Spin Polarization Simulations for the Future Circular Collider e+e- using BMAD

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Swiss Accelerator Research and Technology

Outline

Motivation

- FCC-ee, the first step of the FCC project, will offer high precision explorations of physics at four center-of-mass energies.
- The high precision center-of-mass energy calibration is feasible at Z and W energies by means of resonant depolarization.
- Spin simulations for the validation of the energy calibration method
- Effects of lattice perturbations on spin polarization should be investigated.
- Sufficient polarization levels under various possible lattice conditions
- BMAD, a simulation tool that allows full lattice control and the spin simulations, being actively developed and sustains an active group of users and developers.

BMAD Home Page, https://www.classe.cornell.edu/bmad/

Beam Energy Measurement in FCC-ee

- Resonant depolarisation as a beam energy measurement method relies on the relationship between beam energy and spin tune.

$$\nu \approx a \frac{E}{mc^2}$$

- Possible bias in beam energy due to machine imperfections.
- Latest precision target is 4 keV at Z and 100 keV at W
- Proposed running mode: around 200 non-colliding pilot bunches per beam will be injected first and polarized by wigglers, then inject bunches for luminosity running ⇒ frequent measurement of beam energy during luminosity data taking

FCC collaboration. "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2." European Physical Journal: Special Topics 228.2 (2019): 261-623.

Polarization Build-Up with Radiative Depolarization

- ST effect + radiative depolarization → equilibrium polarization
- Derbenev–Kondratenko–Mane (DKM) formula when radiative depolarization is considered

$$P_{DK} = -\frac{8}{5\sqrt{3}} \times \frac{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \hat{b} \cdot \left(\hat{n} - \frac{\partial \hat{n}}{\partial \delta} \right) \right\rangle_s}{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \left(1 - \frac{2}{9} \left(\hat{n} \cdot \hat{s} \right)^2 + \frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2 \right) \right\rangle_s}$$

$$\tau_{DK}^{-1} = \tau_{BKS}^{-1} + \tau_{dep}^{-1}$$

$$\tau_{dep}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_e} \frac{1}{C} \oint \mathrm{d}s \left\langle \frac{\frac{11}{18} \left(\frac{\partial \hat{n}}{\partial \delta} \right)^2}{|\rho(s)|^3} \right\rangle_s$$

- $\partial \hat{n} / \partial \delta$: the spin-orbit coupling function

Desmond Barber, Radiative Polarization in Electron Storage Rings

Linear Polarization Calculation

SLIM formalism for linearized orbital and spin motions

- 6×6 orbital transfer matrix $\rightarrow 8 \times 8$ spin-orbit transfer matrix

$$\mathbf{T}_{8\times 8} = \begin{pmatrix} \mathbf{M}_{6\times 6} & \mathbf{0}_{6\times 2} \\ \mathbf{G}_{2\times 6} & \mathbf{D}_{2\times 2} \end{pmatrix}$$

- spin-orbit vector $(x, x', y, y', z, \delta, \alpha, \beta)$ with respect to the closed orbit
- $\vec{S} \approx \hat{n}_0 + \alpha \hat{m} + \beta \hat{l}$, unit along $\hat{n}_0$, small deviation from $\hat{n}_0$

Tao (BMAD)

SLIM formalism from A.W. Chao, Evaluation of radiative spin polarization in an electron storage ring

Main Lattice Parameters

Sequence 217 at Z energy is used in the simulations

Circumference (km)	97.756
Beam energy (GeV)	45.6
β_x^* (m)	0.15
β_y^* (mm)	0.8
ϵ_x (nm)	0.27
ϵ_y (pm)	1
Synchrotron tune Q_z	0.025
Horizontal tune Q_x	269.139
Vertical tune Q_y	269.219

Table: Main parameters at Z energy

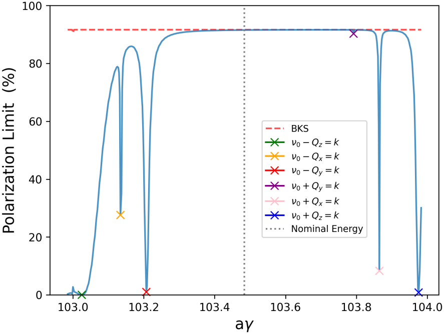
Old version lattice with 60° FODO cells

FCC collaboration. (2019). FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2. European Physical Journal: Special Topics, 228(2), 261-623.

