

# $B \rightarrow D^* \ell \bar{\nu}_\ell$ Angular Analysis

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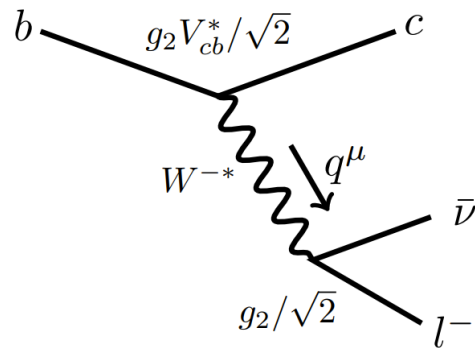
Bundesministerium  
für Bildung  
und Forschung


9/26/2023

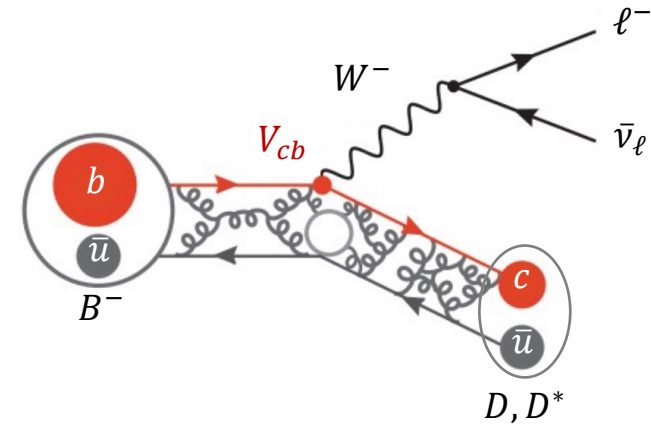
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# The $b \rightarrow c \ell \bar{\nu}_\ell$ Laboratory



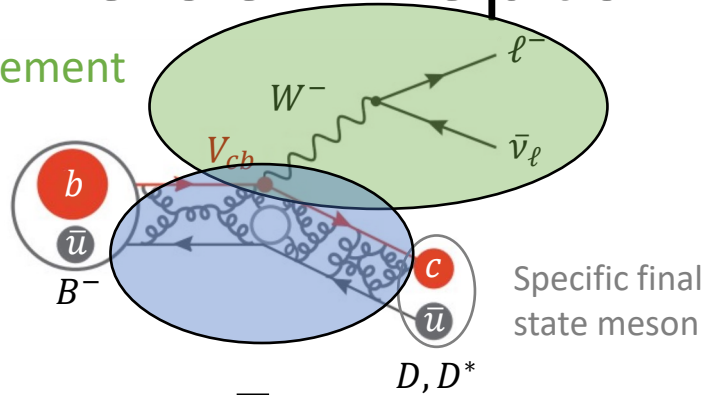
  
 Main Theory Challenge



- Tree level process: large branching fraction (10%), theoretically relatively clean
- Universal lepton gauge coupling in the SM
- Experimentally only access to bound states  $B \rightarrow D^{(*)}, D^{**}, \dots, \Lambda_b \rightarrow \Lambda_c^*, \dots$
- Precision description of the hadronic matrix element required for precision measurements

# Exclusive Semileptonic $B \rightarrow D^* \ell \bar{\nu}_\ell$

Leptonic Matrix Element



$$\Gamma(B \rightarrow D^* \ell \bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{F}(1) \quad \mathcal{F}(1) = h_{A_1}(1)$$



**Hadronic Matrix Elements** cannot be calculated from first principles

→ Can be parameterized with **form factors**  $h_X = h_X(w)$  and extracted from data

→ Lattice QCD must provide (at least) inputs on their **normalization**

$$\frac{\langle D^*(p') | \bar{c} \gamma^\mu b | B(p) \rangle}{\sqrt{m_B m_{D^*}}} = h_V \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* v'_\alpha v_\beta$$

$$\frac{\langle D^*(p') | \bar{c} \gamma^\mu \gamma^5 b | B(p) \rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1} (w + 1) \epsilon^{*\mu} - h_{A_2} (\epsilon^* \cdot v) v^\mu - h_{A_3} (\epsilon^* \cdot v) v'^\mu$$

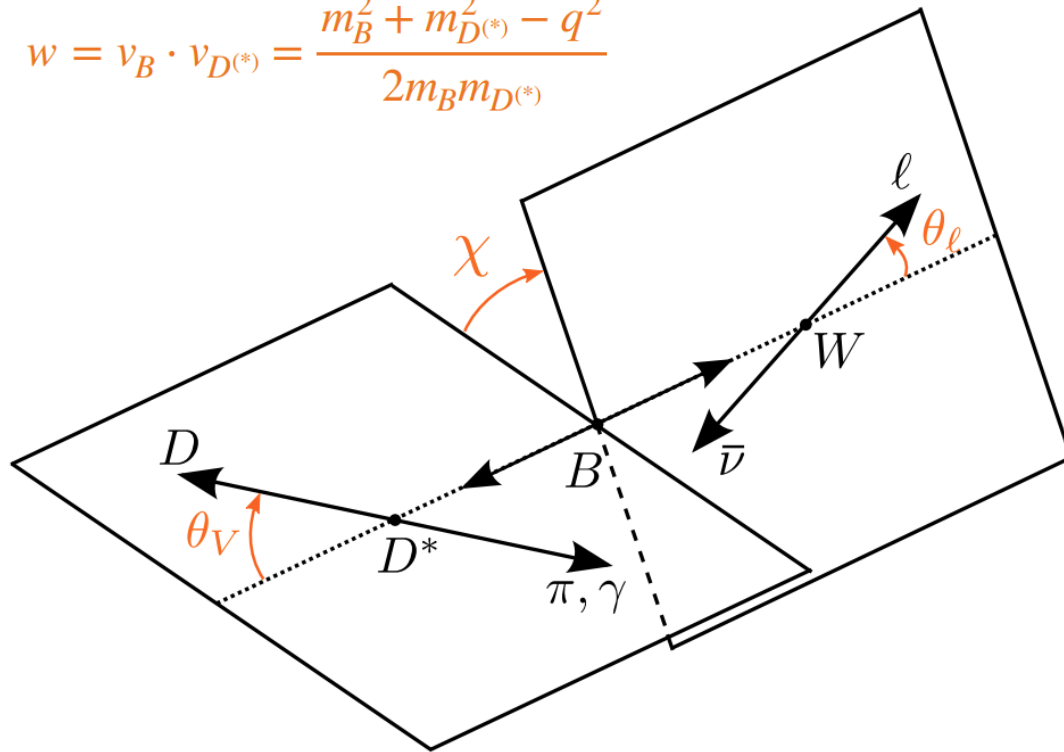
Heavy Quark Symmetry Basis

Common parameterizations for the **form factors**:

