Lattice QCD for flavor physics

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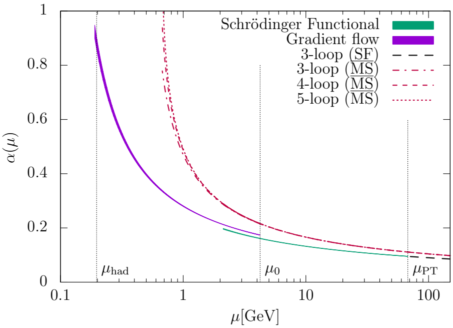
QCD: THE THEORY OF STRONG INTERACTIONS

Very simple theory

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F_{\mu\nu} + \sum_{i=1}^{N_f} \bar{\psi}(\gamma_\mu D_\mu + m_i)\psi$$

- Only $N_f + 1$ free parameters: g^2, m_i
- Incredibly rich phenomena: full hadron spectrum



QCD: THE THEORY OF STRONG INTERACTIONS

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$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F_{\mu\nu} + \sum_{i=1}^{N_\mathrm{f}} \bar{\psi}(\gamma_\mu D_\mu + m_i)\psi$$

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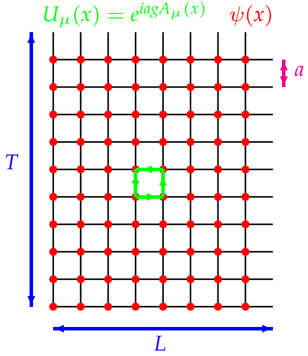
Difficult theory

- At high energies $g^2(\mu) \to 0$: perturbation theory is reliable
- At low energies $\alpha(\mu) \equiv g^2(\mu)/(12\pi) \approx 1$: perturbation theory breaks down
- We need "non-perturbative framework" to describe low energy physics

Computing path integrals: Lattice field theory

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



- Discretize space-time in an hyper-cubic lattice (spacing a)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\rightarrow$ Monte Carlo sampling.

$$\langle O \rangle = \frac{1}{N_{\rm conf}} \sum_{i=1}^{N_{\rm conf}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\rm conf}})$$

Observable computed averaging over samples

- This works both in the perturbative and non-perturbative regimes!

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} Tr(1 - U_p - U_p^+) \xrightarrow[a\to 0]{} -\frac{1}{2}\int d^4x \, \text{Tr}(F_{\mu\nu}F_{\mu\nu})$$

Lattice QCD Timeline

- 1974 First formulation of a non-Abelian gauge theory on a space-time lattice [Wilson. Phys.Rev D10 (1974)].
- 1980 First lattice simulation: Pure $SU(2)$ gauge theory in a lattice up to 10^4. [Creutz. Phys.Rev D21 (1980)].
- 1985 Firsts unquenched simulations: 2×4^3 to 4×8^3 lattices. [Duane, Kogut. Phys.Rev.Lett. 55 (1985)].
- '90s Quenched lattice QCD reign. Formally large N_c limit of QCD. Error $\sim 1/N_c \approx 30\% \rightarrow$ Uncontrolled systematics.
- '00s Unquenched simulation comes up.
- Present Start of precision lattice QCD era: Large volumes, physical quark masses, QED, etc…

Reaching the physical point

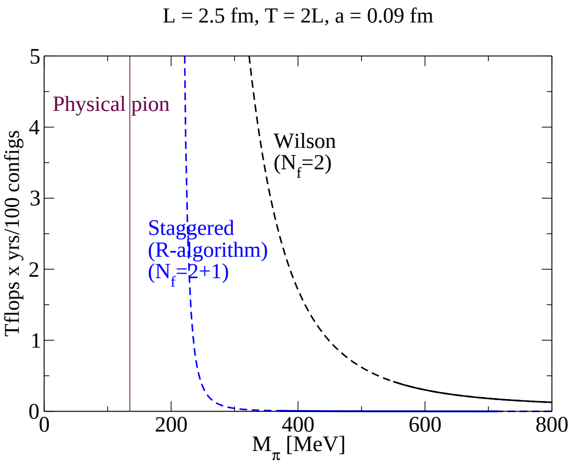
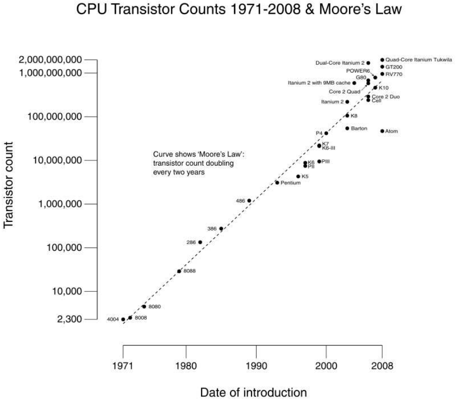


