KEK Linac polarization studies

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1 Introduction

This report is to study the electron polarization in the KEK Linac. Figure 1 shows the layout of the linear accelerator including the acceleration and injection section. The electron beam is prepared at 8.5 MeV at the exit of the source, then accelerated to 7 GeV by RF cavities, passes through four pairs of vertical bends, and enters the tunnel where the ring locates. The proposed polarization upgrade for the SuperKEKB HER is to install a pair of rotators to polarize the electron beam in the longitudinal direction at the interaction point (IP), and it requires the beam to be vertically polarized at the injection point. The vertical spin is preserved in the horizontal bending field but not in the vertical bending field. In staging this project, we are considering installing the polarized source with vertically polarized electrons in the HER as a first step and to study the spin lifetime and other properties before proceeding with the installation of the spin rotator. As Figure 2 shows, the vertical polarization in the HER has a long lifetime, which is about 12 hours based on the linear regression result. To test the behaviour of the spin in the Linac, the initial spin is set to be vertical in the BMAD simulation. As Figure 3 and 4 show, the vertical spin is well preserved before it reaches the vertical bends prior to the HER injection point. We can see that the vertical spin is rotated to other direction and re-established. It was found that those vertical bend pairs have an anti-symmetric structure, as shown in Figure 5. The bends in each pair have the same field strength but in the opposite direction. The spin rotation from the vertical direction is canceled out, which explains the re-establishment of the vertical spin. Thus, if the electron beam is vertically polarized at the source, we can guarantee the vertical polarization at the injection point. Given that we have shown that the transverse spin at the source is also present at the HER injection point, and that BMAD shows that a transverse spin has a long lifetime in the HER, we now focus on how to produce a vertically polarized the beam at the source.

