



Full Event Interpretation and beyond

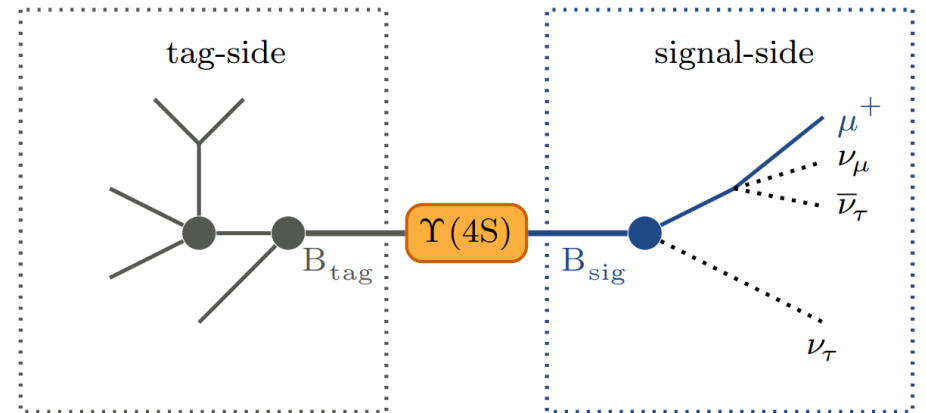
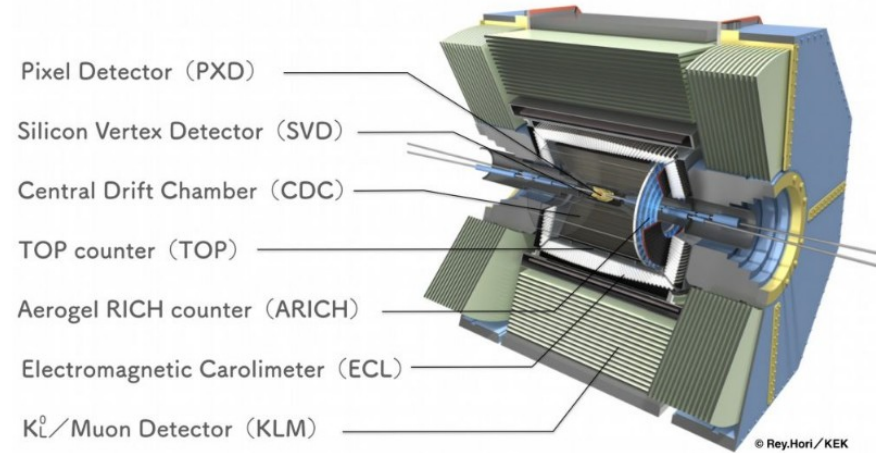
**Belle II Summer Workshop
Duke University – 26/07/2023**

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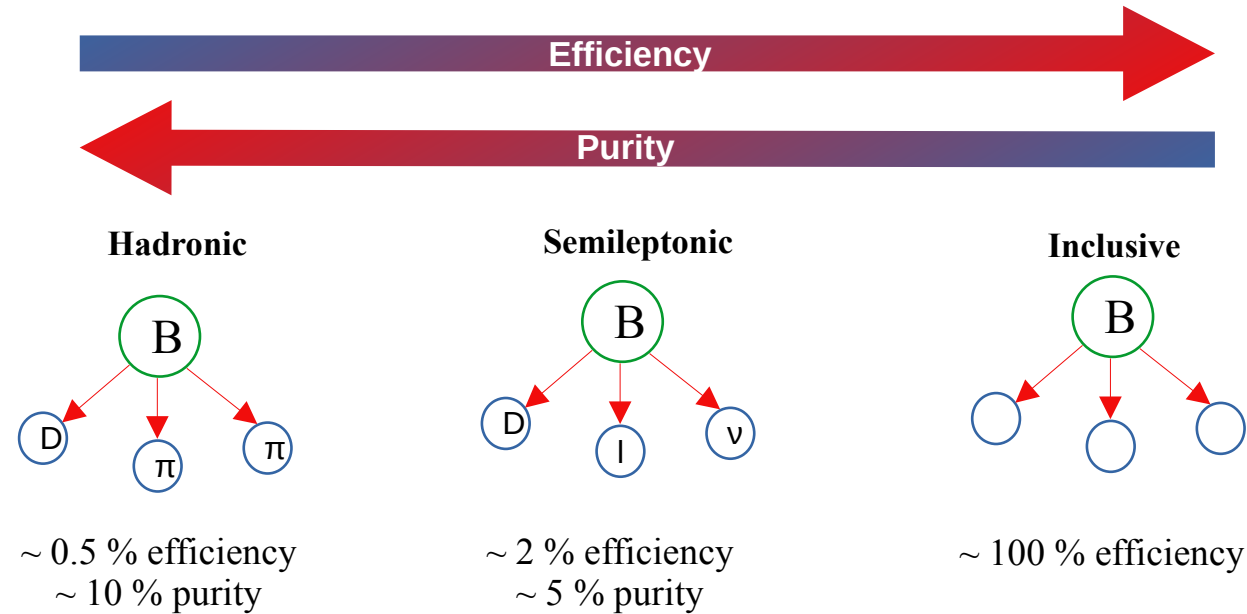
Introduction

- Many Belle II measurements involve processes with **missing energy**:
 - $B \rightarrow K^{(*)} \nu \nu$
 - $B \rightarrow D \tau (\rightarrow X \nu) \nu$
 - $B \rightarrow l \nu \gamma$
 - ...
- Quite some unique features at Belle II:
 - Knowledge of **initial 4-momentum**
 - Good **detector hermeticity**
 - **$\text{BR}(Y(4S) \rightarrow B B) \sim 100\%$**

- \rightarrow Reconstruct *tag-side* B to constrain kinematics on signal-side



How to reconstruct the B_{tag} ?



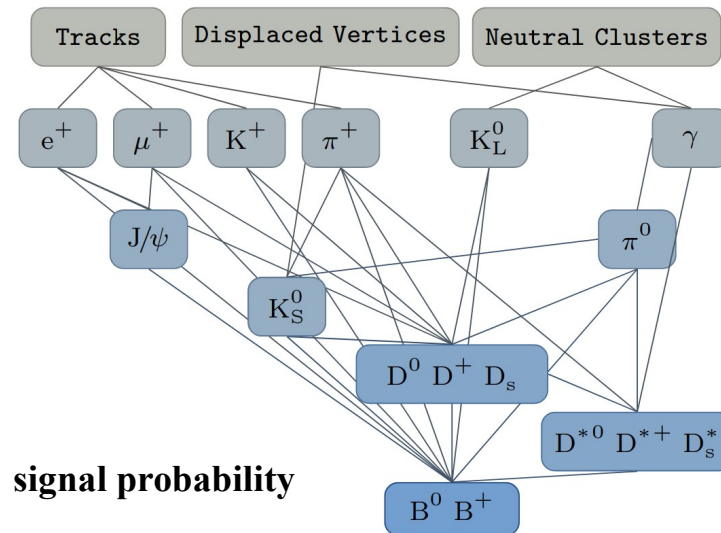
- **Full Event Interpretation (FEI)** algorithm for HAD/SL tagging at Belle II:
 - Hierarchical approach based on BDTs
 - Trained on MC $Y(4S) \rightarrow B B$ events

Full Event Interpretation Comput Softw Big Sci 3, 6 (2019)

1. Reconstruct final state particles using detector information
2. Combine final state particles into intermediates
3. Combine intermediates and FSPs into B candidates

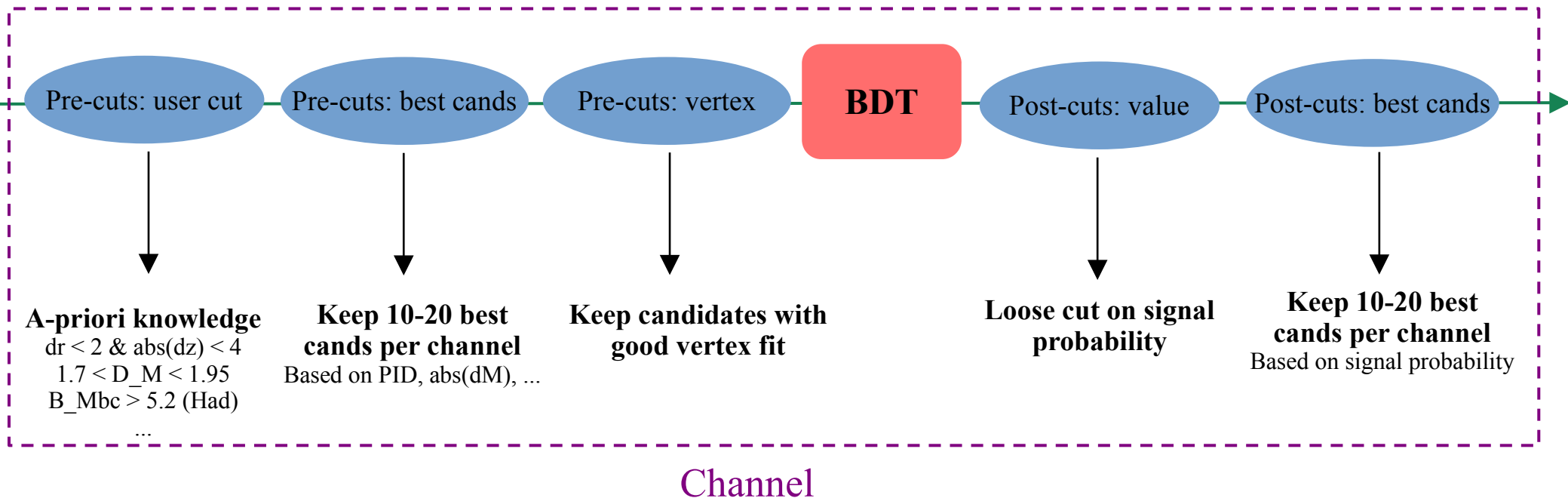
- At each stage a BDT combines the information on candidate into a single number: **signal probability**
- Candidates from different decays are treated equally in following steps
- **Signal probability is available to the next BDT**

