Unravelling the mysteries of CKM matrix

Belle Analysis Workshop 2022

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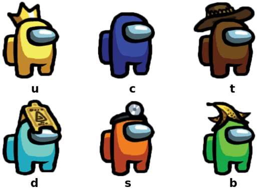


Outline

Flavor in SM

- Flavour in the SM
- Quark Model History
- The CKM matrix

metrology of CKM elements

- CKM elements $|V_{CKM}|$
- CKM phases

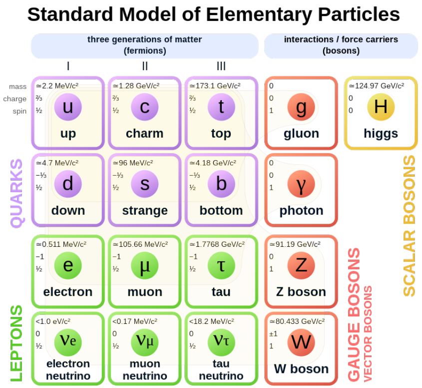
Section I

Flavor physics (of quarks) in the SM

Flavour in the SM

Flavour and Colour

Just as ice cream has both color and flavor so do quarks. - Murray Gell-Mann



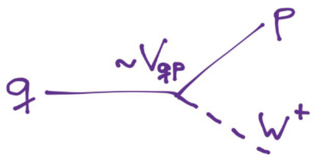
Flavour in the SM

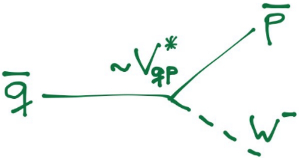
- CKM matrix transforms the mass eigenstate basis to the flavour eigenstate basis
  - and brings with it a rich variety of observable phenomena

mass eigenstates $\neq$ weak eigenstates

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{13}$$

- The up-type quark to down-type quark transition probability proportional to the squared magnitude of the CKM matrix elements, $|V_{ij}|^2$





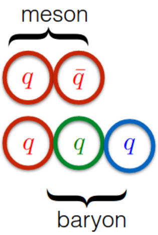
$$\frac{g}{\sqrt{2}} \bar{u}_{Li} V_{ij} \gamma_\mu W^{\mu +} d_{Lj}$$

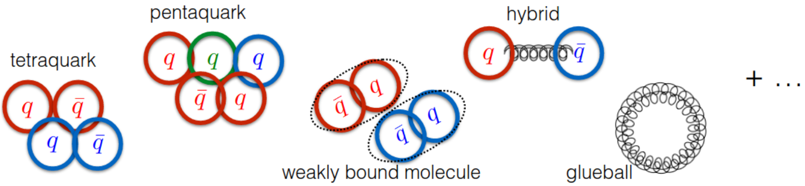
The Quark Model

- Many new particles (a "zoo") discovered in the 60s
- Gell-Mann, Nishijima and Ne'eman introduced the quark "model" (u, d, s) which could elegantly categorise them (the "eight-fold way" - flavour SU(3) symmetry)
- Gell-Mann and Pais
  - Strangeness conserved in strong interactions (production)
  - Strangeness violated in weak interactions (decay)

The Quark Model

- Can only make colour neutral objects
  - Quark anti-quark mesons ($q\bar{q}$) or three quark baryons (qqq). Nearly all known states fall into one of these two categories
  - Can also build colour neutral states containing more quarks (e.g. 4 or 5 quark states). Only quite recently confirmed (and still not entirely understood).





Cabibo angle

- Compare rates of:

  $s \to u$: $K^+ \to \mu^+ \nu_\mu$ $(\Lambda^0 \to p\pi^-,\ \Sigma^+ \to n e^+ \nu_e)$

  $d \to u$: $\pi^+ \to \mu^+ \nu_\mu$ $(n \to p e^+ \nu_e)$

  - Apparent that $s \to u$ transitions are suppressed by a factor ~ 20
- Cabibbo (1963) suggested that "down-type" is some ad-mixture of d and s
  - The first suggestion of quark mixing
  - Physical state is an admixture of flavour states

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos(\theta_C) + s\sin(\theta_C) \end{pmatrix} \tag{14}$$

- The mixing angle is determined experimentally to be $\sin(\theta_C) = 0.22$.

GIM mechanism

- Cabibbo's solution opened up a new experimental problem
  - $K^+ \to \mu^+ \nu_\mu$ had been seen but not $K^0_L \to \mu^+ \mu^-$
    - $\mathcal{B}(K^0_L \to \mu^+ \mu^-) \approx 7 \times 10^{-9}$
    - $\mathcal{B}(K^0_L \to e^+ e^-) \approx 1 \times 10^{-11}$
  - $K^+ \to \pi^0 \mu^+ \nu_\mu$ had been seen but not $K^0_L \to \pi^0 \mu^+ \mu^-$
    - $\mathcal{B}(K^0_L \to \pi^0 \mu^+ \mu^-) \approx 1 \times 10^{-10}$
- If the doublet of the weak interaction is the one Cabibbo suggested, Eq. (14), then one can have neutral currents

$$J^0_\mu = \bar{d}' \gamma_\mu (1 - \gamma_5) d' \tag{15}$$

  which introduces tree level FCNCs (which we don't see)
- Glashow, Iliopoulos and Maiani (1970) provided a solution by adding a second doublet

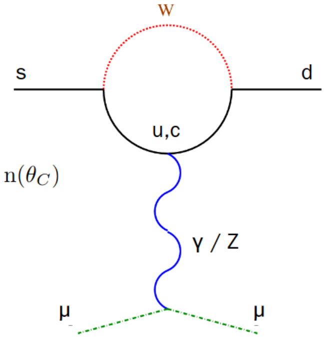
$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -d \sin(\theta_C) + s \cos(\theta_C) \end{pmatrix} \tag{16}$$

  - This exactly cancels the term above, Eq. (15)
  - Thus FCNC contributions are suppressed via loops

GIM suppression

- Consider the $s \to d$ transition required for $K_L^0 \to \mu^+ \mu^-$
- Given that $m_u, m_c \ll m_W$

$$\begin{aligned}\mathcal{A} &\approx V_{us}V_{ud}^* + V_{cs}V_{cd}^* \\ &= \sin(\theta_C)\cos(\theta_C) - \cos(\theta_C)\sin(\theta_C) \\ &= 0\end{aligned}$$



- Indeed 2×2 unitarity implies that

$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* = 0$$

- **Predicts the existence of the charm quark:**
  - Kaon mixing
  - Low branching fractions for FCNC decays

Parameters of the CKM matrix

- 3×3 complex matrix
  - 18 parameters
- Unitary
  - 9 parameters (3 mixing angles, 6 complex phases)
- Quark fields absorb 5 of these (unobservable) phases
- Left with:
  - 3 mixing angles $(\theta_{12}, \theta_{23}, \theta 13)$
  - one complex phase (δ) which gives rise to CP-violation in the SM

The CKM Matrix

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- A highly predictive theory