- BGL
- CLN
- BLPRXP

# Exclusive Semileptonic $B \rightarrow D^* \ell \bar{\nu}_\ell$

$$w = v_B \cdot v_{D^*} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

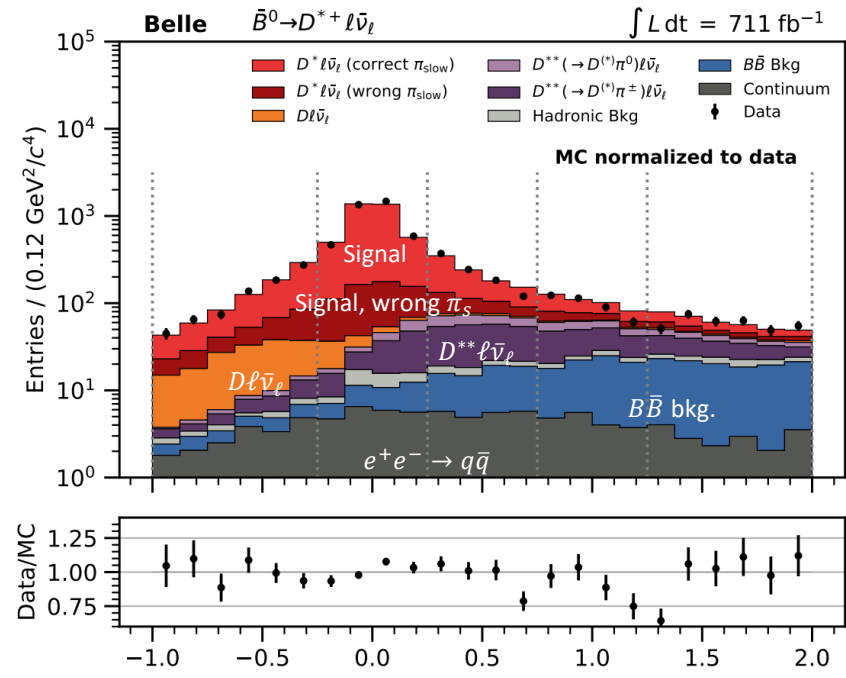
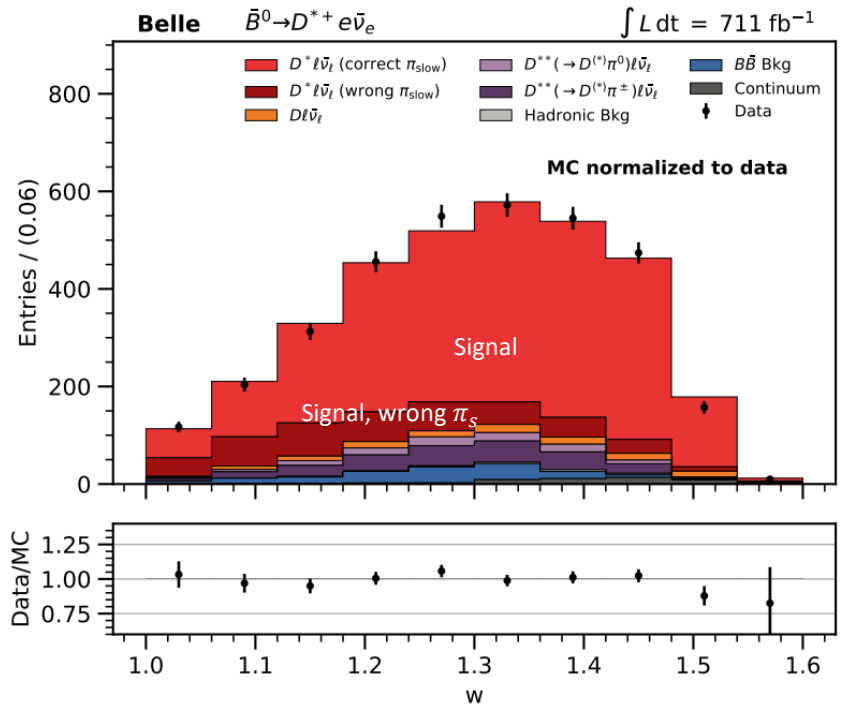
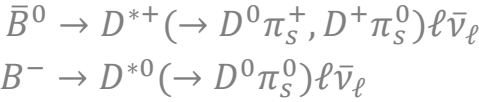


- Form factors are a function of  $w$  only
- **Angles** provide information on, e.g.
  - Forward-backward asymmetry
  - Longitudinal polarization fraction
  - “S” observables sensitive to new physics
- Measure the 4 marginal distributions of the 4D differential decay rate

$$\frac{d\Gamma(B \rightarrow D^* \ell \bar{\nu}_\ell)}{dw d\cos\theta_\ell d\cos\theta_V d\chi} = \frac{6m_B m_{D^*}^2}{8(4\pi)^4} \sqrt{w^2 - 1} (1 - 2wr + r^2) G_F^2 \eta_{EW}^2 |V_{cb}|^2$$

$$\times \left( (1 - \cos\theta_\ell)^2 \sin^2\theta_V H_+^2 + (1 + \cos\theta_\ell)^2 \sin^2\theta_V H_-^2 \right. \\ \left. + 4 \sin^2\theta_\ell \cos^2\theta_V H_0^2 - 2 \sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+ H_- \right. \\ \left. - 4 \sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+ H_0 \right. \\ \left. + 4 \sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_- H_0 \right),$$

# Differential Distributions of $B \rightarrow D^* \ell \bar{\nu}_\ell$



Background subtraction in independent variable to reduce model dependency.