- FEI reconstructs B decays in **$\sim 10\text{k modes}$**
- **Last BDT interpreted as “B probability”**
- Overall performances: $\sim 1\text{-}2\%$ efficiency at $\sim 5\text{-}10\%$ purity



Reducing the combinatoric

- Intermediate cuts applied to reduce combinatoric and save computing time



Training

- Each BDT trained to **discriminate signal from incorrect candidates**
- Latest training performed with 200 fb⁻¹ run independent MC15 (~ 220 M *BB* pairs)

Input variables

FSPs (charged + photons)	Intermediates	B candidates
<ul style="list-style-type: none">• PIDs• p, p_T, p_z• dr, dz• chiProb• Pre-cut ranking based on PID• clusterReg• clusterNHits• clusterTiming• clusterE9E25• p_T, E, p_z• Pre-cut ranking based on E	<ul style="list-style-type: none">• Combination of inv. masses of decay products• Angle between decay products• Momenta of decay products• chiProb of decay products• Signal probability of decay products• Angle between momentum and vertex vector• Vertex χ^2• Energy released Q, dQ• Mass difference wrt nominal value	<ul style="list-style-type: none">• All variables used for intermediates (including signal probability)• ΔE• Flight distance and its significance• dr, dz, dx, dy

Example from basf2 tutorial

Trevor's slides:

- Analysis-oriented data and MC information than mdst, but have less events
 - MC15rd partially available (almost done!) * Imberto if interested
 - Each WG has a skim liaison
- Takeaway: use skims!

⚡ See tomorrow!

- $B^0 \rightarrow \pi/\rho^- l^+ \nu$ MC with **FEI skim**
- Signal-side $B^0 \rightarrow \pi^- \mu^+ \nu$ selection
- Y(4S) reconstruction
- Rest of event building

https://software.belle2.org/development/sphinx/online_book/basf2/fei.html

```
import basf2 as b2
import modularAnalysis as ma
from variables import variables as vm
```

```
main = b2.Path()
```

```
ma.inputMdst(
    "./fei_skimmed_xulnu.udst.root",
    path=main,
)
```

```
good_track = (
    "dr < 0.5 and abs(dz) < 2 and nCDCHits > 20 and thetaInCDCAcceptance"
)
```

```
ma.fillParticleList("mu-", "muonID > 0.9 and " + good_track, path=main)
ma.fillParticleList("pi-", "pionID > 0.5 and " + good_track, path=main)
```

```
ma.reconstructDecay("B0:signal -> pi- mu+ ?nu", cut="", path=main)
```

```
ma.reconstructDecay(
    "Upsilon(4S):opposite_cp -> B0:generic anti-B0:signal", cut="", path=main
)
```

```
ma.reconstructDecay(
    decayString="Upsilon(4S):same_cp -> B0:generic B0:signal",
    cut="",
    path=main,
)
```

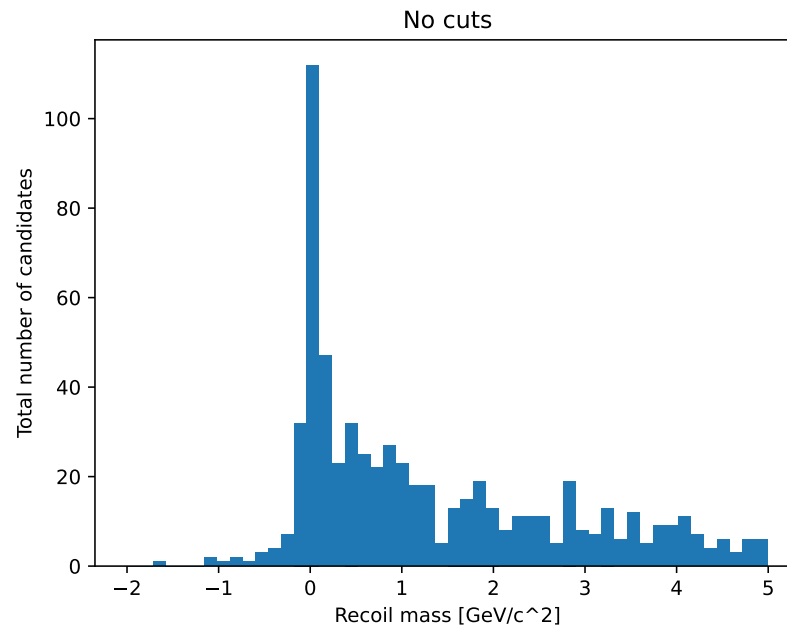
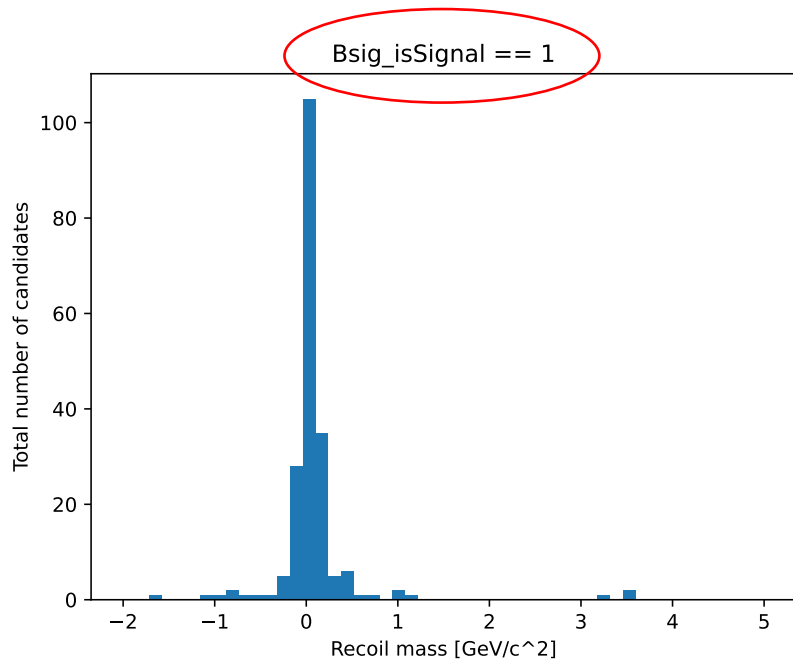
```
# Combine the two Upsilon(4S) lists to one. Note: Duplicates are removed.
ma.copyLists(
    outputListName="Upsilon(4S)",
    inputListNames=["Upsilon(4S):opposite_cp", "Upsilon(4S):same_cp"],
    path=main,
)
```

```
ma.buildRestOfEvent("Upsilon(4S)", path=main)
track_based_cuts = "thetaInCDCAcceptance and pt > 0.075 and dr < 2 and abs(dz) < 4"
ecl_based_cuts = "thetaInCDCAcceptance and E > 0.05"
roe_mask = ("my_mask", track_based_cuts, ecl_based_cuts)
ma.appendROEMasks("Upsilon(4S)", [roe_mask], path=main)
```

```
ma.matchMCTruth(list_name="Upsilon(4S)", path=main)
```

Example from basf2 tutorial

- We look at the B^0 recoil mass
- Should peak at 0 for signal events (neutrino)
- Broad tail in full sample (background events)



