Flavor physics: theory perspective

Table of Contents:

Belle II Summer Workshop (Duke U)

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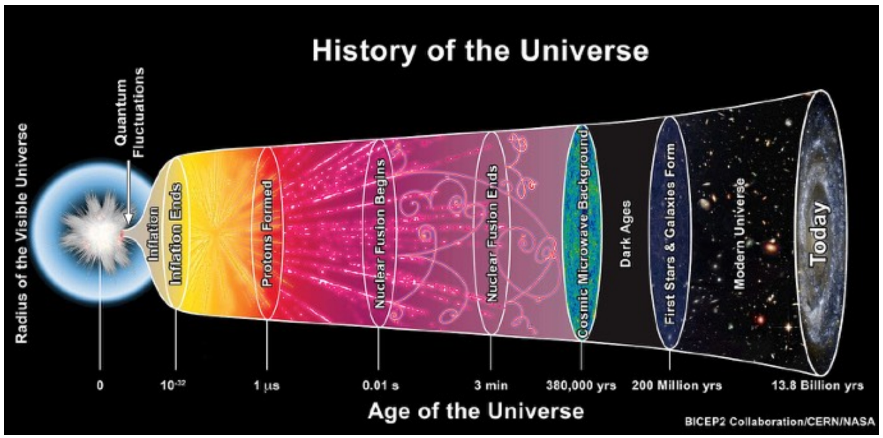
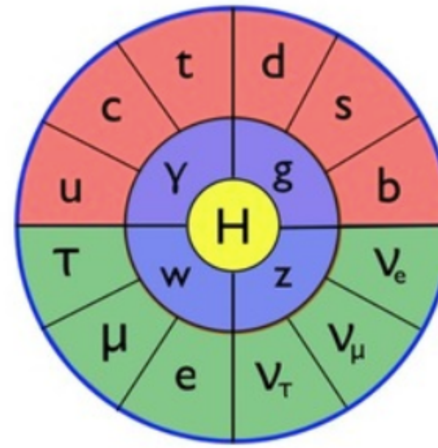
★ Is it possible to build the Universe using the Standard Model as a tool?

- no, but maybe it can tell us where to look for new tools





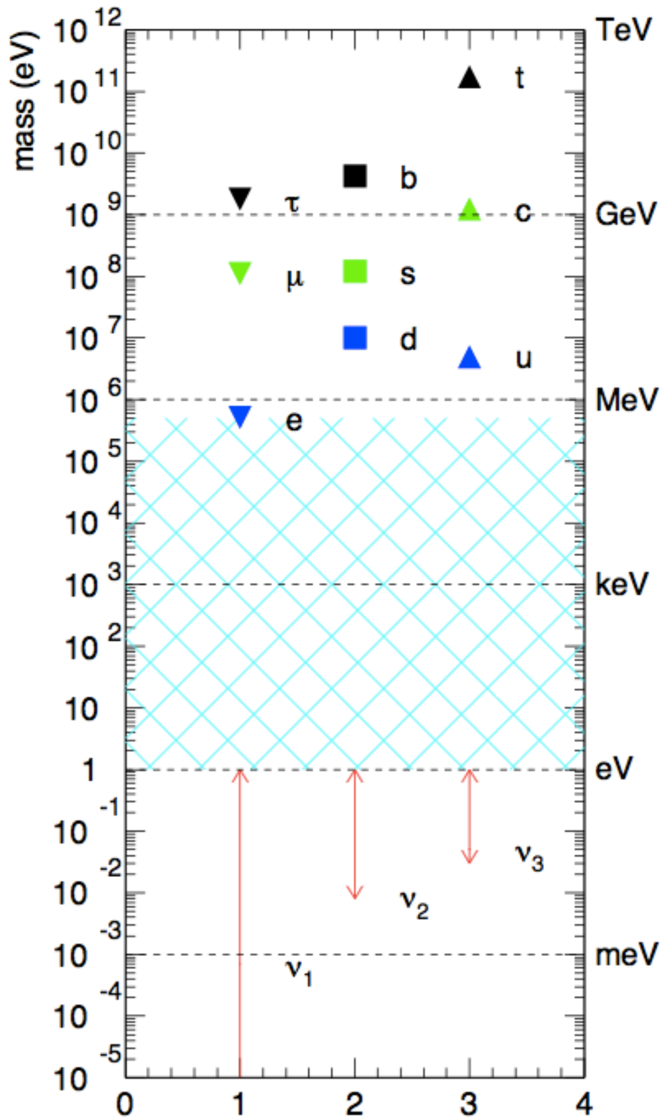
★ The era of "guaranteed discoveries" is over (top quark, electroweak breaking)

- new experiments designed to study rare decays or perform precision studies of various processes might point us in the right direction

★ What about New Physics?

- no new elementary particles so far at the LHC
- neutrinos oscillations: ν's have mass and so CLFV transitions are guaranteed
- use sphaleron mechanism: baryogenesis via leptogenesis Fukugita, Yanagida
- new sources of CP-violation in the lepton sector

Fundamental physics: flavor problem



★ SM and BSM Flavor problem

★ Flavor problem: patterns of masses of particles

- quarks

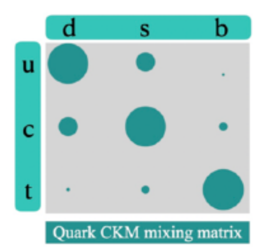
$$\frac{m_d}{m_u} \simeq 2 \ , \quad \frac{m_s}{m_d} \simeq 21 \ ,$$
$$\frac{m_t}{m_c} \simeq 267 \ , \quad \frac{m_c}{m_u} \simeq 431 \ , \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5 \ .$$

- leptons

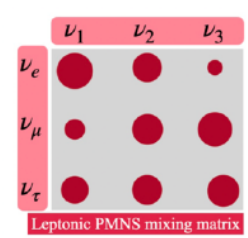
$$\frac{m_\tau}{m_\mu} \simeq 17 \ , \quad \frac{m_\mu}{m_e} \simeq 207 \ .$$

★ Flavor problem: pattern of fermion mixing

- why is the quark mixing matrix so different from the neutrino mixing matrix?



vs



S. Cao, et al.

★ Flavor problem: nature of neutrino mass?

Is flavor “problem” actually a problem?

★ Yukawa couplings are protected by a chiral symmetry:

$$\frac{dy}{d\log\mu} \propto y \quad \Longrightarrow$$

Small couplings remain small:
“Technical Naturalness”

★ So, why is it a problem?

The reason there is a problem is that all these couplings appear to come from the same physics. Therefore they should all start at the same order of magnitude at some UV scale, and the hierarchy should come from RG effects. This is why gauge couplings are not considered hierarchal.

Now the above “technically natural” condition actually HURTS!

Fundamental physics: flavor problem

★ Flavor problem: flavor-changing neutral currents (FCNC)

- there is no term in the SM Lagrangian that leads to FCNC effects: quantum effects (one loop process)
- quarks: massive quarks and non-zero mixing parameters automatically lead to FCNC processes: $b \to s\gamma, c \to u\ell\bar{\ell}, B^0 - \bar{B}^0$-mixing, etc.
- leptons: massive neutrinos and non-zero mixing parameters automatically lead to FCNC processes: $\tau \to e\gamma, \tau \to eee, \mu A \to eA$, etc.

★ Flavor problem: patterns of masses of particles and neutrino mass: new symmetry?

- there could be a mechanism generating mass patterns (Froggatt-Nielsen, etc.)…

A. Blechman, AAP, G.K. Yeghiyan

288 C.D. Froggatt, H.B. Nielsen / Hierarchy of quark masses

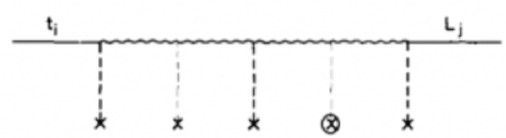
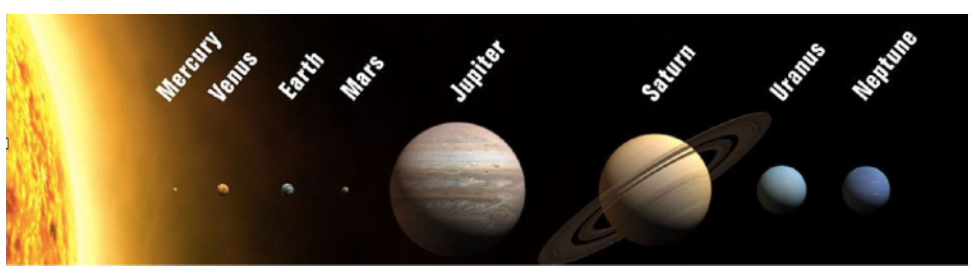


Fig. 1. Feynman diagram which generates the quark mass matrix element $M_{t,ij}$. Full lines represent quarks and wavy lines represent super heavy fermions. The dashed lines represent Higgs tadpoles as follows: – – – × $\langle\phi_1\rangle$, and – – – ⊗ $\langle\phi_2\rangle$.

- … or maybe not (a “just so” solution?)



★ Flavor in the Standard Model: mass generation and CP-violation

- masses are generated through Yukawa terms (quarks)

$$-\mathcal{L}_Y = Y^d_{ij}\overline{Q^f_{Li}} H D^f_{Rj} + Y^u_{ij}\overline{Q^f_{Li}}\widetilde{H} U^f_{Rj} + h.c. \quad \text{with} \quad Q^f_{Li} = \begin{pmatrix} U^f_{Li} \\ D^f_{Li} \end{pmatrix}$$

- after spontaneous symmetry breaking $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix}$

$$-\mathcal{L}_M = (M_d)_{ij}\,\overline{D^f_{Li}} D^f_{Rj} + (M_u)_{ij}\,\overline{U^f_{Li}} U^f_{Rj} + h.c. \quad \text{with} \quad (M_q)_{ij} = \frac{v}{\sqrt{2}}(Y^q)_{ij}$$

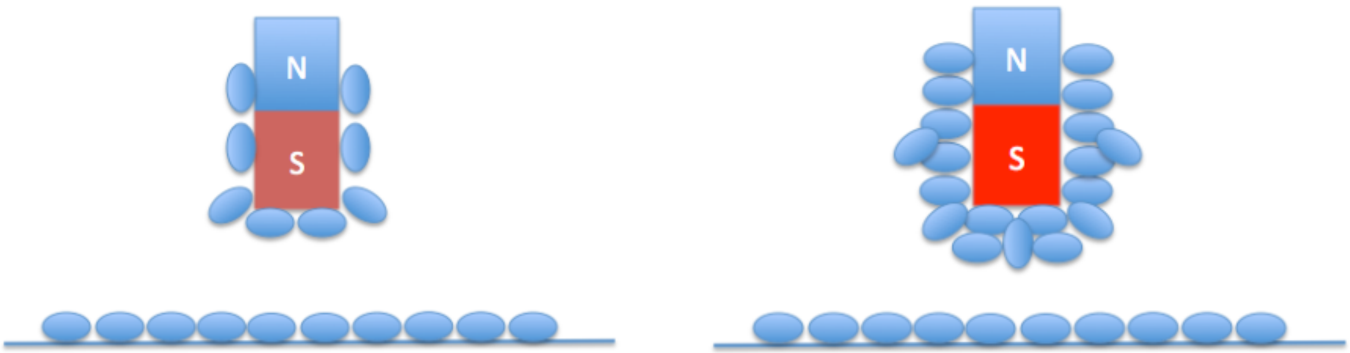
- … but mass matrices above are NOT diagonal! For for both q = {u,d}:

$$V_{qL} M_q V^\dagger_{qR} = M^{\text{diag}}_q \quad \text{with} \quad \begin{aligned} q_{Li} &= (V_{qL})_{ij}\, q^f_{Lj} \\ q_{Ri} &= (V_{qR})_{ij}\, q^f_{Rj} \end{aligned}$$

What is the physical effect of this diagonalization?