Parameters of the CKM matrix

- Absorbing quark phases can be done because under a quark phase transformation

$$u_L^i \to e^{i\phi_u^i} u_L^i, \quad d_L^i \to e^{i\phi_d^i} d_L^i \tag{20}$$

and a simultaneous rephasing of the CKM matrix ($V_{jk} \to e^{i(\phi_j - \phi_k)} V_{jk}$)

$$V_{\rm CKM} \to \begin{pmatrix} e^{i\phi_u} & & \\ & e^{i\phi_c} & \\ & & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi_d} & & \\ & e^{i\phi_s} & \\ & & e^{i\phi_b} \end{pmatrix} \tag{21}$$

the charged current $J^\mu = \bar{u}_{Li} V_{ij} \gamma^\mu d_{Lj}$ is left invariant

- So all additional quark phases are rephased to be relative to just one

Degrees of freedom in an N generation CKM matrix

Number of generations	**2**	**3**	N
Number of real parameters	4	9	N^2
Number of imaginary parameters	4	9	N^2
Number of constraints ($VV^\dagger = \mathbb{1}$)	-4	-9	$-N^2$
Number of relative quark phases	-3	-5	$-(2N-1)$
Total degrees of freedom	1	4	$(N-1)^2$
Number of Euler angles	1	3	$N(N-1)/2$
Number of CP phases	0	1	$(N-1)(N-2)/2$

CKM parameterisations

- The standard form is to express the CKM matrix in terms of three rotation matrices and one CP-violating phase (δ)

$$V_{\rm CKM} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{2nd and 3rd gen. mixing}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{+i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{1st and 3rd gen. mixing + CPV phase}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{1st and 2nd gen. mixing}} \quad (22)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{13}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (23)$$

where

$$c_{ij} = \cos(\theta_{ij}) \quad \text{and} \quad s_{ij} = \sin(\theta_{ij})$$

CKM parameterisations

- Emprically $s_{12} \sim 0.2$, $s_{23} \sim 0.04$, $s_{13} \sim 0.004$
- CKM matrix exhibits a very clear hierarchy
- The so-called Wolfenstein parameterisation exploits this
- Expand in powers of $\lambda = \sin(\theta_{12})$
- Use four real parameters which are all $\sim O(1)$, (A, λ, ρ, η)

The CKM Wolfenstein parameterisation

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (24)$$

- The CKM matrix is almost diagonal
  - Provides strong constraints on NP models in the flavour sector
- Have seen already that quark masses also exhibit a clear hierarchy
- The flavour hierarchy problem
  - Where does this structure come from?

CKM Unitarity Constraints

- The unitary nature of the CKM matrix provides several constraints, $VV^\dagger = \mathbb{1}$
- The ones for off-diagonal elements consist of three complex numbers summing to 0
  - Hence why these are often represented as triangles in the real / imaginary plane (see next slide)

<u>Constraints along diagonal</u>

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1$$
$$|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 = 1$$

$$|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 = 1$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 1$$
$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

<u>Constraints off-diagonal</u>

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$

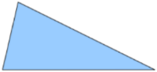
$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$$
$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$$
$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$$

CKM Unitarity Triangles and the Jarlskog Invariant

- The off-diagonal constraints can be represented as triangles in the complex plane

$$V_{ud}V_{us}^{*}+V_{cd}V_{cs}^{*}+V_{td}V_{ts}^{*}=0$$
$$\lambda \quad + \quad \lambda \quad + \quad \lambda^5$$

$$V_{ud}V_{ub}^{*}+V_{cd}V_{cb}^{*}+V_{td}V_{tb}^{*}=0$$
$$\lambda^3 \quad + \quad \lambda^3 \quad + \quad \lambda^3$$



$$V_{us}V_{ub}^{*}+V_{cs}V_{cb}^{*}+V_{ts}V_{tb}^{*}=0$$
$$\lambda^4 \quad + \quad \lambda^2 \quad + \quad \lambda^2$$

- All the triangles have the equivalent area (known as the Jarlskog invariant), $J/2$
- J is a phase convention independent measure of CP-violation in the quark sector

$$|J| = \mathcal{I}m(V_{ij}V_{kl}V_{kj}^{*}V_{il}^{*}) \quad \text{for } i \neq k \text{ and } j \neq k \tag{25}$$

- In the standard notation

$$J = c_{12}c_{13}^{2}c_{23}s_{12}s_{23}s_{13}\sin(\delta) \tag{26}$$

- The small size of the Euler angles means J (and CP-violation) is small in the SM

Section III

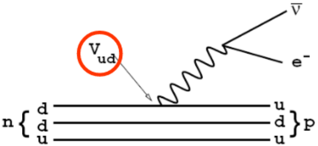
Metrology of CKM matrix

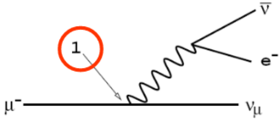
Measuring CKM matrix elements: $|V_{ud}|$

Measuring V_{ud}

- Compare rates of neutron, n^0, and muon, μ^-, decays
- The ratio is proportional to $|V_{ud}|^2$
- $|V_{ud}| = 0.947417 \pm 0.00021$
- $|V_{ud}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





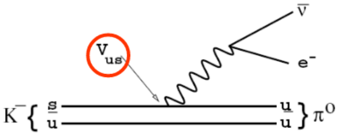
$$\frac{d\Gamma(n \to pe^-\overline{\nu}_e)}{dx_p} = \frac{G_F^2 m_n^2}{192\pi^2}|V_{ud}|^2 f(q^2)^2 \left(x_p^2 - 4\frac{m_p^2}{m_n^2}\right)^{3/2}, \quad \text{where} \quad x_p = \frac{2E_p}{m_n}$$

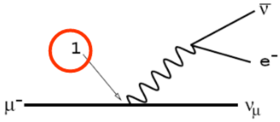
Measuring CKM matrix elements: $|V_{us}|$

Measuring V_{us}

- Compare rates of kaon, K^-, and muon, μ^-, decays
- The ratio is proportional to $|V_{us}|^2$
- $|V_{us}| = 0.2248 \pm 0.0006$
- $|V_{us}| \approx \sin(\theta_C) \approx \lambda$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





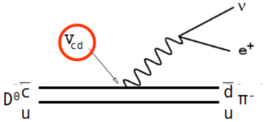
$$\frac{d\Gamma(\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e)}{dx_\pi} = \frac{G_F^2 m_K^2}{192\pi^2} |V_{us}|^2 f(q^2)^2 \left(x_\pi^2 - 4\frac{m_\pi^2}{m_K^2} \right)^{3/2}, \quad \text{where} \quad x_\pi = \frac{2E_\pi}{m_K}$$

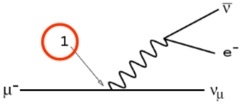
Measuring CKM matrix elements: $|V_{cd}|$ and $|V_{cs}|$

Measuring V_{cd} and V_{cs}

- Early measurements used neutrino DIS
- Now use semi-leptonic charm decays, $D^0 \rightarrow \pi^- \ell^+ \nu_\ell$ (V_{cd}) and $D^0 \rightarrow K^- \ell^+ \nu_\ell$ (V_{cs})
- $|V_{cd}| = 0.220 \pm 0.005$
- $|V_{cs}| = 0.995 \pm 0.016$
- $|V_{cd}| \approx \sin(\theta_C) \approx \lambda$
- $|V_{cs}| \approx 1$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



