

# Amplitude analysis of $\Xi_c^0 \rightarrow \Lambda^0 K^- \pi^+$

Belle II Summer workshop

July 24-28, 2023

Duke University

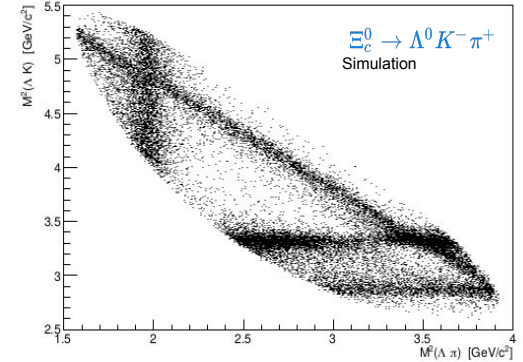
**Saroj Pokharel, Jake Bennett**

The University of Mississippi

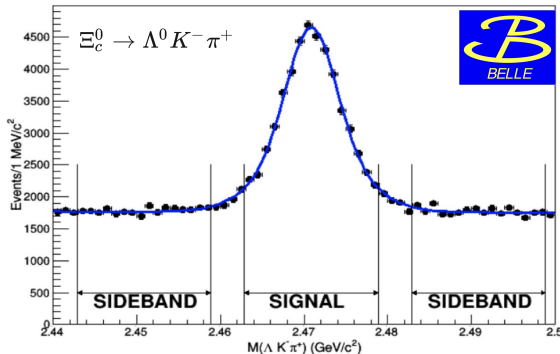


# Motivation

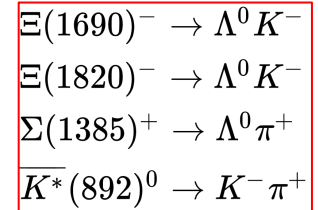
- Belle observed decays of  $\Lambda$ ,  $\Xi_c^0$ ,  $\Xi_c^+$  and  $\Omega_c$  but few have been subjected to a full amplitude analysis
  - Shed light on the existence of hyperon resonances
  - Quark structure of candidate exotic states may be better understood through the hadronic decays of charmed baryons via charm-to-strange quark transitions



$\Xi(1690)^-$  and  $\Xi(1820)^-$  are relatively poorly understood, 3-star states in the PDG



Potential resonant substructure:



- Charmed baryon decays display rich substructure of hyperon resonances
  - Belle studies of  $\Xi(1620)^0$  and  $\Xi(1690)^0$  in  $\Xi_c^+ \rightarrow (\Xi^- \pi^+) \pi^+$
  - Additional analyses looking for  $\Xi(1690)^0$  in  $\Lambda_c^+$  decays
  - Amplitude analysis of  $\Xi_c^0 \rightarrow \Xi^0 (K^+ K^-)$
  - Here we are studying  $\Xi_c^0 \rightarrow \Lambda^0 K^- \pi^+$

# Sample and selection criteria

Sample: MC15ri 1/ab  
release: light-2212-foldex

## Cuts

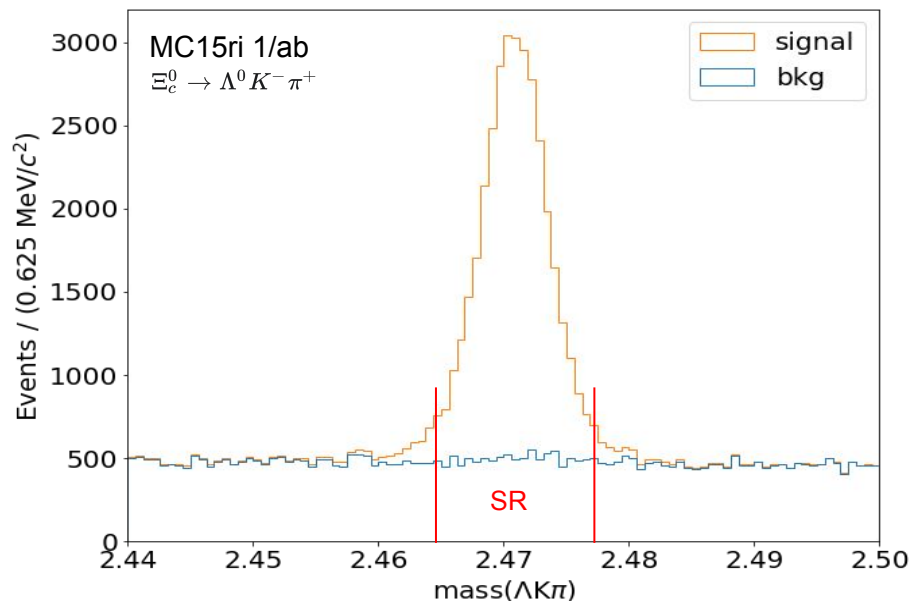
- $X_{ic\_CMS\_p} > 2.8$
- For tracks:
  - $\theta_{inCDCAcceptance}$
  - $n_{CDCHits} > 20$
  - $dr < 1$  and  $abs(dz) < 4$  (prompt only)
- For proton decaying from Lambda:
  - $protonID > 0.5$
- Treefit  $\chi Prob > 0.001$
- Binary K/ $\pi$  ID  $> 0.3$  (kaons)
- Binary  $\pi$ /K ID  $> 0.2$  (pions)
- Kaon  $pt > 0.4$
- Pion  $pt > 0.1$
- $\Lambda^0$  flight significance  $> 10$
- Trinary (K/ $\pi$  & p) ID  $> 0.3$  (kaons)