Extraction Method: Missing Mass Squared

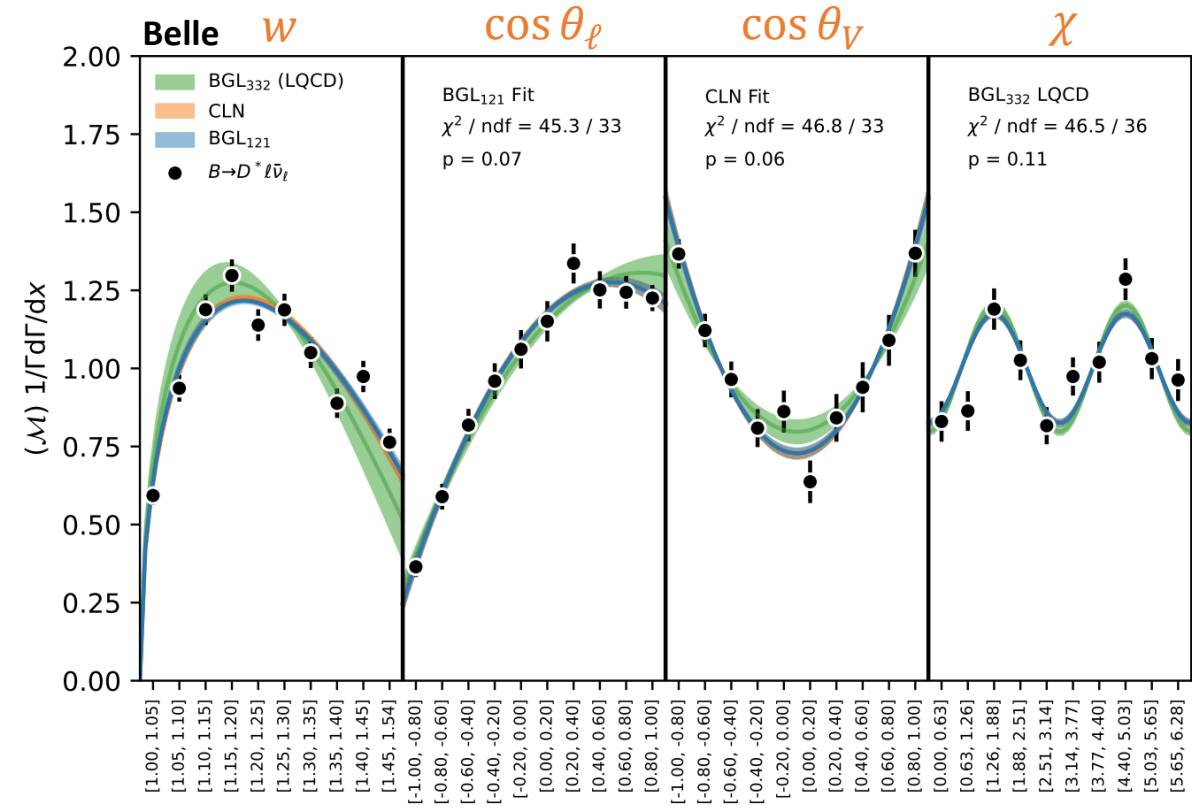
$$0 = m_\nu^2 = M_{\text{miss}}^2 = (p_{e^+e^-} - p_B - p_{D^*} - p_\ell)^2$$



+ Unfolding and Acceptance Correction  
(not today)

# Differential Distributions of $B \rightarrow D^* \ell \bar{\nu}_\ell$

Belle, Prim, et al  
*Phys.Rev.D* 108 (2023) 1, 012002  
 arXiv:2301.07529



$$|V_{cb}|^{\text{BGL}} = (40.6 \pm 0.9) \times 10^{-3}$$

$$|V_{cb}|^{\text{CLN}} = (40.1 \pm 0.9) \times 10^{-3}$$

With lattice QCD result at zero recoil

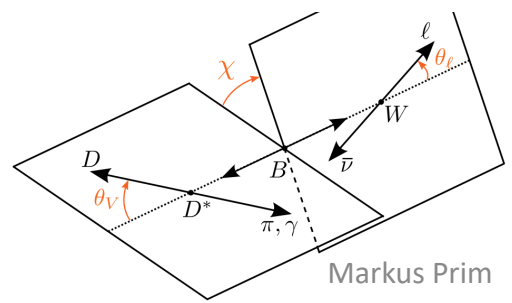
Lepton Flavor Universality Observables

$$\Delta A_{FB} = A_{FB}^\mu - A_{FB}^e = 0.022 \pm 0.027$$

$$\Delta F_L = F_L^\mu - F_L^e = 0.034 \pm 0.024$$

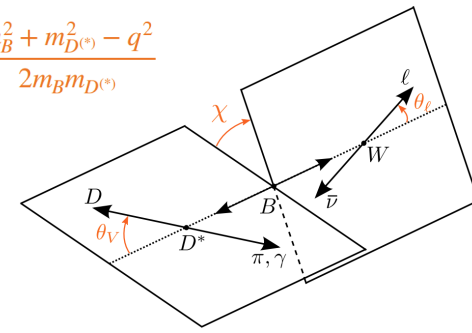
$$R_{e\mu} = \frac{\mathcal{B}(B \rightarrow D^* e \bar{\nu}_e)}{\mathcal{B}(B \rightarrow D^* \mu \bar{\nu}_\mu)} = 0.990 \pm 0.031$$

$$w = v_B \cdot v_{D^*} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$



# Angular Coefficients of $B \rightarrow D^* \ell \bar{\nu}_\ell$

$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$



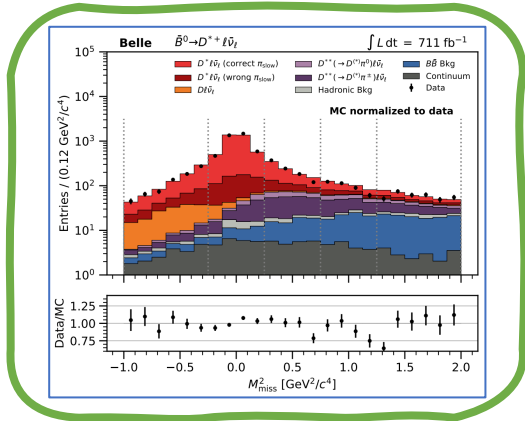
Instead of binning in  $w, \cos \theta_\ell, \cos \theta_V, \chi$ , we now bin the data to determine the angular coefficients in bins of  $w$  and:

Phys.Rev.D 90 (2014) 9, 094003

$$J_i = J_i(w) = \frac{1}{N_i} \sum_{j=1}^8 \sum_{k,l=1}^4 \eta_{i,j}^\chi \eta_{i,k}^{\theta_\ell} \eta_{i,l}^{\theta_V} \left( \chi^{(j)} \otimes \chi^{(k)} \otimes \chi^{(l)} \right)$$

Normalization      Weights      Unfolded Yields

| $J_i$    | $\eta_i^\chi$            | $\eta_i^{\theta_\ell}$ | $\eta_i^{\theta_V}$ | normalization $N_i$ |
|----------|--------------------------|------------------------|---------------------|---------------------|
| $J_{1s}$ | {+}                      | {+, a, a, +}           | {-, c, c, -}        | $2\pi(1)2$          |
| $J_{1c}$ | {+}                      | {+, a, a, +}           | {+, d, d, +}        | $2\pi(1)(2/5)$      |
| $J_{2s}$ | {+}                      | {-, b, b, -}           | {-, c, c, -}        | $2\pi(-2/3)2$       |
| $J_{2c}$ | {+}                      | {-, b, b, -}           | {+, d, d, +}        | $2\pi(-2/3)(2/5)$   |
| $J_3$    | {+, -, -, +, +, -, -, +} | {+}                    | {+}                 | $4(4/3)^2$          |
| $J_4$    | {+, +, -, -, -, -, +, +} | {+, +, -, -}           | {+, +, -, -}        | $4(4/3)^2$          |
| $J_5$    | {+, +, -, -, -, -, +, +} | {+}                    | {+, +, -, -}        | $4(\pi/2)(4/3)$     |
| $J_{6s}$ | {+}                      | {+, +, -, -}           | {-, c, c, -}        | $2\pi(1)2$          |
| $J_{6c}$ | {+}                      | {+, +, -, -}           | {+, d, d, +}        | $2\pi(1)(2/5)$      |
| $J_7$    | {+, +, +, +, -, -, -, -} | {+}                    | {+, +, -, -}        | $4(\pi/2)(4/3)$     |
| $J_8$    | {+, +, +, +, -, -, -, -} | {+, +, -, -}           | {+, +, -, -}        | $4(4/3)^2$          |
| $J_9$    | {+, +, -, -, +, +, -, -} | {+}                    | {+}                 | $4(4/3)^2$          |

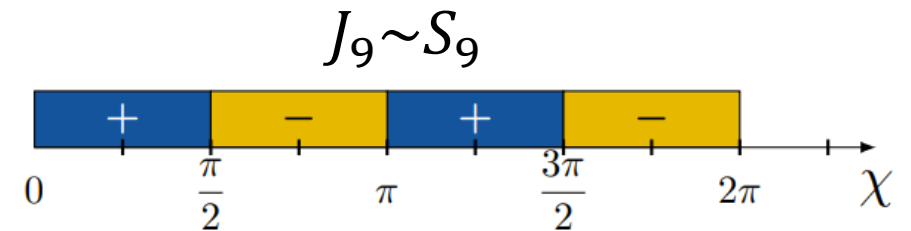
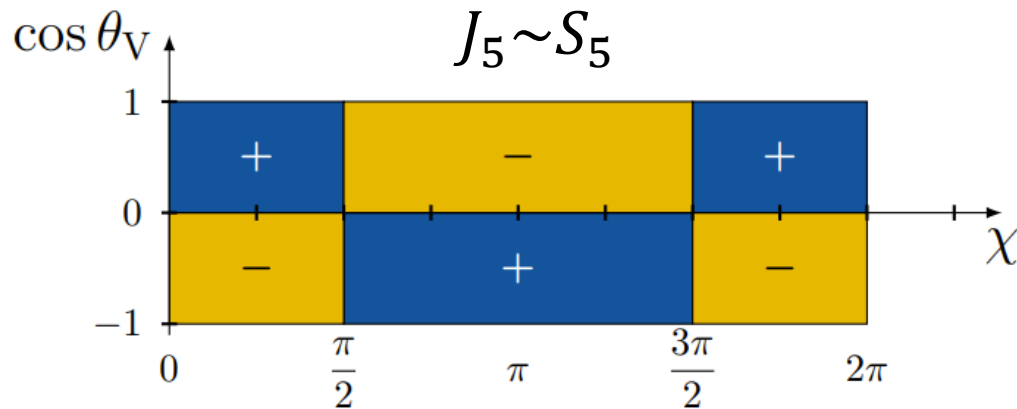
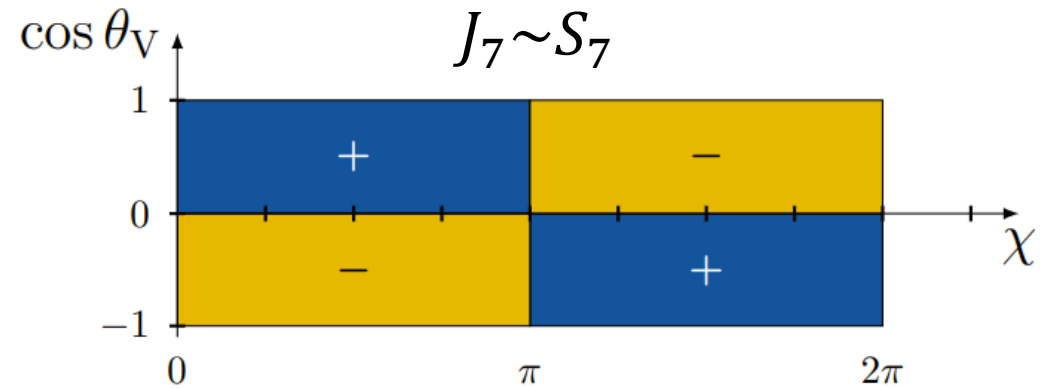
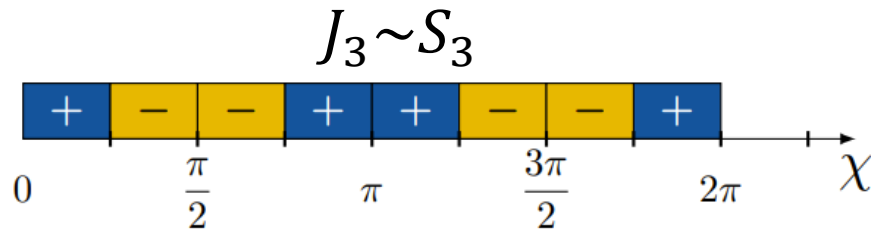