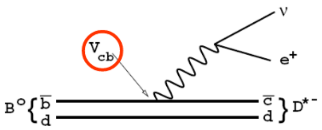


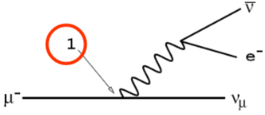
Measuring CKM matrix elements: $|V_{cb}|$

Measuring V_{cb}

- Compare rates of $B^0 \to D^{*-}\ell^+\nu_\ell$ and muon decays
- Ratio is proportional to $|V_{cb}|^2$
- $|V_{cb}| = 0.0405 \pm 0.0013$
- $|V_{cd}| \approx \sin^2(\theta_C) \approx \lambda^2$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$





$$\frac{d\Gamma(b \to u_\alpha \ell^- \overline{\nu}_\ell)}{dx} = \frac{G_F^2 m_b^5}{192\pi^2} |V_{\alpha b}|^2 \left(2x^2 \left(\frac{1-x-\xi}{1-x} \right)^2 \left(3 - 2x + \xi + \frac{2\xi}{1-x} \right) \right)$$

where $\alpha = u, c, \quad \xi = \frac{m_\alpha^2}{m_b^2}, \quad x = \frac{2E_l}{m_b}$

Measuring CKM matrix elements: $|V_{ub}|$

- There are three ways to determine V_{ub}
  1. "Inclusive" decays of $b \to u\ell^- \overline{\nu}_\ell$
     - Of course there are no bare quarks so we are really looking at a sum of exclusive decays of the form $B^{0(-)}_{(s)} \to \pi^{0(-)}\ell^- \overline{\nu}_\ell X$
  2. "Exclusive" decays *e.g.* $\overline{B}^0 \to \pi^+ \ell^- \overline{\nu}_\ell$
  3. Leptonic "annhilation" decays *e.g.* $B^+ \to \ell^+ \nu_\ell$
- These each come with various requirements on theory (form factors) and the results have historically been rather inconsistent
  - This is typical in flavour physics
  - Is the discrepancy a theory issue, an experimental issue or New Physics (or some combination)?

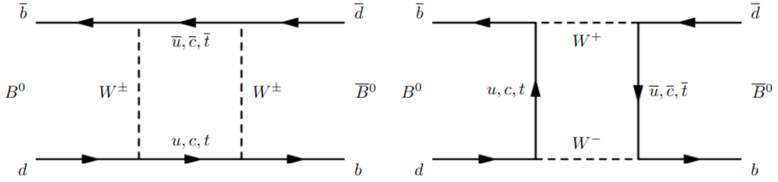
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Measuring CKM matrix elements: $|V_{ts}|$ and $|V_{td}|$

- There is no top decay but can obtain indirect measurements from the loops which appear in B^0 and B_s^0 mixing

- $|V_{ts}| = 0.0082 \pm 0.0006$
- $|V_{td}| = 0.0400 \pm 0.0027$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



- Ratio of frequencies for B^0 and B_s^0:

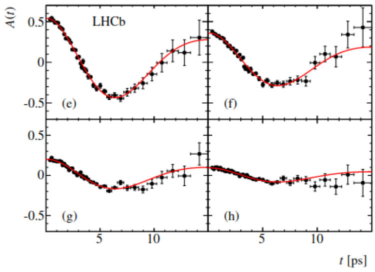
$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s^0}}{m_{B^0}} \frac{f^2_{B_s^0}}{f^2_{B^0}} \frac{B^2_{B_s^0}}{B_{B^0}} \frac{|V_{ts}|^2}{|V_{td}|^2} = \frac{m_{B_s^0}}{m_{B^0}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2} \tag{9}$$

Measuring CKM matrix elements: $|V_{ts}|$ and $|V_{td}|$

- B^0 and B_s^0 oscillation frequencies (which we use to get constraints on V_{td} and V_{ts}) measured at LEP, Tevatron, B-factories and LHCb
- Most precise measurements now come from LHCb

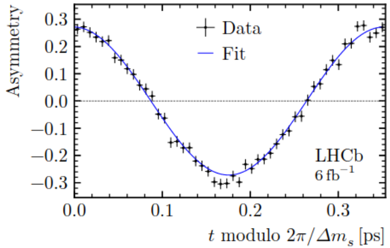
Δm_d from $B^0 \to D^{(*)-}\mu^+\nu_\mu X$

[arXiv:1604.03475]



Δm_s from $B_s^0 \to D_s^-\pi^+$

[arXiv:2104.04421]

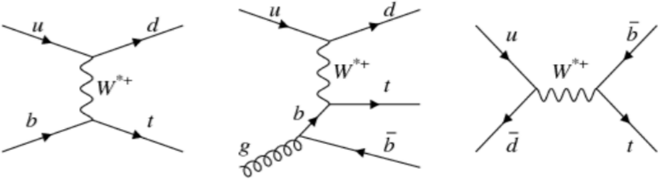


Measuring CKM matrix elements: : $|V_{tb}|$

Measuring V_{tb}

- Use single top production at the Tevatron
- Ratio is proportional to $|V_{tb}|^2$
- $|V_{tb}| = 1.009 \pm 0.0031$

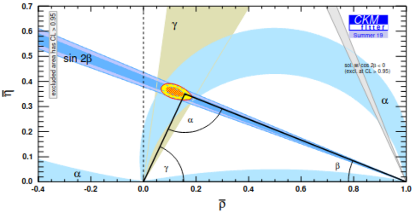
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$R = \frac{\mathcal{B}(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{\sum_q |V_{tq}|^2}$$

Measuring CKM phases

Amplitude	Rel. magnitude	phase
$b \to c$	Dominant	0
$b \to u$	Supressed	γ
$t \to d$	Time-dependent	2β
$t \to s$	Time-dependent	$-2\beta_s$

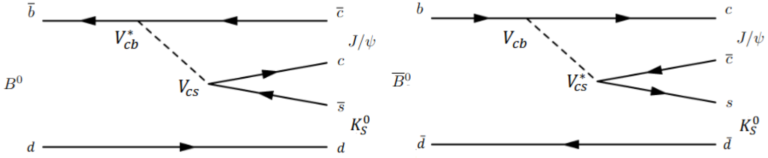


- γ in interference between $b \to u$ and $b \to c$ transitions
- β in interference between B^0 mixing and decay
- $\beta_s \approx \phi_s$ in interference between B_s^0 mixing and decay
- α arises in the interference between different $b \to u$ transitions

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{-i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

Measuring CKM phase: β

- Arises in the interference between $B^0 \to f_{CP}$ and $B^0 \to \overline{B}^0 \to f_{CP}$
- The golden mode is $B^0 \to J/\psi K^0_S$ because the master equations (see Lecture 2) simplify considerably
  1. For a B^0 we have no (or at least negligible) CPV in mixing
  $$\left|\frac{q}{p}\right| \approx 1$$
  2. For the $J/\psi K^0_S$ we have a CP-even final state so $f = \bar{f}$ therefore
  $$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} \equiv \lambda_{\bar{f}}$$
  3. The B^0 and $\overline{B}^0$ amplitudes to f are (almost) identical (can you think what makes them unequal?)
  $$|A_f| = |\bar{A}_f|$$