Aside: "Higgs mechanism"

Imagine all fermions as tiny (almost) massless magnets and Higgs vev as (slightly magnetized) iron filings laying on a table…



Fermion masses depend on the strength of our "magnets"!

Moreover, since the filings are self-interacting, they would clump into bunches ("particles") if disturbed: just like Higgs bosons!

Flavor in the Standard Model

★ Charged current interactions: the only source of flavor violation in SM

- since left and right matrices are different: charge current part of $\mathcal{L}$:

$$-\mathcal{L}^q_{W^\pm} = \frac{g}{\sqrt{2}} \overline{u}_{Li} \gamma^\mu \left[V_{uL} V^\dagger_{qR} \right]_{ij} d_{Lj} W^+_\mu + h.c.$$

$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

(CKM matrix)

- Cabibbo-Kobayashi-Maskawa (CKM) matrix is unitary: $VV^\dagger = 1$ **(N^2 relations)**
- Counting the number of parameters: N×N
  - N×N complex matrix contains $2N^2$ real parameters
  - N×N unitary matrix contains $2N^2 - N^2 = N^2$ real parameters (phases and angles)
  - can rephrase up and down quarks: 2N-1 relations: $N^2 - (2N-1) = (N-1)^2$ parameters
  - … which represent ${}_NC_2 = N(N-1)/2$ angles and $(N-1)(N-2)/2$ phases

2 generations: 1 angle and 0 phases; 3 generations: 3 angles and 1 phase!

(No CPV) **(CPV)**

CKM picture of CP-violation

★ There is a single phase of the CKM matrix for 3-generation SM

- … but there are MULTIPLE ways to parameterize CKM matrix

  - Wolfenstein parameterization (parameters: λ ~ 0.22, A ~ 0.83, ρ ~ 0.15, η ~ 0.35)

$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

  - Buras-Wolfenstein parameterization (with $\bar{\rho} = \rho(1-\lambda^2/2)$ and $\bar{\eta} = \eta(1-\lambda^2/2)$)

$$V = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{bmatrix}$$

(note $\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$)

  - "PDG" parameterization (in terms of rotation angles)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

★ There is a single phase of the CKM matrix for 3-generation SM

- Even though there are MULTIPLE ways to parameterize CKM matrix

$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} = \begin{bmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}$$

(Wolfenstein)

- ...there exists a parameterization-independent quantity,

$$Im\left[V_{ij}V_{kl}V_{il}^{\dagger}V_{kj}^{\dagger}\right] = J_{CKM}\sum_{m,n=1}^{3}\epsilon_{ilkm}\epsilon_{jlkm} \quad \text{with} \quad J_{CKM} \simeq \lambda^6 A^2 \eta$$

- Since CP-violation appears from imaginary parts of the Yukawas, there is a condition for CP-violation to be present in the SM: **(Jarlskog)**

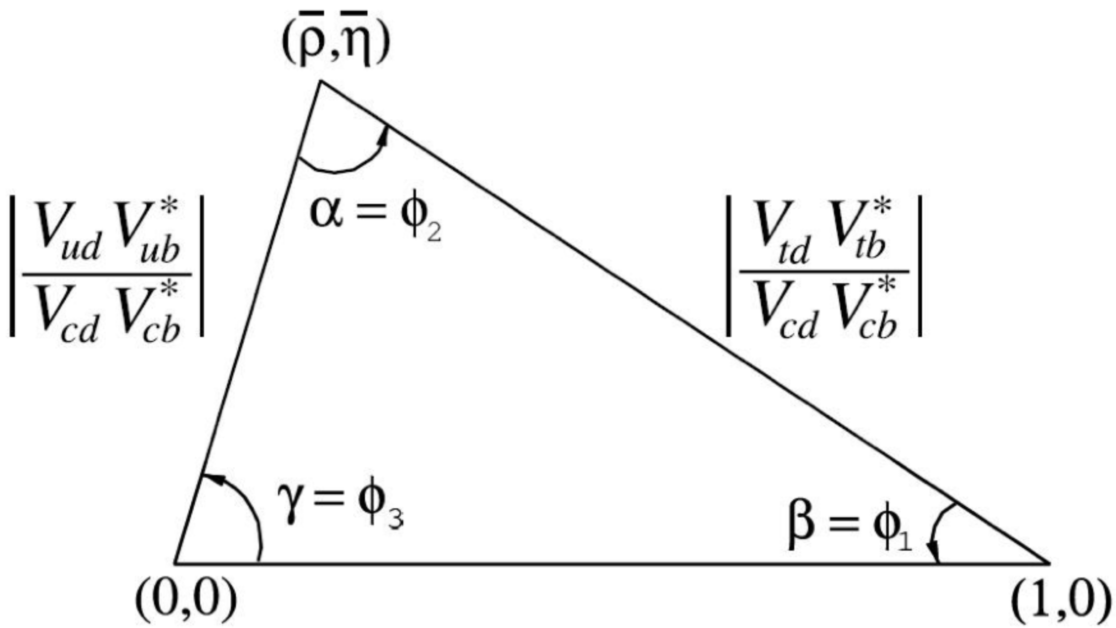
$$\Delta m_{tc}^2 \Delta m_{tu}^2 \Delta m_{cu}^2 \Delta m_{bs}^2 \Delta m_{bd}^2 \Delta m_{sd}^2 J_{CKM} \neq 0 \quad \text{with} \quad \Delta m_{ij}^2 = m_i^2 - m_j^2$$

i.e. no mass degeneracies or zero (or π) angles/phases

CKM picture of CP-violation

★ There is a single phase of the CKM matrix for 3-generation SM

- off-diagonal terms in unitarity relations VV⁺=1 look like triangles in a complex plane (ρ,η), e.g. $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ Each term is $\mathcal{O}(\lambda^3)$



$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- angles are

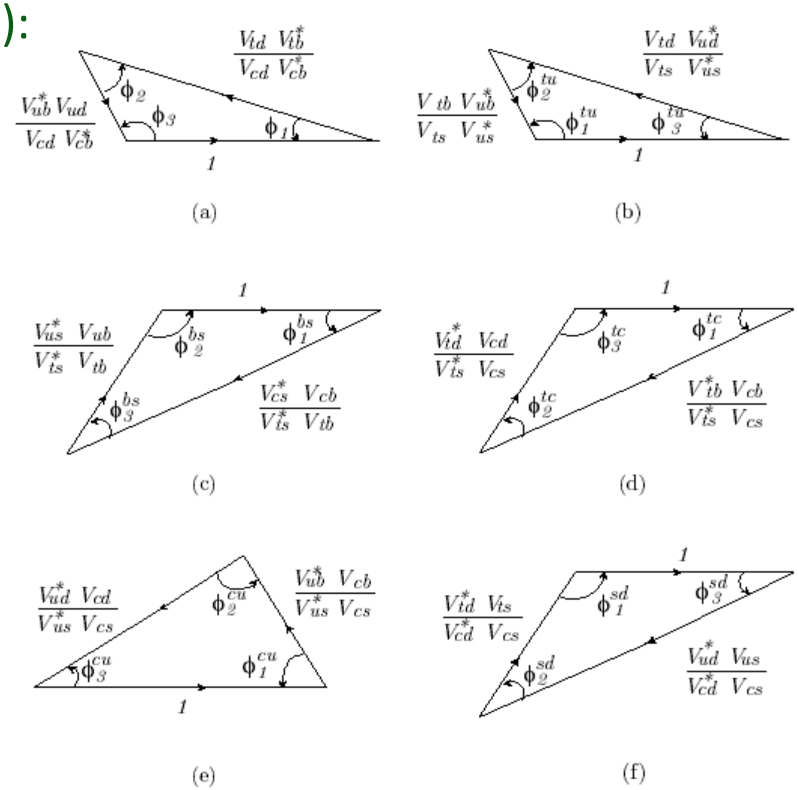
$$\phi_1(\beta) = arg\left[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*\right]$$ phase of V_{td} in Wolfenstein param

$$\phi_2(\alpha) = arg\left[-V_{td}V_{tb}^*/V_{ud}V_{ub}^*\right]$$

$$\phi_3(\gamma) = arg\left[-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*\right]$$ phase of V_{ub} in Wolfenstein param

★ There is a single phase of the CKM matrix for 3-generation SM

- off-diagonal terms in unitarity relations $VV^{+}=1$ look like triangles in a complex plane (ρ,η):



$$V \equiv \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

- … but regardless of the lines/columns used all these triangles have the same area $A = J_{CKM}/2$ (useful cross-check for NP studies)!

CP-violation could be different with NP

★ In any quantum field theory CP-symmetry can be broken

1. Explicitly through dimension-4 (or higher) operators ("hard")

   Example: Standard Model (CKM): $\bar{\psi}_i \psi_k \overset{CP}{\Rightarrow} \bar{\psi}_k \psi_i, \ \varphi \overset{CP}{\Rightarrow} \varphi$

   $$\mathcal{L}_{Yuk} = \zeta_{ik} \bar{\psi}_i \psi_k \varphi + H.c. \ \overset{CP}{\nRightarrow} \ \mathcal{L}_{Yuk}$$

2. Explicitly through dimension <4 operators ("soft")

   Example: SUSY, 2HDM, ...