Energy Scan in Tao (BMAD)

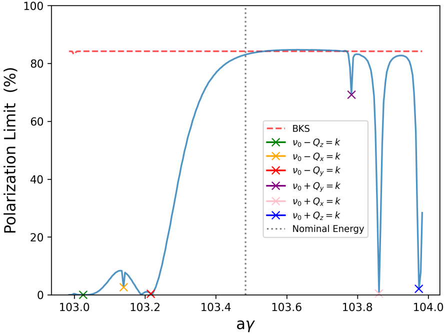
- Use two small error seeds generated from truncated Gaussian distributions
- Six first order spin-orbit resonances between two integer spin tunes
- Polarization curves are sensitive to imperfections

$(\Delta y)_{\text{rms}} = 43.7\,\mu\text{m}$



91.6% near nominal energy

$(\Delta y)_{\text{rms}} = 148\,\mu\text{m}$

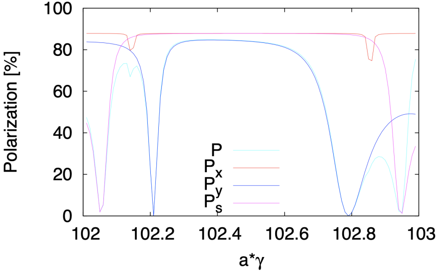


84.6% near nominal energy

Benchmark between Tao (BMAD) and SITF

EPFL

- SITF, the linear spin simulation module in SITROS
- Underlying differences between two codes exist → check step by step



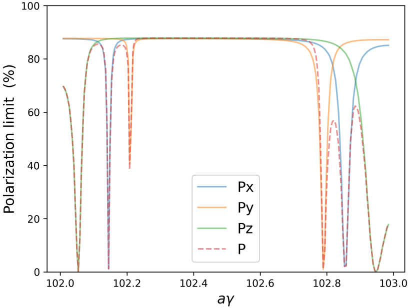


Figure: Energy scan using sequence version 213 seed 13 in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Benchmark between Tao, SITF and SLIM

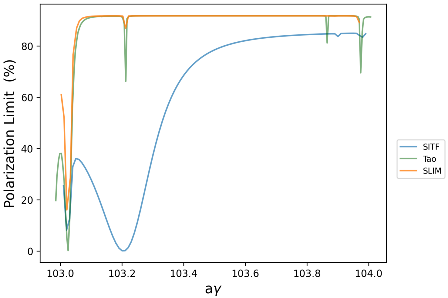


Figure: Energy scan of the equilibrium polarization (left) and the vertical mode polarization (right) by three codes

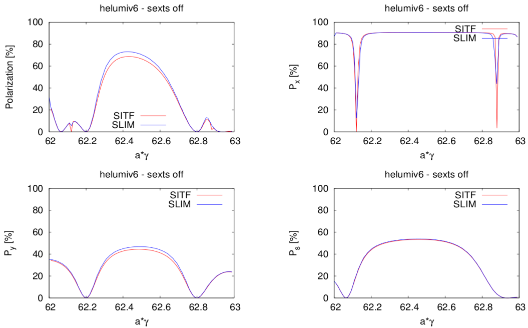
SITF produces much stronger depolarization effects near vertical spin-orbit resonance

SITF and SLIM data are from Eliana Gianfelice-Wendt

Discussions Regarding the Damping in Transport Matrix

Thanks to Eliana Gianfelice-Wendt and Desmond Barber!