Measuring CKM phase: β

- Recall from the master equations (Lecture 2) that

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}$$

- Giving a time-dependent asymmetry of

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{C_f\cos(\Delta m t) - S_f\sin(\Delta m t)}{\cosh(\frac{1}{2}\Delta\Gamma t) + D_f\sinh(\frac{1}{2}\Delta\Gamma t)}} \tag{10}$$

- In the case of $B^0 \to J/\psi K^0_S$ this hugely simplifies as $|\lambda_f| = 1$ and $\Delta\Gamma = 0$ so that

$$\boxed{\mathcal{A}_{CP}(t) = -\mathcal{I}m(\lambda_f)\sin(\Delta m t)} \tag{11}$$

Measuring CKM phase: β

- Looking into more detail at what λ_f is in the case of $B^0 \to J/\psi K^0_S$

$$\lambda_{J/\psi K^0_S} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0_S}}{A_{J/\psi K^0_S}} = \left(\frac{q}{p}\right)_{B^0} \frac{\bar{A}_{J/\psi K^0}}{A_{J/\psi K^0}} \left(\frac{p}{q}\right)_{K^0} \tag{12}$$

$$= -\underbrace{\left(\frac{V^*_{tb}V_{td}}{V_{tb}V^*_{td}}\right)}_{B^0 \text{ mixing}} \underbrace{\left(\frac{V_{cb}V^*_{cs}}{V^*_{cb}V_{cs}}\right)}_{B^0 \to J/\psi K^0} \underbrace{\left(\frac{V_{cs}V^*_{cd}}{V^*_{cs}V_{cd}}\right)}_{K^0 \text{ mixing}} \tag{13}$$

$$= -e^{-2i\beta} \tag{14}$$

it's a useful exercise to show this using the equations from Lecture 2

- So that the time-dependent asymmetry is

$$\boxed{\mathcal{A}_{CP}(t) = \pm \sin(2\beta)\sin(\Delta m t)} \tag{15}$$

the $\pm$ is for CP-even (*e.g.* $J/\psi K^0_L$) or CP-odd (*e.g.* $J/\psi K^0_S$) final states

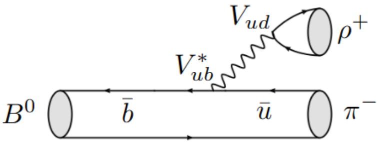
- A theoretically and experimentally clean signature
- Also has a relatively large branching fraction, $O(10^{-4})$

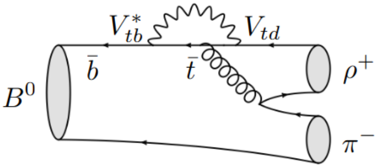
Measuring CKM phase: α

- Following a similar logic to that of $B^0 \to J/\psi K^0_S$ for β one finds that α arises in the time-dependent asymmetry for modes containing a $b \to u\bar{u}d$ transition
  - For example $B^0 \to \pi^+\pi^-$ or $B^0 \to \rho^+\rho^-$
- Recalling the master equations with $\Delta\Gamma = 0$
- Nominally we should have $C_f = 0$ and $S_f = \sin(2\alpha)$ to give

$$\mathcal{A}_{CP}(t) = \pm\sin(2\alpha)\sin(\Delta m t) \tag{23}$$

  exactly equivalent to the extraction of β
- However, in this case there is a **non-negligible contribution from penguin decays** of $b \to d\bar{u}u$
  - Similar in magnitude to the $b \to u\bar{u}q$ transition but has a different weak phase
  - Therefore $C \neq 0$ and $S \neq \pm\sin(2\alpha)$
  - How do we deal with the penguin contamination?





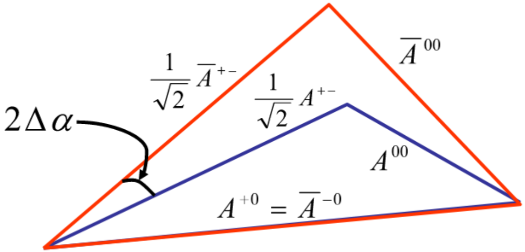
Measuring CKM phase: α

- The contributions from the penguin amplitudes can be accounted for using an "isopsin analysis"
  - Relate the amplitudes for isospin partners

$$A^{+-} \text{ for } B^0 \to \pi^+\pi^-, \quad A^{+0} \text{ for } B^+ \to \pi^+\pi^0, \quad A^{00} \text{ for } B^0 \to \pi^0\pi^0, \tag{24}$$

  - There is no penguin contribution to A^{+0} and $\bar{A}^{-0}$ because $\pi^\pm\pi^0$ is a pure isospin-2 state and the QCD-penguin ($\Delta I = 1/2$) only contributes to the isospin-0 final states
  - Obtain isospin triangle relations

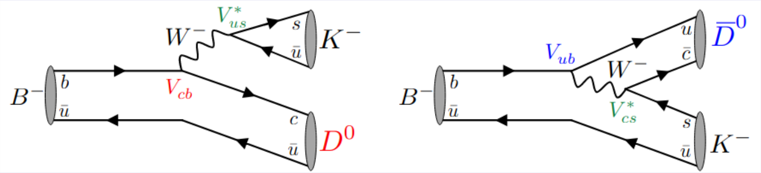
$$A^{+0} = \frac{1}{\sqrt{2}} A^{+-} + A^{00}, \text{ and } \bar{A}^{-0} = \frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} \tag{25}$$



Measuring CKM phase: γ

- γ is the phase between $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$
  - Require interference between $b \to cW$ and $b \to uW$ to access it
  - No dependence on CKM elements involving the top
  - **Can be measured using tree level B decays**
- The "textbook" case is $B^\pm \to \overset{(-)}{D}{}^0 K^\pm$:
  - Transitions themselves have different final states (D^0 and $\bar{D}^0$)
  - Interference occurs when D^0 and $\bar{D}^0$ decay to the same final state f

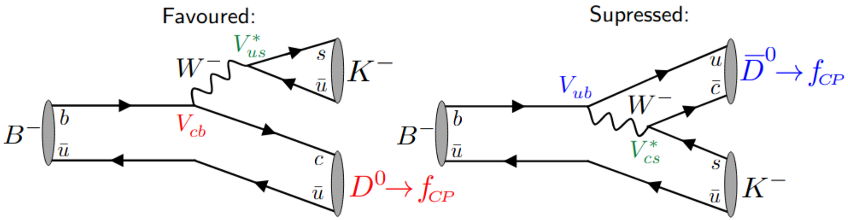
Reconstruct the $D^0/\bar{D}^0$ in a final state accessible to both to acheive interference



- **The crucial feature of these (and similar) decays is that the D^0 can be reconstructed in several different final states** [all have same weak phase γ]

Measuring CKM phase: γ

- Use the $B^{\pm} \to \overset{(-)}{D}{}^0 K^{\pm}$ case as an example:
  - **Consider only D decays to CP eigenstates, f_{CP}**
  - Favoured: $b \to c$ with strong phase δ_F and weak phase ϕ_F
  - Supressed: $b \to u$ with strong phase δ_S and weak phase ϕ_S