3. Spontaneously (CP is a symmetry of the Lagrangian, but not of the ground state)

   Example: multi-Higgs models, left-right models $\langle \Phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' e^{i\eta} \end{pmatrix}$

★ These mechanisms can be probed in quark transitions

BTW: no spontaneous CP-violation in SM

★ One can show that SM (or other 1HDMs) cannot spontaneously break CP

- In order to spontaneously break CP, a scalar doublet (Higgs) must have a VEV, which is independent of $\vec{r}$ and t

- One can perform an SU(2) rotation to bring the doublet to be

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ ve^{i\theta} \end{pmatrix}$$

- Recall that under CP transformation

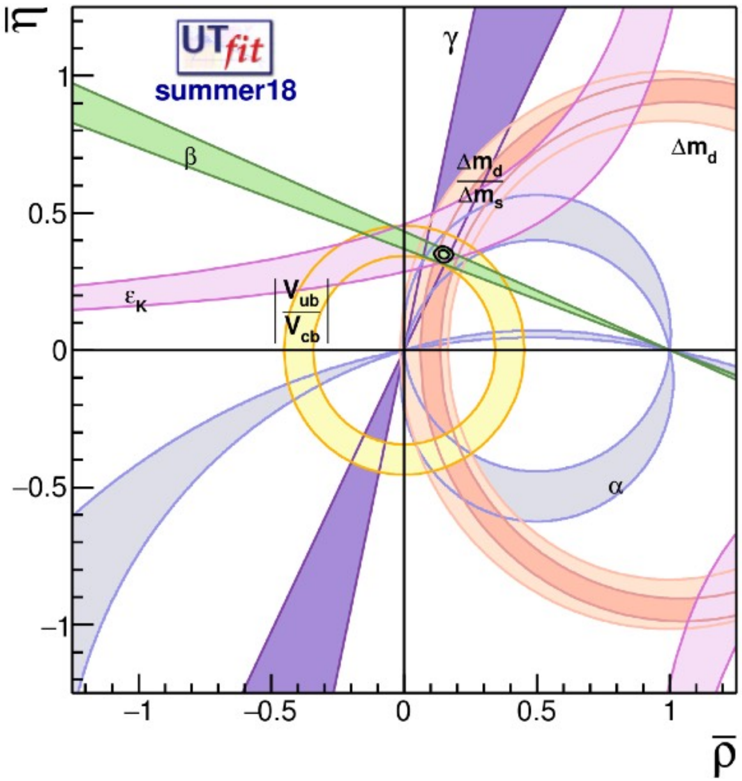
$$[CP]\phi(\vec{r},t)[CP]^\dagger = exp(i\alpha)\phi^\dagger(-\vec{r},t)$$

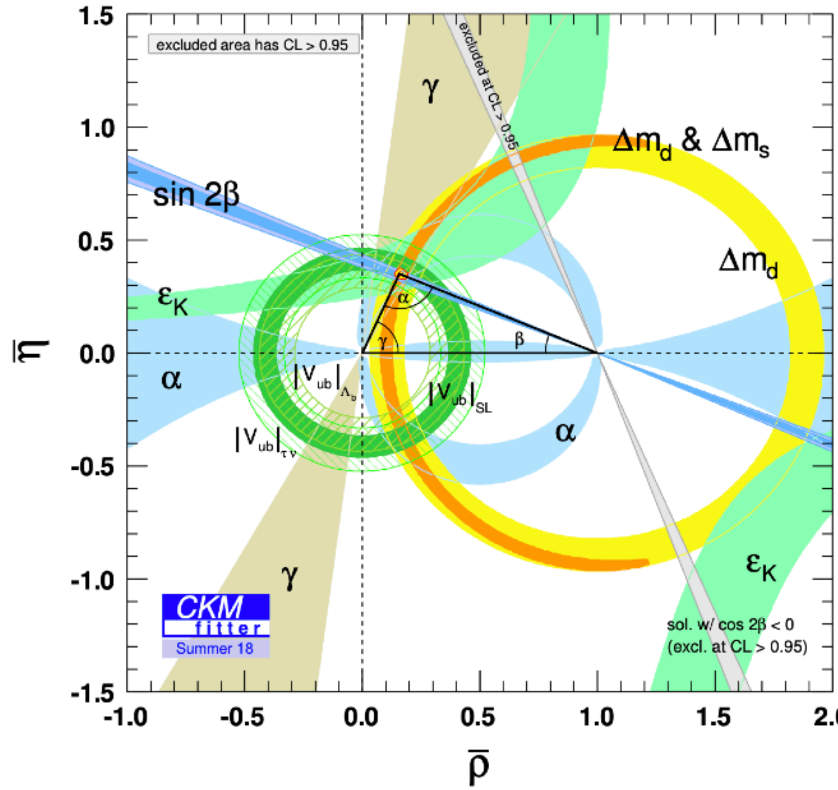
- Choosing $\alpha = 2\theta$ we can always make it invariant under CP-transformation!

★ Thus we need multi-Higgs doublet models to realize spontaneous CP breaking

★ There is a single phase of the CKM matrix for 3-generation SM

- triangle parameters can be determined via a variety of ways…





- … and even though any triangle can be completely defined by two measurements: an angle and two sides (or 3 sides or 3 angles)

A recipe for searches for New Physics

★ Flavor can be used to search for NP, not just new flavor physics!

1. Measure as many processes that depend on CKM parameters independently
2. Interpret those measurements assuming there is no NP contribution and extract the CKM parameters
3. Build CKM triangles out of those CKM parameters. If a triangle does not close, then no-NP assumption was incorrect and there is a (possible) presence of New Physics

We are NOT checking if the CKM matrix is unitary!
We are searching for NP using the CKM matrix unitarity!

Measuring CKM angles: ϕ_3 example

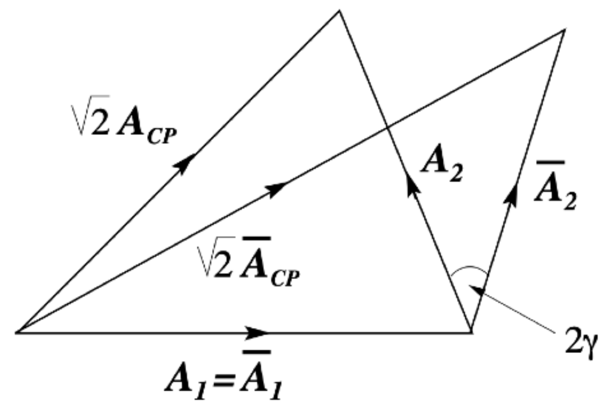
★ Many different methods: see lectures by T. Browder and S. Prell

★ One can also use a fact that initial state at Belle II is quantum coherent

- which means that initial state can be CP-tagged
- can be done for both B_d (at $\Upsilon(4S)$) or B_s (at $\Upsilon(5S)$). For B_s

$$A_{\rm CP} = A(B_s^{\rm CP} \to D_s^- K^+) = (A_1 + A_2)/\sqrt{2}\,,$$
$$\overline{A}_{\rm CP} = A(B_s^{\rm CP} \to D_s^+ K^-) = (\overline{A}_1 + \overline{A}_2)/\sqrt{2}\,.$$

with $A_1 = A(B_s \to D_s^- K^+)$ and $A_2 = A(\bar{B}_s \to D_s^- K^+)$



Falk, AAP

- measuring all amplitudes,

$$\alpha = \frac{2|A_{\rm CP}|^2 - |A_1|^2 - |A_2|^2}{2|A_1||A_2|}\,,$$
$$\overline{\alpha} = \frac{2|\overline{A}_{\rm CP}|^2 - |\overline{A}_1|^2 - |\overline{A}_2|^2}{2|\overline{A}_1||\overline{A}_2|}\,,$$

$$\sin 2\gamma = \pm\left(\alpha\sqrt{1-\overline{\alpha}^2} - \overline{\alpha}\sqrt{1-\alpha^2}\right)$$

- analysis is similar for $B_d \to D\pi$ is similar, but coefficients are time-dependent

Do we need to search for New Physics?

★ Standard Model has intrinsic problem related to Higgs mechanism (stabilization of quantum effects)

- BSM stabilization (e.g. SUSY), other mechanisms of EWSB

★ Standard Model does not have enough CP-violation to describe generation of baryon asymmetry

- need for BSM sources of CP-violation

★ Standard Model adequately describes experimental FCNC data, but does not provide solution to the flavor puzzle

- BSM solution to the flavor problem?

3. New Physics models and their consequences

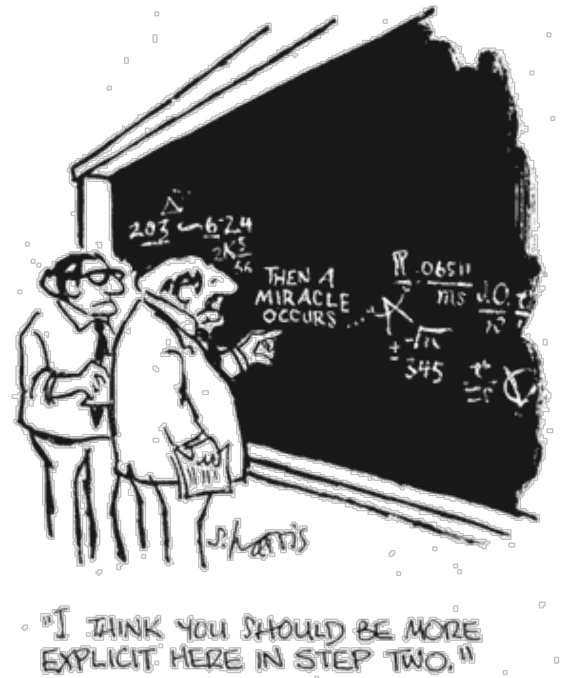
Nature

Theorist's model

(Number of possible models) > (number of model builders). How do we proceed?

NP: modify the SM solution

1. Why generations?
   - Why only 3?
   - Are there only 3?
2. Why hierarchies of masses and mixings?



S. Harris

$$\mathcal{L}_1 = -y_\psi \bar{\psi}_L \psi_R \phi + h.c. \to -\frac{y_\psi v}{\sqrt{2}} \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right),$$

$$m_\psi = y_\psi v / \sqrt{2}$$

No explanation of the hierarchy, but mass hierarchy is related to the hierarchy of Yukawa couplings

$$y_u \sim 10^{-5}, \quad y_c \sim 10^{-2}, \quad y_t \sim 1,$$
$$y_d \sim 10^{-5}, \quad y_s \sim 10^{-3}, \quad y_b \sim 10^{-2},$$
$$y_e \sim 10^{-6}, \quad y_\mu \sim 10^{-3}, \quad y_\tau \sim 10^{-2}.$$

NP flavor model building

★ GUT models: leptonic/quark Yukawas are related

★ Flavor symmetries

SM Lagrangian is $SU(3)^5$-invariant in the limit $y_i \to 0$

- Yukawas arise as a result of spontaneous breaking of a subgroup of $SU(3)^5$?

