pyhf introduction Belle-II pyhf workshop ()

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Introduction

Thanks a lot for the invitation & organization of this workshop!

Particularly thanks to Sally & Florian!



CERN

LHC [Large Hadron Collider]









Mt. Tsukuba

BELLEII

7.5 km

Measure the Standard Model



Goals

Look for the Unknown

Measure the Standard Model



The language in which we communicate our science is statistics

Goals

Look for the Unknown



The Key Ingredient

The most important object in statistics is the description of the measurement as a data generating process

parameters $p(\mathcal{D}|\theta)$ experimental data

The Key Ingredient

The statistical model $p(x | \theta)$ is where the physics lives. Once we have it, we can do all kinds of statistics

$p(\mathcal{D} | \theta) \rightarrow \hat{\theta}, \ [\theta_{-}, \theta_{+}]$

Frequentist Inference

interesting discussion, which to use, but remember physics content is the same (!) and defined by the **common** $p(x \mid \theta)$

Bayesian Inference

 $p(\theta | \mathcal{D}) = \frac{p(\mathcal{D} | \theta)p(\theta)}{p(\mathcal{D})}$



Repeated Experiments

The core of HEP experiments is the collection of an



We can build the dataset-wide model from the per-event model

i.i.d. sample

identically distributed



The bad news

The problem in HEP: we do not know the detector-level perevent model in closed form, such that we could evaluate $p(x | \theta)$



"Simulation"

sum over all possible histories

$p(x \mid \theta) = \int dz \ p(x \mid z_h) \ p(z_h \mid z_p) \ p(z_p \mid \theta)$

But: we can sample from this model without any problems (MC simulation)

 $x \sim p(x \mid \theta)$



Empirical Density Estimate

empirical density estimate

e.g. using a histogram as an approximation



With samples from a density $x \sim p(x \mid \theta)$ we can construct an

$\{x_i\} \rightarrow \hat{p}(x \mid \theta)$



Problem: We can't fill a histogram in 100M dimensions

Summary Statistics

Most of what we call Reconstruction & Analysis is about good low-dimensional observables, for which we can fill histograms



Summary Statistics

How to build a statistical model using histograms is what **HistFactory** is all about



A powerful and flexible statistical modelling approach

A bit of Nomenclature

HistFactory denotes a mathematical modelling approach for describing typical HEP situations



Standard Model

SUSY





Belle-II Analyses

A bit of Nomenclature HistFactory was first implemented in ROOT / RooFit (2010)



for a while the only implementation

HistFactory: A tool for creating statistical models for use with RooFit and RooStats

Kyle Cranmer, George Lewis, Lorenzo Moneta, Akira Shibata, Wouter Verkerke

June 20, 2012

Contents

1	Tert	aduation	•						
T	Intr	-oduction	2						
	1.1	Preliminaries	2						
	1.2	Generalizations and Use-Cases	3						
2	The	he Likelihood Template							
	2.1	Index Convention	4						
	2.2	The Template	4						
		2.2.1 Incorporating Monte Carlo statistical uncertainty on the histogram templates	5						
3	Using HistFactory								
	3.1	The HistFactory XML	7						
	3.2	Normalization Conventions	8						
	3.3	Usage of the HistFactory	10						
	3.4	Usage with RooStats tools	10						
4	Inte	erpolation & Constraints	12						
	4.1	Interpolation Options	13						
		4.1.1 Defaults in ROOT 5.32	15						
	4.2	Constraint Terms (+ global observables and nuisance parameter priors) \ldots	16						
		4.2.1 Consistent Bayesian and Frequentist modeling	16						
		4.2.2 Options for Constraint Terms	17						
5	Exa	mples	22						
	5.1	A Simple Example	22						
	5.2	ABCD	23						
6	\mathbf{The}	e HistFactory XML Schema in DTD Format	25						
7	Manual entries								



A bit of Nomenclature

pyhf is a newer implementation within the scientific python ecosystem of the HistFactory model (2018)

Based on the Scientific Python and ML ecosystems











The HistFactory model starts from considering space of events.