Subsequent amplitude to final state f_{CP} is:

$$B^- : A_f = |F|e^{i(\delta_F - \phi_F)} + |S|e^{i(\delta_S - \phi_S)} \tag{26}$$

$$B^+ : \bar{A}_f = |F|e^{i(\delta_F + \phi_F)} + |S|e^{i(\delta_S + \phi_S)} \tag{27}$$

because strong phases (δ) don't change sign under CP while weak phases (ϕ) do

Measuring CKM phase: γ

- Can define the sum and difference of rates with B^+ and B^-

Rate difference and sum

$$|\bar{A}_{\bar{f}}|^2 - |A_f|^2 = 2|F||S|\sin(\delta_F - \delta_S)\sin(\phi_F - \phi_S) \quad (28)$$

$$|\bar{A}_{\bar{f}}|^2 + |A_f|^2 = |F|^2 + |S|^2 + 2|F||S|\cos(\delta_F - \delta_S)\cos(\phi_F - \phi_S) \quad (29)$$

- Choose $r_B = \frac{|S|}{|F|}$ (so that $r < 1$) and use strong phase difference $\delta_B = \delta_F - \delta_S$
- γ is the weak phase difference $\phi_F - \phi_S$
- Subsequently have two **experimental observables** which are

GLW CP asymmetry

$$\mathcal{A}_{CP} = \frac{\pm 2 r_B \sin(\delta_B)\sin(\gamma)}{1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)}$$

GLW total rate

$$\mathcal{R}_{CP} = 1 + r_B^2 \pm 2 r_B \cos(\delta_B)\cos(\gamma)$$

- The $+(-)$ sign corresponds to CP-even (-odd) final states
- Note that r_B and δ_B (ratio and strong phase difference of favoured and supressed modes) are different for each B decay
- **The value of γ is shared by all such decays**

- We discussed a myriad of topics under the umbrella of Flavor physics.
- First half focused on the SM, quark model history, and CKM matrix.
- In the second half we talked about metrology of CKM parameters.
- The talk is based on the course taken by Prof. Mathew Kenzie.

Flavour in the SM

A brief theoretical interlude which we will flesh out with some history afterwards

- Particle physics can be described to excellent precision by a relatively straightforward and very beautiful theory (we all know and love the SM):

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm Gauge}(A_a, \psi_i) + \mathcal{L}_{\rm Higgs}(\phi, A_a, \psi_i) \tag{1}$$

- It contains:
  - Gauge terms that deal with the free fields and their interactions via the strong and electroweak interactions
  - Higgs terms that give rise to the masses of the SM fermions and weak bosons

Flavour in the SM

- The Gauge part of the Lagrangian is well verified

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D}\psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a} \quad (2)$$

- Parity is violated by electroweak interactions
- Fields are arranged as left-handed doublets and right-handed singlets

$$\psi = \boxed{Q_L, u_R, d_R, c_R, s_R, t_R, b_R} \text{ quarks} \quad (3)$$

$$\boxed{L_L, e_R, \mu_R, \tau_R} \text{ leptons} \quad (4)$$

with

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \text{ and } L_L = \begin{pmatrix} e_L \\ \nu_{eL} \end{pmatrix}, \begin{pmatrix} \mu_L \\ \nu_{\mu L} \end{pmatrix}, \begin{pmatrix} \tau_L \\ \nu_{\tau L} \end{pmatrix} \quad (5)$$

- The Lagrangian is invariant under a specific set of symmetry groups: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$

Quark Gauge Couplings

- Without the Higgs we have flavour universal gauge couplings equal for all three generations (huge degeneracy)

$$\mathcal{L}_{\text{quarks}} = \sum_{j}^{3} \underbrace{i\bar{Q}_j \not{D}_Q Q_j}_{\text{left-handed doublets}} + \underbrace{i\bar{U}_j \not{D}_U U_j + i\bar{D}_j \not{D}_D D_j}_{\text{right-handed singlets}} \qquad (6)$$

leptons have been omitted for simplicity

- with the covariant derivatives

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + ig_s\lambda_\alpha G_\mu^\alpha + ig\sigma_i W_\mu^i + iY_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + ig_s\lambda_\alpha G_\mu^\alpha + iY_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + ig_s\lambda_\alpha G_\mu^\alpha + iY_D g' B_\mu \end{aligned}$$

and $Y_Q = 1/6$, $Y_U = 2/3$, $Y_D = -1/3$

Yukawa couplings

- In order to realise fermion masses we introduce "Yukawa couplings"
- This is rather ad-hoc. It is necessary to understand the data but is not stable with respect to quantum corrections (the Hierarchy problem).
- By doing this we introduce flavour non-universality via the Yukawa couplings between the Higgs and the quarks

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i,j}^{3} (-\bar{Q}_L^i Y_U^{ij} \tilde{H} u_R^j - \bar{Q}_L^i Y_D^{ij} H d_R^j + h.c.) \qquad (7)$$

leptons have been omitted for simplicity

- Replace H by its vacuum expectation value, $\langle H \rangle = (0, \nu)^T$, and we obtain the quark mass terms

$$\sum_{i,j}^{3} (-\bar{u}_L^i m_U^{ij} u_R^j - \bar{d}_L^i m_D^{ij} d_R^j) \qquad (8)$$

with the quark mass matrices given by $m_A = \nu Y_A$ with $A = (U, D, L)$

Diagonalising the mass matrices

- Quark mass matrices, m_U, m_D, m_L, are 3×3 complex matrices in "flavour space" with *a priori arbitary values*.
  - We can diagonalise them via a field redefintion

$$u_L = \hat{U}_L u_L^m, \;\; u_R = \hat{U}_R u_R^m, \;\; d_L = \hat{D}_L d_L^m, \;\; d_R = \hat{D}_R d_R^m \tag{9}$$

  - such that in the mass eigenstate basis the matrices are diagonal

$$m_U^{\text{diag}} = \hat{U}_L^\dagger m_U \hat{U}_R, \quad m_D^{\text{diag}} = \hat{D}_L^\dagger m_D \hat{D}_R \tag{10}$$

- The right-handed SU(2) singlet is invariant but recall the left-handed SU(2) doublet gives rise to terms like

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \gamma_\mu W^\mu d_L^i \tag{11}$$

- In the mass basis this then becomes

$$\frac{g}{\sqrt{2}} \bar{u}_L^i \underbrace{\hat{U}_L^{\dagger ij} \hat{D}_L^{jk}}_{\hat{V}_{\text{CKM}}} \gamma_\mu W^\mu d_L^k \tag{12}$$

This combination, $\hat{V}_{\text{CKM}} = \hat{U}_L^{\dagger ij} \hat{D}_L^{jk}$, **is the** physical CKM matrix and generates flavour violating charged current interactions. It is complex and unitary, $VV^\dagger = \mathbb{1}$

Flavour in the SM

- The gauge part of the SM Lagrangian is invariant under U(3) symmetries of the left-handed doublets and right-handed singlets if the fermions are massless

$$\mathcal{L}_{\text{Gauge}} = \sum_j i\bar{\psi}_j \not{D} \psi_j - \sum_a \frac{1}{4g_a^2} F^a_{\mu\nu} F^{\mu\nu,a}$$