Same signal extraction strategy as before!

$$\frac{d\Gamma(B \rightarrow D^* \ell \bar{\nu}_\ell)}{dw d\cos \theta_\ell d\cos \theta_V d\chi} = \frac{2G_F^2 \eta_{EW}^2 |V_{cb}|^2 m_B^4 m_{D^*}}{2\pi^4} \times \left( J_{1s} \sin^2 \theta_V + J_{1c} \cos^2 \theta_V \right. \\
+ (J_{2s} \sin^2 \theta_V + J_{2c} \cos^2 \theta_V) \cos 2\theta_\ell + J_3 \sin^2 \theta_V \sin^2 \theta_\ell \cos 2\chi \\
+ J_4 \sin 2\theta_V \sin 2\theta_\ell \cos \chi + J_5 \sin 2\theta_V \sin \theta_\ell \cos \chi + (J_{6s} \sin^2 \theta_V + J_{6c} \cos^2 \theta_V) \cos \theta_\ell \\
\left. + J_7 \sin 2\theta_V \sin \theta_\ell \sin \chi + J_8 \sin 2\theta_V \sin 2\theta_\ell \sin \chi + J_9 \sin^2 \theta_V \sin^2 \theta_\ell \sin 2\chi \right)$$

# Angular Asymmetries in $B \rightarrow D^* \ell \bar{\nu}_\ell$

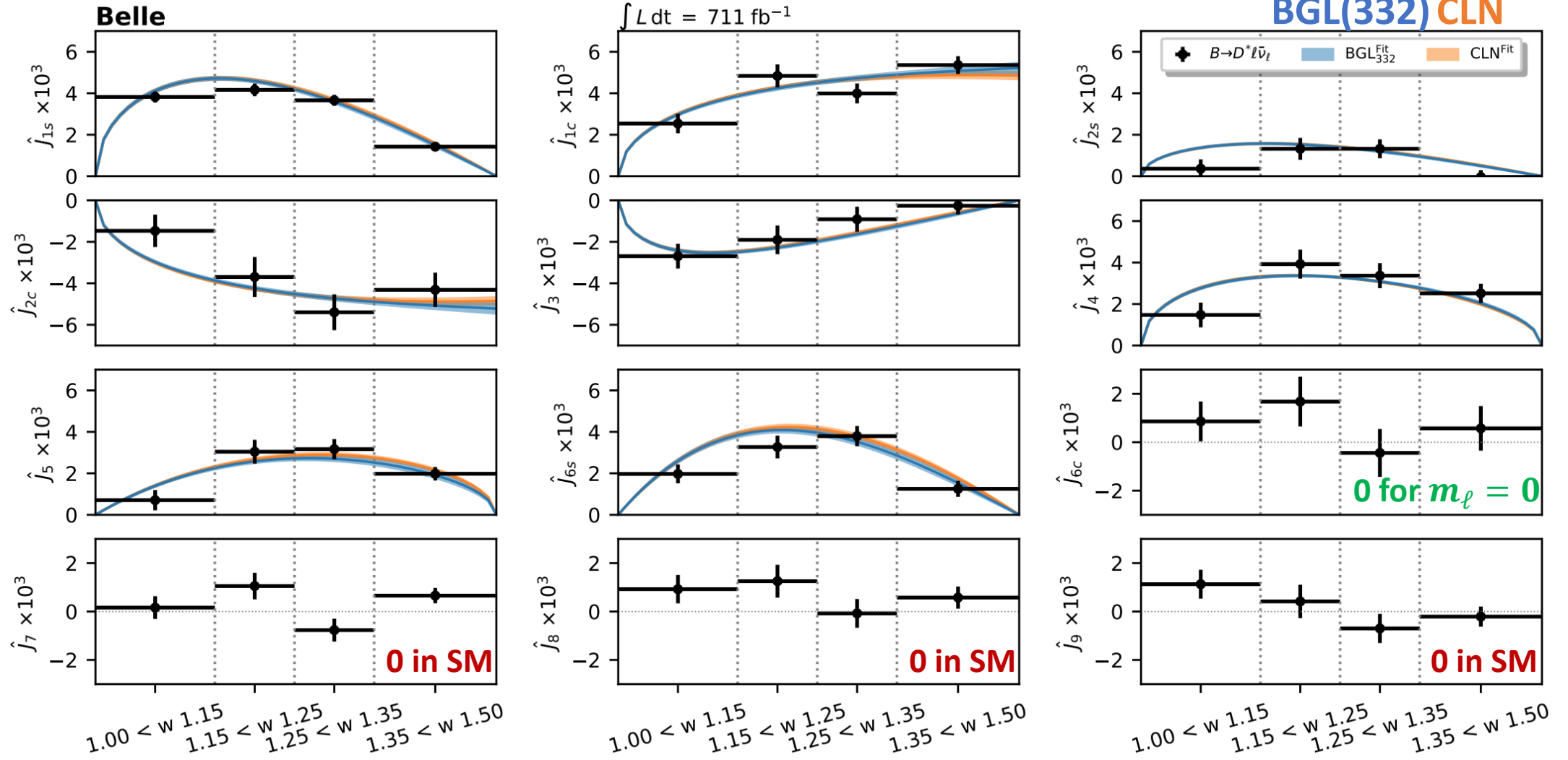
$$\mathcal{A}_x(w) = \frac{N_x^+(w) - N_x^-(w)}{N_x^+(w) + N_x^-(w)}$$