HEP typically cannot look at all events but a subset. Often a handful of orthogonal "regions" or "channels"





For a single channel, we describe project events onto a single summary statistic $x \rightarrow s(x)$ for which we estimate $\hat{p}(s \mid \theta)$



projection $x \rightarrow s(x)$

 $p(x \mid \theta)$ Changes in expected data distribution as function of parameters translate accordingly



Describing a 1-D observable in a HEP setting can be done by describing each bin in the density as a Poisson measurement



$\int_{b} \text{Pois}(n_b \mid \lambda_b(\theta))$

s(x)





In a given region of phase-space, typically a number of physics processes ("samples") contribute: we model the total as a sum.





$c \in \text{channels } b \in \text{bins}_{c}$

In a given region of phase-space, typically a number of physics processes ("samples") contribute: we model the total as a sum.



Caputing the Information

In order to build a model like this, we need to keep track of a bunch of numbers.

How? A JSON File

Why? A ubiquitous format that is easy to share / validate.

(see later today topic of "likelihood preservation)

Caputing the Information

What we discussed so far would be captured with something very simple like this: **2 Channels - 2 Samples**

```
spec = {
    'channels': [
        ٦
            'name': 'channel1',
            'samples': [
                {'name': 'sample1', 'data': [50,60,70]},
                {'name': 'sample2', 'data': [10,5,2]}
        },
            'name': 'channel2',
            'samples': [
                {'name': 'sample1', 'data': [150,160,170]},
                {'name': 'sample2', 'data': [20,10,4]}
```

Remaining questions:

- How do we model effect of varying $\boldsymbol{\theta}$
- How do we incorporate prior information on θ

varying θ or information on θ

The distribution of s(x) depends on parameters θ - includes core physics parameters and nuisance parameters



projection $x \rightarrow s(x)$

 $p(x \mid \theta)$



The distribution of s(x) depends on parameters θ - includes core physics parameters and nuisance parameters



projection $x \rightarrow s(x)$





The distribution of s(x) depends on parameters θ - includes core physics parameters and nuisance parameters



projection $x \rightarrow s(x)$





very expensive. Need a fast way to approximate this.



- Estimating the low-D distribution with samples from high-D is
 - **Parametrization in Histogram space**

HistFactory provides a few standard building blocks in order to model the effect of parameters to the distribution



"Modifiers"

Normalization

normfactor

Correlated Scaling

normfactor

Uncorrelated Scaling

shapefactor

Luminosity

lumi



Simplest modifier is a scaling $\lambda_{csb}(\theta) = \theta \lambda_{csb}^0$



Example Normfactor

2x



To capture how each λ_{csh} is a function of various parameters, $\lambda_{csh} = \lambda_{csh}(\theta)$ we can add a number of such modifiers in JSON

```
spec = {
    'channels': [
            'name': 'channel1',
            'samples': [
                {'name': 'sample1', 'data': [50,60,70], 'modifiers': []},
                {'name': 'sample2', 'data': [10,5,2], 'modifiers': [
                    {'type': 'normfactor', 'name': 'mu', 'data': None}
                ]}
        },
            'name': 'channel2',
            'samples': [
                {'name': 'sample1', 'data': [150,160,170], 'modifiers': []},
                {'name': 'sample2', 'data': [20,10,4], 'modifiers': [
                    {'type': 'normfactor', 'name': 'mu', 'data': None}
                ]}
```

Example Normfactor









Histosys builds a parametrization of histograms, based on three input histograms ("nominal", "up", "down")