- In SLIM/Tao linear calculation undamped 8×8 transport matrix is used for polarization.
- In SITF/SITROS tracking the damped transport matrix is used between emission points.
- Two codes agree when the undamped matrix is used



Nonlinear Spin Tracking

- The higher order resonances may become prominent at high energies and affect the achievable polarization level
- Reveal all effects of lattice imperfections on spin polarization
- Long-Term Tracking module in BMAD
- Track the polarization level turn by turn and extract τ_{dep}

$$P(t) = P_{DK}\left[1 - e^{-t/\tau_{DK}}\right] + P_0 e^{-t/\tau_{DK}} \simeq P_0 e^{-t/\tau_{dep}}$$

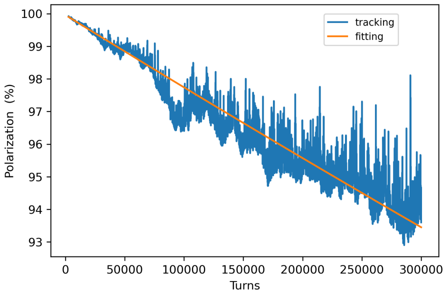
$$P_{eq} \simeq P_{BKS}\frac{\tau_{dep}}{\tau_{BKS} + \tau_{dep}}$$

Long-Term Tracking in BMAD

PTC, $\nu_0 = m + Q_y - Q_s$

10 electrons

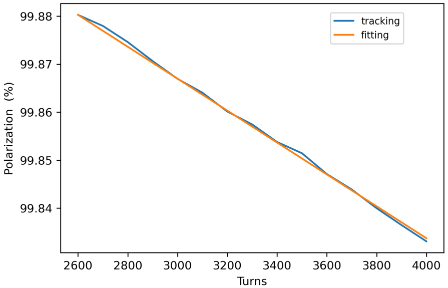
$P_{eq} = 0.15\%$



RMSE$=0.59$
$R^2 = 0.89$

500 electrons

$P_{eq} = 0.099\%$



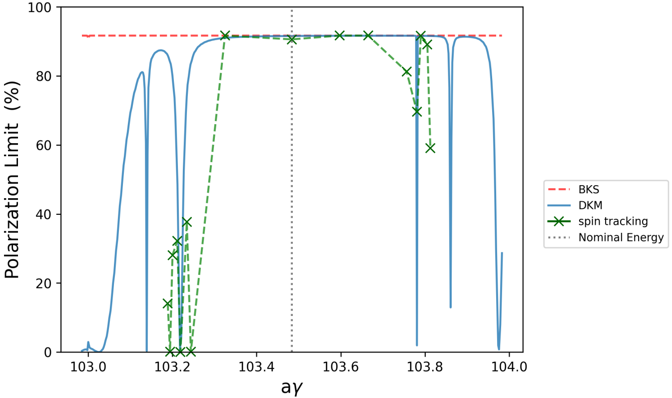
RMSE$=4.4 \times 10^{-4}$
$R^2 = 0.999$

Using more particles improves the fitting precision but needs more time.

Root-mean-square error, RMSE $= \sqrt{\sum_{i=1}^{N}(P - P*)^2/N}$

Preliminary Results of Nonlinear Spin Tracking

nonlinear: 1000 particles, 7000 turns, PTC BMAD



More data points are needed near nominal energy

Harmonic spin matching (HSM)

- $\hat{n}_0(s)$: Periodic and stable spin direction on the closed orbit
- In a perfectly aligned flat ring, $\hat{n}_0(s)$ is vertical, $P_{eq} \approx 92.4\%$
- Misalignments in the flat ring $\rightarrow$ vertical misalignments of quadrupoles $\rightarrow$ $\hat{n}_0(s)$ deviation from vertical $\rightarrow$ stronger spin diffusion
- Vertical quadrupole misalignments are difficult to control
- After conventional orbit correction, harmonic spin matching is needed to correct the $\hat{n}_0(s)$ deviation and reduce the polarization loss due to spin diffusion

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

$\hat{n}_0(s)$ tilt correction

$\hat{m}_0(s)$ and $\hat{l}_0(s)$ are two solutions of the T-BMT equation on the closed orbit, $(\hat{n}_0, \hat{m}_0, \hat{l}_0)$ form a right-handed orthonormal basis

$$\delta\hat{n}_0 = \alpha\hat{m}_0 + \beta\hat{l}_0$$

Expand α and β to Fourier series

$$(\alpha - i\beta)(s) = -i\frac{C}{2\pi}\sum_k \frac{f_k}{k - \tilde{\nu}} e^{i2\pi ks/C}$$

f_k is related with the closed orbit and perturbing fields

Make additional orbit corrections using vertical correction magnets, and reduce the rms tilt by minimizing the Fourier coefficients