- These U(3) symmetries are broken by the Yukawa terms. The only remaining symmetries correspond to lepton number and baryon number conservation
- These are "accidental" symmetries, coming from the particle content, rather than being explicitly imposed

We will return to the CKM matrix and CKM metrology later!

particle zoo

SU(2) flavour mixing

- Four possible combinations from two quarks (u and d)

$$u\bar{u}, d\bar{d}, u\bar{d}, \bar{u}d$$

- Under SU(2) symmetry the π^0 and η states are members of an isospin triplet and singlet respectively

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \eta = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

SU(3) flavour mixing

- Introducing the strange quark (under SU(3) symmetry) we now have an octuplet and a singlet

$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \eta_1 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}), \quad \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

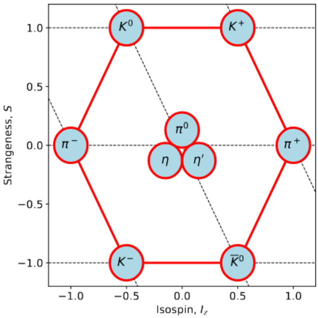
- The physical states involve a further mixing

$$\eta = \eta_1 \cos\theta + \eta_8 \sin\theta, \quad \eta' = -\eta_1 \sin\theta + \eta_8 \cos\theta$$

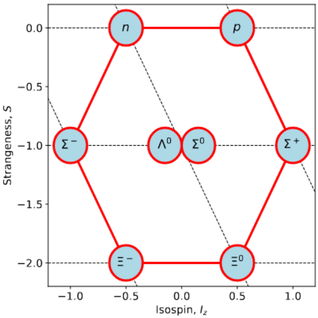
Particle zoo

- Can elegantly categorise states by isospin (up/downess) and strangeness
- Also get the excited states which can be categorised in the same way

Spin-0 Mesons



Spin-1/2 Baryons



Homework

- What is the quark content of these states?
- Do you know the spin-1 (spin-3/2) states?

CKM mechanism

- In 1973 Kobayashi and Maskawa introduce the CKM mechanism to explain CP-violation
- As we will see this requires a third generation of quark and so they predict the existence of b and t quarks

CP Violation in the Renormalizable Theory of Weak Interaction

Makoto Kobayashi, Toshihide Maskawa (Kyoto U.)

Feb 1973 - 6 pages

Prog.Theor.Phys. 49 (1973) 652-657

Also in *Lichtenberg, D. B. (Ed.), Rosen, S. P. (Ed.): Developments In The Quark Theory Of Hadrons, Vol. 1*, 218-223.

DOI: 10.1143/PTP.49.652

KUNS-242

Abstract (Oxford Journals)

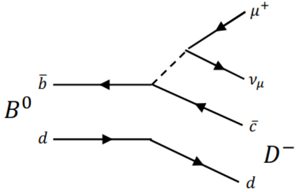
In a framework of the renormalizable theory of weak interaction, problems of CP-violation are studied. It is concluded that no realistic models of CP-violation exist in the quartet scheme without introducing any other new fields. Some possible models of CP-violation are also discussed.

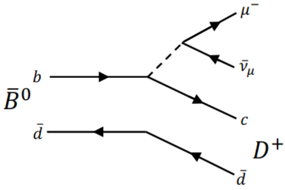
Section II

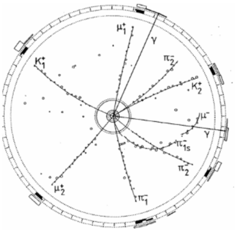
Meson mixing and CP violation in the SM

Neutral Meson Mixing

- In 1987 the ARGUS experiment observed coherently produced $B^0 - \overline{B}^0$ pairs and observed them decaying to same sign leptons
- How is this possible?
  - Semileptonic decays "tag" the flavour of the initial state



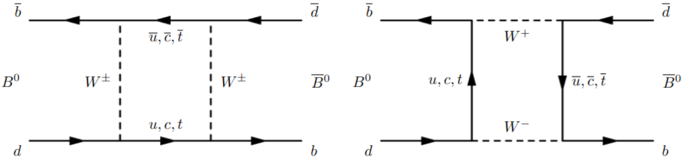




- The only explanation is that B^0–$\overline{B}^0$ can oscillate
- Rate of mixing is large $\rightarrow$ top quark must be heavy

Neutral Meson Mixing

- In the SM occurs via box diagrams involving a charged current ($W^{\pm}$) interaction
- Weak eigenstates are not the same as the physical mass eigenstates
  - The particle and antiparticle flavour states (via CPT theorem) have equal and opposite charge, identical mass and identical lifetimes
  - But the mixed states (*i.e.* the physical B^0_L and B^0_H) can have $\Delta m, \Delta\Gamma \neq 0$



- In the SM we have four possible neutral meson states
  - K^0, D^0, B^0, B^0_s (mixing has been observed in all four)
  - Although they all have rather different properties (as we will see in a second)

Coupled meson systems

- A single particle system evolves according to the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}|X(t)\rangle = \mathcal{H}|X(t)\rangle = \left(M - i\frac{\Gamma}{2}\right)|M(t)\rangle \tag{3}$$

- For neutral mesons, mixing leads to a coupled system

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} \tag{4}$$

$$= \begin{pmatrix}M_{11} - i\Gamma_{11}/2 & M_{12} - i\Gamma_{12}/2\\ M_{12}^* - i\Gamma_{12}^*/2 & M_{22} - i\Gamma_{22}/2\end{pmatrix}\begin{pmatrix}|B^0\rangle\\|\overline{B}^0\rangle\end{pmatrix} \tag{5}$$

where

$$M_{12} = \frac{1}{2M}\mathcal{A}(B^0 \to \overline{B}^0) = \langle\overline{B}^0|\mathcal{H}(\Delta B = 2)|B^0\rangle \tag{6}$$

Coupled meson systems

- To start with we will neglect CP-violation in mixing (approximately the case for all four neutral meson species)
- Neglecting CP-violation, the physical states are an equal mixture of the flavour states

$$|B_L^0\rangle = \frac{|B^0\rangle + |\overline{B}^0\rangle}{2}, \qquad |B_H^0\rangle = \frac{|B^0\rangle - |\overline{B}^0\rangle}{2}$$

with mass and width differences

$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|\Gamma_{12}|, \quad \Delta M = M_H - M_L = 2|M_{12}|$$

so that the physical system evolves as

$$i\frac{\partial}{\partial t}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \mathcal{H}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} = \left(\boldsymbol{M} - i\frac{\boldsymbol{\Gamma}}{2}\right)\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{7}$$

$$= \begin{pmatrix}M_L - i\Gamma_L/2 & 0 \\ 0 & M_H - i\Gamma_H/2\end{pmatrix}\begin{pmatrix}|B_L^0\rangle \\ |B_H^0\rangle\end{pmatrix} \tag{8}$$

