
# The CKM Matrix

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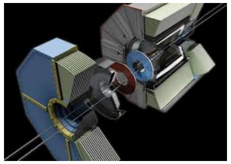
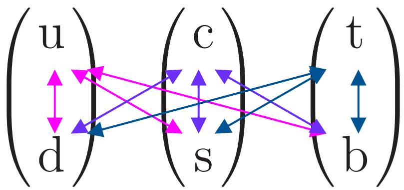
*28 July 2023*

- *the CKM matrix and Unitarity triangle*
- *semileptonic decays: formalism*
- *exclusive measurements of $|V_{ub}|$*
- *exclusive measurements of $|V_{cb}|$*
- *inclusive measurements of $|V_{cb}|$*
- *inclusive measurements of $|V_{ub}|$*
- *putting it all together*
- *summary*



# The CKM Matrix and Unitarity Triangle

*All flavor coupling constants ("coupling strengths") can be arranged in a matrix:*

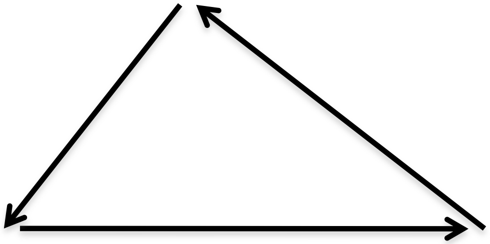


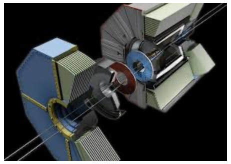

$$
\begin{array}{cc}
 & \begin{array}{ccc} \textbf{\textit{d}} & \textbf{\textit{s}} & \textbf{\textit{b}} \end{array} \\
U \equiv & \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{array}{c} \textbf{\textit{u}} \\ \textbf{\textit{c}} \\ \textbf{\textit{t}} \end{array}
\end{array}
$$

*Unitarity ($U^{\dagger}U$=1) prescribes 6 complex equations:*

$$
\begin{aligned}
V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} &= 0 \\
V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} &= 0 \\
V_{cd}^* V_{td} + V_{cs}^* V_{ts} + V_{cb}^* V_{tb} &= 0 \\
V_{us}^* V_{ud} + V_{cs}^* V_{cd} + V_{ts}^* V_{td} &= 0 \\
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} &= 0 \\
V_{ub}^* V_{us} + V_{cb}^* V_{cs} + V_{tb}^* V_{ts} &= 0
\end{aligned}
$$

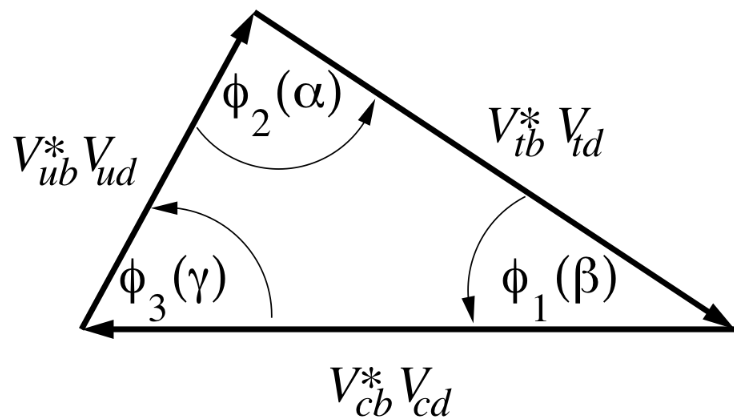
*Each equation can be plotted in the complex plane as the sum of three vectors:*





# *The Unitarity Triangle*

$$\boxed{V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} \;=\; 0}$$



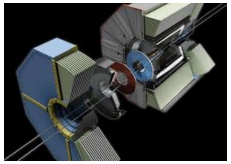

*The internal angles of this triangle are phase differences, which can be measured:*

$$\phi_1\ (\beta) \;=\; \arg\left(\frac{V_{cb}^* V_{cd}}{-V_{tb}^* V_{td}}\right)$$

$$\phi_2\ (\alpha) \;=\; \arg\left(\frac{V_{tb}^* V_{td}}{-V_{ub}^* V_{ud}}\right)$$

$$\phi_3\ (\gamma) \;=\; \arg\left(\frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}}\right)$$

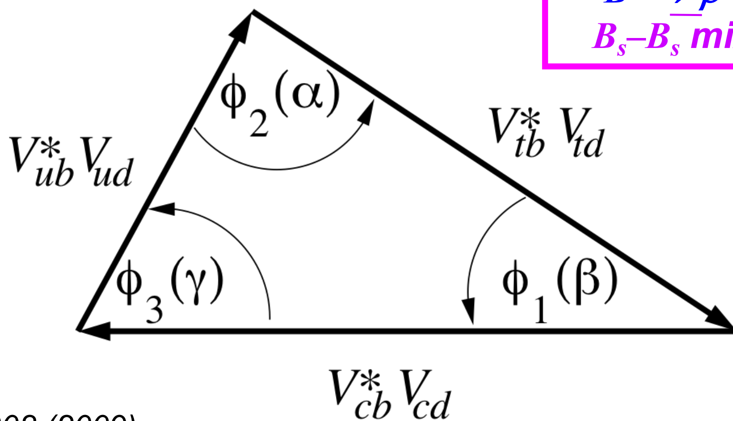
***Convention:***
*$V_{td}$ and $V_{ub}$ are taken to be complex, others real*



# Unitarity triangle – determining the sides

Belle
LHCb

$B^0 \to \rho^0 \gamma$
$B_s$–$\bar{B}_s$ mixing




$B^0 \to \pi \ell^+ \nu$
$B^0 \to X_u \ell \nu$
$B^+ \to \tau^+ \nu$
$\Lambda_b \to p \ell^+ \nu$

Jubb et al., Nucl. Phys. B 915, 431 (2017)
Artuso et al., RMP 88, 045002 (2016)
Lenz, Nierste, arXiv:1102.4274 (2011)
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Detmold et al., PRD 92, 034503 (2015)
Faustov and Galkin, PRD 94, 073008 (2016)

Lange et al. (BLNP), PRD 72, 073006 (2005)
Andersen, Gardi (DGE), JHEP 601, 97 (2006)
Gambino et al. (GGOU), JHEP 10, 058 (2007)
Aglietti et al. (ADFR), EPJ C59 (2009)
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$B^0 \to D^{(*)} \ell \nu$
$B^0 \to X_c \ell \nu$ ($\ell$ energy, hadron mass moments)
$B^0 \to X_s \gamma$ ($\gamma$ energy moments)