- continuous flavor symmetries
- discrete flavor symmetries
- accidental flavor symmetries

- numerology?

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$$

Koide formula (also "works" for heavy quarks)

★ Dynamical approaches

★ Geometric approaches (localization in extra dimension)

Notice that an extra scalar boson can help to solve the flavor puzzle:

$$\mathcal{L}_2 = -y_\psi \bar{\psi}_L \psi_R \phi_1 - y_\chi \bar{\chi}_L \chi_R \phi_2 + \text{h.c.}$$

Then assuming $\tan\beta \gg 1$

$$\frac{m_\chi}{m_\psi} = \frac{y_\chi}{y_\psi}\frac{v_2}{v_1} = \frac{y_\chi}{y_\psi}\tan\beta \gg 1$$

So it looks like we can solve the flavor puzzle by just having more scalar bosons, letting all Yukawa couplings be $\mathcal{O}(1)$ and $\tan\beta \gg 1$

Top quark: Das, Kao, Phys. Lett. B 392 (1996) 106.
Xu, Phys. Rev. D44, R590 (1991).
Blechman, AAP, Yeghiyan, JHEP 1011 (2010) 075

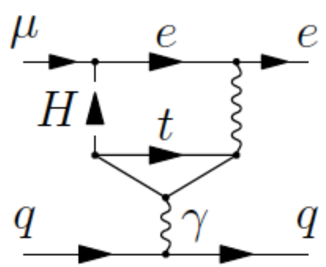
NP models and high energy processes

★ Leptonic FCNC could be generated by New Physics

✦ Ex.1 FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$ Harnik, Kopp, Zupan

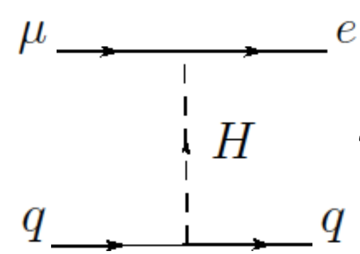
‣ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

✦ Ex.2 Exceptional couplings of (flavor-diagonal) NP to third generation $\mathcal{H}_{\rm NP} = G\bar{b}'_L\gamma^\lambda b'_L\bar{\tau}'_L\gamma_\lambda\tau'_L$ flavor "anomalies" Glashow, Guadagnoli, Lane

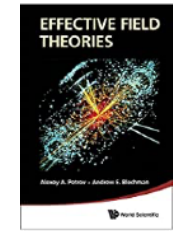
✦ Ex.3 Leptoquarks -> flavor "anomalies"
Muon collider?

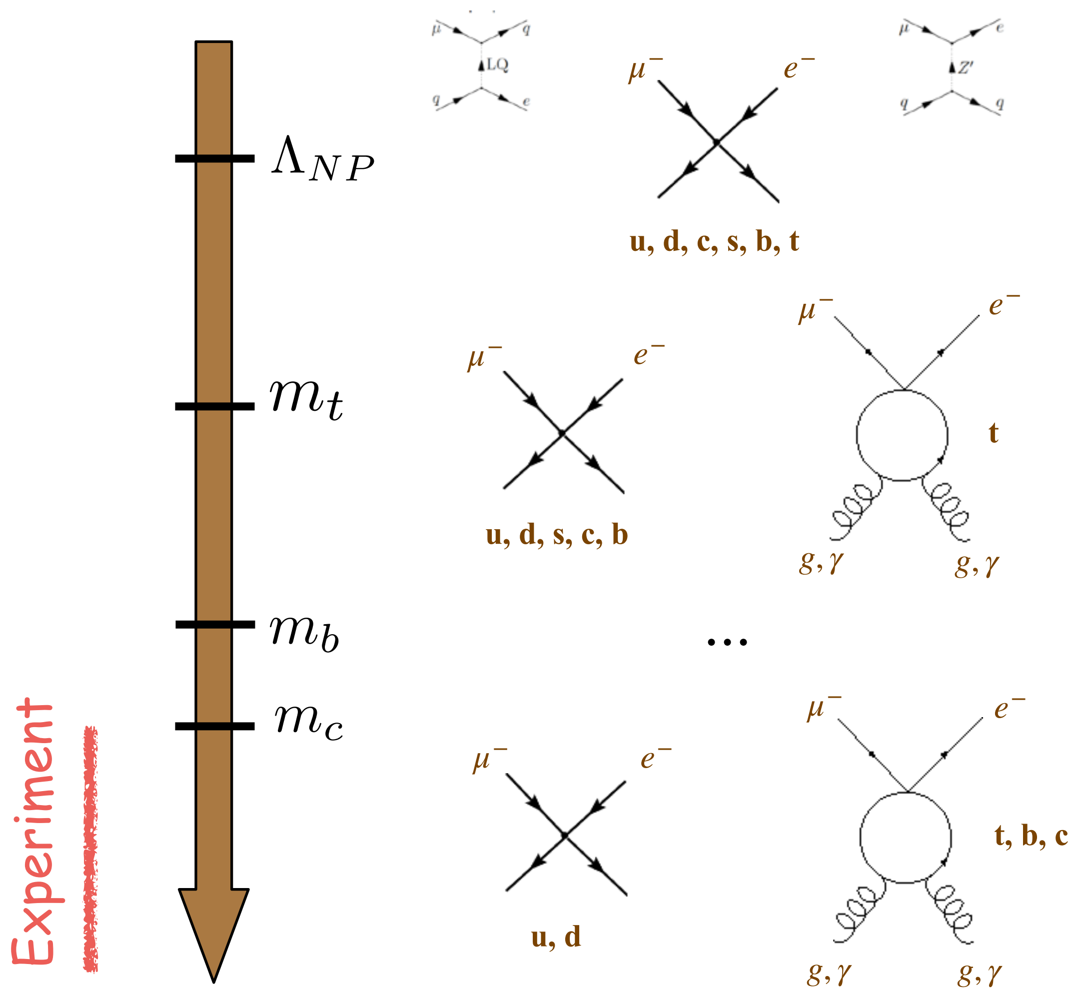
(Number of possible models) > (number of model builders). How do we proceed?

Models and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories

- ★ It is important to understand ALL relevant energy scales for the problem at hand





New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

...

heavy quarks decouple

...

Scales associated with experiment

Possibilities for New Physics

★ There is New Physics around the corner

low-energy SUSY, KK modes of extra-D theories, etc,

★ There is New Physics far away

Left-Right models, (also anything from above), desert

★ There is no New Physics

SM plus a right-handed neutrino are perfectly fine by themselves

★ There are no new scales

- conformal symmetry, anyone?

Which one gives the right solution?

Effective Lagrangians: probing all NP models

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and $\psi^2 H^3$.

X^3		H^6 and H^4D^2		ψ^2H^3 + h.c.	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, ψ^2XH, and ψ^2H^2D.

X^2H^2		ψ^2XH + h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p\gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p\tau^I\gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{H} G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p\gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\tau^I\widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p\gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p\tau^I\gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A d_r\right) H G_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p\gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p\gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right) H B_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p\gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

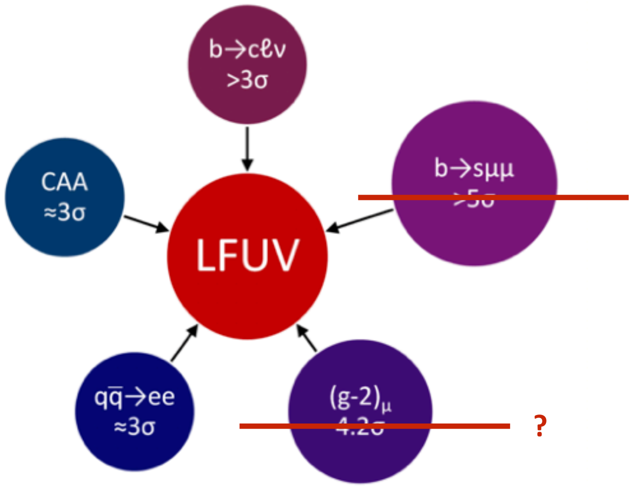
$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{L}_s\gamma^\mu L_t\right)$	Q_{ee}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$	Q_{le}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{uu}	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{lu}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p\gamma^\mu\tau^I Q_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p\gamma^\mu d_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{eu}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{qe}	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p\gamma^\mu\tau^I L_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{ed}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
		$Q_{ud}^{(1)}$	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p\gamma^\mu T^A q_r\right)\left(\overline{u}_s\gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p\gamma^\mu T^A u_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p\gamma^\mu q_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p\gamma^\mu T^A Q_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

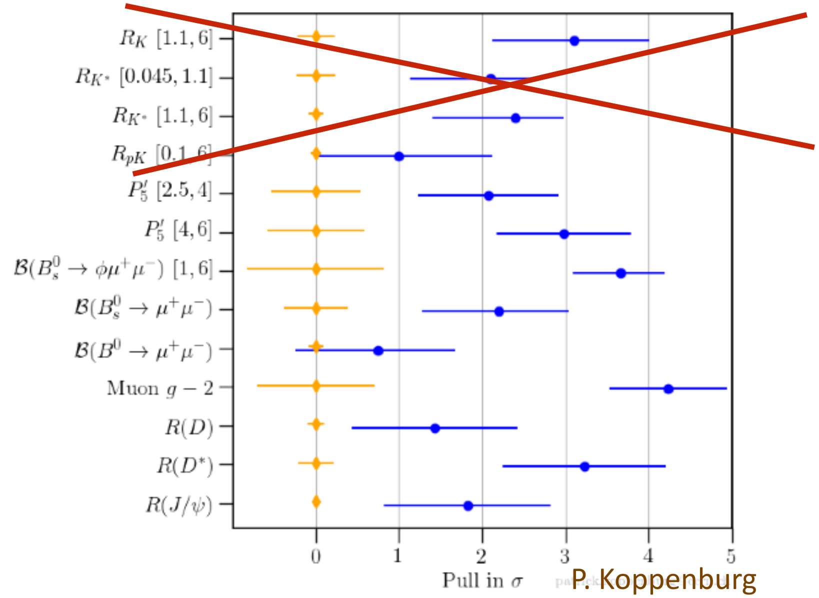
$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left((\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left((\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left((\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left((\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left((\overline{L}_p^j\sigma_{\mu\nu}e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k\sigma^{\mu\nu}u_t\right)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Recent experimental anomalies

★ Many experimental anomalies involve interactions with muons and taus

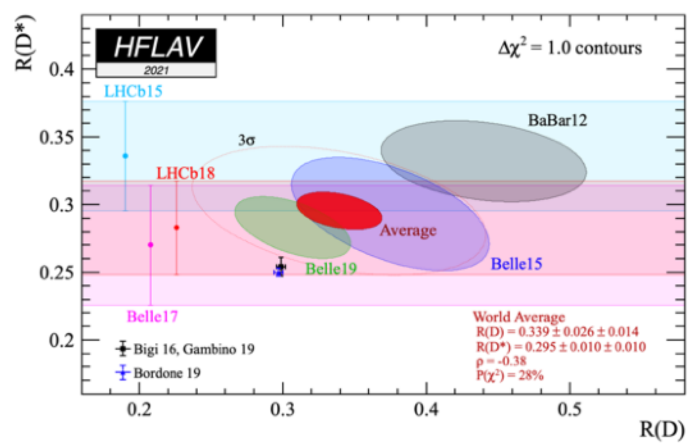


Crivellin, Hoferichter



P. Koppenburg

- other lepton-flavor conserving processes
  - magnetic properties: muon g-2
    - currently a discrepancy theory/exp
  - electric properties: muon EDM
    - probes CP-violation in leptons
  - muonic hydrogen
    - proton size/QED/New Physics



How to search for NP with leptons?