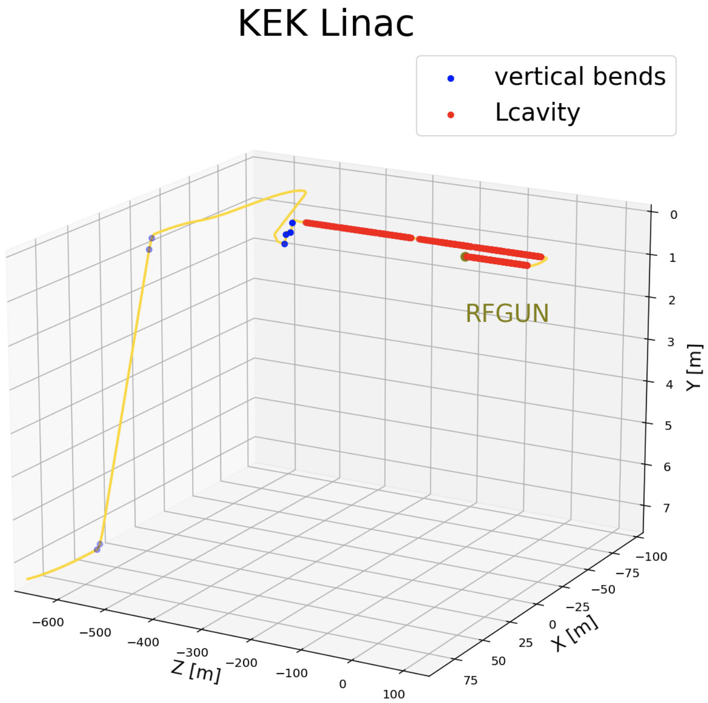

Figure 1: The layout of the KEK Linac

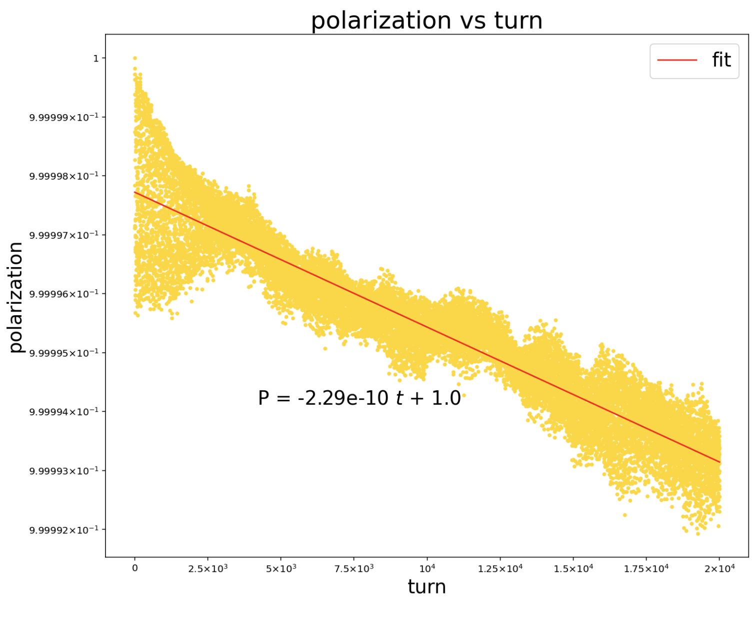

Figure 2: Vertical polarization as a function of number of turns in the HER where one turn is approximately equivalent to 10μs.

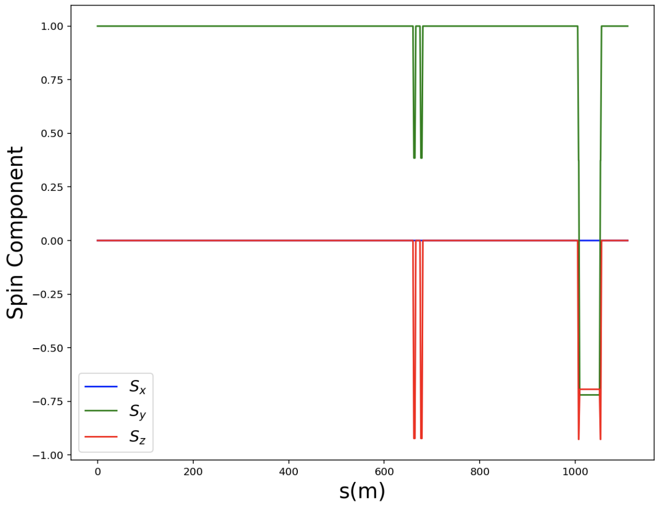

Figure 3: Spin motion in the Linac

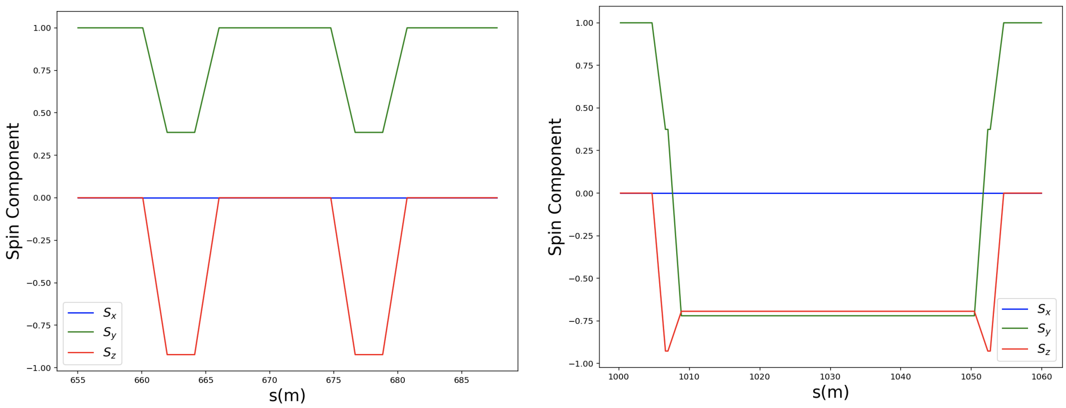

Figure 4: Left: the spin motion of the electron when passing through first two pairs of vertical bends; Right: the spin motion of the electron when passing through second two pairs of vertical bends