Caprini et al., Nucl. Phys. B530, 153 (1998)
FNAL/MILC, PRD 89, 114504 (2014)
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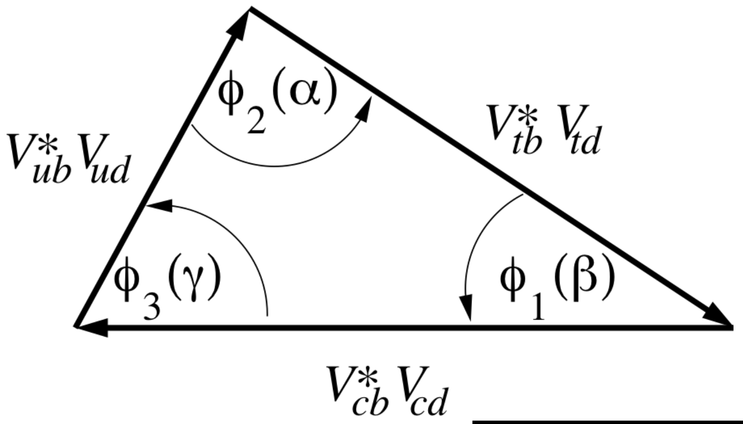
Gambino and Schwanda, PRD 89, 014002 (2014)

# Semileptonic decays "roadmap"

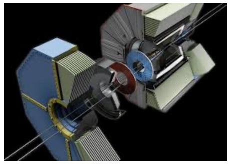
$B^0 \to \pi \ell^+ \nu$
$B^0 \to X_u \ell \nu$
$B^+ \to \tau^+ \nu$
$\Lambda_b \to p \ell^+ \nu$




$B^0 \to D^{(*)} \ell \nu$
$B^0 \to X_c \ell \nu$ ($\ell$ energy, hadron mass moments)
$B^0 \to X_s \gamma$ ($\gamma$ energy moments)

**Exclusive** decays:

- *final state is fully reconstructed*
- *straightforward to measure*
- *significant theory uncertainty to extract $|V_{ub}|$, $|V_{cb}|$ due to initial/final states being hadrons*



# *Semileptonic decays "roadmap"*

$B^0 \to \pi \ell^+ \nu$
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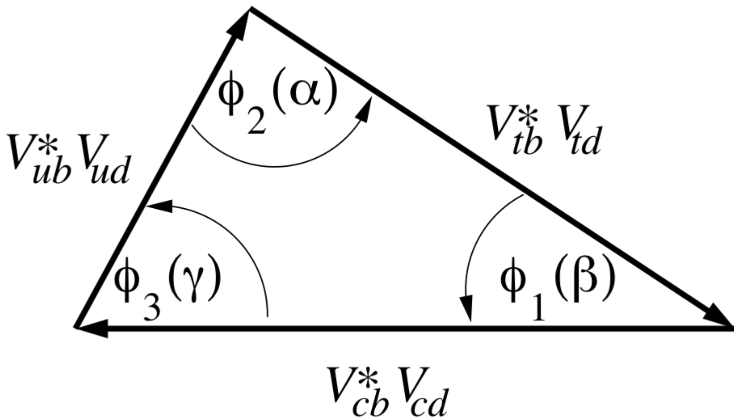
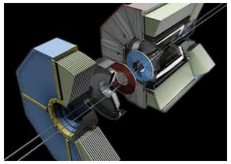



$B^0 \to D^{(*)} \ell \nu$
$B^0 \to X_c \ell \nu$ ($\ell$ energy, $q^2$, hadron mass moments)
$B^0 \to X_s \gamma$ ($\gamma$ energy moments)

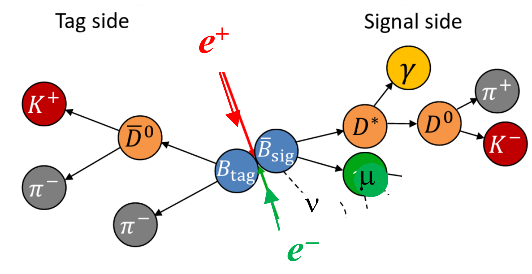
**Exclusive** decays:
- *final state is fully reconstructed*
- *straightforward to measure*
- *significant theory uncertainty to extract $|V_{ub}|$, $|V_{cb}|$ due to initial/final states being hadrons*

**Inclusive** decays:
- *final hadronic state not reconstructed*
- *challenging to measure, large backgrounds (especially $b \to c$ contaminating $b \to u$)*
- *"small" theory uncertainty to extract $|V_{ub}|$, $|V_{cb}|$: can use heavy quark expansion and determine nonperturbative matrix elements from measuring moments*



# *The experimental landscape: $|V_{cb}|$*

## $|V_{cb}|$




**Form factors**

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- B. Aubert et al. (BABAR), A Measurement of the Branching Fractions of Exclusive $B^0 \to D^{(*)}(\pi)\,\ell^-\nu$ Decays in Events with a Fully Reconstructed B Meson, Phys. Rev. Lett. 100, 151802 (2008).
- F. Abudinen et al. (Belle II), Studies of the semileptonic $B^0 \to D^{*+}\ell^-\nu$ and $B^- \to D^0\ell^-\nu$ decay processes with 34.6 fb−1 of Belle II data, arXiv:2008 .07198.
- F. Abudinen et al. (Belle II), Measurement of the semileptonic $B^0 \to D^{*+}\ell^-\nu$ branching fraction with fully reconstructed B meson decays and 34.6 fb−1 of Belle II data, arXiv:2008.10299.
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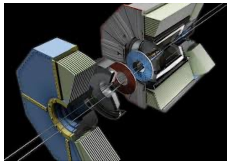
**Hadron moments**

- B. Aubert et al. (BABAR), Measurement and interpretation of moments in inclusive semileptonic decays $B \to X_c\,\ell^-\nu$, Phys. Rev. D 81, 032003 (2010).
- C. Schwanda et al. (Belle), Moments of the hadronic invariant mass spectrum in $B \to X_c\,\ell\,\nu$ decays at Belle, Phys. Rev. D 75, 032005 (2007).

- **Lepton moments, $q^2$ moments**
  B. Aubert et al. (BABAR), Measurement of the electron energy spectrum and its moments in inclusive $B \to X\,e\,\nu$ decays, Phys. Rev. D 69, 111104 (2004).
- P. Urquijo et al. (Belle), Moments of the electron energy spectrum and partial branching fraction of $B \to X_c\,e\,\nu$ decays at Belle, Phys. Rev. D 75, 032001 (2007).
- Abudinén et al. (Belle II), Measurement of lepton mass squared moments in $B \to X_c\,\ell\,\nu$ decays with the Belle II experiment, Phys. Rev. D 107, 072002 (2023).