Example HistoSys

 $\lambda(\alpha = 0) = \lambda^{\text{nom}}$

 $\lambda(\alpha = 1) = \lambda^{dn}$





Example HistoSys

```
spec = {
    'channels': [
             'name': 'channel1',
             'samples': [
                 {'name': 'sample1', 'data': [50,60,70],
                   'modifiers':
                     {'type': 'histosys', 'name': 'mu', 'data': {
                          'hi_data': [55, 65,75],
                          'lo_data': [45, 50,30]
                     }}
                                                          model = pyhf.Model(spec)
                                                          model.expected_actualdata([0.0])
                 ]},
                                                          array([50., 60., 70.])
                                                          model.expected_actualdata([1.0])
                                                          array([55., 65., 75.])
```

model.expected_actualdata([-1.0])

```
array([45., 50., 30.])
```

Example NormSys

Normsys parametrizes a histogram to model a normalization uncertainty ("my background is $X \pm Y$ ")

spec = {

 $\lambda(\alpha = 1) = k_{up}\lambda^{nom}$ $\lambda(\alpha = 0) = \lambda^{nom}$ $\lambda(\alpha = 1) = k_{dn}\lambda^{nom}$

```
'channels': [
       'name': 'channel1',
       'samples': [
          {'name': 'sample1', 'data': [50,60,70],
           'modifiers': [
              {'type': 'normsys', 'name': 'mu', 'data': { 'hi': 1.1, 'lo': 0.8 }}
          ]},
                          model = pyhf.Model(spec)
                          model.expected_actualdata([0.0])
                          array([50., 60., 70.])
                          model.expected_actualdata([1.0])
                          array([55., 66., 77.])
                          model.expected_actualdata([-1.0])
                          array([40., 48., 56.])
```



You can find a detailed description of all available modifiers and what they do in the pyhf documentation

•••	
脅 pyhf ∨0.7.0	 the samples no the absolute u
Search docs	nuisance parameters
Introduction	These values are, in the
Likelihood Specification	needs to go back and
Workspace Channel Sample	The previous example v allocate three nuisance
Modifiers	
Uncorrelated Shape (shapesys)	<pre>{ "name": "mod_name";</pre>
Correlated Shape (histosys)	
Normalisation Uncertainty (normsys)	Correlated Shape
MC Statistical Uncertainty (staterror)	
Luminosity (lumi)	shapes, hence a correla
Unconstrained Normalisation (normfactor)	distributions with a "do
Data-driven Shape (shapefactor)	absolute shape variatio
Data	· · · · ·
Measurements	{ "name": "mod_name";
Observations	
Toy Example	This example specifies
Additional Material	in first bin, 15 events ir
Fundamentals	second bin). This variat
Examples	Normalization
Outreach	Normalisation U
Installation	The normalisation unce
Developing	between downward ("lo

v: v0.7.0 🗸

Display a menu

pyhf.readthedocs.io
 i pyhf.readthedocs.io
 i pyhf.readthedocs.io
 i pyhf.readthedocs.io
 i i pyhf.readthedocs.io

will allocate three nuisance parameters for <a>mod_name. The following example will also e parameters for a 3-bin channel, with the second nuisance parameter fixed to <a>1:

"type": "shapesys", "data": [1.0, 0.0, 2.0] }

(histosys)

ts the same source of uncertainty which has a different effect on the various sample ated shape. To implement an interpolation between sample distribution shapes, the ownward variation" ("lo") associated with $\alpha = -1$ and an "upward variation" ("hi") -1 are provided as arrays of floats. An example of a correlated shape modifier with ons for a 2-bin channel is shown below:

"type": "histosys", "data": {"hi_data": [20,15], "lo_data": [10, 10]} }

the expected event rate for the high-variation of the histosys as [20, 15] (20 events in second bin); for the low-variation as [10, 10] (10 events in first bin, 10 events in fion is absolute (not relative!).

ncertainty (normsys)