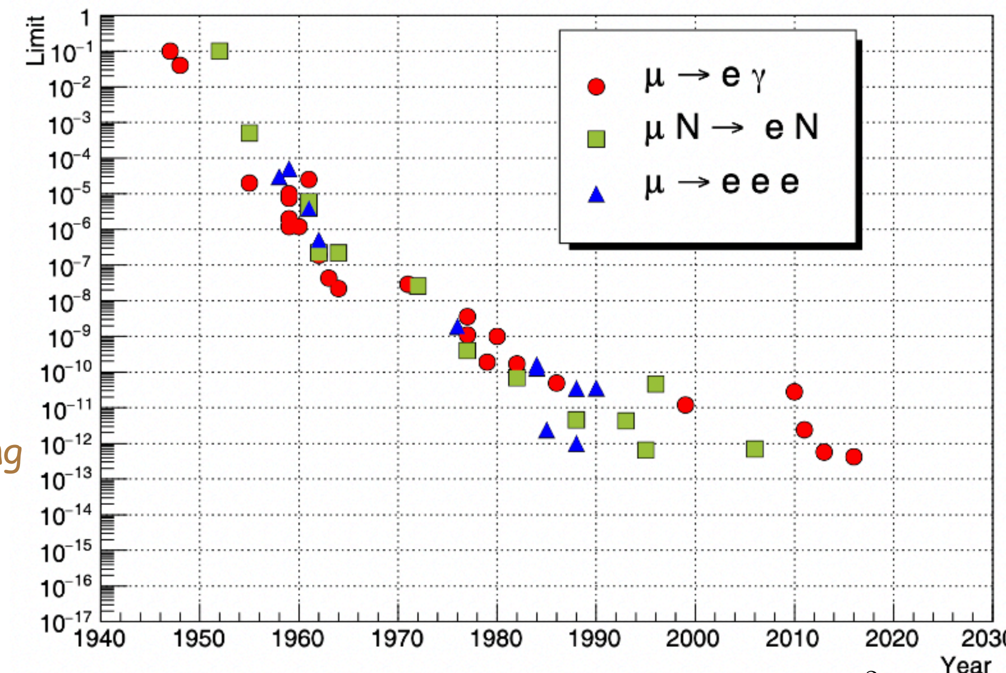
4. Lepton flavor violation

★ Leptons can help solve the most fundamental problems in particle physics! Flavor?

★ Possible experimental searches for Charged Lepton Flavor Violation (CLFV)

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+e^- \to e^-\mu^+$ (muonium oscillations)
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm2) + e^{\mp}e^{\mp}$
  - $\mu^- + (A, Z) \to e^+ + (A, Z-2)$

LORENZO CALIBBI and GIOVANNI SIGNORELLI



★ Decays are highly suppressed in the Standard Model: $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i}U_{ei}\frac{m^2_{\nu_i}}{M^2_W}\right|^2 < 10^{-54}$

★ But: no trivial FCNC vertices in the Standard Model: sensitive tests of New Physics!

Again: effective Lagrangians: probing all NP models

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \dots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and ψ^2H^3.

X^3		H^6 and H^4D^2		ψ^2H^3 + h.c.	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	Q_{eH}	$(H^\dagger H)(\bar{L}_p e_r H)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{uH}	$(H^\dagger H)(\bar{Q}_p u_r \widetilde{H})$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	Q_{HD}	$(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$	Q_{dH}	$(H^\dagger H)(\bar{Q}_p d_r H)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, ψ^2XH, and ψ^2H^2D.

X^2H^2		ψ^2XH + h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{L}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{L}_p\gamma^\mu L_r)$
$Q_{H\widetilde{G}}$	$H^\dagger H\widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{L}_p\sigma^{\mu\nu}e_r) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{L}_p\tau^I\gamma^\mu L_r)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{Q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H} G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{H\widetilde{W}}$	$H^\dagger H\widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{Q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{Q}_p\gamma^\mu Q_r)$
Q_{HB}	$H^\dagger H B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{Q}_p\sigma^{\mu\nu}u_r)\widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{Q}_p\tau^I\gamma^\mu Q_r)$
$Q_{H\widetilde{B}}$	$H^\dagger H\widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{Q}_p\sigma^{\mu\nu}T^A d_r) H G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{Q}_p\sigma^{\mu\nu}d_r)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{Q}_p\sigma^{\mu\nu}d_r) H B_{\mu\nu}$	Q_{Hud}	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

TABLE 2.5 Four-fermion operators, classes $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$, and $(\bar{L}L)(\bar{R}R)$.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{L}_p\gamma^\mu L_r)(\bar{L}_s\gamma^\mu L_t)$	Q_{ee}	$(\bar{e}_p\gamma^\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{L}_p\gamma^\mu L_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{Q}_p\gamma^\mu Q_r)(\bar{Q}_s\gamma^\mu Q_t)$	Q_{uu}	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{L}_p\gamma^\mu L_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{Q}_p\gamma^\mu\tau^I Q_r)(\bar{Q}_s\gamma^\mu\tau^I Q_t)$	Q_{dd}	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{L}_p\gamma^\mu L_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{L}_p\gamma^\mu L_r)(\bar{Q}_s\gamma^\mu Q_t)$	Q_{eu}	$(\bar{e}_p\gamma^\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{Q}_p\gamma^\mu Q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{L}_p\gamma^\mu\tau^I L_r)(\bar{Q}_s\gamma^\mu\tau^I Q_t)$	Q_{ed}	$(\bar{e}_p\gamma^\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{Q}_p\gamma^\mu Q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{Q}_p\gamma^\mu T^A Q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\bar{L}R)(\bar{R}L)$		B-violating	
Q_{ledq}	$((\bar{L}_p^j e_r)(\bar{d}_s Q_t^j)$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(Q_s^{\gamma j})^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$((\bar{Q}_p^j u_r)\epsilon_{jk}(\bar{Q}_s^k d_t)$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(Q_p^{\alpha j})^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$((\bar{Q}_p^j T^A u_r)\epsilon_{jk}(\bar{Q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[(Q_p^{\alpha j})^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$((\bar{L}_p^j e_r)\epsilon_{jk}(\bar{Q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}(\tau^I\epsilon)_{jk}(\tau^I\epsilon)_{mn}\left[(Q_p^{\alpha j})^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$((\bar{L}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{Q}_s^k\sigma^{\mu\nu}u_t)$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

Flavor violation and effective Lagrangians

★ Radiative FCNC decays of leptons $\ell_1 \to \ell_2 + \gamma$ (tau decays at Belle II)

- the most general amplitude is

$$A_{\ell_1 \to \ell_2 \gamma}(p,p') = \frac{i}{m_{\ell_1}} \overline{u}_{\ell_2}(p') \left[A_L P_L + A_R P_R\right] \sigma_{\mu\nu} q^\nu u_{\ell_1}(p) \epsilon^{*\mu},$$

- which leads to the decay rate

$$\Gamma(\ell_1 \to \ell_2 \gamma) = \frac{m_{\ell_1}}{16\pi} \left(|A_L|^2 + |A_R|^2\right)$$

with $$A_R = A_L^* = \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} \left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi}\right) \equiv \sqrt{2}\, \frac{v m_i^2}{\Lambda^2} C_\gamma^{fi}$$

Effective coupling (example)	Bounds on Λ (TeV) (for $\lvert C_{ij}^6\rvert = 1$)	Bounds on $\lvert C_{ij}^6\rvert$ (for $\Lambda = 1$ TeV)	Observable
$C_{e\gamma}^{\mu e}$	6.3×10^4	2.5×10^{-10}	$\mu \to e\gamma$
$C_{e\gamma}^{\tau e}$	6.5×10^2	2.4×10^{-6}	$\tau \to e\gamma$
$C_{e\gamma}^{\tau\mu}$	6.1×10^2	2.7×10^{-6}	$\tau \to \mu\gamma$
$C_{\ell\ell,ee}^{\mu eee}$	207	2.3×10^{-5}	$\mu \to 3e$
$C_{\ell\ell,ee}^{e\tau ee}$	10.4	9.2×10^{-5}	$\tau \to 3e$
$C_{\ell\ell,ee}^{\mu\tau\mu\mu}$	11.3	7.8×10^{-5}	$\tau \to 3\mu$
$C_{(1,3)H\ell}^{\mu e}, C_{He}^{\mu e}$	160	4×10^{-5}	$\mu \to 3e$
$C_{(1,3)H\ell}^{\tau e}, C_{He}^{\tau e}$	≈ 8	1.5×10^{-2}	$\tau \to 3e$
$C_{(1,3)H\ell}^{\tau\mu}, C_{He}^{\tau\mu}$	≈ 9	$\approx 10^{-2}$	$\tau \to 3\mu$

Teixeira; Feruglio, Paradisi, Pattori

Other interesting modes that probe similar couplings: $\ell_1 \to \ell_2\gamma\gamma$, $\ell_1 \to 3\ell_2$, and others

Lepton-flavor violation with meson decays

★ There are many effective operators, so a single operator dominance hypothesis (SODH) is usually applied to get constraints on relevant Wilson coefficients.

$$
\begin{aligned}
\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_{q_1,q_2} \Big[& \left(C_{VR}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \gamma^\mu P_L \ell_2 \right)\ \overline{q}_1 \gamma_\mu q_2 \\
& + \left(C_{AR}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \gamma^\mu P_L \ell_2 \right)\ \overline{q}_1 \gamma_\mu \gamma_5 q_2 \\
& + m_2 m_{q_H} G_F \left(C_{SR}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 P_L \ell_2 + C_{SL}^{q \ell_1 \ell_2}\ \overline{\ell}_1 P_R \ell_2 \right)\ \overline{q}_1 q_2 \\
& + m_2 m_{q_H} G_F \left(C_{PR}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 P_L \ell_2 + C_{PL}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 P_R \ell_2 \right)\ \overline{q}_1 \gamma_5 q_2 \\
& + m_2 m_{q_H} G_F \left(C_{TR}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q_1 q_2 \ell_1 \ell_2}\ \overline{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right)\ \overline{q}_1 \sigma_{\mu\nu} q_2 \ + \ h.c. \Big].
\end{aligned}
$$

- Can (partially) do away with SODH if designer initial/final states are used
- This can be done in case of restricted kinematics (e.g. 2-body decays)…
- … except for the choice of initial states for $q_1 \neq q_2$ is limited (only pseudoscalars)

Kinematically-constrained B/D/... decays

★ Constraints are available for quark off-diagonal currents from B/D → μe, τe, etc.