# The experimental landscape: $|V_{ub}|$

$|V_{ub}|$

**Form factors**

- H. Ha et al. (Belle), Measurement of the decay $B^0 \to \pi^- \ell^+ \nu$ and determination of $|V_{ub}|$, Phys. Rev. D 83, 071101 (2011).
- A. Sibidanov et al. (Belle), Study of exclusive $B \to X_u \ell \nu$ decays and extraction of $|V_{ub}|$ using full reconstruction tagging at the Belle experiment, Phys. Rev. D 88, 032005 (2013).
- P. del Amo Sanchez et al. (BABAR), Study of $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ decays and determination of $|V_{ub}|$, Phys. Rev. D 83, 032007 (2011).
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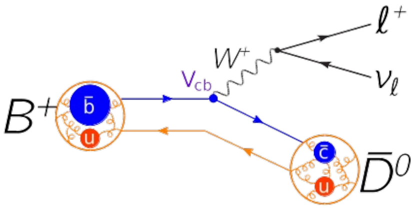
**$M_X$, $q^2$, $E_e$, $p^*_\ell$ moments**

- J. P. Lees et al. (BABAR), Study of $B \to X_u \ell \nu$ decays in BB events tagged by a fully reconstructed B-meson decay and determination of $|V_{ub}|$, Phys. Rev. D 86, 032004 (2012).
- J. P. Lees et al. (BABAR), Measurement of the inclusive electron spectrum from B meson decays and determination of $|V_{ub}|$, Phys. Rev. D 95, 072001 (2017).
- B. Aubert et al. (BABAR), Determination of $|V_{ub}|$ from Measurements of the Electron and Neutrino Momenta in Inclusive Semileptonic B Decays, Phys. Rev. Lett. 95, 111801 (2005).
- A. Limosani et al. (Belle), Measurement of inclusive charmless semileptonic B-meson decays at the endpoint of the electron momentum spectrum, Phys. Lett. B 621, 28 (2005).
- H. Kakuno et al. (Belle), Measurement of $|V_{ub}|$ Using Inclusive $B \to X_u \ell \nu$ Decays with a Novel $X_u$ Reconstruction Method, Phys. Rev. Lett. 92, 101801 (2004).
- I. Bizjak et al. (Belle), Measurement of the Inclusive Charmless Semileptonic Partial Branching Fraction of B Mesons and Determination of $|V_{ub}|$ Using the Full Reconstruction Tag, Phys. Rev. Lett. 95, 241801 (2005).
- L. Cao et al. (Belle), Measurements of partial branching fractions of inclusive $B \to X_u \ell^+ \nu$ decays with hadronic tagging, Phys. Rev. D 104, 012008 (2021).

# *Semileptonic Decays: some formalism*




$$d\Gamma \;\propto\; |\mathcal{A}|^2 \;=\; G_F^2 |V_{cb}^2| \cdot |H^\mu L_\mu|^2$$

$$L_\mu \;=\; \langle P_\ell P_\nu | \bar{\ell}\gamma_\mu (1-\gamma^5)\nu_\ell | 0\rangle \quad \text{(leptonic current)}$$

$$H^\mu \;=\; \langle D | \bar{c}\gamma^\mu b | B\rangle \quad \text{(hadronic current)}$$

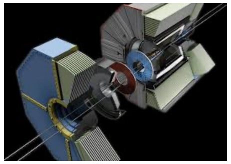
*As the leptons are "point" particles, we can evaluate the leptonic current using spinor wave functions. But $D$ and $B$ cannot be represented by spinors, i.e., the hadronic current is non-perturbative. However, it must transform as a 4-vector, and only two 4-vectors are available: $P_B{}^\mu$ and $P_D{}^\mu$. Thus:*

$$\begin{aligned}
\langle D | \bar{c}\gamma^\mu b | B\rangle &= A \cdot P_B^\mu + B \cdot P_D^\mu \\
&\to f_+(P_B + P_D)^\mu + f_-(P_B - P_D)^\mu \qquad \text{(form factors)} \\
&= f_+(q^2)(P_B + P_D)^\mu + f_-(q^2) q^\mu \qquad \text{where } q^\mu \equiv (P_B - P_D)^\mu
\end{aligned}$$

*Contracting this with the leptonic current gives:*

$$\begin{aligned}
q^\mu \bar{\ell}\gamma_\mu (1-\gamma^5)\nu &= (P_B - P_D)^\mu \bar{\ell}\gamma_\mu (1-\gamma^5)\nu \;=\; (P_\ell + P_\nu)^\mu \bar{\ell}\gamma_\mu (1-\gamma^5)\nu \\
&= (P_\ell + P_\nu)^\mu \bar{\ell}\gamma_\mu \nu \;-\; (P_\ell + P_\nu)^\mu \bar{\ell}\gamma_\mu \gamma^5 \nu \\
&= \bar{\ell}(\not{p}_\ell + \not{p}_\nu)\nu \;-\; \bar{\ell}(\not{p}_\ell + \not{p}_\nu)\gamma^5 \nu \\
&= (-m_\ell + m_\nu)\,\bar{\ell}\nu \;-\; (-m_\ell - m_\nu)\,\bar{\ell}\gamma^5\nu \\
&\qquad [\text{applying the Dirac equations } (\not{p} - m)\psi = 0 \text{ and } \bar{\psi}(\not{p} + m) = 0] \\
&= (-m_\ell + m_\nu)\,\bar{\ell}\nu \;+\; (m_\ell + m_\nu)\,\bar{\ell}\gamma^5\nu \\
&\approx 0 \qquad [\text{since } \mathrm{m}_\nu \simeq 0 \text{ and } \mathrm{m}_\ell \ll \mathrm{M_B}, \mathrm{M_D}]
\end{aligned}$$

⇒ *for $\ell = e, \mu$, the contribution from $f_-(q^2)$ is negligible, and decay rate depends only on $f_+(q^2)$ form factor*



# $|V_{ub}|$ via exclusive $B \rightarrow \pi l \nu$

$$\frac{d\Gamma(B\rightarrow \pi\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}\, p^{*3}\, |V_{ub}|^2\, f_+^2(q^2)$$

$$\boxed{f_+(q^2) = \frac{1}{(1-q^2/M_{B^*}^2)} \sum_{k=0}^{3} b_k \left[ z^k - (-1)^k \frac{k}{4} z^4 \right]}$$

*Bourrely, Caprini, Lellouch, PRD 79, 013008 (2009)*

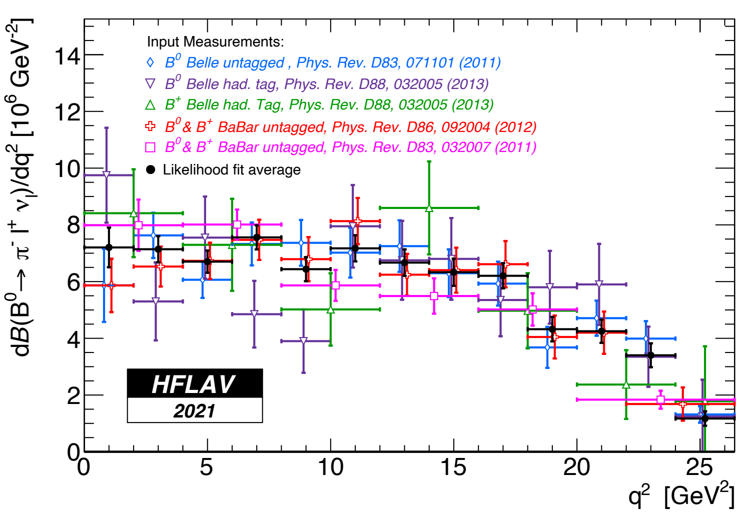
$$\text{where } z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