$\delta\hat{n}_0$ is sensitive when k is close to $\tilde{\nu}$

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

$\hat{n}_0(s)$ tilt correction in the FCC-ee

- Harmonics of -1, 0, 1, 2 should be controlled
- At least 8 closed bumps will be used
- Each bump contains at least three vertical correctors
- Bump installment locations shall be optimized

First trials of adding harmonic bumps

- Using lattice version 22 with 4 IPs
- Small residual errors generated to simulate the orbits after lattice correction
- Control harmonics 0 and 1 using at least 4 bumps
- Expand $n_{0x}(s) + in_{0z}(s)$ to Fourier series

$$n_{0x} + in_{0z} \approx \sum_{k=-N}^{N} c_k \cdot e^{i2\pi ks/C}$$

$$c_k = \frac{1}{C}\int (n_{0x} + in_{0z}) \cdot e^{-i2\pi ks/C} ds \approx \frac{\sum_{j=1}^{M}[n_{0x}(s_j) + in_{0z}(s_j)] \cdot e^{-i2\pi ks_j/C}}{M}$$

The coefficient is estimated with a Riemann sum of M data points

Harmonic bumps

1st corrector: give a vertical kick

2nd corrector: kick y back to initial value at the 3rd corrector

3rd corrector: kick py back to initial value

The kicks of 2nd and 3rd correctors are adjusted to make the bump closed

one independent variable for each bump

Matrix construction

Since each bump has an independent and linear contribution to the harmonics coefficients, a matrix is constructed to express the linear relation, and estimate the total contribution from all bumps

$$\mathbf{MK} = \mathbf{C}$$

K: amplitudes (the first kick value) of the bumps
C: real and imaginary parts of the required harmonics coefficients
$[c_{0\mathrm{real}}, c_{0\mathrm{imag}}, c_{1\mathrm{real}}, c_{1\mathrm{imag}}]$

For arbitrary misaligned lattice, **C** can be obtained by taking the opposite of the harmonics coefficients of misaligned lattice, and the required amplitudes of bumps **M** can be calculated via

$$\mathbf{K} = \mathbf{M}^{-1}\mathbf{C}$$

D. P. Barber, et al. "High spin polarization at the HERA electron storage ring." Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 338.2-3 (1994): 166-184.

Adding bumps in the arcs

What to do:

- Use a perfectly aligned lattice
- Add one bump by adding three vertical correctors in front of three consecutive vertically focusing quadrupoles
- Correct 2nd and 3rd kicks to make y and yp be all 0 outside the bump
- Analyze its contribution to harmonics 0 and 1
  (real and imaginary part of c_0 and $c_1 \rightarrow 4$ real numbers)
- Change the bump position and redo the analysis
- Determine the bumps to use and construct relations between bump amplitudes and Fourier coefficients

Example for HSM

At 45.82 GeV ($a\gamma = 103.983$)

$kick = 6 \times 10^{-6}$

Bumps num	c_0 real	c_0 imag	c_1 real	c_1 imag
1	1.16×10^{-4}	-3.74×10^{-4}	-5.51×10^{-7}	-5.25×10^{-6}
2	3.92×10^{-4}	-6.54×10^{-5}	3.73×10^{-6}	-1.00×10^{-5}
3	1.16×10^{-4}	-3.74×10^{-4}	-4.95×10^{-6}	-1.47×10^{-7}
4	3.92×10^{-4}	-6.53×10^{-5}	-9.25×10^{-6}	-3.76×10^{-6}
5	3.27×10^{-4}	1.87×10^{-4}	-4.98×10^{-6}	4.38×10^{-7}
6	-2.92×10^{-4}	2.66×10^{-4}	-2.31×10^{-6}	-7.23×10^{-6}
7	-3.90×10^{-4}	-3.79×10^{-6}	-2.57×10^{-6}	-2.43×10^{-6}
8	-2.92×10^{-4}	2.66×10^{-4}	-7.87×10^{-6}	2.72×10^{-6}