- most general parameterization, as before:

$$\mathcal{A}(P \to \ell_1 \bar{\ell}_2) = \overline{u}(p_1, s_1) \left[E_P^{q_1 q_2 \ell_1 \ell_2} + i F_P^{q_1 q_2 \ell_1 \ell_2} \gamma_5 \right] v(p_2, s_2)$$

- form-factors depend on Wilson coefficients (no gluonic operators):

$$E_P^{q_1 q_2 \ell_1 \ell_2} = \kappa_P \frac{m_P f_P y}{2\Lambda^2} \left[\left(C_{AL}^{q_1 q_2 \ell_1 \ell_2} + C_{AR}^{q_1 q_2 \ell_1 \ell_2} \right) + m_P^2 G_F \left(C_{PL}^{q_1 q_2 \ell_1 \ell_2} + C_{PR}^{q_1 q_2 \ell_1 \ell_2} \right) \right],$$

$$F_P^{q_1 q_2 \ell_1 \ell_2} = i \kappa_P \frac{m_P f_P y}{2\Lambda^2} \left[\left(C_{AL}^{q_1 q_2 \ell_1 \ell_2} - C_{AR}^{q_1 q_2 \ell_1 \ell_2} \right) + m_P^2 G_F \left(C_{PL}^{q_1 q_2 \ell_1 \ell_2} - C_{PR}^{q_1 q_2 \ell_1 \ell_2} \right) \right].$$

★ Experimental data exist for most transitions B/D → μe, τe, etc.

$\ell_1 \ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(B_d^0 \to \ell_1 \ell_2)$	2.2×10^{-5}	2.8×10^{-5}	1.0×10^{-9}
$\mathcal{B}(B_s^0 \to \ell_1 \ell_2)$	$\cdots$	$\cdots$	5.4×10^{-9}
$\mathcal{B}(\bar{D}^0 \to \ell_1 \ell_2)$	FPS	$\cdots$	1.3×10^{-8}
$\mathcal{B}(K_L^0 \to \ell_1 \ell_2)$	FPS	FPS	4.7×10^{-12}

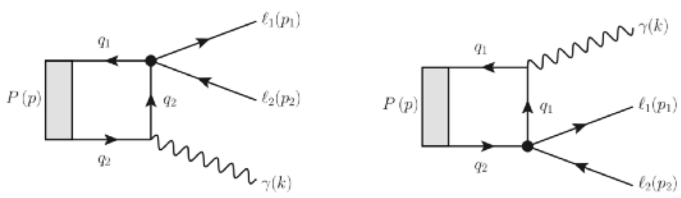
Kinematically-constrained B/D/... decays

★ Constraints from B/D → μe, τe, etc. on WC can be obtained (with SODH)

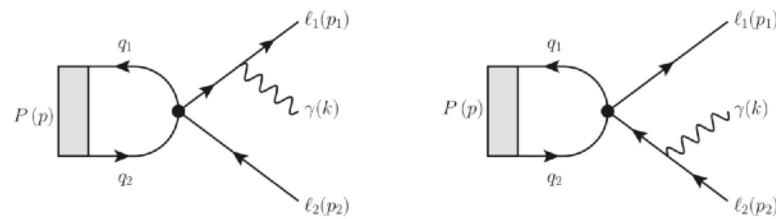
	Leptons	Initial state			
Wilson coefficient	$\ell_1\ell_2$	$B_d^0\ (d\bar{b})$	$B_s^0\ (s\bar{b})$	$\bar{D}^0\ (u\bar{c})$	$K_L^0\ ((d\bar{s}-s\bar{d})/\sqrt{2})$
$\left\|C_{AL}^{q_1q_2\ell_1\ell_2}/\Lambda^2\right\|$	$\mu\tau$	2.3×10^{-8}	$\cdots$	FPS	FPS
	$e\tau$	2.6×10^{-8}	$\cdots$	$\cdots$	FPS
	$e\mu$	2.3×10^{-9}	4.4×10^{-9}	2.4×10^{-8}	5.0×10^{-12}
$\left\|C_{AR}^{q_1q_2\ell_1\ell_2}/\Lambda^2\right\|$	$\mu\tau$	2.3×10^{-8}	$\cdots$	FPS	FPS
	$e\tau$	2.6×10^{-8}	$\cdots$	$\cdots$	FPS
	$e\mu$	2.3×10^{-9}	4.4×10^{-9}	2.4×10^{-8}	5.0×10^{-12}
$\left\|C_{PL}^{q_1q_2\ell_1\ell_2}/\Lambda^2\right\|$	$\mu\tau$	7.1×10^{-5}	$\cdots$	FPS	FPS
	$e\tau$	8.0×10^{-5}	$\cdots$	$\cdots$	FPS
	$e\mu$	7.1×10^{-6}	1.3×10^{-5}	5.9×10^{-4}	1.7×10^{-6}
$\left\|C_{PR}^{q_1q_2\ell_1\ell_2}/\Lambda^2\right\|$	$\mu\tau$	7.1×10^{-5}	$\cdots$	FPS	FPS
	$e\tau$	8.0×10^{-5}	$\cdots$	$\cdots$	FPS
	$e\mu$	7.1×10^{-6}	1.3×10^{-5}	5.9×10^{-4}	1.7×10^{-6}

Can we use other decays to avoid using SODH?

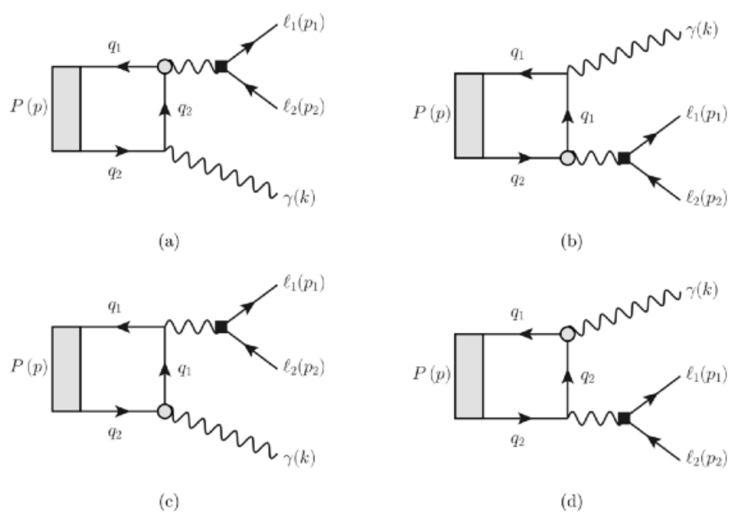
★ In general, lots of possible contributions to the invariant amplitudes (examples)



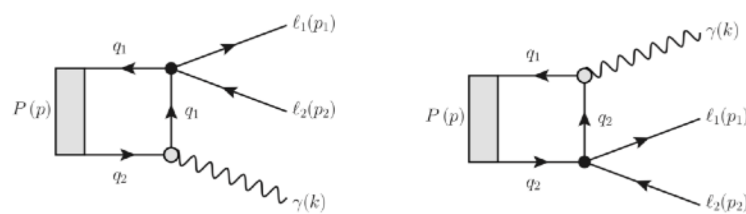
structure dependent ($l_1 l_2 q_1 q_2$)



bremsstrahlung-type ($l_1 l_2 q_1 q_2$)



dipole (small)



SM dipole ($l_1 l_2 qq$)

★ Complicated expression: single operator dominance again? Experiment?

Looking for New Physics with flavor

★ How can one use flavor data to test New Physics models?

1. Processes allowed in the Standard Model at tree level
   - relations, valid in the SM, but not necessarily in general
   - processes where SM rates and uncertainties are known
   - example: CKM triangle relations

2. Processes forbidden in the Standard Model at tree level
   - can be used for testing both heavy and light NP
   - example: penguin-mediated decays, D-mixing, etc.

3. Processes forbidden in the Standard Model to all orders
   - example: $D^0 \to p^+ \pi^- \nu$

★ Even if LHC discovers NP particles, flavor constraints will help identification

Rembrandt "Old Man in Military Costume",
BNL/DESY X-ray study

Accurate analysis of (flavor) data might reveal hidden layers of something previously unknown.

5. Conclusions and things to take home

- Flavor puzzle is still a big problem for particle physics
  - The reason(s) for generations and mass hierarchy are not known
  - Standard Models simply parameterizes the solution
  - New Physics models use flavor as input, not output
- Flavor-changing neutral current transitions provide great opportunities for studies of flavor in the SM and BSM
  - several anomalies in B physics might point to New Physics “around the corner”
  - studies of charmed transitions experience explosive growth
    - unique access to up-type quark sector
    - large available statistics/in many cases small SM background
    - large contributions from New Physics are possible, but not seen
- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Maybe flavor physics will be the first to see glimpses of New Physics
- Maybe flavor physics will be the only game in town to see New Physics...