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**Signal Region** = 2.465 - 2.477 (12 MeV window) ( $5\sigma$ )

Accepted signal = 26,659

Remaining background in SR = 10,200

S/B in SR = 2.61 (Purity = 72.3%)

## Amplitude model

- Construct the amplitude as a piece describing the coupling and propagator and a piece describing the decay:

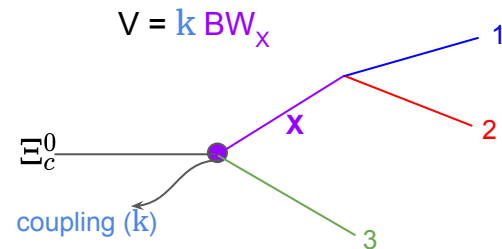
$$U^{M,\lambda_\Lambda}(\vec{x}) = \langle \Lambda K^+ \pi^- | H | \Xi_c \rangle = \sum_{j_X, \lambda_X} T_{j_X, \lambda_X} \Theta_J^{M, \lambda_X}(\theta_X, \phi_X) \times g_{j_X, \lambda_X} \Phi_{j_X}^{\lambda_X, \lambda_\Lambda}(\theta_\Lambda, \phi_\Lambda)$$

- Then treat the **coupling as a free parameter**, so the amplitude becomes

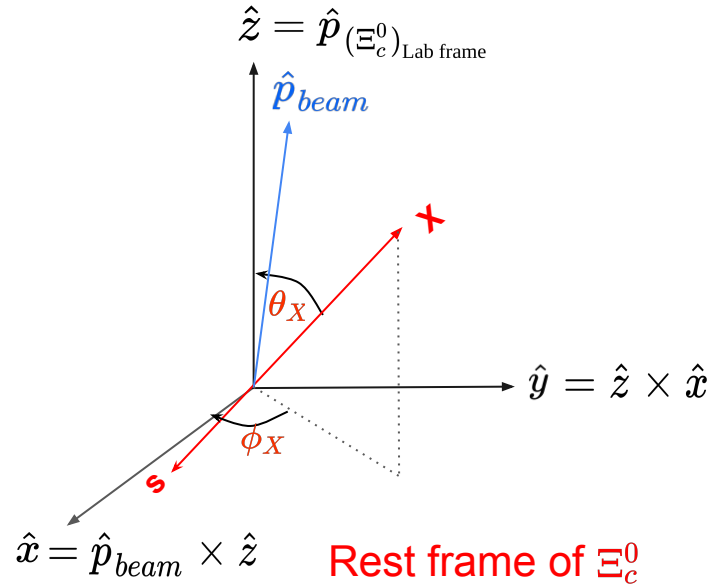
$$U^{M,\lambda_\Lambda}(\vec{x}) = \sum_{j_X, \lambda_X} V_{j_X, \lambda_X} A_{j_X, \lambda_X}^{M, \lambda_\Lambda}(\vec{x})$$

$$V_{j_X, \lambda_X} = k_{j_X, \lambda_X} BW_{j_X, \lambda_X}$$

$$A^{M, \lambda_\Lambda}(\vec{x}) = \sum_{j_X, \lambda_X} N_J D_{M, \lambda_X}^j(\phi_X, \theta_X, 0) N_{j_X} D_{\lambda_X, \lambda_\Lambda}^{j_X}(\phi_\Lambda, \theta_\Lambda, 0)$$