Figure: 2001 Berlin conference

The fall of the Berlin wall

- Schwartz-preconditioned Hybrid MC **[Lüscher ’03-’04; PACS-CS ’08]**
- Multiple time scale integration **[Hasenbusch ’01;BMW ’07]**
- Mass preconditioning **[Urbach ’06; BMW ’07]**
- Deflation acceleration **[Lüscher ’07]**
- Better understanding of the simulations ⇒ more intelligent choice of simulation parameters.



Lattice QCD: How is a simulation done?

Any lattice simulation requires $N_\mathrm{f}+1$ input

- We need to choose g_0^2: "bare" coupling
- We need to choose am_i: "bare" quark masses in lattice units
- Only dimensionless input! $\Longrightarrow$ only dimensionless predictions
- We can obtain $aM_\pi, aM_p, aM_\Omega, \ldots$

Remember renormalization: we do not know what input to choose!

- Quark masses am_i and coupling g_0^2 **renormalize** $\Longrightarrow a\bar{m}_i(\mu), g^2(\mu)$

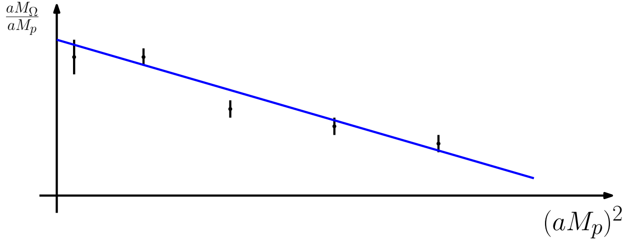
Lattice QCD: How a simulation is done?

Determine input to "match" our world

- Choose a value for the bare coupling g_0 (implicitly this corresponds to a value of a)
- Use N_f **dimensionless** physical input to fix quark masses

$$\frac{M_\pi}{M_p} = 0.144\ldots, \frac{M_K}{M_p} = 0.530\ldots$$

- Tune bare quark masses am_i to match those!
- Repeat for several values of the bare coupling g_0 (i.e. several values of a)
- Make **dimensionless** predictions by performing a continuum extrapolation



Some lattice QCD jargon

- We like to quote **dimensionfull** predictions
- Convert to physical units by using reference scale

$$M_\Omega = M_p^{\exp} \times \lim_{a\to 0} \frac{aM_\Omega(a)}{aM_p(a)}$$

$$M_\Xi = M_p^{\exp} \times \lim_{a\to 0} \frac{aM_\Xi(a)}{aM_p(a)}$$

$$\ldots$$

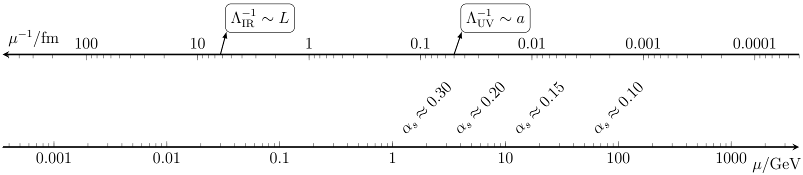
- Usually we say that "We use M_p to set the scale"
- We say that we determine the lattice spacing of our simulation

$$a \equiv \frac{aM_p}{M_p^{\exp}}$$

- We then use

$$M_\Omega = \lim_{a\to 0} \frac{aM_\Omega}{a}$$

Systematic in Lattice QCD



Lattice QCD simulations are performed

- At non-zero lattice spacing $\Longrightarrow$ Continuum extrapolation
- In finite volume $\Longrightarrow$ Finite volume effects extrapolation
- At non-physical values of quark masses $\Longrightarrow$ Chiral extrapolation/interpolation
- Some observables require renormalization $\Longrightarrow$ Non-perturbative/perturbative
- Heavy quarks (i.e. is $am_h \ll 1$??) $\Longleftarrow$ Effective field theory approach