FEDERAL
GALAXY
TOP NEWS
ENLIST
EXIT
Indirect Searches for New Physics
Alexey A. Petrov
CRC Press
EFFECTIVE FIELD THEORIES
Alexey A. Petrov • Andrew E. Blechman
World Scientific
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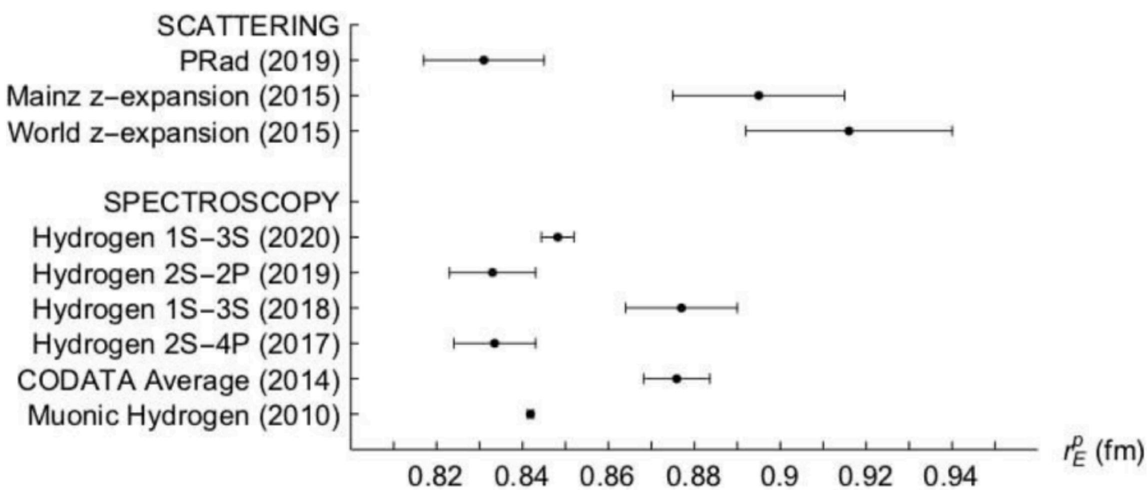
EXTRA!

Muons and recent experimental anomalies

★ Proton's radius from muonic hydrogen: possible New Physics?

★ Level splittings (e.g. Lamb shift) are sensitive to the charge radius of the proton



★ They are also sensitive to QED radiative corrections

★ Are there possible light New Physics particles that are responsible for this difference?

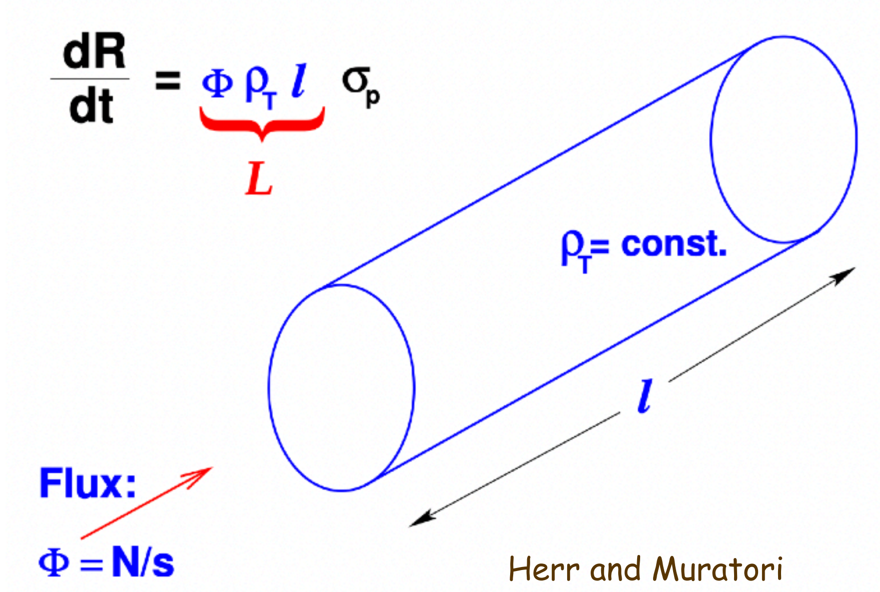
Barger et al, PRL 106 (2011) 153001

Remove proton radius issue from the problem: atomic physics with muonium?

Experimental studies of rare processes: luminosity

★ Need a lot of muons: high luminosity experiments

– Number of events/second



	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$
Tevatron ($p\bar{p}$)	1000x1000	$50\ 10^{30}$
HERA (e^+p)	30x920	$40\ 10^{30}$
LHC (pp)	7000x7000	$10000\ 10^{30}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$
PEP (e^+e^-)	9x3	$3000\ 10^{30}$
KEKB (e^+e^-)	8x3.5	$10000\ 10^{30}$
eRHIC		10^{33}-10^{35}

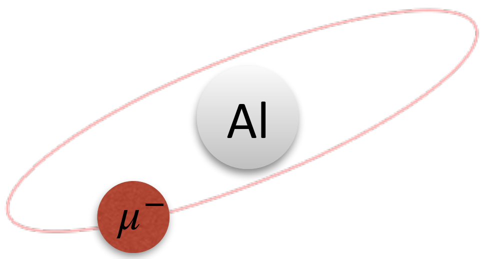
– ... or another way $L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$

What if incident particles formed bound states with target particles?

Bound states: muon conversion

- How effective is this approach compared to scattering?
  - let's compute effective luminosity
  - recall that

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$



  - in this "experiment" the probability density is given by the 1s wave function
  - … and we need to take into account the fact that muon decays
  - Then luminosity = (density)(velocity)(flux of muons)(lifetime)

$$L_{\text{eff}} = |\psi(0)|^2 \times \alpha Z \times \Phi_\mu \times \tau_\mu = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \Phi_\mu \tau_\mu$$

  - For Al target (Z=13), flux of $\Phi_\mu = 10^{10}$ muons/sec and $\tau_\mu = 2\ \mu$sec

$$L_{\text{eff}} = 10^{48} \text{cm}^{-2}\ \text{sec}^{-1}$$

Bernstein, Czarnecki

- A possibility of using muon beams at CMP facilities

Jian Tang, talk at RPPM meeting (Snowmass 2021)

	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

×5 CSNS-II upgrade

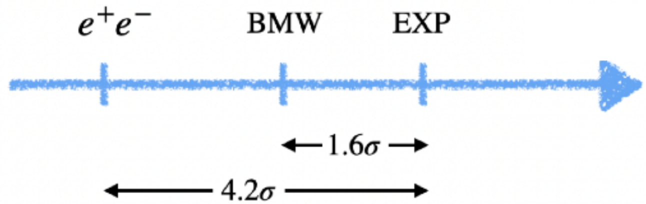
Facility	Source Type	Intensity (μ+/sec)*
ISIS	pulsed	1.5×10^6
J-PARC	continuous	1.8×10^6
PSI	continuous	7.0×10^4
TRIUMF	pulsed	5.0×10^6
SEEMS	pulsed	1.9×10^8

- Muonium Antimuonium Conversion Experiment (MACE) EMuS at CSNS

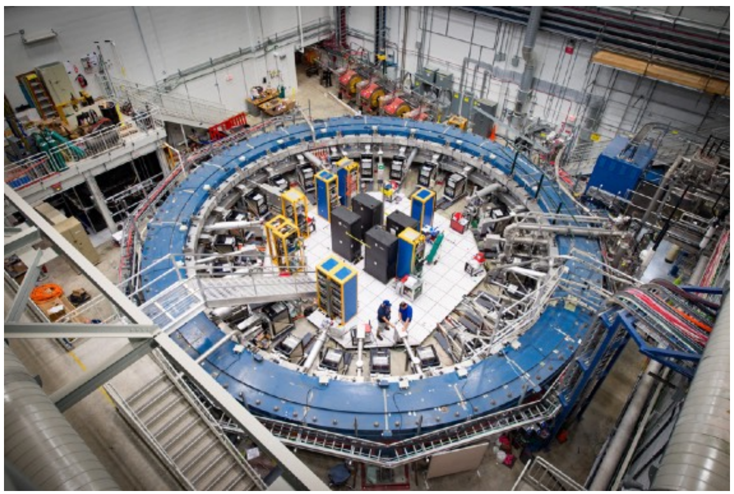
Muons and recent experimental anomalies

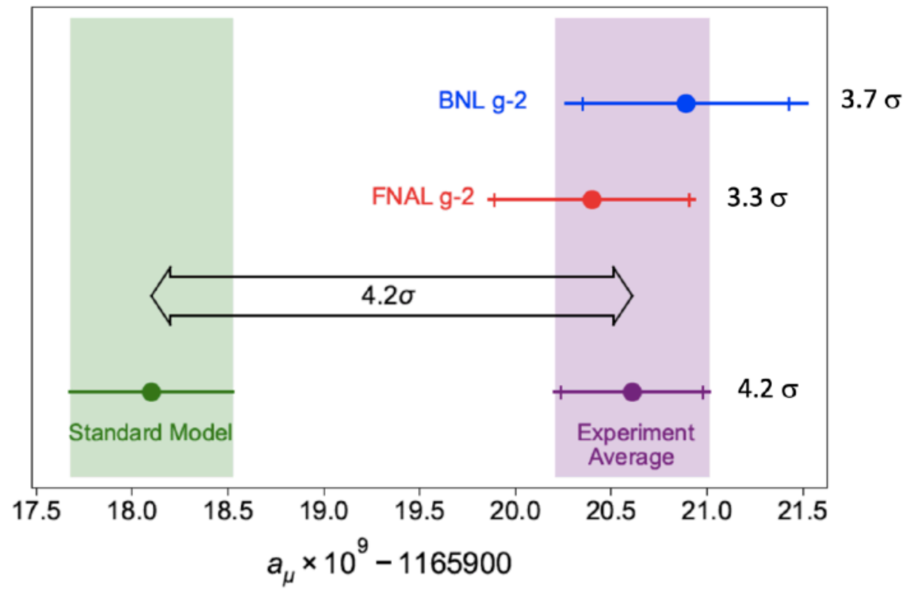
★ Muon's magnetic properties (g-2): $a_\mu = (g-2)/2$ with $\vec{\mu} = g\frac{e}{2m}\vec{s}$

discrepancy



Di Luzio, et al.





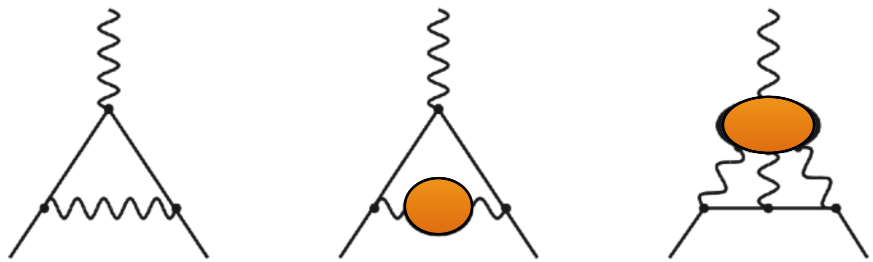
FNAL (g-2): $a_\mu(\mathrm{Exp}) = 116592061(41) \times 10^{-11}$

$a_\mu(\mathrm{Theory}) = 116591810(43) \times 10^{-11}$

$a_\mu(\mathrm{BMW}) = 116591954(55) \times 10^{-11}$

Are there possible New Physics particles that are responsible for this difference?