Example for HSM

Using an error seed that can produce vertical orbit distortion of 72 μm, before HSM

$$(c_0)_{real} = -1.35 \times 10^{-4},\ (c_0)_{imag} = -1.18 \times 10^{-5}$$

$$(c_1)_{real} = -4.06 \times 10^{-7},\ (c_1)_{imag} = 4.27 \times 10^{-7}$$

$$P_{DK} = 10.68\%,\ \delta n_0 = 2.28\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$$

Four bumps should together generate harmonic coefficients of

$$\mathbf{C} = [1.35 \times 10^{-4}, 1.18 \times 10^{-5}, 4.06 \times 10^{-7}, -4.27 \times 10^{-7}]$$

If bumps No.1, 2, 6, 8 are used, the estimated bump amplitudes are

$$\mathbf{K} = [-7.84 \times 10^{-7}, 2.05 \times 10^{-6}, -1.48 \times 10^{-6}, 1.15 \times 10^{-6}]$$

Example for HSM

Before correction

$$(c_0)_{real} = -1.35 \times 10^{-4},\ (c_0)_{imag} = -1.18 \times 10^{-5}$$

$$(c_1)_{real} = -4.06 \times 10^{-7},\ (c_1)_{imag} = 4.27 \times 10^{-7}$$

$$P_{DK} = 10.68\%,\ \delta n_0 = 2.28\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$$

After correction using 4 randomly chosen bumps

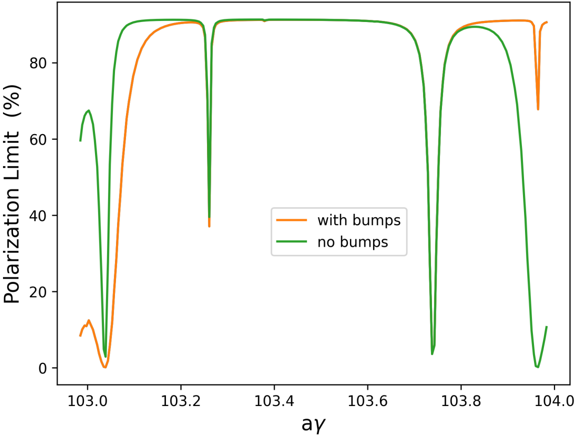
$$(c_0)_{real} = -1.37 \times 10^{-7},\ (c_0)_{imag} = 3.26 \times 10^{-6}$$

$$(c_1)_{real} = -3.90 \times 10^{-7},\ (c_1)_{imag} = 3.73 \times 10^{-6}$$

$$P_{DK} = 90.58\%,\ \delta n_0 = 0.912\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 72.09\,\mu\mathrm{m}$$

Energy scan comparison

Using 4 bumps which are optimized at 45.82 GeV ($a\gamma = 103.983$)



Bump choice optimization

Choose a combination of four bumps out of eight that produce a matrix **M** with the largest determinant, to minimize the vertical orbit distortion within bumps $\Rightarrow$ Bumps 1,2,4,6

Before correction

$$P_{DK} = 10.68\%,\ \delta n_0 = 2.28\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$$

Using one random combination of 4 bumps

$$P_{DK} = 90.58\%,\ \delta n_0 = 0.912\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 72.09\,\mu\mathrm{m}$$

Using an optimized combination of 4 bumps

$$P_{DK} = 90.53\%,\ \delta n_0 = 0.906\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.65\,\mu\mathrm{m}$$

Bump choice optimization

Using 8 bumps to correct harmonics 0 and 1 is a problem of High Dimensional Inference. The least-squares solution to the linear matrix equation is obtained

$$\mathbf{MK} = \mathbf{C}$$

where $\mathbf{M}$ is a 4×8 matrix and returned solution K will minimize the Euclidean 2-norm $||\mathbf{C} - \mathbf{MK}||$. If there are multiple minimizing solutions, the one with the smallest 2-norm $||\mathbf{K}||$ will be returned.