Time evolution

- Solving the Schrödinger equation gives the time evolution of a pure state $|B^0\rangle$ or $|\overline{B}^0\rangle$ at time $t = 0$

$$|B^0(t)\rangle = g_+(t)|B^0\rangle + \frac{q}{p} g_-(t)|\overline{B}^0\rangle$$
$$|\overline{B}^0(t)\rangle = g_+(t)|\overline{B}^0\rangle + \frac{p}{q} g_-(t)|B^0\rangle \qquad (9)$$

where

$$g_+(t) = e^{-iMt} e^{-\Gamma t/2} \left[\cosh\left(\frac{\Delta\Gamma t}{4}\right) \cos\left(\frac{\Delta m t}{2}\right) - i \sinh\left(\frac{\Delta\Gamma t}{4}\right) \sin\left(\frac{\Delta m t}{2}\right)\right]$$
$$g_-(t) = e^{-iMt} e^{-\Gamma t/2} \left[-\sinh\left(\frac{\Delta\Gamma t}{4}\right) \cos\left(\frac{\Delta m t}{2}\right) + i \cosh\left(\frac{\Delta\Gamma t}{4}\right) \sin\left(\frac{\Delta m t}{2}\right)\right] \qquad (10)$$

and $M = (M_L + M_H)/2$ and $\Gamma = (\Gamma_L + \Gamma_H)/2$

- No CP-violation in mixing means that $|p/q| = 1$ (and thus we have equal admixtures)

Time evolution

- Using Eq. (10) flavour remains unchanged $(+)$ or will oscillate $(-)$ with probability

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{11}$$

- With no CP violation in the mixing, the time-integrated mixing probability is

$$\frac{\int |g_{-}(t)|^2 dt}{\int |g_{-}(t)|^2 dt + \int |g_{+}(t)|^2 dt} = \frac{x^2 + y^2}{2(x^2 + 1)} \tag{12}$$

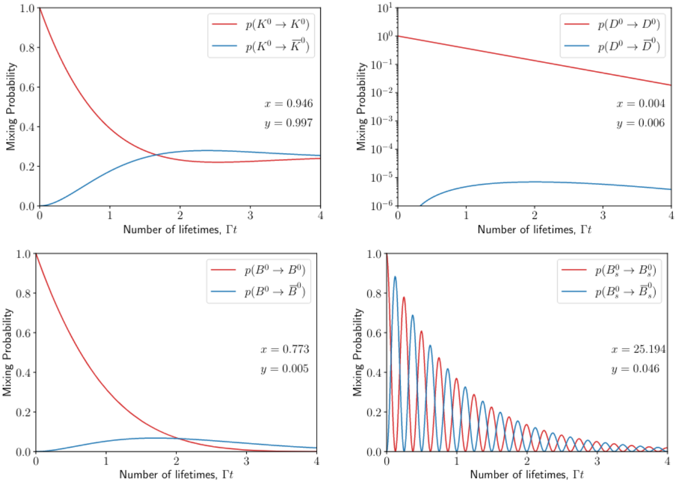
where

$$x = \frac{\Delta m}{\Gamma} \quad \text{and} \quad y = \frac{\Delta\Gamma}{2\Gamma} \tag{13}$$

- The four different neutral meson species which mix have very different values of (x, y) and therefore very different looking time evolution properties

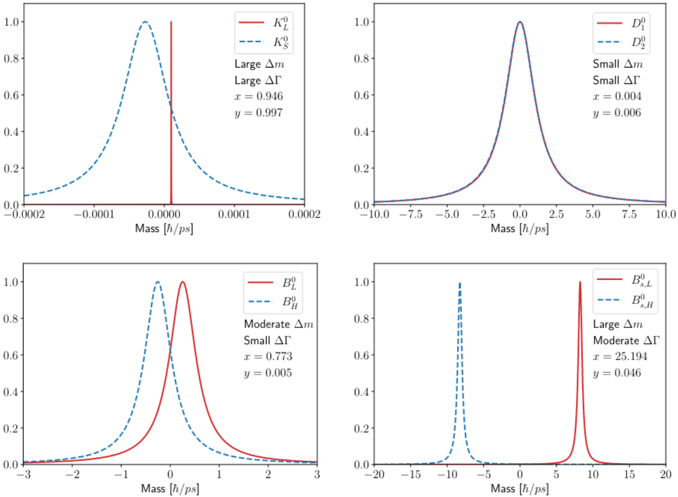
Neutral Meson Mixing

$$|g_{\pm}(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \tag{14}$$



Neutral Meson Mixing

- Mass and width differences of the neutral meson mixing systems



Measuring CP violation

1. Need at least two interfering amplitudes
2. Need two phase differences between them
   - One CP conserving ("strong") phase difference (δ)
   - One CP violating ("weak") phase difference (ϕ)

- If there is only a single path to a final state, f, then we cannot get direct CP violation
- If there is only one path we can write the amplitudes for decay as

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1 + \phi_1)}$$
$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1 - \phi_1)}$$

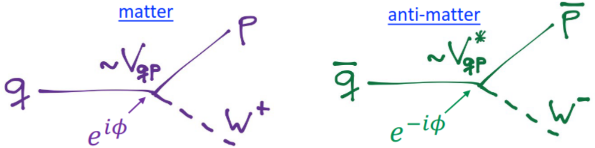
- Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\overline{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\overline{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} = 0 \qquad (17)$$

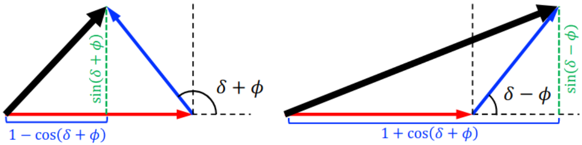
- In order to observe CP-violation we need a second amplitude.
- This is often realised by having interefering tree and penguin amplitudes

Measuring CP violation

- We measure quark couplings which have a complex phase
- This is only visible when there are two amplitudes



- Below we represent two amplitudes (red and blue) with the same magnitude $= 1$
  - The strong phase difference is, $\delta = \pi/2$
  - The weak phase difference is, $\phi = \pi/4$



$$\Gamma(B \to f) = |A_1 + A_2 e^{i(\delta+\phi)}|^2 \qquad \Gamma(\bar{B} \to \bar{f}) = |A_1 + A_2 e^{i(\delta-\phi)}|^2$$

Measuring (direct) CP violation

- Introducing the second amplitude we now have

$$\mathcal{A}(B \to f) = A_1 e^{i(\delta_1+\phi_1)} + A_2 e^{i(\delta_2+\phi_2)} \tag{18}$$

$$\mathcal{A}(\bar{B} \to \bar{f}) = A_1 e^{i(\delta_1-\phi_1)} + A_2 e^{i(\delta_2-\phi_2)} \tag{19}$$

- Which gives an asymmetry of

$$\mathcal{A}_{CP} = \frac{|\mathcal{A}(\overline{B} \to \bar{f})|^2 - |\mathcal{A}(B \to f)|^2}{|\mathcal{A}(\overline{B} \to \bar{f})|^2 + |\mathcal{A}(B \to f)|^2} \tag{20}$$

$$= \frac{4A_1A_2 \sin(\delta_1-\delta_2)\sin(\phi_1-\phi_2)}{2A_1^2 + 2A_2^2 + 4A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)} \tag{21}$$

$$= \frac{2r\sin(\delta)\sin(\phi)}{1+r^2+2r\cos(\delta)\cos(\phi)} \tag{22}$$

where $r = A_1/A_2$, $\delta = \delta_1 - \delta_2$ and $\phi = \phi_1 - \phi_2$

- This is only non-zero if the amplitudes have **different** weak **and** strong phases
- This is CP-violation in decay (often called "direct" CP violation).
  - This is the only possible route of CP violation for a charged initial state
  - We will see now that for a neutral initial state there are other ways of realising CP violation