- Let's see if we can do better with the FEI

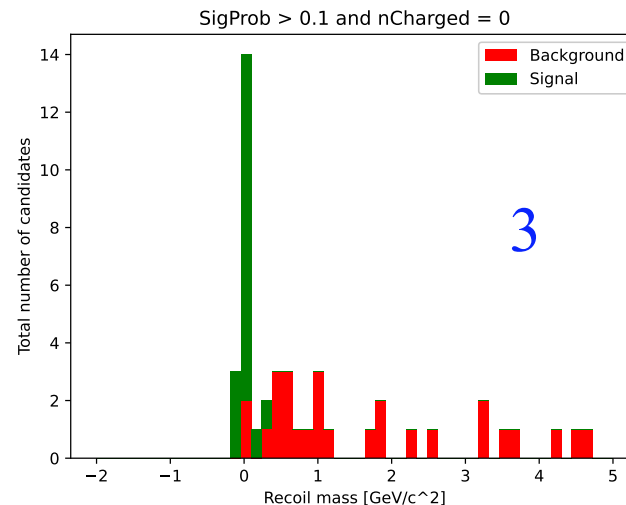
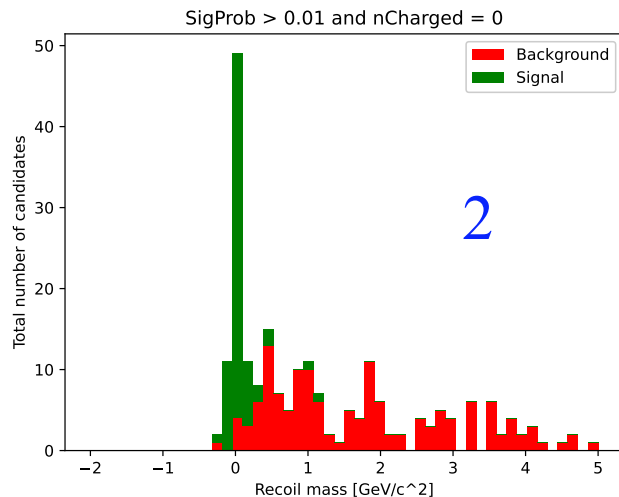
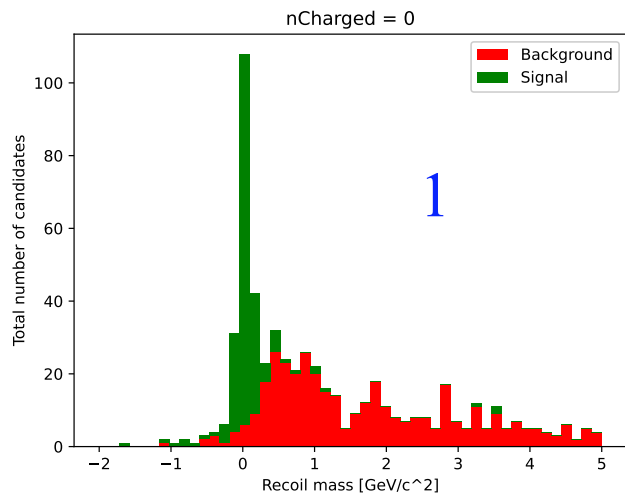
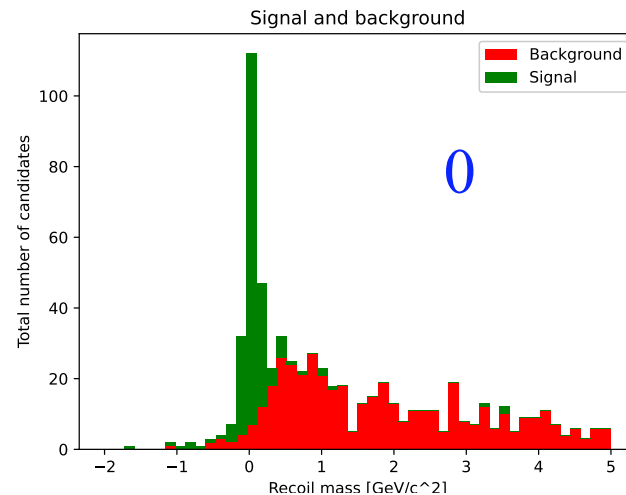
Example from basf2 tutorial

0) Initial dataset

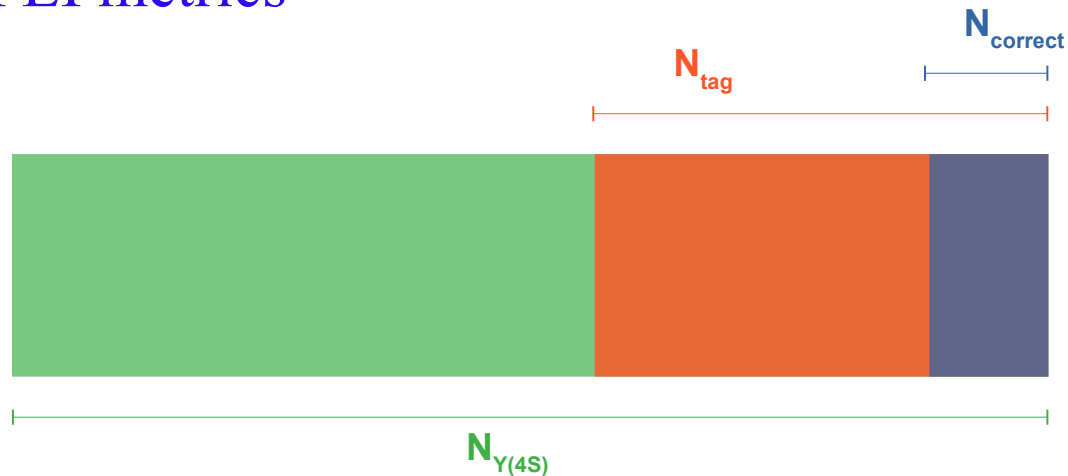
1) We can start requiring 0 charged tracks in the ROE (**completeness constraint**)

2) Then we cut on signal probability > 0.01

3) Finally on signal probability > 0.1



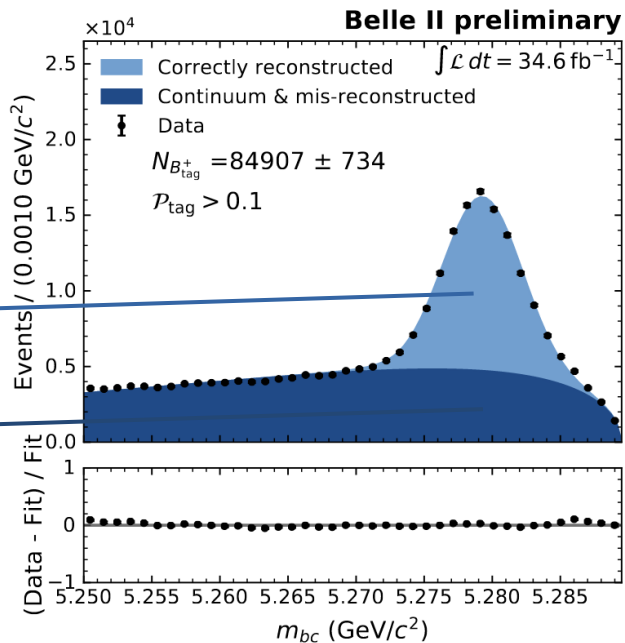
FEI metrics

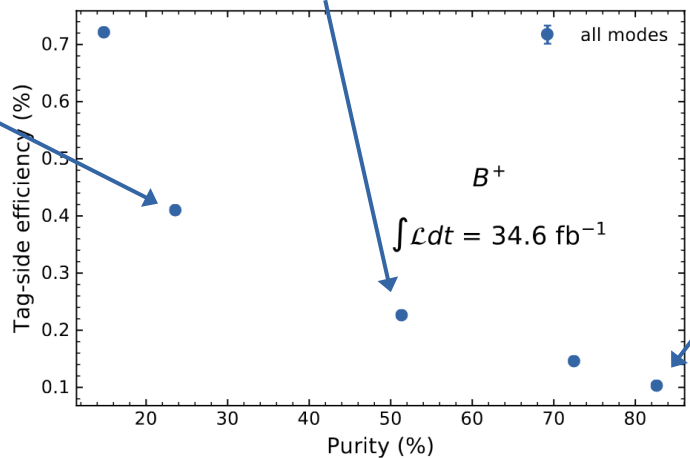
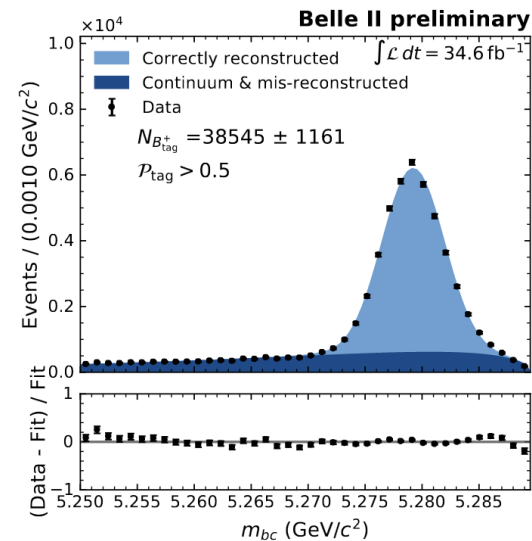
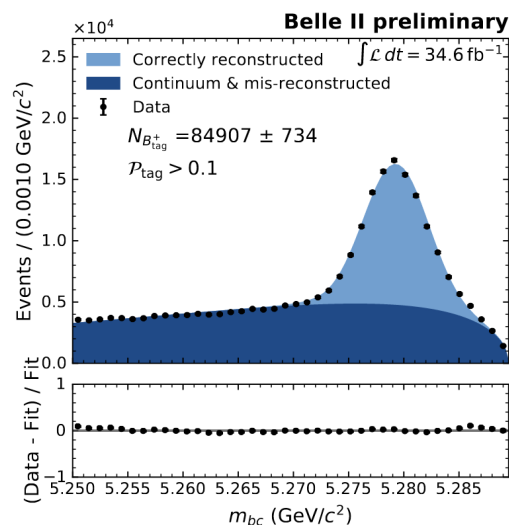
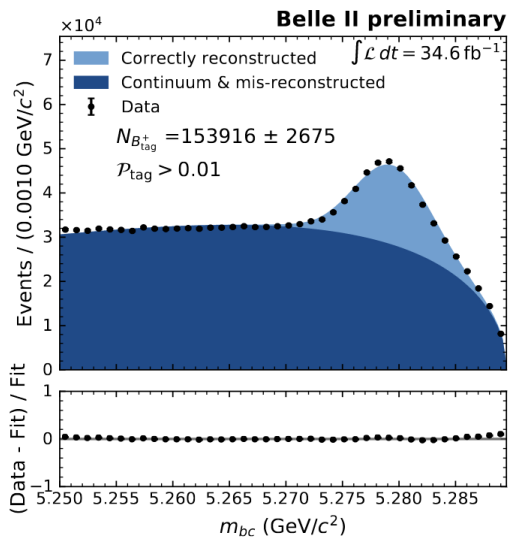