Bump choice optimization

Before correction

$$P_{DK} = 10.68\%,\ \delta n_0 = 2.28\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$$

Using optimized combination of 4 bumps

$$P_{DK} = 90.53\%,\ \delta n_0 = 0.906\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.65\,\mu\mathrm{m}$$

Using all 8 bumps

$$P_{DK} = 90.79\%,\ \delta n_0 = 0.903\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 70.80\,\mu\mathrm{m}$$

Bumps location optimization

Bump location standard:

- three correctors are set next to three consecutive vertically focusing arc quadrupoles
- sextupole within is closer to either the first or the third corrector

$\Rightarrow$ a total of 140 possible positions in the ring

Before correction

$$P_{DK} = 10.68\%,\ \delta n_0 = 2.28\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.96\,\mu\mathrm{m}$$

Using 4 bumps with smallest norm $||\mathbf{K}||$ required

$$P_{DK} = 90.96\%,\ \delta n_0 = 0.9\,\mathrm{mrad},\ (\Delta y)_{\mathrm{rms}} = 71.30\,\mu\mathrm{m}$$

Outlook

- Optimize HSM scheme to cope with various possible orbits
- Possible use of deterministic spin matching
- Benchmark with SITROS in nonlinear spin trackings (Desmond Barber, Eliana Gianfelice-Wendt)
- Resonant depolarization simulation (with David Sagan)
- Possible collaboration with KIT and ESRF

Thank you!

Effective Model

- Use an effective model to simulate realistic orbits after lattice correction
- The errors are randomly distributed obeying the truncated Gaussian distributions (truncated at $2.5\,\sigma$)

Residual errors after lattice correction

Type	$\sigma_{\Delta \mathrm{X}}$ (μm)	$\sigma_{\Delta \mathrm{Y}}$ (μm)	$\sigma_{\Delta \mathrm{S}}$ (μm)	$\sigma_{\Delta \mathrm{PSI}}$ (μrad)	$\sigma_{\Delta \mathrm{THETA}}$ (μrad)	$\sigma_{\Delta \mathrm{PHI}}$ (μrad)
Arc quadrupole	0.1	0.1	0.1	2	2	2
Arc sextupole	0.1	0.1	0.1	2	2	2
Dipoles	0.1	0.1	0.1	2	0	0
IR quadrupole	0.1	0.1	0.1	2	2	2
IR sextupole	0.1	0.1	0.1	2	2	2

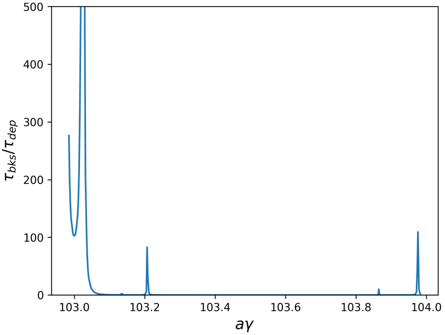
Table: An effective model for the small error generation used in the spin-orbit simulations

Energy Scan in Tao (BMAD)

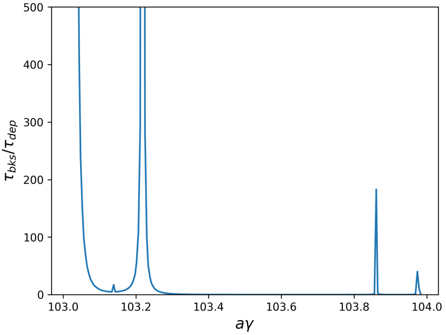
$$P_{eq} \simeq P_{bks} \frac{1}{1 + \tau_{bks}/\tau_{dep}}$$

$$\tau_{tot} = \tau_{bks} \frac{1}{1 + \tau_{bks}/\tau_{dep}}$$

$(\Delta y)_{\mathrm{rms}} = 43.7\,\mu\mathrm{m}$



$(\Delta y)_{\mathrm{rms}} = 148\,\mu\mathrm{m}$



Latest Lattice Parameters for HSM

Sequence 22 at Z energy

Circumference (km)	91.17
Beam energy (GeV)	45.6
β_x^* (m)	0.098
β_y^* (mm)	0.8
ϵ_x (nm)	0.71
ϵ_y (pm)	0.90
Synchrotron tune Q_z	0.037
Horizontal tune Q_x	214.26
Vertical tune Q_y	214.38

Table: Main parameters at Z energy

New version lattice with 90° FODO cells