Classification of CP violation

- First let's consider a generalised form of a neutral meson, X^0, decaying to a final state, f
- There are four possible amplitudes to consider

$$A_f = \langle f|X^0\rangle \qquad \bar{A}_f = \langle f|\bar{X}^0\rangle$$
$$A_{\bar{f}} = \langle \bar{f}|X^0\rangle \qquad \bar{A}_{\bar{f}} = \langle \bar{f}|\bar{X}^0\rangle$$

- Define a complex parameter, λ_f (**not** the Wolfenstein parameter, λ)

$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}, \qquad \bar{\lambda}_f = \frac{1}{\lambda_f}, \qquad \lambda_{\bar{f}} = \frac{q}{p}\frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}}, \qquad \bar{\lambda}_{\bar{f}} = \frac{1}{\lambda_{\bar{f}}}$$

Classification of CP violation

Can realise CP violation in three ways:

1. CP violation in decay
   - For a charged initial state this is only the type possible

$$\Gamma(X^0 \to f) \neq \Gamma(\bar{X}^0 \to \bar{f}) \Longrightarrow \boxed{\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1} \tag{23}$$

2. CP violation in mixing

$$\Gamma(X^0 \to \bar{X}^0) \neq \Gamma(\bar{X}^0 \to X^0) \Longrightarrow \boxed{\left|\frac{p}{q}\right| \neq 1} \tag{24}$$

3. CP violation in the interference between mixing and decay

$$\Gamma(X^0 \to f) \neq \Gamma(X^0 \to \bar{X}^0 \to f) \Longrightarrow \arg(\lambda_f) = \boxed{\arg\left(\frac{q}{p}\frac{\bar{A}_f}{A_f}\right) \neq 0} \tag{25}$$

- We just saw an example of CP violation in decay
- Let's extend our formalism of neutral mixing, Eqs. (9–13), to include CP violation

Neutral Meson Mixing with CP violatio

- Allowing for CP violation, $M_{12} \neq M_{12}^*$ and $\Gamma_{12} \neq \Gamma_{12}^*$
- The physical states can now be unequal mixtures of the weak states

$$
\begin{aligned}
|B_L^0\rangle &= p|B^0\rangle + q|\overline{B}^0\rangle \\
|B_H^0\rangle &= p|B^0\rangle - q|\overline{B}^0\rangle
\end{aligned}
\tag{26}
$$

where

$$
|p|^2 + |q|^2 = 1
$$

- The states now have mass and width differences

$$
|\Delta\Gamma| \approx 2|\Gamma_{12}|\cos(\phi), \quad |\Delta M| \approx 2|M_{12}|, \quad \phi = \arg(-M_{12}/\Gamma_{12}) \tag{27}
$$

- We'll see some examples of this later
- Now to equip ourselves with the formalism for a generalised meson decay

Generalized Meson Decay Formalism

The probability that state X^0 at time t decays to f at time t

- contains terms for CPV in decay, mixing and the interference between the two

$$\Gamma_{X^0\to f}(t) = \boxed{|A_f|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\lambda_f|^2}\boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+^*(t) g_-(t)\right]}\right) \quad (28)$$

$$\Gamma_{X^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2}\boxed{\left|\frac{q}{p}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\lambda_{\bar{f}}|^2}\boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_{\bar{f}} g_+(t) g_-^*(t)\right]}\right) \quad (29)$$

$$\Gamma_{\bar{X}^0\to f}(t) = \boxed{|A_f|^2}\boxed{\left|\frac{p}{q}\right|^2} \left(\boxed{|g_-(t)|^2} + \boxed{|\lambda_f|^2}\boxed{|g_+(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_f g_+(t) g_-^*(t)\right]}\right) \quad (30)$$

$$\Gamma_{\bar{X}^0\to \bar{f}}(t) = \boxed{|\bar{A}_{\bar{f}}|^2} \left(\boxed{|g_+(t)|^2} + \boxed{|\lambda_{\bar{f}}|^2}\boxed{|g_-(t)|^2} + \boxed{2\mathcal{R}e\left[\lambda_{\bar{f}} g_+^*(t) g_-(t)\right]}\right) \quad (31)$$

where the mixing probabilities are as before

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma t}{2}\right) \pm \cos(\Delta m t)\right] \quad (32)$$

$$g_+^* g_-^{(*)} = \frac{e^{-\Gamma t}}{2}\left[\sinh\left(\frac{\Delta\Gamma t}{2}\right) \pm i\sin(\Delta m t)\right] \quad (33)$$

Generalized Meson Decay Formalism

- From the above we get the "master equations" for neutral meson decay

$$\Gamma_{X^0\to f}(t) = |A_f|^2 \quad (1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) + C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) - S_f\sin(\Delta m t)\Big] \quad (34)$$

$$\Gamma_{\overline{X}^0\to f}(t) = |A_f|^2\left|\frac{p}{q}\right|^2(1+|\lambda_f|^2)\frac{e^{-\Gamma t}}{2}\Big[\cosh(\tfrac{1}{2}\Delta\Gamma t) - C_f\cos(\Delta m t) + D_f\sinh(\tfrac{1}{2}\Delta\Gamma t) + S_f\sin(\Delta m t)\Big] \quad (35)$$

where

$$C_f = \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}, \quad D_f = \frac{2\mathcal{R}e(\lambda_f)}{1+|\lambda_f|^2}, \quad S_f = \frac{2\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2} \quad (36)$$

- and equivalents for the CP conjugate final state $\bar{f}$
- **The time-dependent CP asymmetry is** (for non-CP-eigenstates there are two)

$$\mathcal{A}_{CP}(t) = \frac{\Gamma_{X^0\to f}(t) - \Gamma_{\overline{X}^0\to f}(t)}{\Gamma_{X^0\to f}(t) + \Gamma_{\overline{X}^0\to f}(t)} = \boxed{\frac{2C_f\cos(\Delta m t) - 2S_f\sin(\Delta m t)}{2\cosh(\frac{1}{2}\Delta\Gamma t) + 2D_f\sinh(\frac{1}{2}\Delta\Gamma t)}} \quad (37)$$

Specific cases:

$$\mathcal{A}_{CP}(t) = \frac{2\mathcal{C}_f \cos(\Delta m t) - 2\mathcal{S}_f \sin(\Delta m t)}{2\cosh(\frac{1}{2}\Delta\Gamma t) + 2\mathcal{D}_f \sinh(\frac{1}{2}\Delta\Gamma t)}$$

- For B^0, $\Delta\Gamma$ is small $\Rightarrow \mathcal{A}_{CP}(t) = 2\mathcal{C}_f \cos(\Delta m t) - 2\mathcal{S}_f \sin(\Delta m t)$
- For D^0, both $\Delta\Gamma$ and Δm are small $\Rightarrow \mathcal{A}_{CP}(t) = \frac{\mathcal{C}_f - \mathcal{S}_f \Delta m t}{1 + \frac{1}{2}\mathcal{D}_f \Delta\Gamma t}$