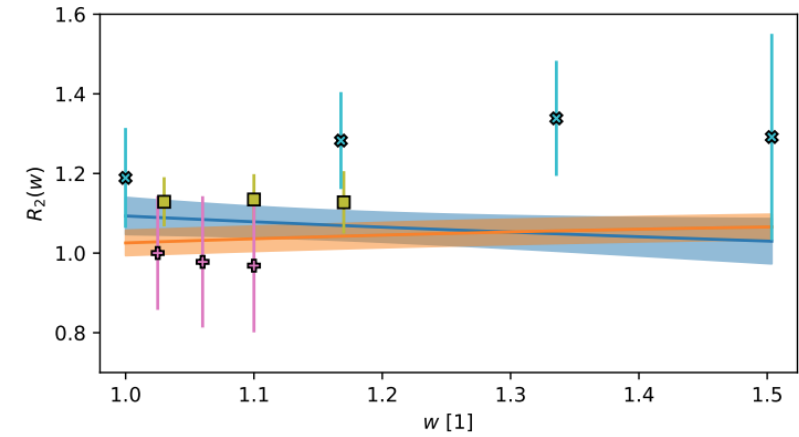
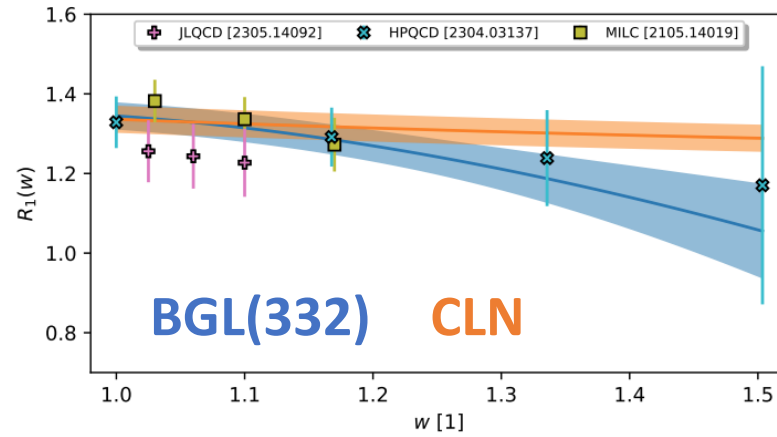
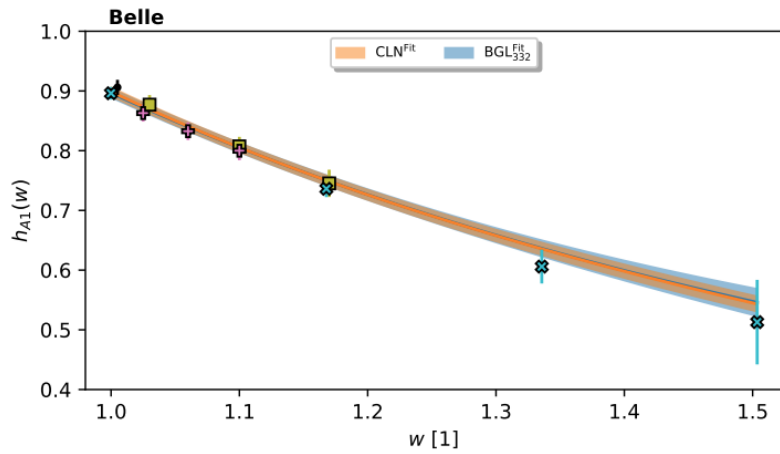
Preliminary