- **Tagging efficiency** = $N_{\text{tag}} / N_{Y(4S)}$
- **Tag-side efficiency** = $N_{\text{correct}} / N_{Y(4S)}$
- **Purity** = $N_{\text{correct}} / N_{\text{tag}}$

[BELLE2-NOTE-PH-2019-031](#)
W. Sutcliffe, F. Bernlochner

$$N_{\text{correct}} + N_{\text{incorrect}} = N_{\text{tag}}$$



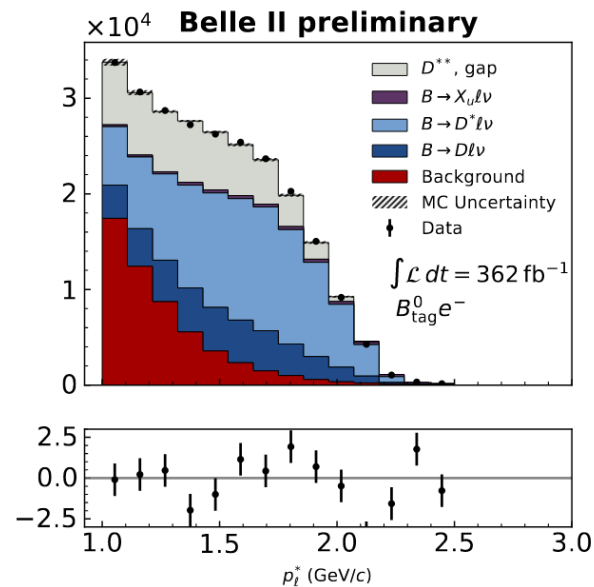
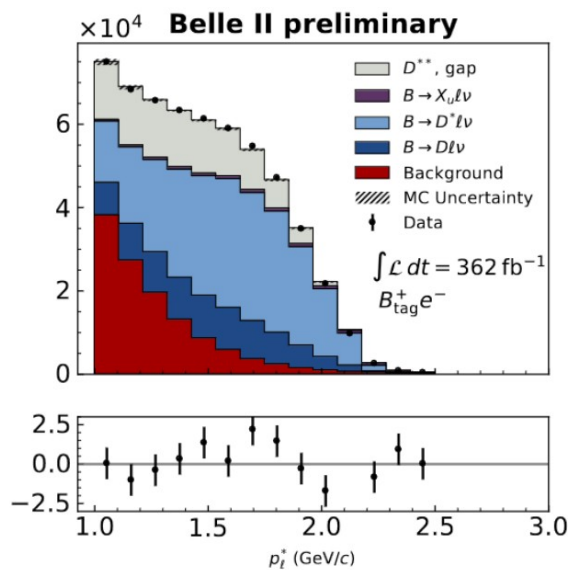


FEI calibration

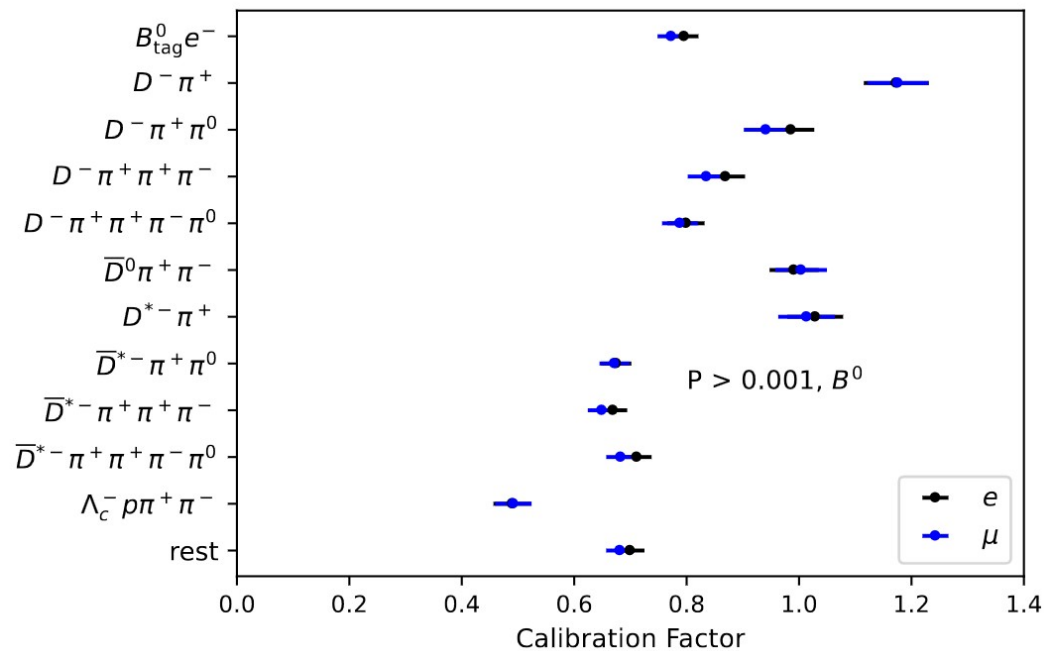
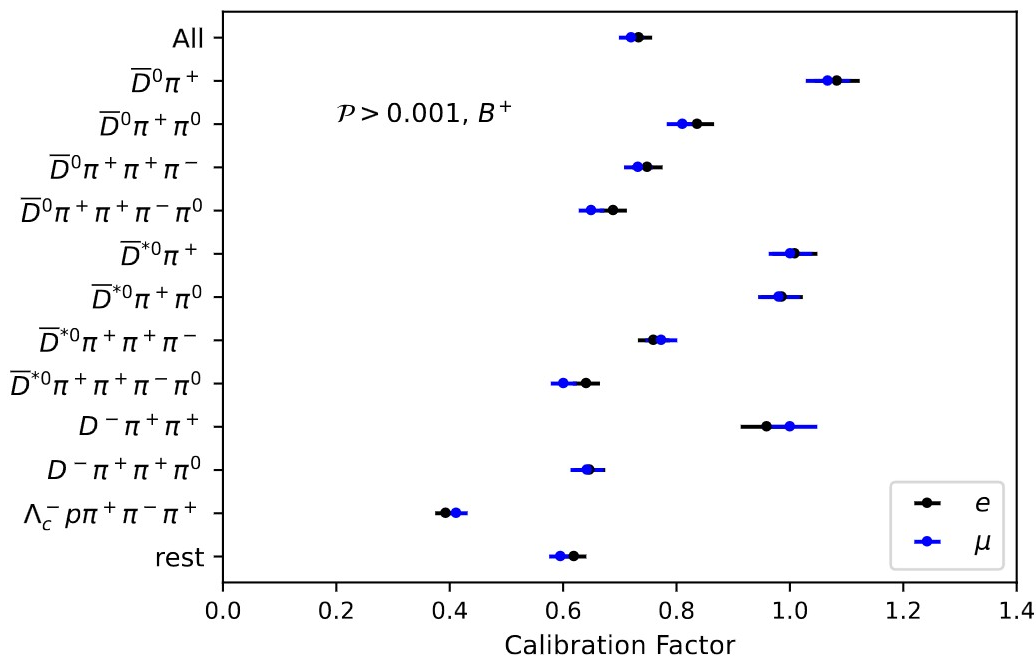
- **Efficiency does not agree between simulation and data** because of:
 - **Branching fractions** of B and intermediate decays not well known
 - **Data-MC differences** of BDT variables
 - Tag-side particles have **no corrections** applied
- → Need to perform a *calibration* of the FEI:
 - Measure the **yield of B decays in data and MC** and extract a correction factor $N_{\text{data}}/N_{\text{MC}}$
 - Perform the calibration for various values of signal probability, tag-side decay mode and possibly other quantities

- Perform calibration using **inclusive** $B \rightarrow X l \nu$ decays ($l = e, \mu$)
 - Perform **B -tagging using hadronic FEI**
 - **Select lepton** from ROE with $p_i^* > 1$ GeV
 - Perform **binned fit to p_i^*** to get yields in data and MC

B^0 modes
Sig prob > 0.001



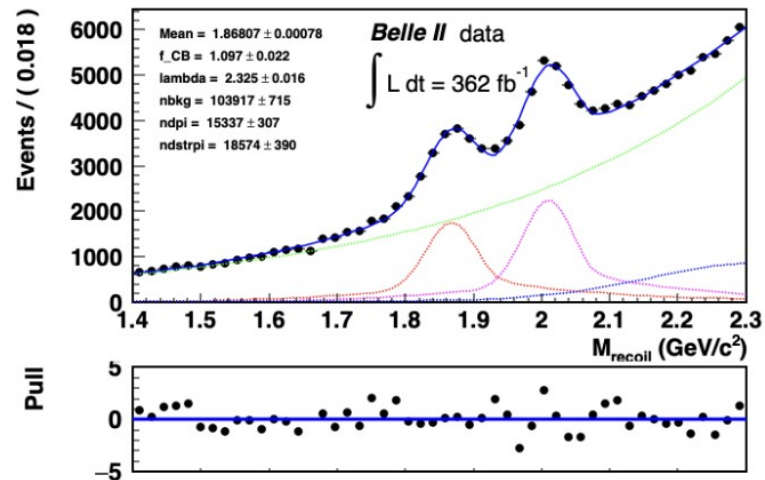
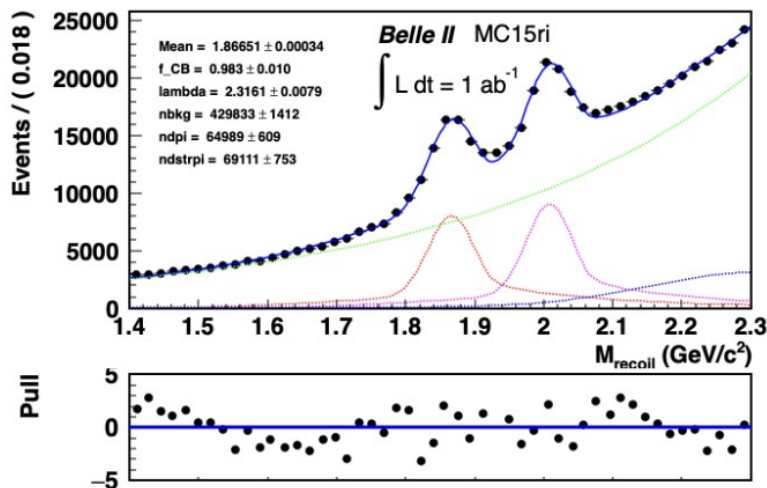
- **Mode-by-mode calibration factors for B^+ and B^0 and $p > 0.001$** , using electron and muon channels
- Systematic and consistent **tension between electron and muon channels**
- Still investigating, maybe due to $K \rightarrow \mu$ fake rate or background in muon channel