# Angular distribution



# Angular distribution

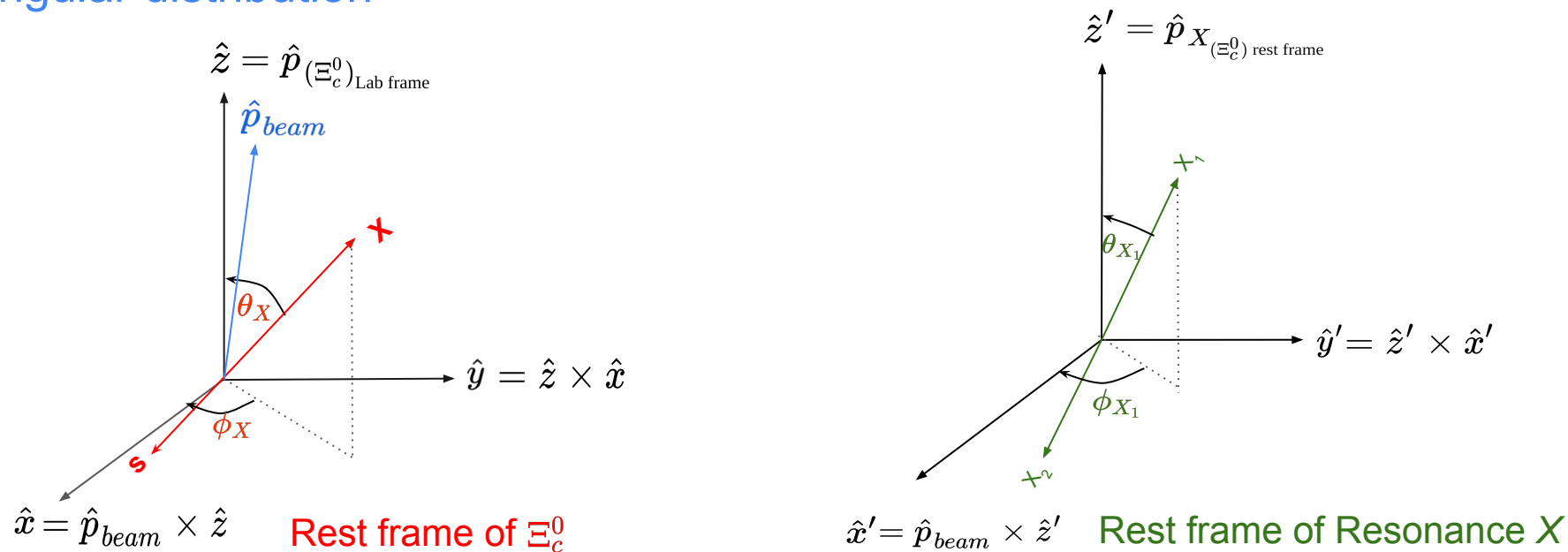


Fig: Definition of the helicity angles given in the rest frame of the parent particles for the  $\Xi_c^0 \rightarrow \Lambda k \pi$

$$A^{M, \lambda_\Lambda}(\vec{x}) = \sum_{j_X, \lambda_X} N_J D_{M, \lambda_X - \lambda_s}^j(\phi_X, \theta_X, 0) N_{j_X} D_{\lambda_X, \lambda_{X1}}^{j_X}(\phi_{X1}, \theta_{X1}, 0)$$

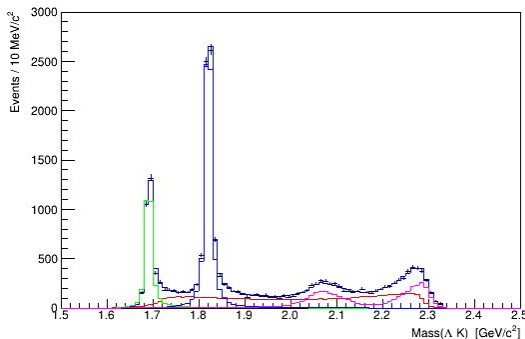
Wigner D-function representing the decay of **mother particle** and **resonant particle**

