, C., Robin, D., Decking, W., Laskar, J., Nadolski, L., & Wu, Y. (2005, May). Measuring and understanding the momentum aperture in a storage ring. In *Proceedings of the 2005 Particle Accelerator Conference* (pp. 645-649). IEEE.

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Extra: NAFF algorithm

- Much more precise method of approximating the quasiperiodic behavior of a particle in the accelerator ring than standard FFT methods
- Simple description of the procedure
  1. Assume the behavior can be (roughly) approximated into $f(t) = \sum_{n=1}^{\infty} a_n e^{i\nu_n t}$
  2. FFT on this signal
  3. Include a secondary function $g(t) = e^{iwt}$
  4. Find the two highest maximum amplitudes of the FFT and secondary function (using Gaussians or **Hanning windows**, etc.)
  5. Take the initial function and subtract the secondary function
  6. Repeat with the new function

May need a ***window function!***

Better than FFT *for precision!*

Nadolski, L. S. (2010-2011). *Frequency Map Analysis.* Beam Dynamics Group, Synchrotron SOLEIL. https://npac2013.lal.in2p3.fr/2012-2013/Cours/AccConcept/fma2011_handout.pdf

Skoufaris. K. (2019). *An update on the NAFF tool NAFF_UV.* CERN. https://indico.cern.ch/event/797108/contributions/3312193/attachments/1801356/2938840/An_update_on_the_NAFF_tool.pdf

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Extra: Useful Documents & Sites

- SuperKEKB related FMA: https://accelconf.web.cern.ch/ipac2016/papers/wepoy040.pdf
- The very useful presentation on FMA by Nadolski: https://npac2013.lal.in2p3.fr/2012-2013/Cours/AccConcept/fma2011_handout.pdf
- FMA used at the ALS: https://accelconf.web.cern.ch/p03/PAPERS/WOAB001.PDF
- Momentum Aperture study of the ALS: https://accelconf.web.cern.ch/P05/PAPERS/WOAC008.PDF
- Github page for FortNAFF, which is used in Bmad: https://github.com/MichaelEhrlichman/FortNAFF
- Bmad's manual: https://www.classe.cornell.edu/bmad/manual.html