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Summary/averages of Lattice QCD results for flavor physics

- ★ the parameter values and ranges used to generate the data sets allow for a satisfactory control of the systematic uncertainties;
- ○ the parameter values and ranges used to generate the data sets allow for a reasonable attempt at estimating systematic uncertainties, which however could be improved;
- ■ the parameter values and ranges used to generate the data sets are unlikely to allow for a reasonable control of systematic uncertainties.

Continuum extrapolation

Decrease as a^2 (with logarithmic corrections)

- ★ at least three lattice spacings and at least two points below 0.1 fm and a range of lattice spacings satisfying $[a_{max}/a_{min}]^2 > 2$
- ○ at least two lattice spacings and at least one point below 0.1 fm and a range of lattice spacings satisfying $[a_{max}/a_{min}]^2 > 1.4$
- ■ otherwise

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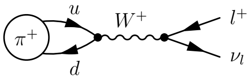
Finite volume effects

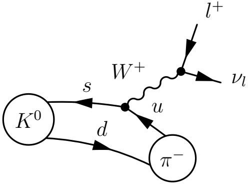
These are exponentially suppressed $\propto e^{-M_\pi L}$

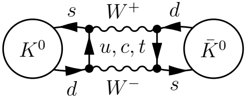
- ★ $[M_{\pi,min}/M_{\pi,fid}]^2 \exp\{4 - M_{\pi,min}[L(M_{\pi,min})]_{max}\} < 1$, or at least three volumes
- ○ $[M_{\pi,min}/M_{\pi,fid}]^2 \exp\{3 - M_{\pi,min}[L(M_{\pi,min})]_{max}\} < 1$, or at least three volumes
- ■ otherwise

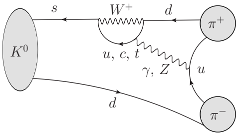
Lattice QCD for flavor physics

- Lattice QCD is an essential ingredient to investigate flavor structure of SM

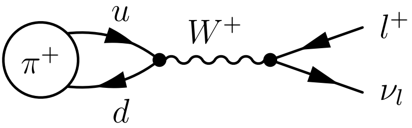
- V_{ud}: 

- $|V_{us}|$: 

- Indirect CP violation in $K \to \pi\pi$ 

- Direct CP violation in $K \to \pi\pi$ 

What we can do very well

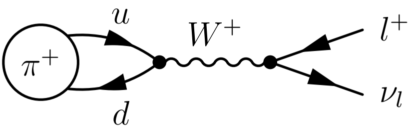


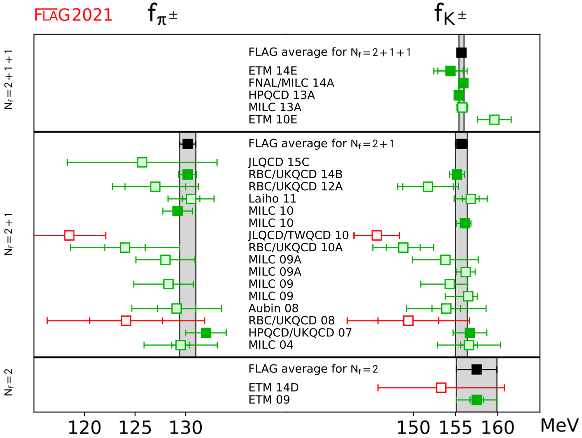
- Only one hadron
- (leptons are not simulated)

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|0\rangle = \imath p_\mu f_\pi$$

- No signal to noise problem (single meson)