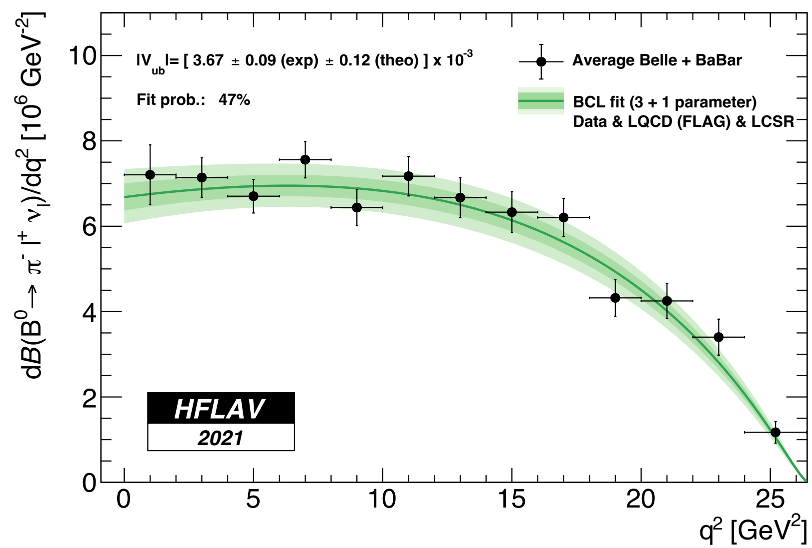
$$t_+ = (M_B + M_\pi)^2 = 29.4 \text{ GeV}^2,$$

$$t_0 = (M_B + M_\pi)\left(\sqrt{M_B} - \sqrt{M_\pi}\right)^2 = 20.1 \text{ GeV}^2$$

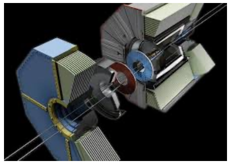
*Fit $q^2$ spectrum + **LCSR** + **LQCD** for BCL parameters and $|V_{ub}|$:*

***LQCD:*** *Aoki (FLAG), EPJC 82 (2022) 869)*
***LCSR:*** *Bharucha, JHEP 05, 092, (2012)*

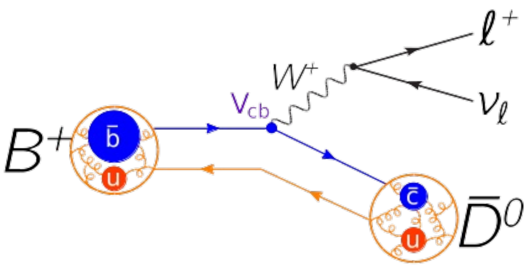



$$|V_{ub}| = (3.67 \pm 0.09_{exp} \pm 0.12_{th}) \times 10^{-3}$$



# $|V_{cb}|$ from $B \to D^{(*)} l \nu$




*New kinematic variable w (rather than $q^2$):*

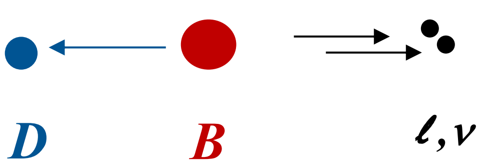
$$w \;\equiv\; \frac{P_B \cdot P_{D^*}}{M_B\, M_{D^*}} \;=\; \frac{-(P_B - P_{D^*})^2 + P_B^2 + P_{D^*}^2}{2\, M_B\, M_{D^*}} \;=\; \frac{M_B^2 + M_{D^*}^2 - q^2}{2\, M_B\, M_{D^*}}$$

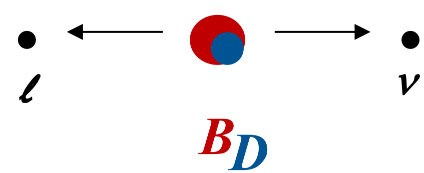
[Recall that $q^2 = (P_B - P_{D^*})^2 = (P_\ell + P_\nu)^2$ ]

*Two extreme situations:*

$q^2 \approx 0 \rightarrow \; w = w_{max}$
$= (M_B^2 + M_{D^*}^2)/(2M_B M_{D^*})$
$= 1.6$

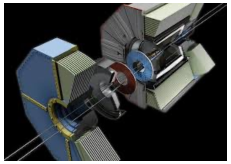
$q^2 = q^2_{max} = (M_B - M_{D^*})^2$
$= 10.69\ (\mathrm{GeV})^2 \rightarrow w_{min} = 1$



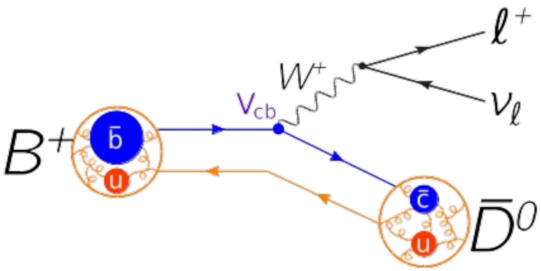




*(LCSR reliable, LQCD not)*

*("zero recoil" : LQCD reliable, LCSR not)*



# $|V_{cb}|$ from $B \to D^{(*)} l \nu$




$$\boxed{w \;\equiv\; v_B \cdot v_D \;=\; \frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}}$$

**$B \to D^* l \nu$** *decay rate:*

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{48\pi^3} M_{D^*}^3 (M_B - M_{D^*})^2 \sqrt{w^2-1}\,(w+1)^2 |V_{cb}|^2 \eta_{EW}^2\, F^2(w)$$

form factor

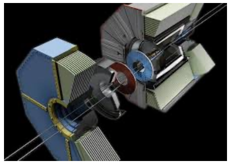
$$F^2(w) = h_{A_1}^2(w) \left\{ 2\left[\frac{1-2wr+r^2}{(1-r)^2}\right]\left[1 + R_1^2(w)(w-1)\right] + \left[1 + (1-R_2(w))\frac{w-1}{1-r}\right]^2 \right\}$$

where $r = M_{D^*}/M_B$

*Caprini, Lelouch, Neubert:*

$$\begin{aligned} h_{A_1}(z) &= h_{A_1}(1)\left[1 - 8\rho^2 z + (53\rho^2 - 15) z^2 - (231\rho^2 - 91) z^3\right] \\ R_1(w) &= R_1(1) - 0.12(w-1) + 0.05(w-1)^2 \\ R_2(w) &= R_2(1) - 0.11(w-1) + 0.06(w-1)^2 \end{aligned}$$

where $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$



# $|V_{cb}|$ from $B \to D^* l \nu$

711 fb$^{-1}$

Waheed at al. (Belle), PRD 100, 052007 (2019)