- Independent lattice computations of HVP
- Data-driven estimates of hadronic vacuum polarization (HVP)
  - discrepancy between KLOE and BaBar data used in HVP

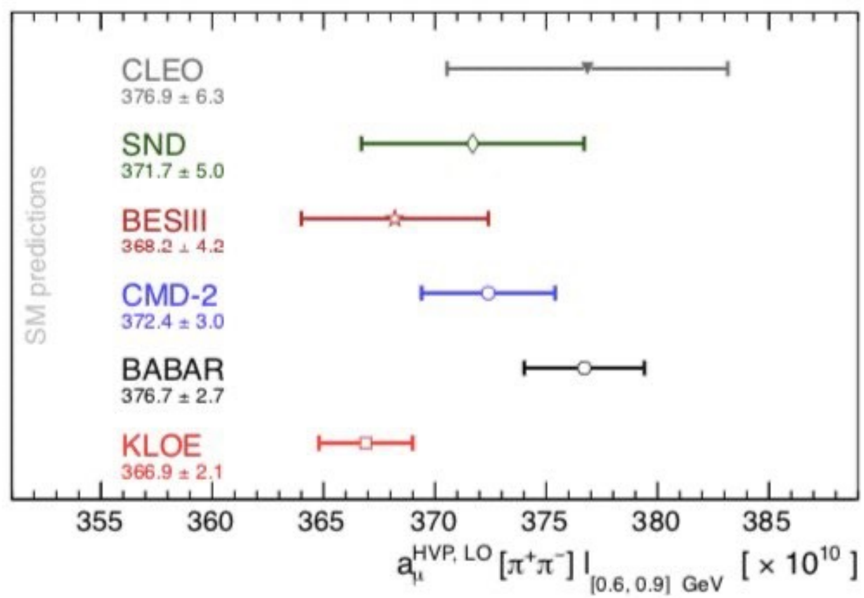


Leading order | Vacuum polarization | Light-by-light scattering

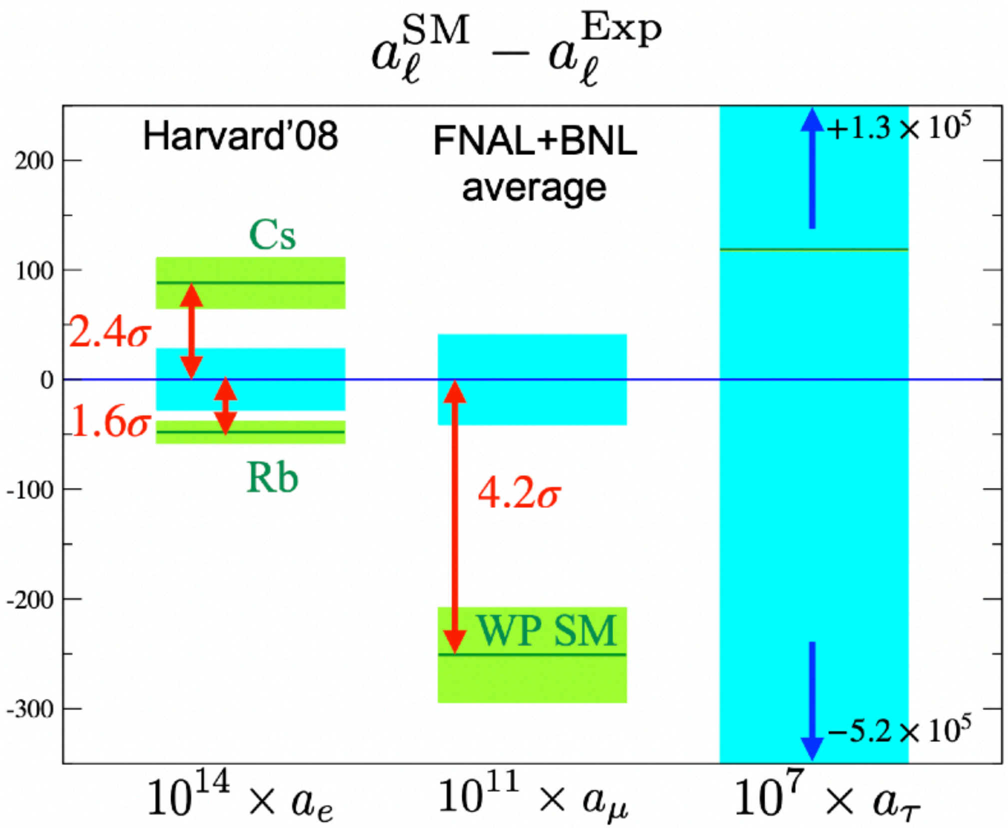
$$a_\mu^{hvp} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$\frac{1}{12\pi} R(s) = \frac{1}{12\pi} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)m_\mu^2}{x^2 m_\mu^2 + (1-x)s}$$



  - need radiative return Belle II data to eliminate the discrepancy
  - τ-decay data is not currently used: Belle II + lattice?



Sensitivity to heavy new physics:

$$a_\ell^{NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4\times10^4$$

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

A. El-Khadra (talk at LP21)

- Muon decay $\mu \to 3e$:

$$\Gamma\ (\mu \to 3e) =$$
$$= \frac{\alpha m_\mu^5}{3\Lambda^4(4\pi)^2}\left(|C_{DL}|^2 + |C_{DR}|^2\right)\left(8\log\left[\frac{m_\mu}{m_e}\right] - 11\right)$$
$$+ \frac{4m_\mu^5}{3\Lambda^4(16\pi)^3}\left(m_e^4 G_F^2\left(|C_{SR}^e|^2 + |C_{SL}^e|^2\right)\right.$$
$$+ 2\left(2\left(|C_{VR}^e|^2 + |C_{VL}^e|^2 + |C_{AR}^e|^2 + |C_{AL}^e|^2\right) + |C_{AR}^e + C_{VR}^e|^2 + |C_{AL}^e - C_{VL}^e|^2\right)\Big)$$
$$- \frac{\sqrt{4\pi\alpha}m_\mu^5}{3\Lambda^4(4\pi)^3}\left(\Re\left[C_{DL}\left(3C_{VR}^e + C_{AR}^e\right)^*\right] + \Re\left[C_R^D\left(3C_{VL}^e - C_{AL}^e\right)^*\right]\right)$$

- Muonium decay $M_\mu^V \to e^+e^-$:

$$\Gamma\left(M_\mu^V \to e^+e^-\right) = \frac{f_M^2 M_M^3}{48\pi\Lambda^4}\left\{\frac{3}{2}|C_{VR}^e + C_{AR}^e|^2 - \frac{3}{2}|C_{VL}^e + C_{AL}^e|^2 + |2C_{VL}^e + C_{VR}^e|^2 + |2C_{AL}^e + C_{AR}^e|^2\right\}$$

- Note: different combination of Wilson coefficients!

R. Conlin, J. Osborne, AAP

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics - and QED!

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Lepton wave functions are taken as solutions of Dirac equation
- with usual substitutions $u_1(r) = r\,g(r)$ and $u_2(r) = r\,f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi_\kappa^\mu = \begin{pmatrix} g(r)\chi_\kappa^\mu(\theta,\phi) \\ i f(r)\chi_{-\kappa}^\mu(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} <$ a few $\times 10^{-17}$

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a general quark operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z\rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle\right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \qquad \int d^3\rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) \ll m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

★ Gluonic contribution can be removed removed using anomaly equation or can be computed

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a gluonic Rayleigh operator

$$\langle N| \frac{\beta_L}{4\alpha_s} G^a_{\mu\nu} G^{a\mu\nu} |N\rangle = -\frac{9}{2} \left[Z G^{(g,p)} \rho^{(p)} + (A-Z) G^{(g,n)} \rho^{(n)} \right],$$

where $G^{(g,\mathcal{N})} = \langle \mathcal{N}| \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} |\mathcal{N}\rangle \approx -189 \text{ MeV}$

★ The (coherent) conversion rate is

$$\Gamma_{(}\mu^- + (A,Z) \to e^- + (A,Z)) = \frac{4 a_N^2}{\Lambda^4} \left(|c_1|^2 + |c_3|^2 \right)$$

with $a_N = G^{(g,p)} S^{(p)} + G^{(g,n)} S^{(n)}$

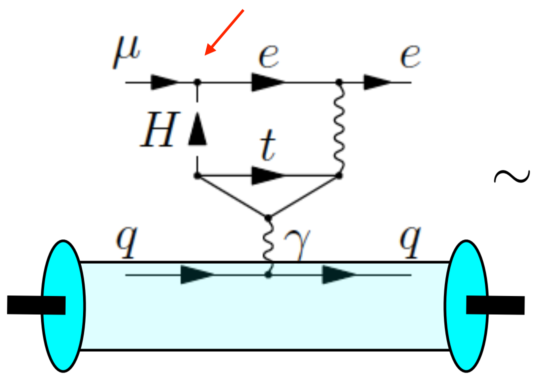
The overlap integrals $S^{(p,n)}$ with muon and electron wave functions are

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 Z \rho^{(p)} (g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr r^2 (A-Z) \rho^{(n)} (g_e^- g_\mu^- - f_e^- f_\mu^-).$$

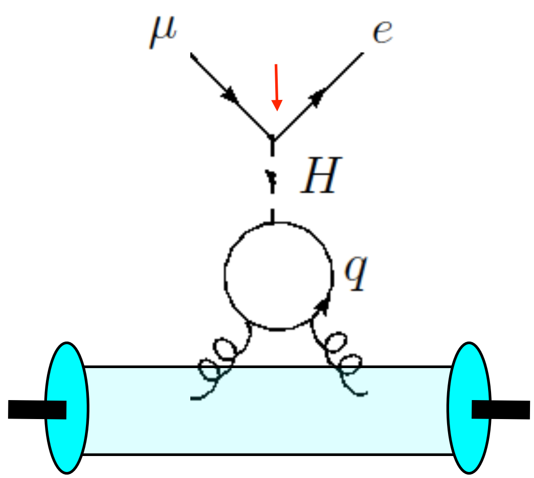
★ Contribution of heavy quarks can, in principle, be large even at low energies

★ Two-loop sensitivity to NP in muon conversion experiment...



$$\sim \mathcal{O}(\alpha^2)$$

★ ... becomes one-loop!



➡ gluonic couplings to hadrons are not (always) suppressed!
➡ NP couplings to heavy quarks are not well constrained and could be large

AAP and D. Zhuridov
PRD89 (2014) 3, 033005