What we can do very well





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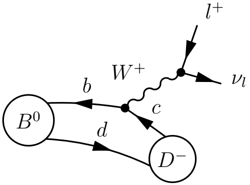
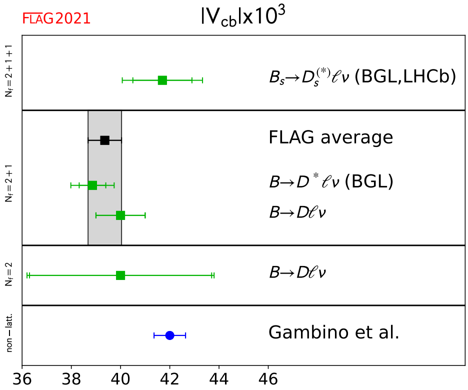
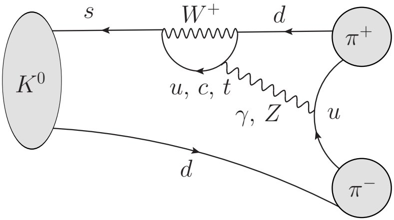




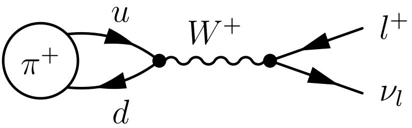
Challenges

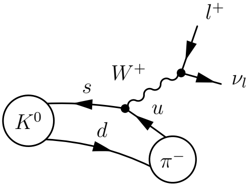


$K \to \pi\pi$ and ϵ'/ϵ

- Two particles in the final state (strongly interacting)
- Impressive progress by RBC collaboration spanning several years
- Still far from “FLAG” level where several collaborations compute a quantity

Challenges





QED corrections

- SM at low energies $\Longrightarrow$ QCD + QED + Effective weak interactions
- QED corrections are relevant in precise determinations (i.e. % effect)
- QED on a finite volume is hard: No charged states!
- Radiation is very hard
- Not only a **Numerical challenge**, but also a **theoretical one**: How to do the computation at all!

QED and the Gauss law in a periodic box $\mathbb{T}^4 = L \times L \times L \times L$

$$S_{\rm QED} = \int_{\mathbb{T}^4} d^4x \left\{ \frac{1}{4e^2} F_{\mu\nu} F_{\mu\nu} + \sum_{f=1}^{N_f} \bar{\psi}_f \left(\gamma_\mu D^f_\mu + m_f \right) \psi_f \right\}$$

$$D^f_\mu = \partial_\mu - \imath q_f A_\mu$$

usual gauge invariance

$$\psi(x) \to e^{\imath \alpha(x)} \psi(x)$$
$$A_\mu(x) \to A_\mu(x) + \partial_\mu \alpha(x)$$

Problems describing the propagation of electrically charged states?

1. e^- propagator is not a gauge invariant quantity

$$\langle \overline{\psi}(x) \psi(y) \rangle \to e^{\imath(\alpha(y) - \alpha(x))} \langle \overline{\psi}(x) \psi(y) \rangle \implies \langle \overline{\psi}(x) \psi(y) \rangle = 0$$

2. Gauss law implies a total zero charge

$$\partial_k E_k = \rho(x) \implies \int_{\mathbb{T}^3} d^3x \, \partial_k E_k = \int_{\mathbb{T}^3} d^3x \, \rho(x) = 0$$

Well known problem in perturbation theory $L = \infty$

Gauge fix before computing propagators not enough in FV

Use for example Coulomb gauge: $\partial_i A_i = 0$

- Gauss constrain is not automatically solved in FV
- Large gauge transformations $\alpha(x) = \frac{2\pi n}{L_\mu} x_\mu$ survive gauge fixing

$$\psi(x) \to e^{2\imath\pi n x/L}\psi(x)$$
$$A_\mu(x) \to A_\mu(x) + \frac{2\pi n}{L}$$

  and $\partial_i A_i = 0$.
- Propagators still vanish (!!)

Summary

In a periodic box Hamiltonian spectra contains only neutral states

Conclusions

- Lattice QCD is in a precision era
  - Physical quark masses
  - Large volumes
  - Non-perturbative renormalization
  - QED effects
- Key for the future of flavor physics
- FLAG is your friend!