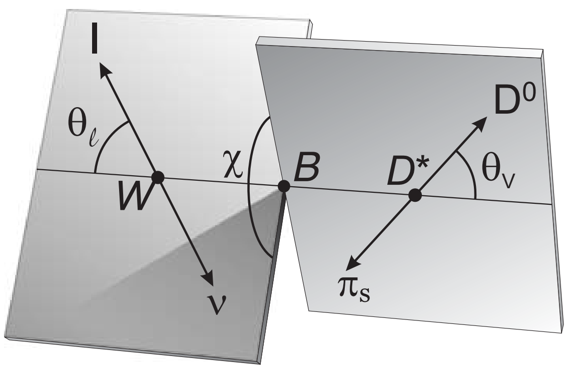
**Advantages over $B \to D l \nu$ :**

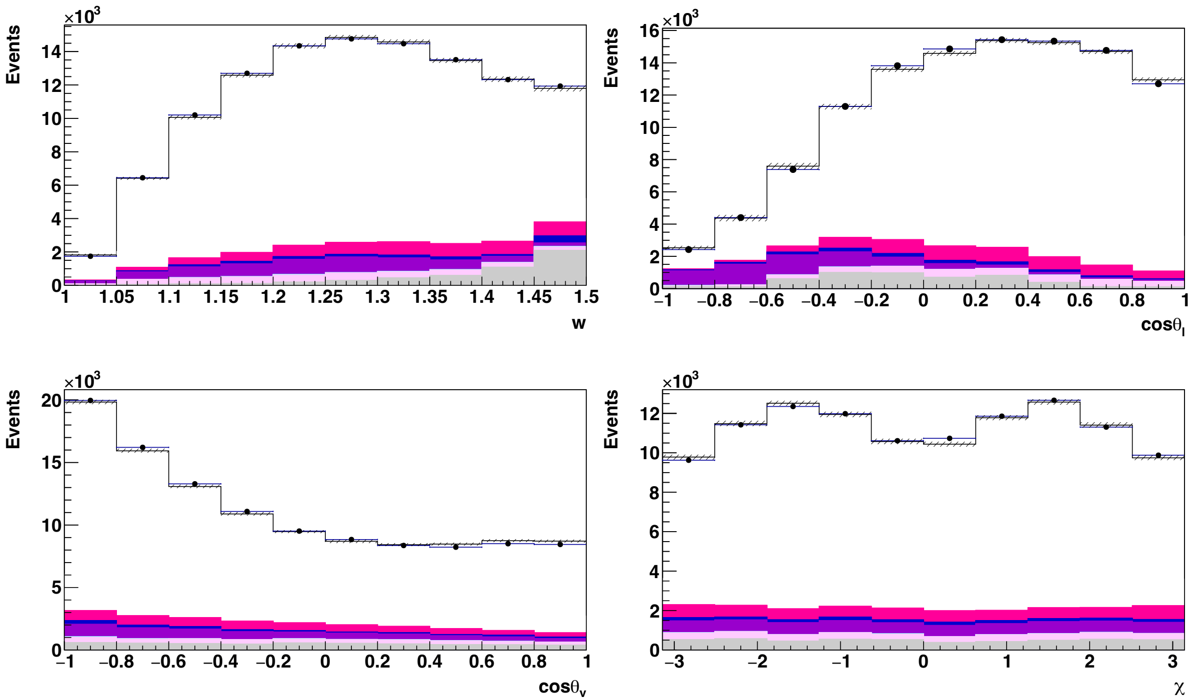
- (2.2–2.4)x larger branching fraction
- hadronic tag reconstruction not needed due to $D^*$

⇒ much higher statistics (180k signal events, vs. 17k for $B \to D l \nu$)

Statistics are high enough to fit the $w$, cos$\theta_\ell$, cos$\theta_V$, $\chi$ distributions

to fully differential decay rate $$\frac{d\Gamma(B^0 \to D^{*-}\ell^+\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi}$$



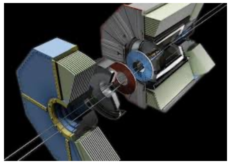




**Result:**

$\eta_{EW}\, F(1)\, |V_{cb}| = (35.06 \pm 0.58) \times 10^{-3}$

Using $F(1) = 0.906 \pm 0.013$ [MILC, PRD 89, 114504, (2014)]
$\eta_{EW} = 1.0066 \pm 0.0050$ [Sirlin, Nucl. Phy. B196, 83 (1982)]

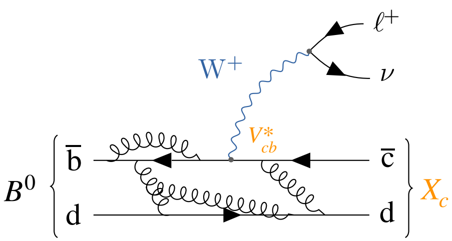
$|V_{cb}| = (38.4 \pm 0.63_{exp} \pm 0.6_{theor}) \times 10^{-3}$



# *Inclusive $|V_{cb}|$*

*Gambino and Schwanda, PRD 89, 014022 (2014)*

*Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)*

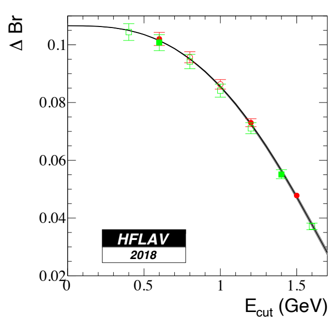


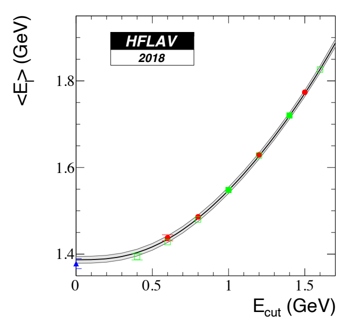

*An **"inclusive" search** means $B \to X_c\, l\nu$ :, where $X_c$ denotes final state hadrons containing charm.*

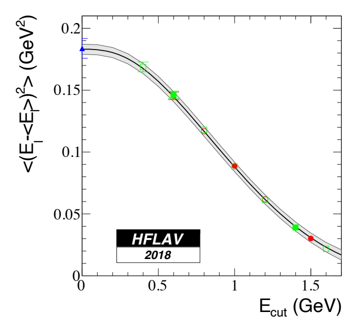
- *Experimentally, no specific final state is reconstructed. Statistics are high, but backgrounds are high*
- *Theoretically, one calculate a b $\to$ c transition, not a $\langle D^*|\mathcal{H}|B\rangle$ matrix element (parameterized by form factors). Typically this gives less theoretical uncertainty*
- *a decay mode with a specific final state is called an **"exclusive"** decay*