Had FEI calibration with $D^{(*)}\pi$ samples

- Calibrate the FEI by partially reconstructing $B \rightarrow D^{(*)}\pi$ decays
 - Perform **B -tagging using hadronic FEI**
 - **Select pion with highest momentum** from ROE
 - Fit **recoil mass** of the system

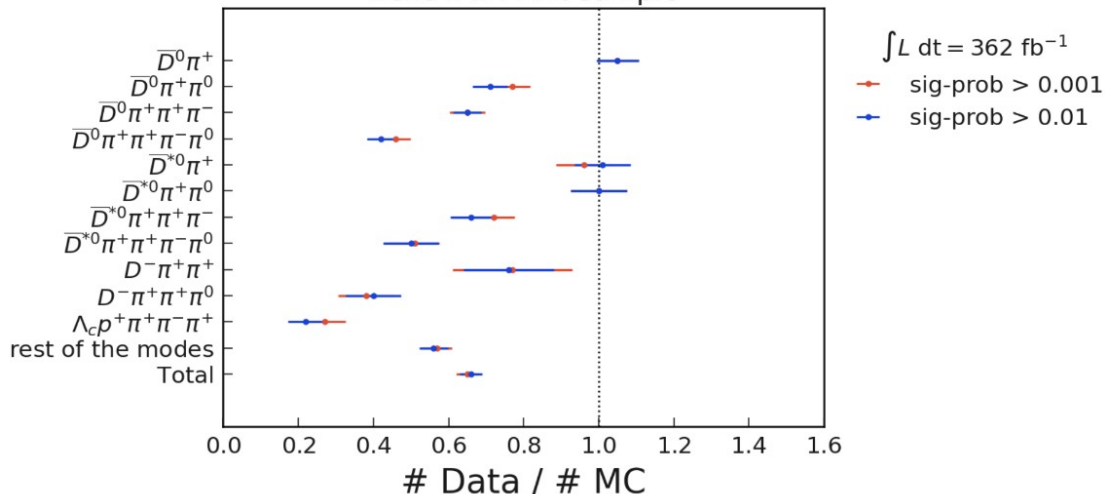
B^+ modes
Sig prob > 0.001



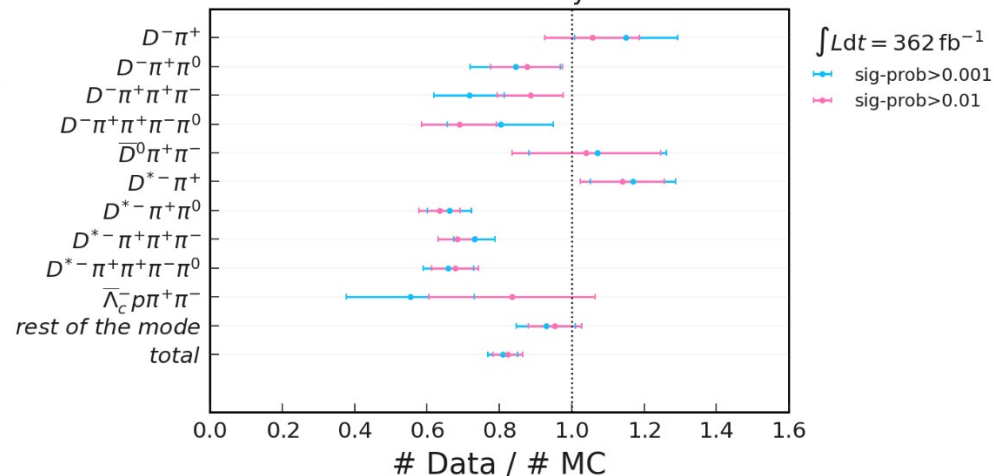
Had FEI calibration with $D^{(*)}\pi$ samples

- Mode-by-mode calibration factors for B^+ and B^0 using $D\pi$ samples

Belle II B^+ : $D\pi$ sample



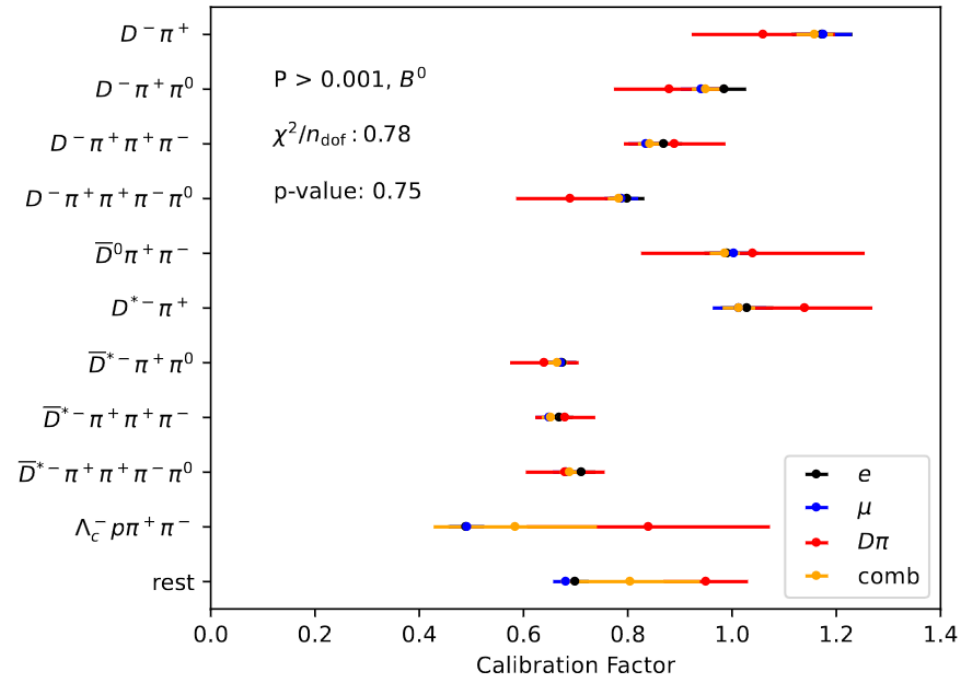
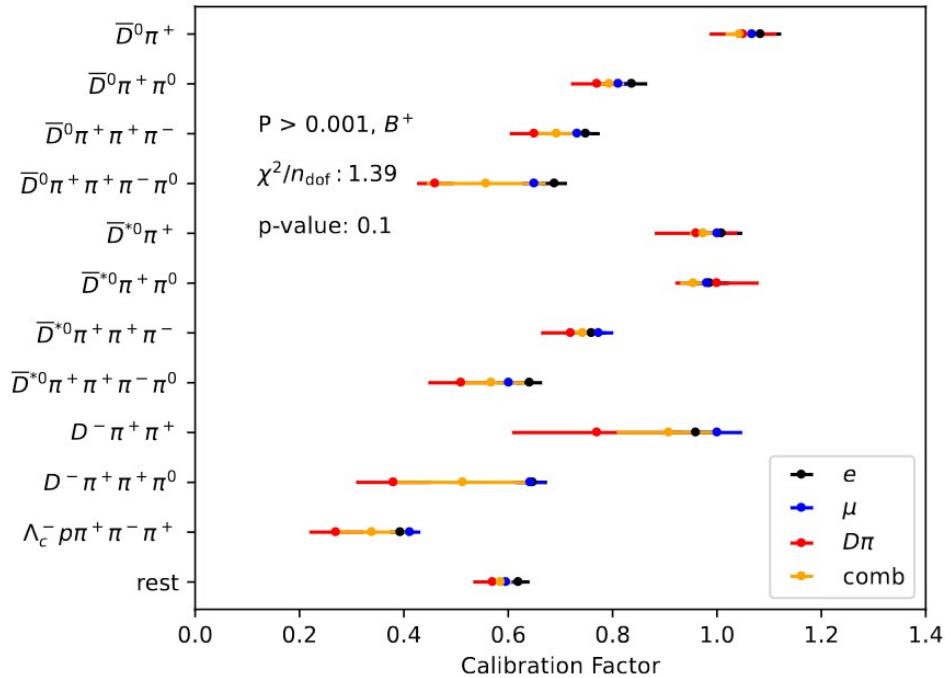
Belle II B^0 FEI fit mode by mode of $D\pi$



Combination

- Combined calibration factors and example application jupyter notebook can be found on kekcc:

`/hsm/belle2/bdata/users/sutclw/fei_calibration/hadronic_FEI_calibration_factors`