	Index	name	key	s(m)	l(m)	REF_TILT_TOT	B_field	floor.y	spin.x	spin.y	spin.z
0	3499	BV1UE	Sbend	661.981	1.906	-1.5708	-0.90687	0.070517	-5.656700e-16	0.38464	-9.230700e-01
1	3505	BV1DE	Sbend	666.036	1.906	1.5708	-0.90687	0.300000	-4.302700e-16	1.00000	7.612600e-16
2	3538	BV1UE	Sbend	676.670	1.906	-1.5708	-0.90687	0.370520	1.266100e-15	0.38464	-9.230700e-01
3	3544	BV1DE	Sbend	680.726	1.906	1.5708	-0.90687	0.600010	1.398400e-15	1.00000	-1.354100e-17
4	4134	BV2UE	Sbend	1006.619	1.906	-1.5708	-0.91564	0.671210	2.237800e-15	0.37411	-9.273800e-01
5	4139	BV2UE	Sbend	1008.875	1.906	-1.5708	-0.91564	0.910540	2.627600e-15	-0.72008	-6.938900e-01
6	4219	BV2DE	Sbend	1052.333	1.906	1.5708	-0.91564	7.312200	2.435100e-15	0.37411	-9.273800e-01
7	4224	BV2DE	Sbend	1054.589	1.906	1.5708	-0.91564	7.409500	2.182700e-15	1.00000	-4.510300e-16

Figure 5: Vertical bends

2 Wien Filter

The electron beam is generated and longitudinally polarized at the source. A spin rotator must be installed to rotate the spin from the longitudinal to the vertical direction. Also, this rotator should not change the trajectory of the electron beam. When charged particles travel in the electromagnetic field, they experience the Lorentz force. The Wien Filter consists of perpendicular electric and magnetic fields. A charged particle with a velocity $\vec{v}$ which follows a straight path, as shown in Figure 6, experiences zero Lorentz force. The Wien Filter can be used as a spin rotator that preserves the straight trajectory of the particle.

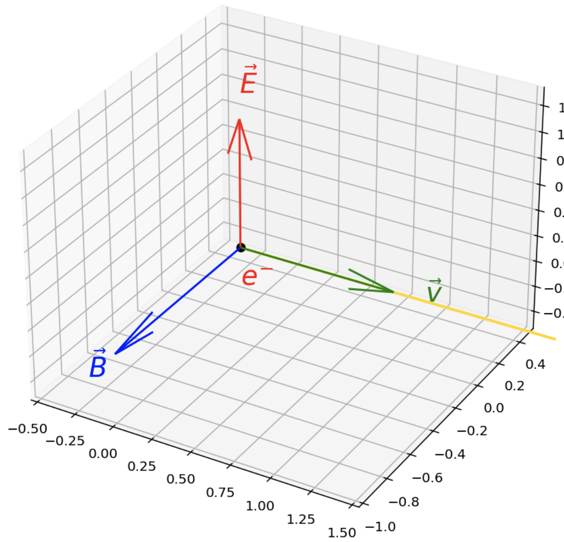

Figure 6: Wien Filter

The Lorentz force due to the E field and the B field should cancel out each other:

$$q\vec{\mathrm{E}} + q\vec{\mathrm{v}} \times \vec{\mathrm{B}}_{\perp} = 0, \tag{1}$$

which requires

$$\vec{\mathrm{E}} = -\vec{\mathrm{v}} \times \vec{\mathrm{B}}_{\perp}. \tag{2}$$