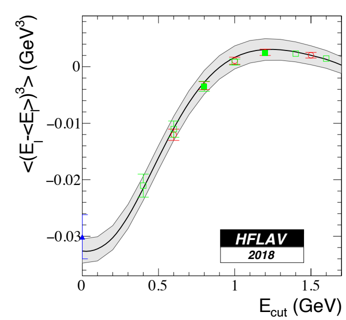
***Strategy:** the inclusive b $\to$ clv decay rate is calculated using the Heavy Quark Expansion. This is a double expansion in small (perturbative) parameters $\alpha_s$ and $(\Lambda_{QCD}/m_b)$. The expansion depends on unknown $B$ matrix elements of local operators. However, these matrix elements also determine moments of the lepton energy and recoil hadronic mass in $B \to X\, l\nu$ decays. The moment distributions have been measured (Belle, Babar), and thus one can fit the moment distributions and the measured width for $B \to X\, l\nu$ to extract $|V_{cb}|$*

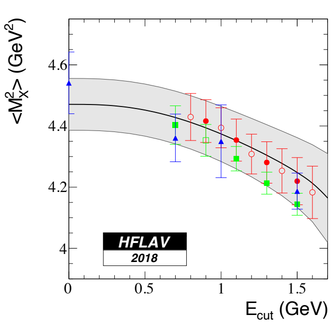
$$\langle E_\ell^n \rangle = \frac{\int_{E_{\rm cut}}^{E_{\rm max}} dE_\ell\, (E_\ell)^n \frac{d\Gamma}{dE_\ell}}{\int_{E_{\rm cut}}^{E_{\rm max}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

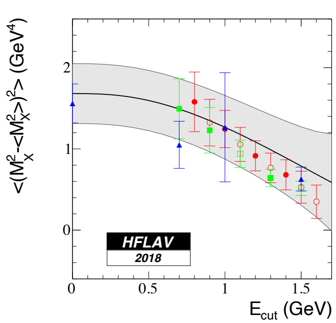


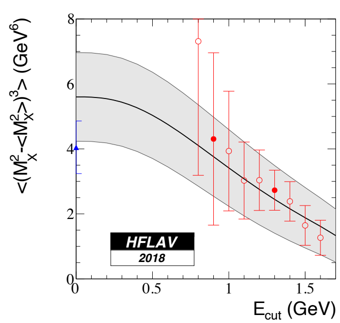



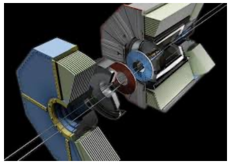



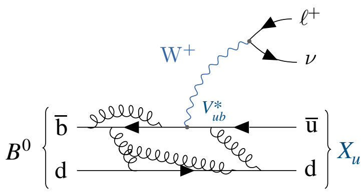

$|V_{cb}| = (42.19 \pm 0.78) \times 10^{-3}$ *(kinetic scheme)*

$|V_{cb}| = (41.98 \pm 0.45) \times 10^{-3}$ *(1S scheme)*



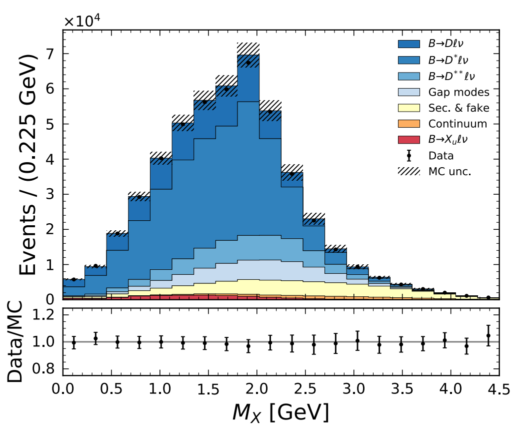
# *Inclusive $|V_{ub}|$*

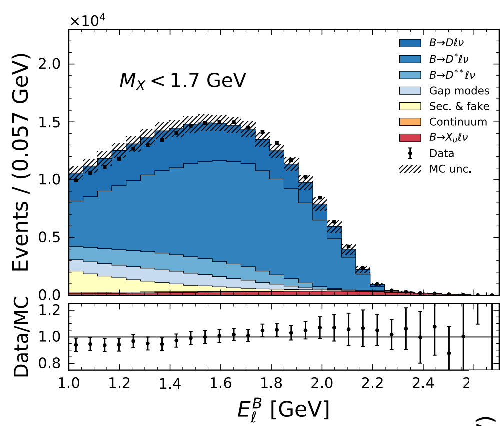
*Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)*




*Very challenging to measure $B \to X_u \, l \nu$ ($X_u$ denotes final state hadrons not coming charm), because $B \to X_c \, l \nu$ background is ~50x larger and swamps the signal.*

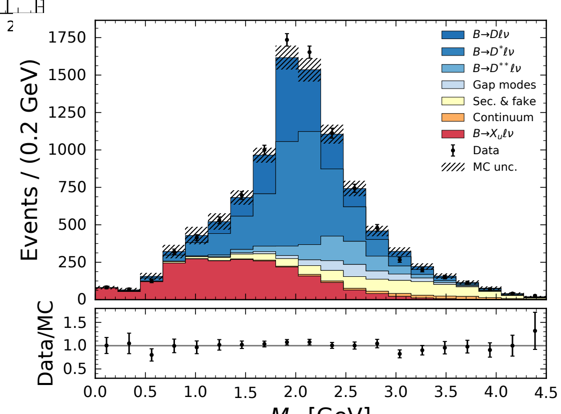
***Strategy:*** *fit data in limited regions of $M_X$, $E_\ell$, and $q^2$ where $B \to X_c \, l \nu$ background is suppressed, e.g., at lower values of $M_X$, higher values of $E_\ell$, and higher values of $q^2$. Requiring such limited phase space regions complicates the perturbatve QCD calculations needed to extract $|V_{ub}|$ from the measured rate. Different theoretical models use different parameterizations of the "shape functions" needed to evaluate the unmeasured regions of phase space. Five theory models are commonly used: BLNP, DGE, GGOU, ADFR, and BLL, but no theoretical approach is preferred over the others.*