# Angular Coefficients of $B \rightarrow D^* \ell \bar{\nu}_\ell$



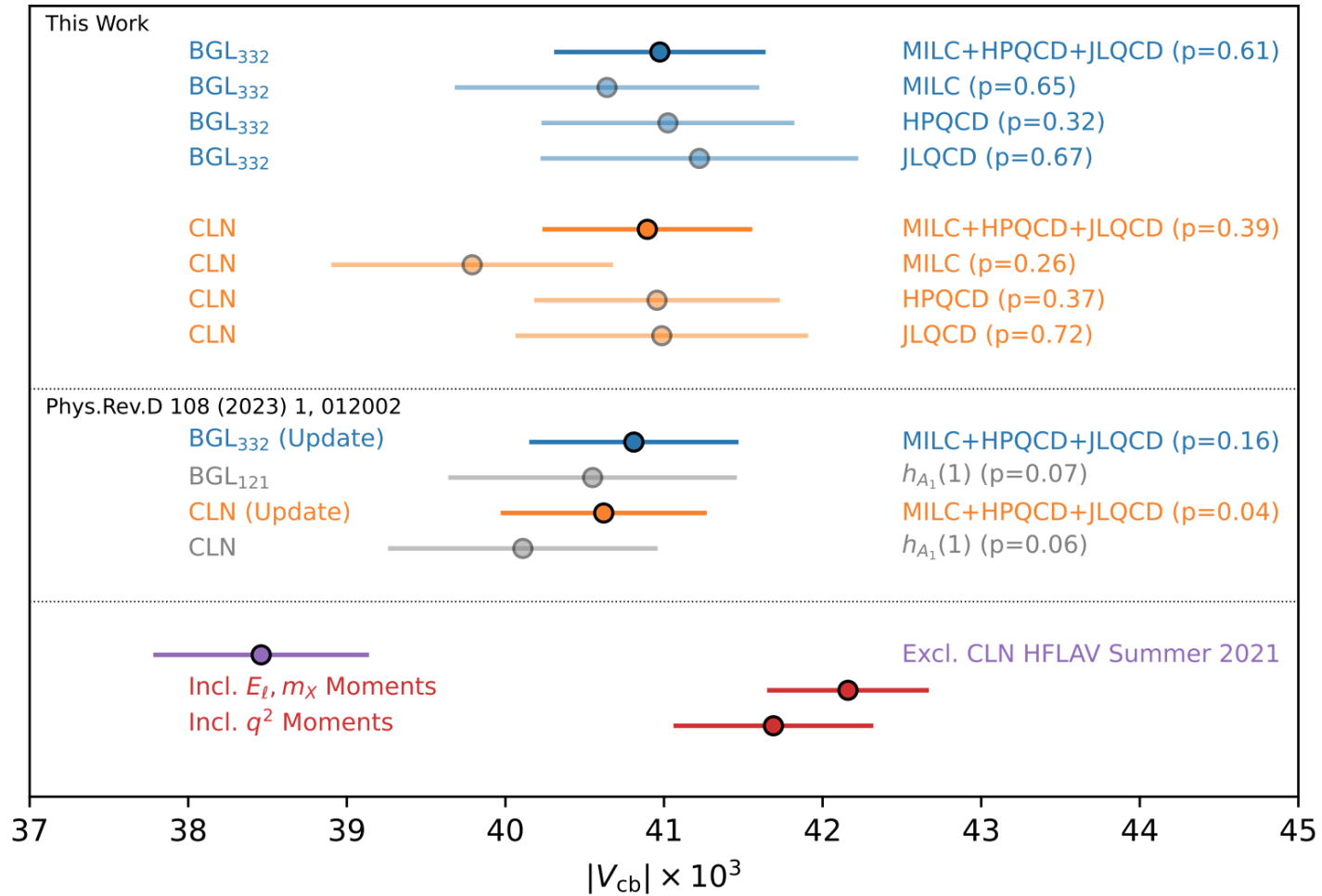
Preliminary

# Angular Coefficients of $B \rightarrow D^* \ell \bar{\nu}_\ell$



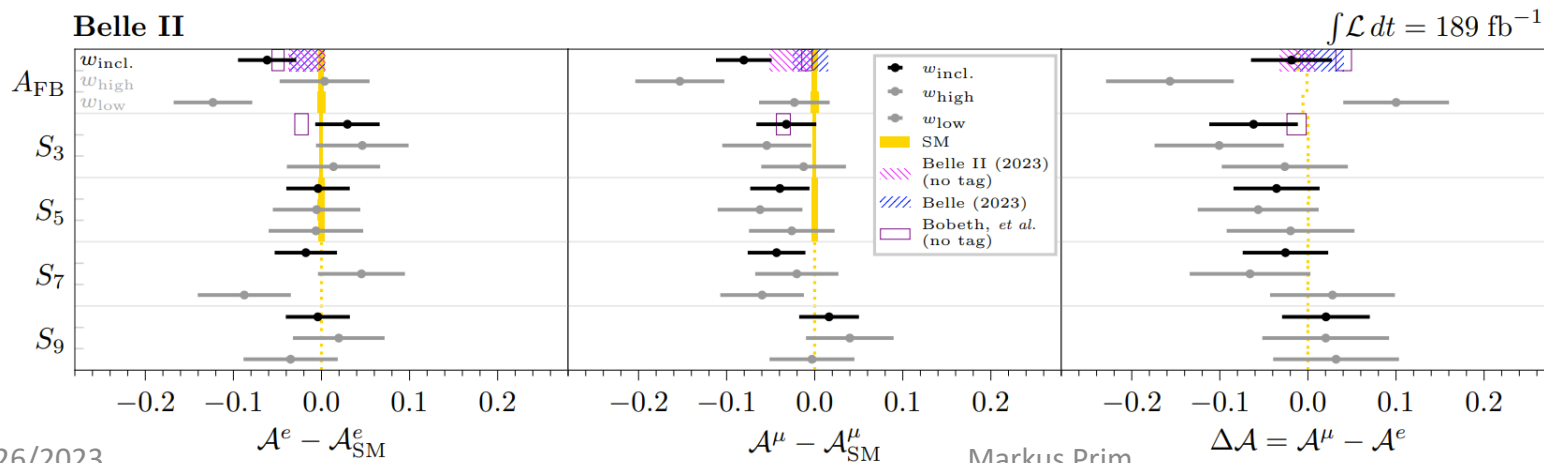
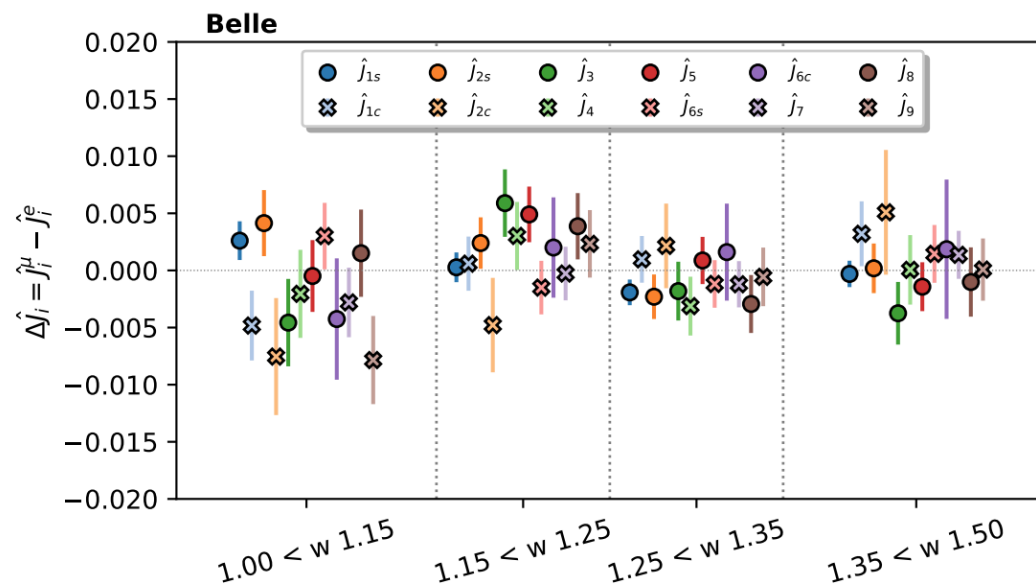
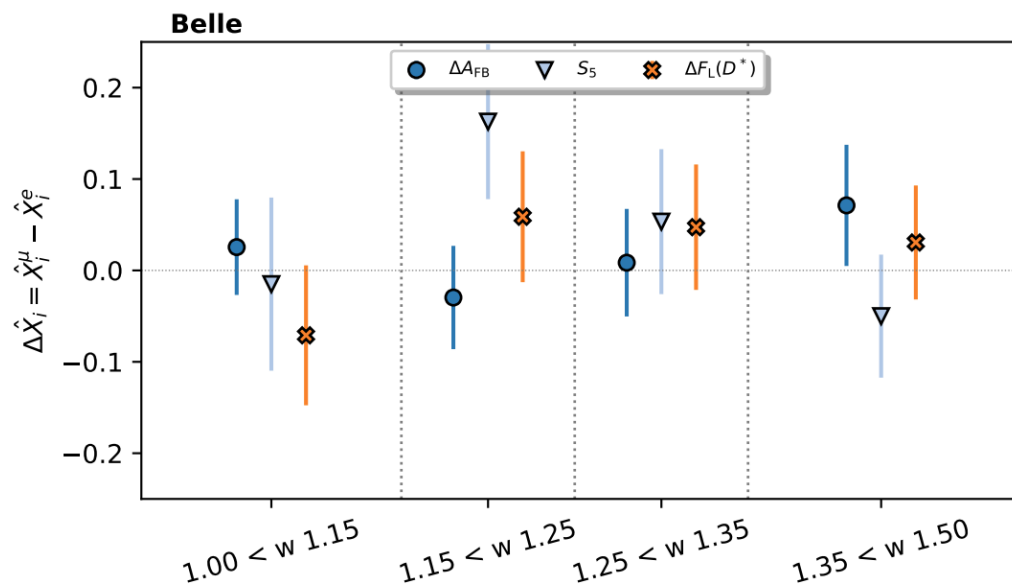
# Preliminary