# Fitting results: Toy model

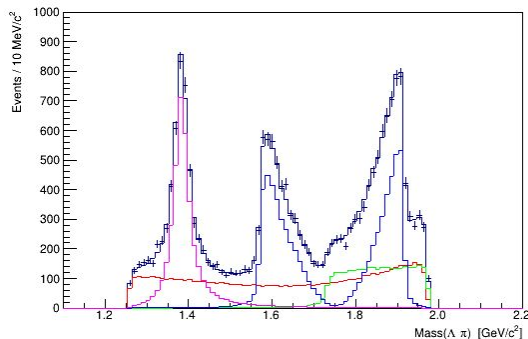
Generated amplitude	Fitted amplitude
$\bar{K}^*(892)^0 = 1.0, 0$	1.0, 0
$\Xi(1690)^- = 0.8, 0$	0.8141, -0.0478
$\Xi(1820)^- = 1.0, 0.5$	1.0063, 0.5042
$\Sigma(1385)^+ = 1.0, -0.5$	0.9917, -0.5219

Sample generated and fitted by AmpTools

$\Xi(1690)^-$   
 $M = 1.690 \text{ GeV}/c^2$   
 $W = 0.01 \text{ GeV}$   
 $J = 1/2$



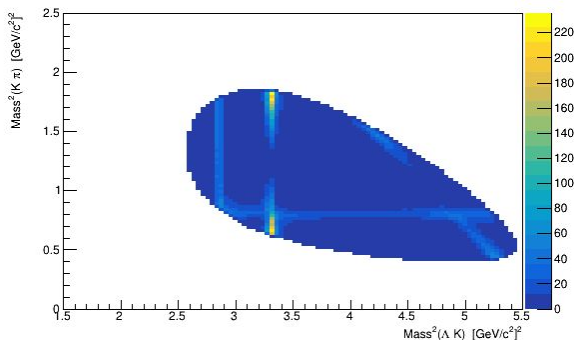
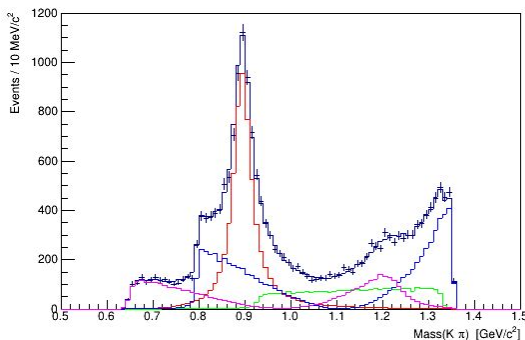
$\Xi(1820)^-$   
 $M = 1.820 \text{ GeV}/c^2$   
 $W = 0.01 \text{ GeV}$   
 $J = 3/2$



$-2\ln(L) = -412021.3178$

$\Sigma(1385)^-$   
 $M = 1.3833 \text{ GeV}/c^2$   
 $W = 0.0385 \text{ GeV}$   
 $J = 3/2$

$\bar{K}^*(892)$   
 $M = 0.8947 \text{ GeV}/c^2$   
 $W = 0.0445 \text{ GeV}$   
 $J = 1$





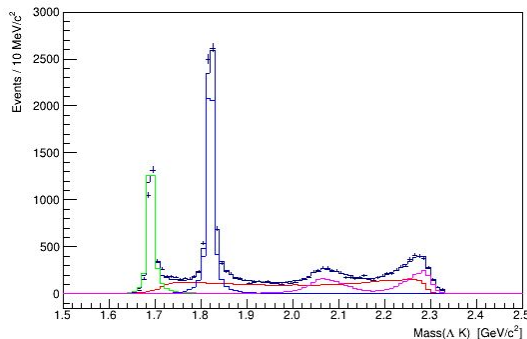
# Fitting results: Toy model

Testing alternate hypothesis:

Sample generated and fitted by AmpTools

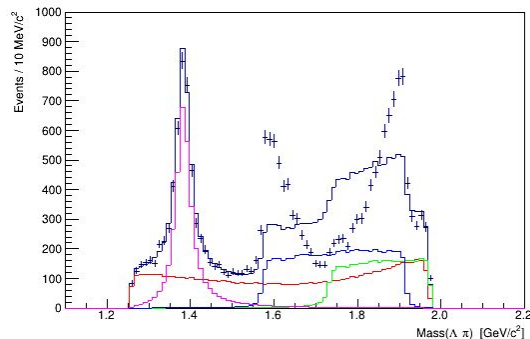
Generated amplitude	Fitted amplitude
$\bar{K}^*(892)^0 = 1.0, 0$	1.0, 0
$\Xi(1690)^- = 0.8, 0$	0.1717, -0.8295
$\Xi(1820)^- = 1.0, 0.5$	0.7502, -0.3169
$\Sigma(1385)^+ = 1.0, -0.5$	0.7771, -0.7114