The normalisation uncertainty modifies the sample rate by a overall factor $\kappa(\alpha)$ constructed as the interpolation between downward ("lo") and upward ("hi") as well as the nominal setting, i.e. $\kappa(-1) = \kappa_{\alpha=-1}$, $\kappa(0) = 1$ and $\kappa(+1) = \kappa_{\alpha=+1}$. In the modifier definition we record $\kappa_{\alpha=+1}$ and $\kappa_{\alpha=-1}$ as floats. An example of a normalisation uncertainty modifier with scale factors recorded for the up/down variations of an *n*-bin channel is

Adding prior Information

With our toolbox we can quickly compose different modifiers to build up complex

$c \in \text{channels } b \in \text{bins}_{c}$

We could already start fitting those

But we can do more: add prior information

 $p(\text{data} | \theta) = \sum \lambda_{csb}(\theta)$ S

Adding prior Information

Just parametrizing is not enough, often we want to give additional information about possible parameter values

Example:

When I say: the background is 50 ± 10 events, I ofen mean it as a type of statistical interval

- b = 50 most prefered given what I know
- b = 40,60 possible
- b = 10,80 disfavored

How do we express this in our model

such prior information - it's not true, as we will see. But it's true: in Bayesian it's very simple

Our Experiment

$$p(\text{data} | \theta) = \prod_{c \in \text{channels}} \prod_{b \in \text{bins}_{c}} \text{Pois}(n_{cb} | \lambda_b = \sum_{s} \lambda_{csb}(\theta))$$

$$p(\theta \mid \mathscr{D})$$

- There is a myth that only Bayesian procedures allow you to add



Prior Beliefs over parameter values

But where does our prior belief come from? Most likely from a prior measurement!

 $p(\theta) = p(\theta | \text{data}_{a})$

Plugging this in gives us:

$$p_{\text{uux}} = \frac{p(\text{data}_{\text{aux}} | \theta) p_{\text{ur}}(\theta)}{p(\text{data}_{\text{aux}})}$$

$$p(\theta | \text{data}) = \frac{p(\text{data} | \theta)}{p(\text{data})} \frac{p(\text{data}_{\text{aux}} | \theta)}{p(\text{data})} p_{\text{ur}}(\theta)$$

prior measurement!

full information after all measurements

But where does our prior belief come from? Most likely from a



This short Bayesian treatment gives us a hint how to add prior information also in the Frequentist case:

$$p(\theta | \text{data}) = \frac{p(\text{data} | \theta) p(\text{data}_{\text{aux}} | \theta)}{p(\text{data})} p_{\text{ur}}(\theta)$$

If our prior information is driven by a prior indepdentnt measurement, we can just use a statistical combination to incorporate it

 $p(\text{data} | \theta) \rightarrow p(\text{data} | \theta)p(\text{data}_{\text{aux}} | \theta)$

Example Aux Measurement

Let's go back to: "my background is X ± Y"

 interpret: there is a prior measurement of data such that $\hat{\alpha} = 0 \pm 1$ would be favored (in some units)

$$\lambda(\alpha = 1) = k_{up}\lambda^{nom}$$
$$\lambda(\alpha = 0) = \lambda^{nom}$$
$$\lambda(\alpha = 1) = k_{up}\lambda^{nom}$$



The solution HistFactory uses is to add "proxy models" that would produce a similar likelihood for the combination a = 0



Solution





Some HistFactory modifiers that are associated with "systematic uncertainties" - will automatically add such a term





The full HistFactory Model

Some HistFactory modifiers that are associated with



 $c \in \text{channels } b \in \text{bins}_c$ S

"systematic uncertainties" - will automatically add such a term

The full HistFactory Model

```
spec = {
    'channels': [
            'name': 'channel1',
            'samples': [
                {'name': 'signal', 'data': [5,10],
                 'modifiers': [
                    {'type': 'normfactor', 'name': 'mu', 'data': None}
                ]},
                {'name': 'background', 'data': [100,50],
                 'modifiers': [
                    {'type': 'normsys', 'name': 'uncrt', 'data': { 'hi': 1.1, 'lo': 0.9 }}
                ]},
```

model.expected_actualdata([1.0,-1])

model = pyhf.Model(spec) model.expected_actualdata([1.0,0.0])

array([105., 60.])

model.expected_actualdata([1.0,1])

array([115., 65.])

array([95., 55.])