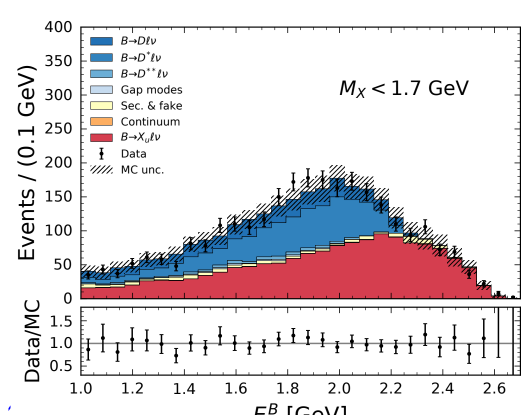


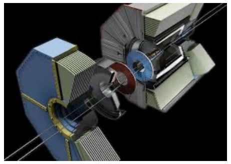

*To beat down $B \to X_c l \nu$, Belle uses a sophisticated BDT based on $M_{miss}{}^2$, finding a soft $\pi^+$ from $D^*$ decay, number of kaons, $B_{sig}$ vertex, and $Q_{tot}$. Cutting on BDT output rejects 98.7% of $X_c l \nu$, keeping 18% of $X_u l \nu$:*
*[Cao et al. (Belle), PRD 104, 012008 (2021)]*

***BDT***




*A. J. Schwartz*



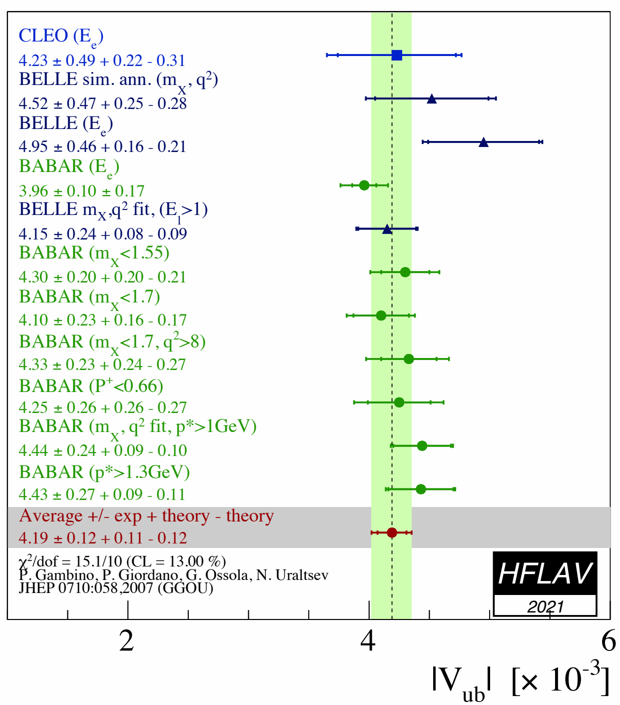
# Inclusive $|V_{ub}|$

Y. Amhis et al. (Heavy Flavor Averaging Group), EPJC 81, 226 (2021)

| Measurement | Accepted region | $\Delta\mathcal{B}[10^{-4}]$ | Notes |
|---|---|---|---|
| CLEO [564] | $E_e > 2.1\,\mathrm{GeV}$ | $3.3 \pm 0.2 \pm 0.7$ | |
| *BABAR* [563] | $E_e > 2.0\,\mathrm{GeV},\ s_{\mathrm{h}}^{\max} < 3.5\,\mathrm{GeV}^2$ | $4.4 \pm 0.4 \pm 0.4$ | |
| *BABAR* [560] | $E_e > 1.0\,\mathrm{GeV}$ | $1.55 \pm 0.08 \pm 0.09$ | Using the GGOU model |
| Belle [565] | $E_e > 1.9\,\mathrm{GeV}$ | $8.5 \pm 0.4 \pm 1.5$ | |
| *BABAR* [555] | $M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^4$ | $6.9 \pm 0.6 \pm 0.4$ | |
| Belle [566] | $M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^4$ | $7.4 \pm 0.9 \pm 1.3$ | |
| Belle [567] | $M_X < 1.7\,\mathrm{GeV}/c^2, q^2 > 8\,\mathrm{GeV}^2/c^4$ | $8.5 \pm 0.9 \pm 1.0$ | Used only in BLL average |
| *BABAR* [555] | $P_+ < 0.66\,\mathrm{GeV}$ | $9.9 \pm 0.9 \pm 0.8$ | |
| *BABAR* [555] | $M_X < 1.7\,\mathrm{GeV}/c^2$ | $11.6 \pm 1.0 \pm 0.8$ | |
| *BABAR* [555] | $M_X < 1.55\,\mathrm{GeV}/c^2$ | $10.9 \pm 0.8 \pm 0.6$ | |
| Belle [554] | $(M_X, q^2)$ fit, $p_\ell^* > 1\ \mathrm{GeV}/c$ | $19.6 \pm 1.7 \pm 1.6$ | |
| *BABAR* [555] | $(M_X, q^2)$ fit, $p_\ell^* > 1\ \mathrm{GeV}/c$ | $18.2 \pm 1.3 \pm 1.5$ | |
| *BABAR* [555] | $p_\ell^* > 1.3\ \mathrm{GeV}/c$ | $15.5 \pm 1.3 \pm 1.4$ | |
| Belle (2021) | $E_\ell > 1.0\ \mathrm{GeV}$ | $15.9 \pm 0.7 \pm 1.6$ | |

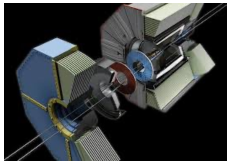
$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B\to X_u\ell^+\nu)}{\tau_B \cdot \Delta\Gamma_{\mathrm{th}}(B\to X_u\ell^+\nu)}}$$