$\Xi(1690)^-$   
 $M = 1.690 \text{ GeV}/c^2$   
 $W = 0.01 \text{ GeV}$   
 $J = 1/2$

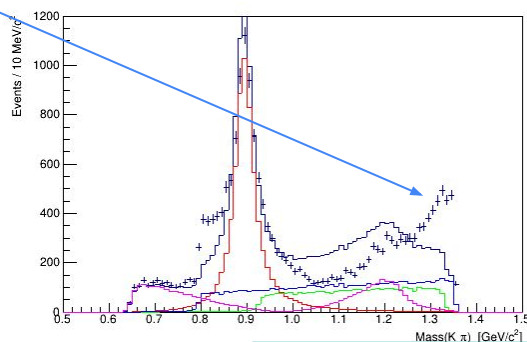


$\Xi(1820)^-$   
 $M = 1.820 \text{ GeV}/c^2$   
 $W = 0.01 \text{ GeV}$   
 $J = 1/2$

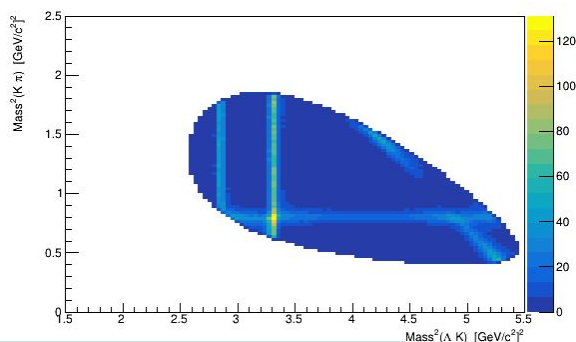
$-2\ln(L) = -404073.9963$   
 Much worse!



$\Sigma(1385)^-$   
 $M = 1.3833 \text{ GeV}/c^2$   
 $W = 0.0385 \text{ GeV}$   
 $J = 3/2$



$\bar{K}^*(892)$   
 $M = 0.8947 \text{ GeV}/c^2$   
 $W = 0.0445 \text{ GeV}$   
 $J = 1$



AmpTools can extract correct parameters

# Fitting result: basf2 reconstructed sample

Sample: Four resonances with equal weight (25%)

## Fit fraction per amplitude

$$\bar{K}^*(892)^0 = 0.2427 \pm 0.0031$$

$$\Xi(1690)^- = 0.2619 \pm 0.0027$$

$$\Xi(1820)^- = 0.2442 \pm 0.0028$$

$$\Sigma(1385)^+ = 0.2137 \pm 0.0024$$

$$-2\ln(L) = -1151795.8079$$

Spin of resonances while fitting:

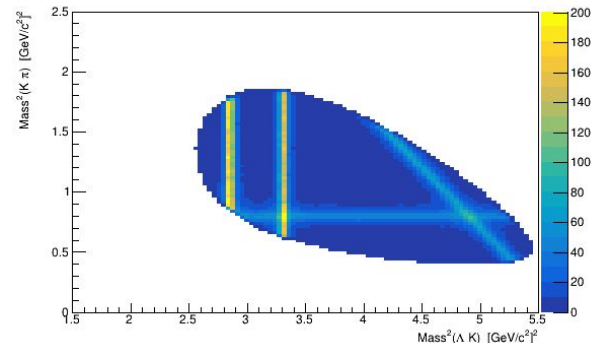
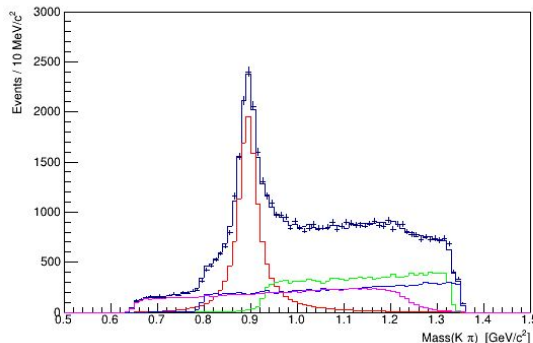
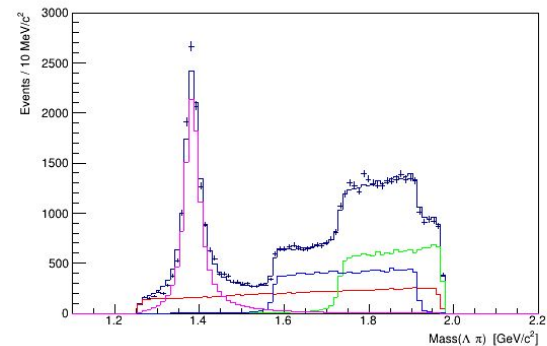
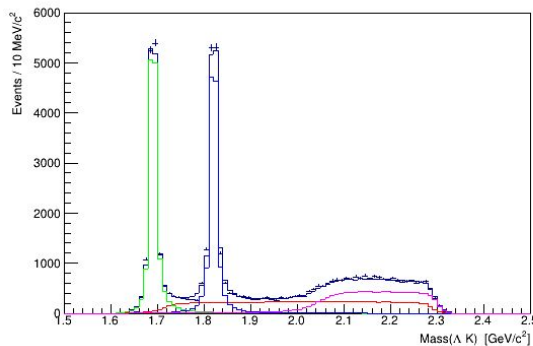
$$\bar{K}^*(892)^0 = 0$$