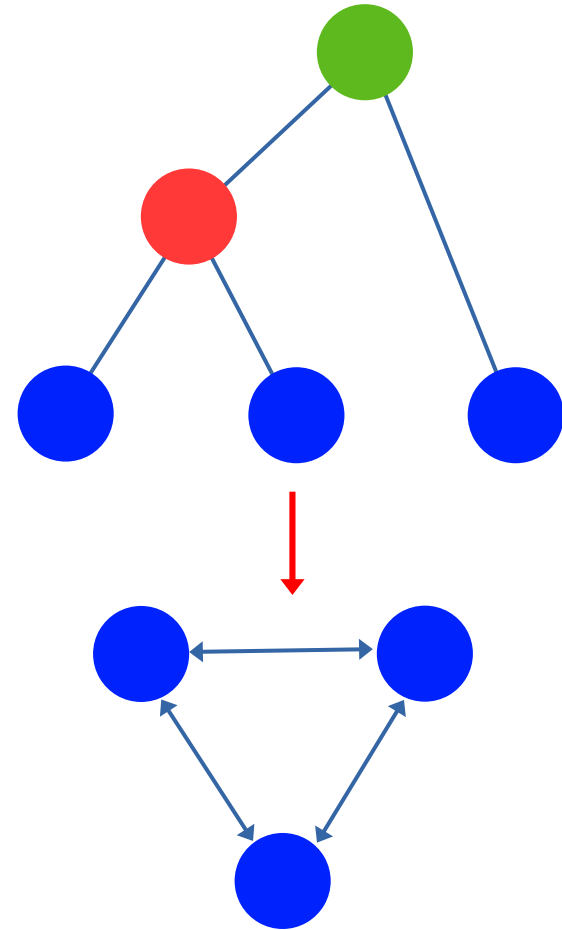


What comes next?

B reconstruction using Graph Neural Networks

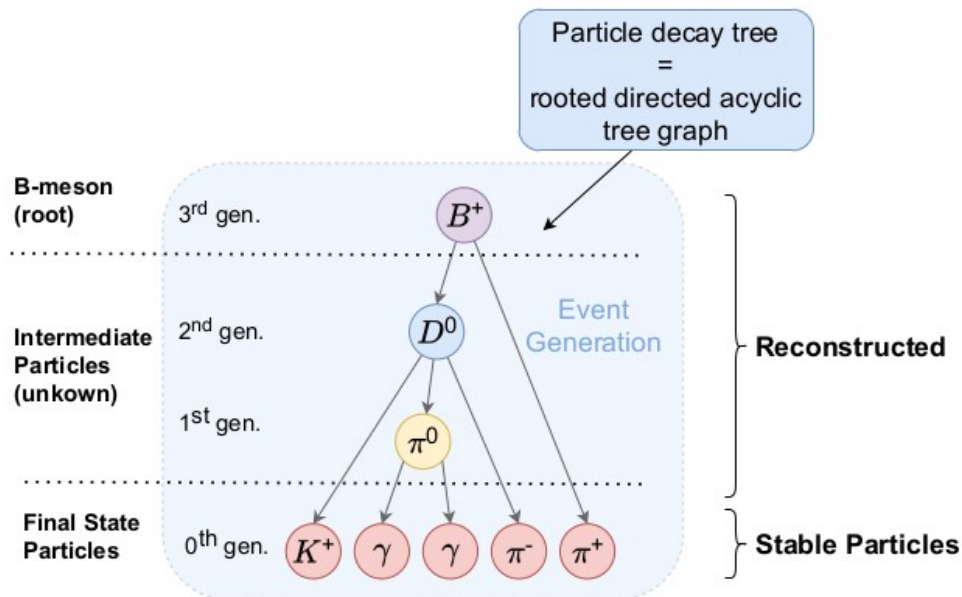
- Main limitation of FEI: **channels need to be hard-coded**
- Decay tree can be encoded in **rooted directed acyclic tree graph**
- Goal: use graph neural networks to inclusively reconstruct B_{tag}
 - → **Graph-based Full Event Interpretation (graFEI)**

- Challenges:
 - We only have information on FSPs
 - Variable number of FSPs
 - Unknown number of intermediates
- **Solution: encode decay tree as FSP relations**



Lowest Common Ancestor (LCA) matrix

- Based on:
 - [Ilias Tsaklidis](#)' and [Lea Reuter](#)'s master theses
 - Learning tree structures from leaves for particle decay reconstruction*, Kahn et al 2022 [Mach. Learn.: Sci. Technol. 3 035012](#)



Adjacency Matrix

	B^+	D^0	π^0	K^+	γ	γ	π^-	π^+
B^+	0	1	0	0	0	0	0	1
D^0	1	0	1	1	0	0	1	0
π^0	0	1	0	0	1	1	0	0
K^+	0	1	0	0	0	0	0	0
γ	0	0	1	0	0	0	0	0
γ	0	0	1	0	0	0	0	0
π^-	0	1	0	0	0	0	0	0
π^+	1	0	0	0	0	0	0	0

Lowest Common Ancestor (LCA) Matrix

	K^+	γ	γ	π^-	π^+
K^+	K^+	D^0	D^0	D^0	B^+
γ	D^0	γ	π^0	D^0	B^+
γ	D^0	π^0	γ	D^0	B^+
π^-	D^0	D^0	D^0	π^-	B^+
π^+	B^+	B^+	B^+	B^+	π^+



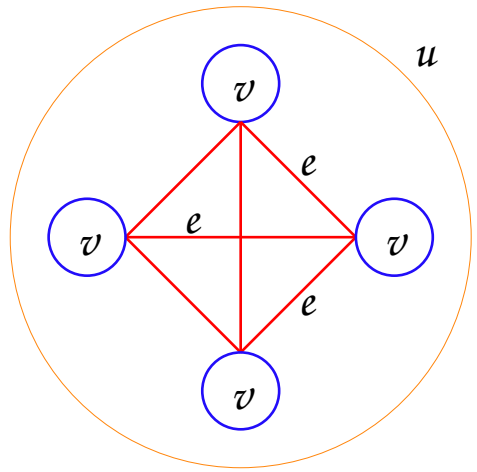
LCAS

	K^+	γ	γ	π^-	π^+
K^+	0	3	3	3	5
γ	3	0	1	3	5
γ	3	1	0	3	5
π^-	3	3	3	0	5
π^+	5	5	5	5	0

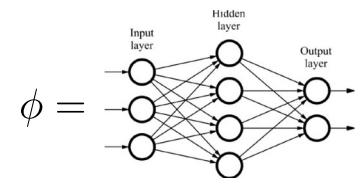
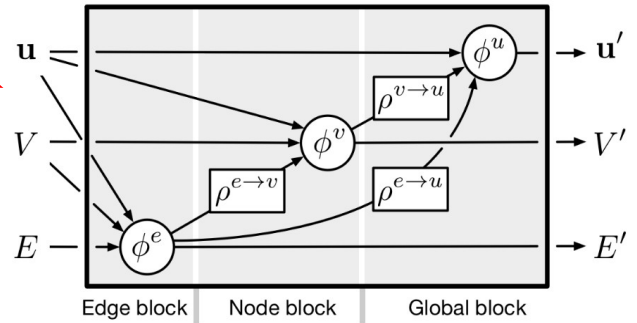
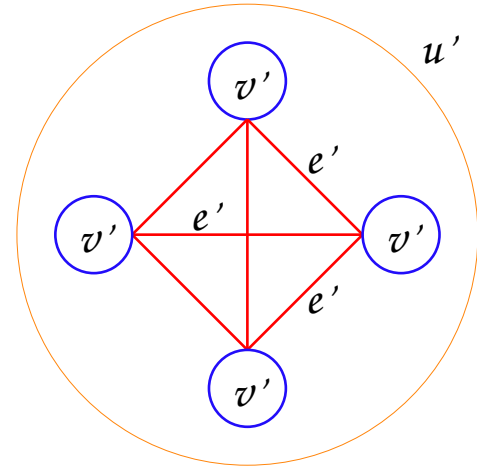
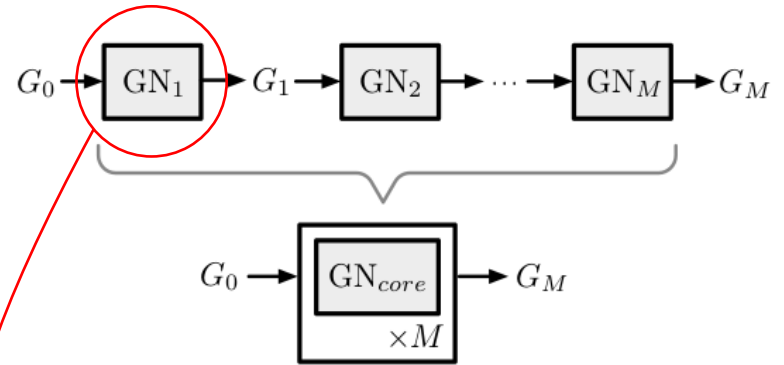
5	B^0, B^+
4	D^+, D^+, D_s^{*+}
3	D^0, D^+, D_s^+
2	K_S^0
1	$\pi^0, J/\psi$

GraFEI – Model description

- Model based on *graph network blocks* [arXiv:1806.01261](https://arxiv.org/abs/1806.01261)
- We input a **fully connected graph**, output graph has same structure with updated attributes