The full HistFactory Model We can actually see the auxiliary data in action:

model = pyhf.Model(spec) model.expected_actualdata([1.0,0.0])

array([105., 60.])

model.expected_actualdata([1.0,1])

array([115., 65.])

model.expected_actualdata([1.0,-1])

array([95., 55.])

model.expected_data([1.0,0.0])

array([105., 60., 0.])

model.expected_data([1.0,1.0])

array([115., 65., 1.])

array([95., 55., -1.])

model.expected_data([1.0,-1.0])

model = pyhf.Model(spec) model.config.auxdata

[0.0]

Evaluating the PDF Take this example: what is the value of the likelihood at $\theta = (\mu = 0, \alpha = 1)$ if we observe N = [102,48]?



 $Pois([102,48] | \lambda(0,1)) = Pois([102,48] | \lambda(0,1))$ = Pois([102,48] | [110,55])

```
{'type': 'normfactor', 'name': 'mu', 'data': None}
{'type': 'normsys', 'name': 'uncrt', 'data': { 'hi': 1.1, 'lo': 0.9 }}
```

Norm($0 \mid \alpha = 1$)

Evaluating the PDF Take this example: what is the value of the likelihood at $\theta = (\mu = 0, \alpha = 1)$ if we observe N = [102, 48]?



 $Pois([102,48] | \lambda(0,1)) = Pois([102,48] | \lambda(0,1))$ = Pois([102,48] | [110,55])

Evaluating the PDF

Upshot: HistFactory is not magic, it's just a useful toolbox to help you model your measurement

pyhf: implementation of HistFactory in python

(interpolation algorithms, bookkeeping of constraint terms, ...)

Beyond just the Model Beyond the model building, pyhf also comes with "batteries included" for some basic things • Fitting / Limit Setting, Basic Plotting

```
>>> import pyhf
>>> pyhf.set_backend("numpy")
>>> model = pyhf.simplemodels.uncorrelated_background(
        signal=[12.0, 11.0], bkg=[50.0, 52.0], bkg_uncertainty=[3.0, 7.0]
. . .
. . .
>>> data = [51, 48] + model.config.auxdata
>>> test_mu = 1.0
>>> CLs_obs, CLs_exp = pyhf.infer.hypotest(
        test_mu, data, model, test_stat="qtilde", return_expected=True
. . .
. . .
>>> print(f"Observed: {CLs_obs:.8f}, Expected: {CLs_exp:.8f}")
Observed: 0.05251497, Expected: 0.06445321
```

But for more advanced use, best to use a library around pyhf e.g. cabinetry (next talk)



Next Steps in pyhf

- The core HistFactory model has been stable for >10 years we know a huge amount of physics can be modelled with it
- But some use-cases need to go beyond (see Lorenz' talk) working on making this easy to to
- custom modifiers (possibly ML-based interpolation)
- pyhf is just the model $p(x \mid \theta)$ no reason to be frequentist only working on fully consistent Bayesian APIs

Resources

https://pyhf.github.io/pyhf-tutorial/introduction.html



LEARN FUNDAMENTALS

pyhf Tutorial

Welcome!



Welcome to the pyhf tutorial! We'll first poin (pyhf.readthedocs.io/) and recommend that Let's dive right in.

We won't review the full pedagogy of HistF 2020.



	53	0	E Contents
			Welcome!
			Installation
- differentiable			
\mathscr{L} ikelihoods			
Int you towards our documentation website	davar	aplac	
it you visit it for much more detailed explanations an	u exan	ipies.	
Factory, so instead we'll point you to the pyhf talk a	t SciPy	/	
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