The spin motion of a charged particle traveling in the electromagnetic field follows the Thomas-BMT equation:

$$\frac{d\vec{\mathrm{S}}}{dt} = \vec{\Omega} \times \vec{\mathrm{S}} = -\frac{q}{m\gamma}\left[(1+\mathrm{G}\gamma)\vec{\mathrm{B}}_{\perp} + (1+\mathrm{G})\vec{\mathrm{B}}_{//} + (\mathrm{G}\gamma + \frac{\gamma}{1+\gamma})\frac{\vec{\mathrm{E}} \times \vec{\beta}}{c}\right] \times \vec{\mathrm{S}}, \tag{3}$$

where $\vec{\mathrm{S}}$ is the spin vector in the rest frame of the particle; t is the time in the lab frame; q is the charge of the particle; $\vec{\mathrm{B}}_{\perp}$ and $\vec{\mathrm{B}}_{//}$ are the transverse and longitudinal component of the magnetic field with respect to the velocity of the particle in the lab frame; $\vec{\mathrm{E}}$ is the electric field in the lab frame; the velocity of the particle is given by $\vec{\beta}c$ and c is the speed of light; γ is the Lorentz factor; G = $\frac{g-2}{2}$ is the anomalous gyromagnetic g-factor; $\vec{\Omega}$ is the axial vector of the precession.

For the electron, q = -e, and there is no longitudinal B field in the Wien Filter, $\vec{\mathrm{B}}_{//} = 0$. The expression of the axial vector $\vec{\Omega}$ can be expressed as:

$$\vec{\Omega} = \frac{e}{m\gamma}\left[(1+\mathrm{G}\gamma)\vec{\mathrm{B}}_{\perp} + (\mathrm{G}\gamma + \frac{\gamma}{\gamma+1})\frac{\vec{\mathrm{E}}\times\vec{\beta}}{c}\right]. \tag{4}$$

Plugging Eq.2 into Eq.4 can reduce it to following form:

$$\vec{\Omega} = \frac{1+G}{\gamma^2 m}e\vec{\mathrm{B}}_{\perp}. \tag{5}$$

Take the approximation that $\frac{ds}{dt} \simeq v$ and re-express the equation of the spin motion:

$$\frac{d\vec{\mathrm{S}}}{ds} = \frac{d\vec{\mathrm{S}}}{dt}\frac{dt}{ds} = \frac{1}{v}\frac{d\vec{\mathrm{S}}}{ds} = \frac{1}{v}\vec{\Omega}\times\vec{\mathrm{S}}, \tag{6}$$

where s is the s-position in the co-moving coordinate, v is the speed of the particle.

For a Wien Filter with length L, the rotation angle of the spin vector can be determined as following:

$$\theta = \int \frac{\Omega}{v}ds = \frac{1+G}{\gamma}\frac{e}{p}\mathrm{B}_{\perp}\mathrm{L} = \frac{1+G}{\gamma}\frac{\mathrm{B}_{\perp}\mathrm{L}}{\mathrm{B}\rho}, \tag{7}$$

where Ω is the magnitude of the axial vector, and Bρ is the magnetic rigidity.

The target is to rotate the spin vector from the longitudinal to the vertical direction, the total amount of rotation is $\frac{\pi}{2}$, thus can find:

$$\mathrm{B}_{\perp}\mathrm{L} = \frac{\gamma\pi}{2(1+G)}\mathrm{B}\rho. \tag{8}$$

The Wien Filter is not suitable for MeV electrons, the required E field is too strong (>10 MeV/m) which is unrealistic to build and install. There is an acceleration stage in the source in which the electron beam is initially accelerated to 8.5 MeV before it reaches the exit. Thus, the Wien Filter must be installed before the acceleration stage. Based on Eq.8, it can be found that for a 250 keV electron, the required magnetic field is 44 gauss (i.e. 4.4mT), and the electric field is about 1 MV/m for a 1 m long Wien Filter. (We note that the earth's magnetic field is 0.25-0.65 gauss, which represents a 1% effect on the Wien filter magnetic field.) It's necessary to study the structure of the source and figure out how much space can be used between the RF gun and the acceleration stage.