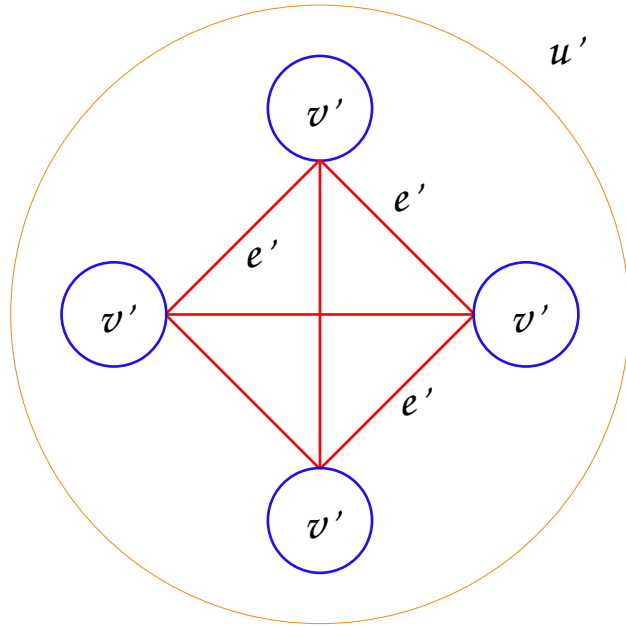


u = global features
 v = node features
 e = edge features

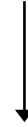


$$\rho = \frac{1}{N} \sum_i x^i$$

GraFEI - Loss function



$$\text{Loss} = \text{LCA} + \alpha \cdot \text{Particle IDs}$$



6-classes cross-entropy:

- 5 : B^0
- 4 : $D^{(\pm)}_{(S)}$ *
- 3 : $D^{(\pm)}_{(S)}$
- 2 : K_S^0
- 1 : $\pi^0, J/\psi$
- 0 : background

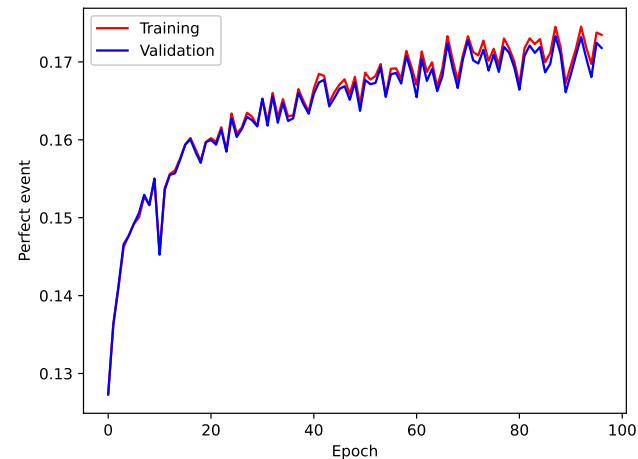
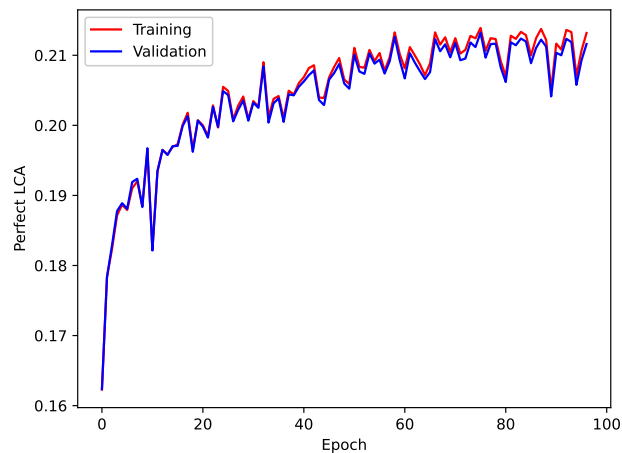
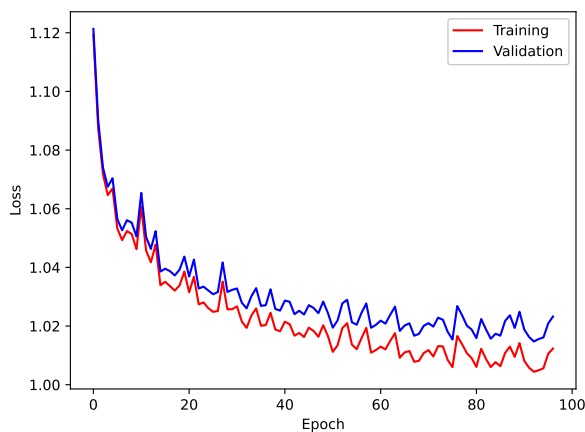


7-classes cross-entropy:

- 6 : γ
- 5 : p
- 4 : K
- 3 : π
- 2 : μ
- 1 : e
- 0 : other

GraFEI – Training

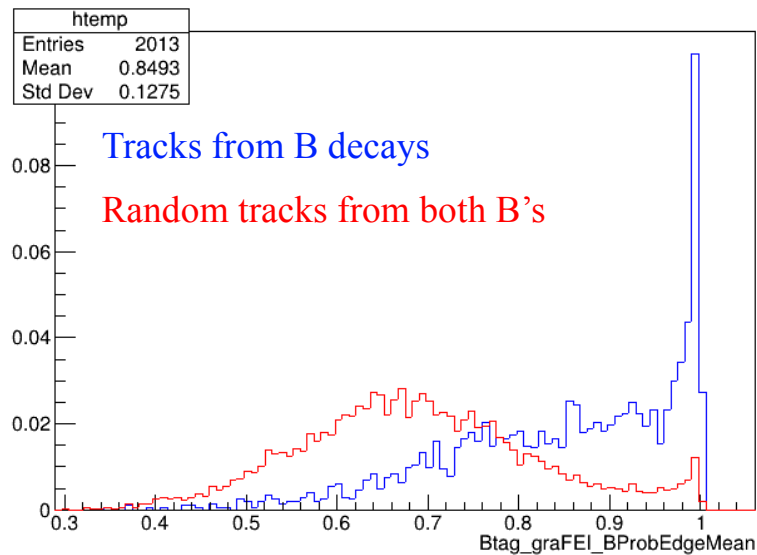
- Model trained on mixed MC: $\sim 3\text{M}$ decays for training + $\sim 150\text{k}$ for evaluation
- Input features:
 - Node-level: particle IDs, pt, pz, charge, dr, dz, clusterNHits, clusterTiming, clusterCharge
 - Edge-level: $\cos(\theta)$ between momenta, DOCA between tracks



GraFEI - B probability

- Having a **definition of “B probability”** analogous to FEI is needed
 - Each LCA element has a corresponding probability of belonging to the predicted class
 - Arithmetic mean of class probabilities defined as B probability

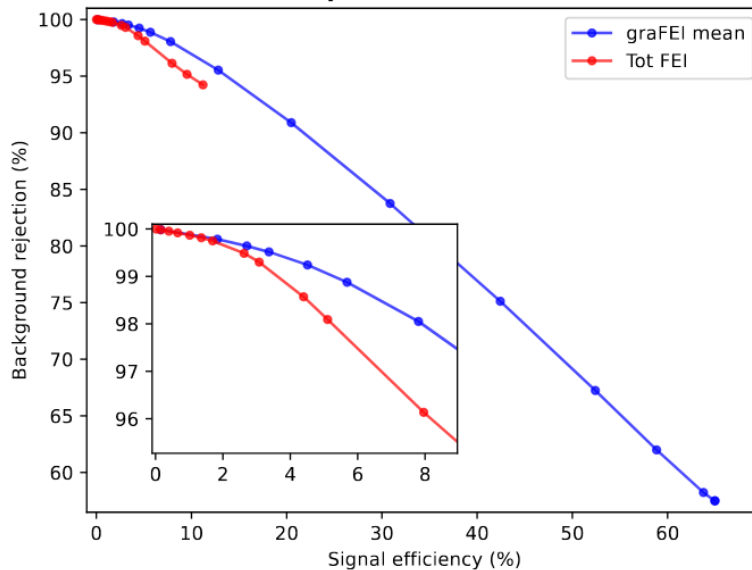
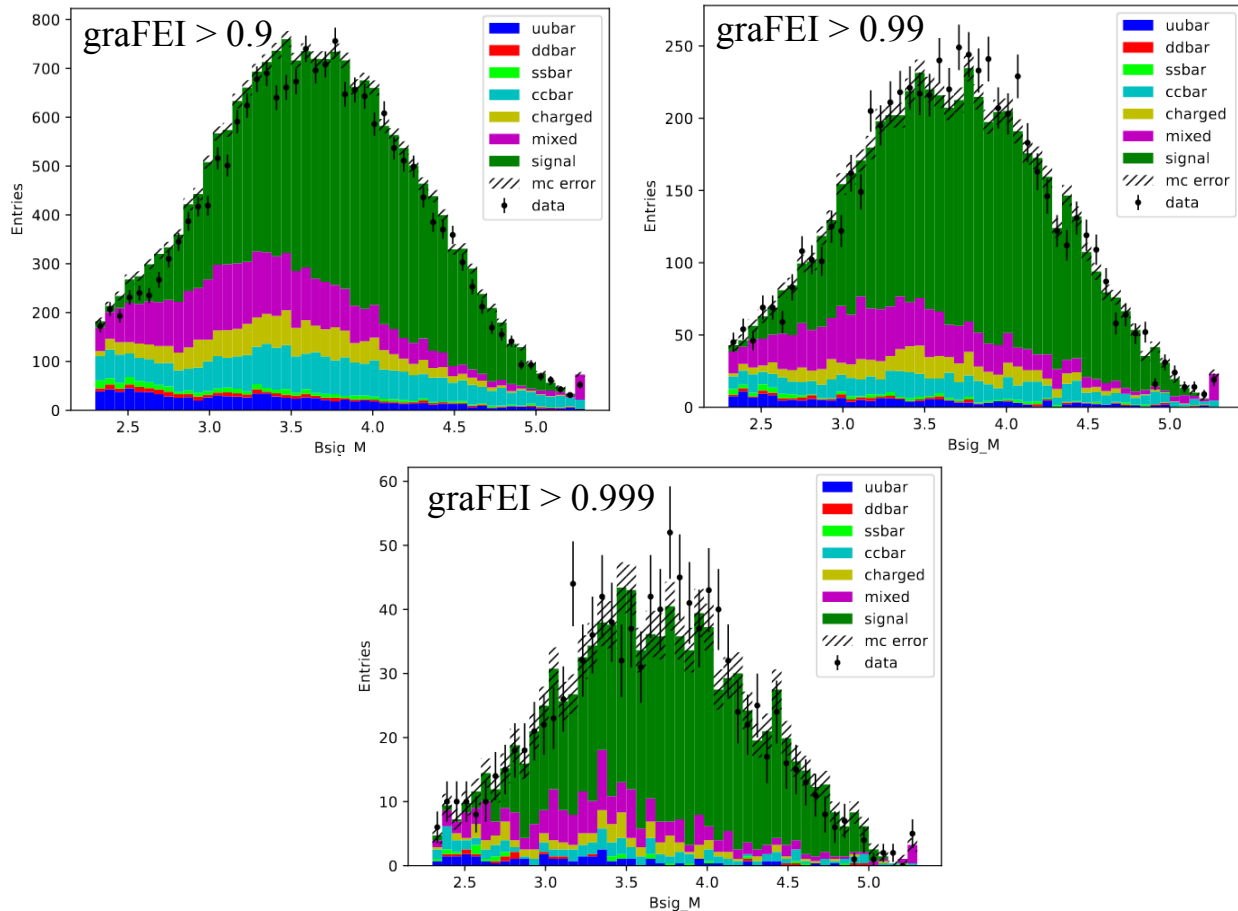
$$\text{LCA} = \begin{pmatrix} 0 & 3 & 5 \\ 3 & 0 & 5 \\ 5 & 5 & 0 \end{pmatrix} \longleftrightarrow \begin{pmatrix} 0 & 0.62 & 0.31 \\ 0.62 & 0 & 0.76 \\ 0.31 & 0.76 & 0 \end{pmatrix} \rightarrow 0.563$$