**Using GGOU for $\Delta\Gamma_{th}$:**

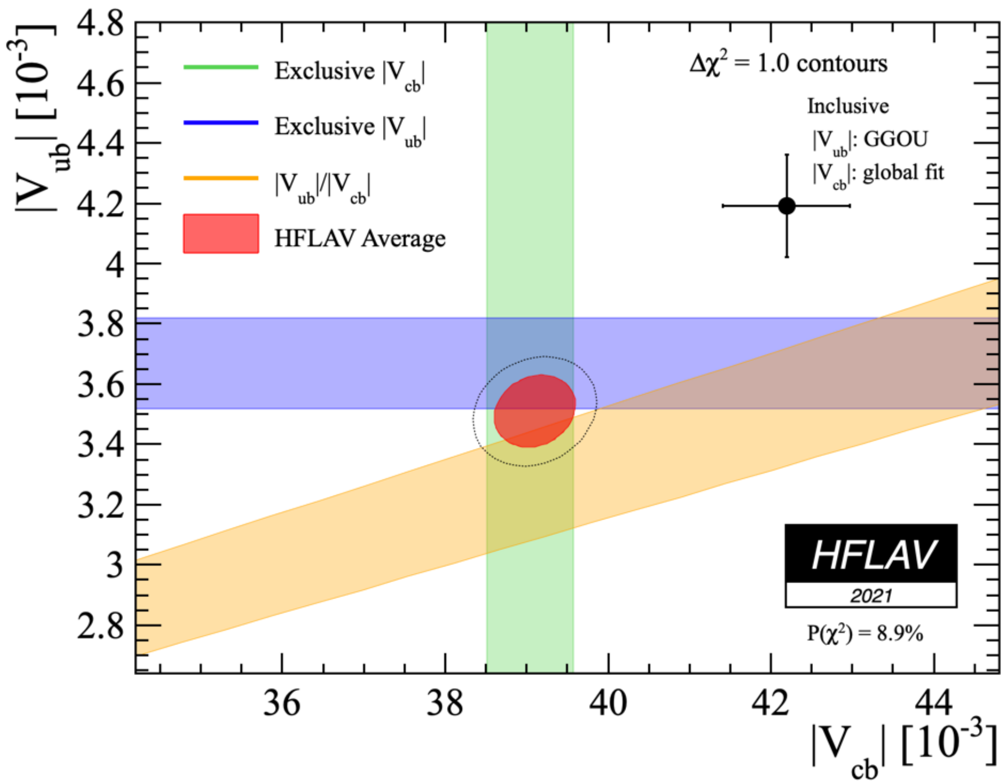



Cao et al. (Belle), PRD 104, 012008 (2021):

$$|V_{ub}|\ (\mathrm{BLNP}) = \left(4.05 \pm 0.09\,^{+0.20}_{-0.21}\,^{+0.18}_{-0.20}\right) \times 10^{-3}$$
$$|V_{ub}|\ (\mathrm{DGE}) = \left(4.16 \pm 0.09\,^{+0.21}_{-0.22}\,^{+0.11}_{-0.12}\right) \times 10^{-3}$$
$$|V_{ub}|\ (\mathrm{GGOU}) = \left(4.15 \pm 0.09\,^{+0.21}_{-0.22}\,^{+0.08}_{-0.09}\right) \times 10^{-3}$$
$$|V_{ub}|\ (\mathrm{ADFR}) = \left(4.05 \pm 0.09\,^{+0.20}_{-0.21} \pm 0.18\right) \times 10^{-3}$$



# *Putting all together: Inclusive vs. Exclusive $|V_{cb}|$, $|V_{ub}|$*

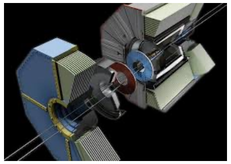



***Lattice results used:***

*Bailey et al. (MILC), PRD 89, 114504 (2014)*
*Bailey et al. (MILC), PRD 92, 034506 (2015)*
*Bailey et al. (MILC), PRD 92, 014024 (2015)*
*Flynn et al., (RBC/UKQCD) PRD 91, 074510 (2015)*
*Harrison et al. (HPQCD), PRD 97, 054502 (2018)*

*Aoki (FLAG), EPJC 82 (2022) 869*

| | Exclusive (x $10^{-2}$) | Inclusive (x $10^{-2}$) | Difference |
|---|---|---|---|
| $\lvert V_{cb}\rvert$ | $3.846 \pm 0.040 \pm 0.055$ ($D^*\ell\nu$ CLN)<br>$3.83 \pm 0.07 \pm 0.06$ ($D^*\ell\nu$ BGL [Belle])<br>$3.958 \pm 0.094 \pm 0.037$ ($D\ell\nu$) | $4.219 \pm 0.078$ (kinetic scheme)<br>$4.198 \pm 0.045$ (1S scheme) | 2.2–3.3 σ |
| $\lvert V_{ub}\rvert$ | $0.367 \pm 0.015$ ($\pi\ell\nu$) | $0.419 \pm 0.012 \pm 0.012$ (GGOU)<br>$0.428 \pm 0.013 \pm 0.020$ (BLNP) | 2.2–2.3 σ |

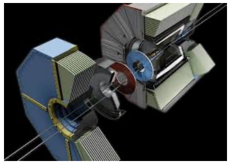


# Summary of CKM measurements

- $|V_{cb}|$ is measured via exclusive $B \to D^* \ell\nu$ and $B \to D\ell\nu$ decays. Uncertainty arises from form factors, of which there are two common choices: CLN and BGL
- $|V_{cb}|$ is measured via inclusive $B \to X_c \ell\nu$ decays and using HQE. Uncertainty arises from matrix elements of local operators. These are determined by fitting moment distributions. Two theory schemes available: kinetic scheme and 1S scheme.
- The measurements differ: inclusive $|V_{cb}|$ is higher than exclusive by 2.2–3.3$\sigma$

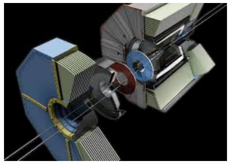
- $|V_{ub}|$ is measured via exclusive $B \to \pi \ell\nu$ decays. Uncertainty arises from form factors, of which there is one common choice: BCL
- $|V_{cb}|$ is measured via inclusive $B \to X_u \ell\nu$ decays. Many cuts are made to reduce huge $B \to X_c \ell\nu$ background, and this makes it challenging to theoretically predict the rate. Five theory schemes available: BLNP, DGE, GGOU, ADFR, and BLL.
- The measurements differ: inclusive $|V_{ub}|$ is higher than exclusive by 2.2–2.3$\sigma$

- $|V_{cs}|$ is measured via exclusive $D_s^+ \to \ell^+ \nu$ and $D \to K \ell\nu$ decays. Uncertainty arises from decay constants and form factors, respectively. Results agree. $D \to K \ell\nu$ has much higher statistics, but theory error from form factors ~~is~~ was larger, so overall precision ~~is~~ was worse.
- $|V_{cd}|$ is measured via exclusive $D^+ \to \ell^+ \nu$ and $D \to \pi \ell\nu$ decays. Uncertainty arises from decay constants and form factors, respectively. Results agree. $D \to \pi \ell\nu$ has much higher statistics, but theory error from form factors ~~is~~ was larger, so overall precision ~~is~~ was worse.
- Strong competition from BESIII (!)



# Extra

Extra Slides



# $|V_{cb}|$ from $B\to Dl\nu$

711 fb$^{-1}$

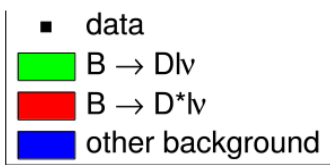
Glattauer at al. (Belle), PRD 93, 032006 (2016)

## $B\to D\ell\nu$ Reconstruction:

*After tag side reconstructed, tracks are "removed" and signal side $D$ reconstructed. After $D$ reconstructed, $e$ or $\mu$ is added to decay and missing mass calculated:*

$$M^2_{\rm miss} \;=\; \left(P_{\rm beam} - P_D - P_\ell\right)^2$$

*Missing mass spectrum (in bins of $w$) is fit for signal yield; from signal yield one calculates $\Delta\Gamma/\Delta w$.*




$B^0 \to D^+e^- \nu$ *(2848 signal events)*