## Overview on $|V_{cb}|$



Preliminary

# Angular Coefficients of $B \rightarrow D^* \ell \bar{\nu}_\ell$



$$\langle S_i^{(\ell)} \rangle \equiv \frac{\langle J_i^{(\ell)} \rangle + \langle \bar{J}_i^{(\ell)} \rangle}{\Gamma^{(\ell)} + \bar{\Gamma}^{(\ell)}}$$

# Backup

# $B \rightarrow D^* \ell \bar{\nu}_\ell$ Form Factor Parameterizations

$m_\ell = 0$ :  $h_{A_1}(w)$  and the form factor ratios  $R_1(w) = \frac{h_V}{h_{A_1}}$  and  $R_2(w) = \frac{h_{A_3} + r^* h_{A_2}}{h_{A_1}}$

## Boyd-Grinstein-Lebed (BGL)

- applies a conformal transformation to approximate the form factors as a series expansion
- parameterize the form factors in terms of  $\{a_n, b_n, c_n\}$  expansion coefficients

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^{n_a} a_n z^n$$

$$f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^{n_b} b_n z^n$$

$$F_1(z) = \frac{1}{P_{F_1}(z)\phi_{F_1}(z)} \sum_{n=0}^{n_c} c_n z^n$$

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

$$c_0 = \left( \frac{(m_B - m_{D^*})\phi_{F_1}(0)}{\phi_f(0)} \right) b_0$$

## Caprini-Lellouch-Neubert (CLN)

- incorporates quark model inputs from QCD sum rules
- obtains a prediction for a  $z$  expansion of  $h_{A_1}$ , with coefficients depending only on a slope parameter  $\rho^2$ , and normalizations  $R_1(1)$  and  $R_2(1)$

$$h_{A_1}(z) = h_{A_1}(w=1) \times (1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3)$$

$$R_1(w) = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

$$\frac{d\hat{\Gamma}^{(\ell)}}{dw} \equiv \frac{1}{2} \frac{d(\Gamma^{(\ell)} + \bar{\Gamma}^{(\ell)})}{dw}, \quad (3)$$

$$\begin{aligned} \frac{1}{\hat{\Gamma}^{(\ell)}} \frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_\ell} &= \frac{1}{2} + \left\langle A_{\text{FB}}^{(\ell)} \right\rangle \cos\theta_\ell \\ &\quad + \frac{1}{4} \left( 1 - 3 \left\langle \tilde{F}_L^{(\ell)} \right\rangle \right) \frac{3\cos^2\theta_\ell - 1}{2}, \end{aligned} \quad (4)$$

$$\frac{1}{\hat{\Gamma}^{(\ell)}} \frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_D} = \frac{3}{4} \left( 1 - \left\langle F_L^{(\ell)} \right\rangle \right) \sin^2\theta_D + \frac{3}{2} \left\langle F_L^{(\ell)} \right\rangle \cos^2\theta_D, \quad (5)$$

$$\frac{1}{\hat{\Gamma}^{(\ell)}} \frac{d\hat{\Gamma}^{(\ell)}}{d\chi} = \frac{1}{2\pi} + \frac{2}{3\pi} \left\langle S_3^{(\ell)} \right\rangle \cos 2\chi + \frac{2}{3\pi} \left\langle S_9^{(\ell)} \right\rangle \sin 2\chi, \quad (6)$$