GraFEI in action – Comparison with FEI

- Applied on tag-side of $B^0 \rightarrow D^- (\rightarrow K^+ \pi^- \pi^-) \mu^+ \nu$ candidates

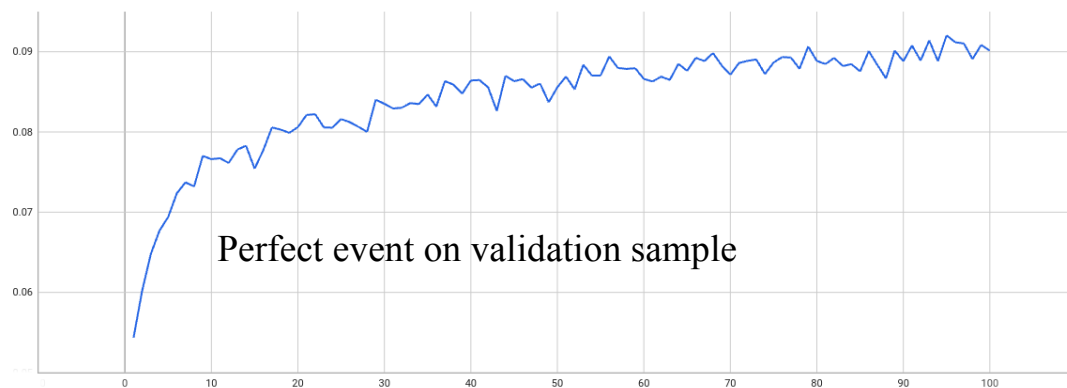
Preliminary!



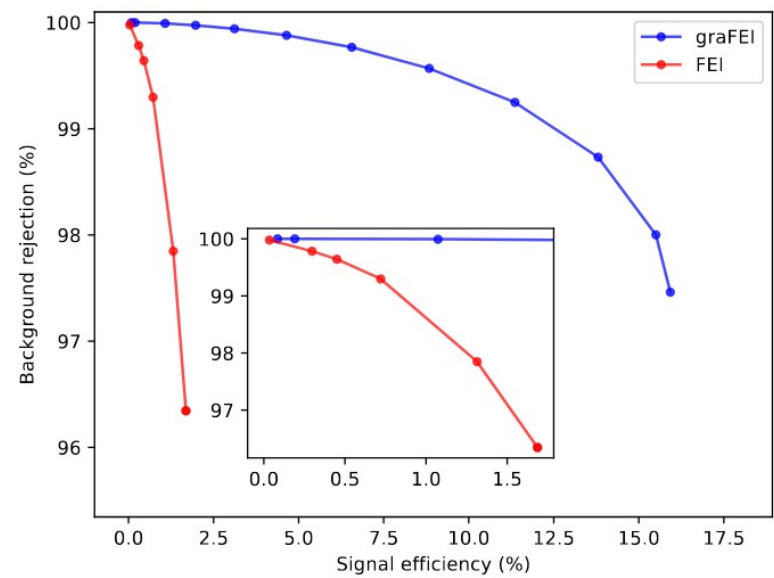
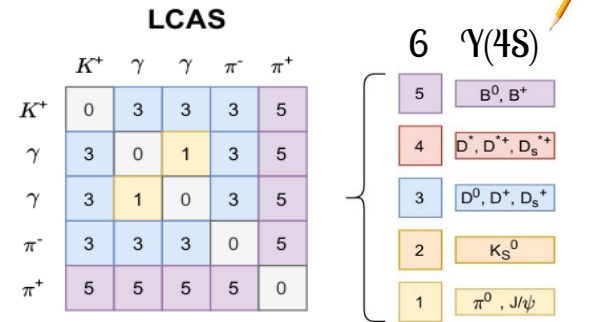
GraFEI – Full $Y(4S)$ reconstruction

Preliminary!


- Train the model on signal MC to **reconstruct $Y(4S)$ candidates**
 - Used $\sim 3M B^0 \rightarrow K^{*0} \nu \nu$ signal MC events
 - ~ 2 days training on a GPU Nvidia V100



- Currently writing an internal note and implementing the code into basf2
- **If you want to join us just send me a mail!**



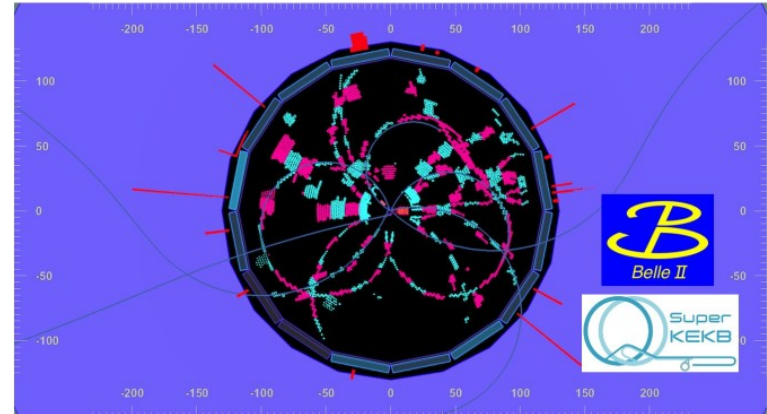
Conclusions

- Take-home messages:
 - Belle II features allow to **reconstruct the partner B produced in the event**
 - Reconstruction performed with **Full Event Interpretation** (use FEI skim )
 - Overall performances: $\sim 1-2\%$ efficiency at $\sim 5-10\%$ purity
 - Calibration needed, performed as a function of decay mode and signal probability cut
- **GraFEI could be a possible extension:**
 - Based on graph neural network, reconstructs decay topology and particle IDs
 - Seems powerful when trained on signal MC, more investigation and documentation ongoing

BACKUP

Belle II [\[arXiv:1011.0352\]](https://arxiv.org/abs/1011.0352)

- **Multi-purpose detector @ SuperKEKB** accelerator
- Focus on B, charm and τ physics
- Collisions at center-of-mass energy of 10.58 GeV
 - $\sigma(e^+e^- \rightarrow \Upsilon(4S)) \sim 1 \text{ nb}$
 - $\mathcal{B}(\Upsilon(4S) \rightarrow B\bar{B}) \gtrsim 96\%$
- **Will collect 50 ab^{-1}** at the end of operation (now $\sim 430 \text{ fb}^{-1}$)
- Instantaneous luminosity world record: **$4.7 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$** (June 2022)



graFEI on phasespace dataset [\[arXiv:2208.14924\]](https://arxiv.org/abs/2208.14924)

- “Perfect world” simulation generated with phasespace library
- Comparison of two GNN models:
 - Neural Relational Inference (NRI) [\[arXiv:1802.04687\]](https://arxiv.org/abs/1802.04687)
 - Transformer encoder
- Hyperparameter optimisation with **Optuna**
- **4-momentum** used as input feature
- **Average 47.7 % perfectly predicted LCAG with NRI**
 - 60.9 % for decays with up to 10 leaves, 94.2 % up to 6 leaves