$$\Xi(1690)^- = 1/2$$

$$\Xi(1820)^- = 1/2$$

$$\Sigma(1385)^+ = 1/2$$

- Mass and width of each resonance fixed



# Bootstrap to check uncertainties

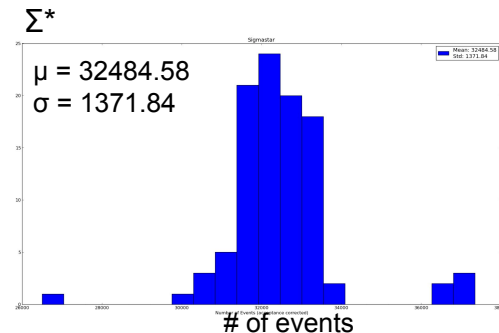
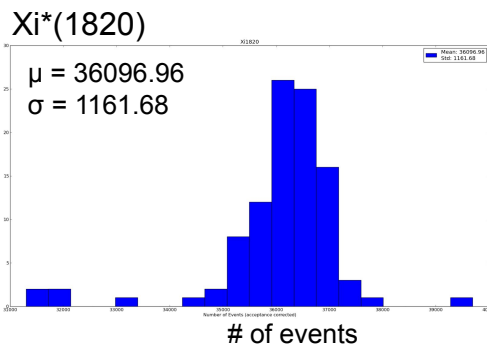
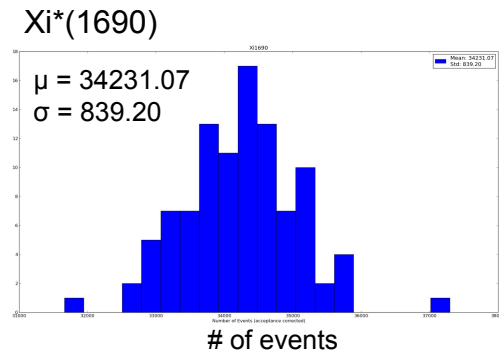
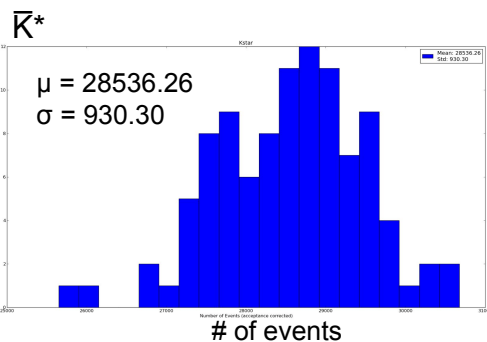
\*Requires larger statistics sample (pending), but uncertainties underestimated (~few percent)

AmpTools uncertainties computed from matrix of second derivatives and **may not be reliable in all cases**

To test: bootstrap (from large sample), **compare the standard deviation of the fitted parameter with uncertainties from nominal fit**

Initial test

- Bootstrap 10,373 events (comparable to expectations in data) from a sample of 29,973 signal events
- Mass and width of resonances = fixed
- Constrain  $K^*$  amplitude to be real
- Free parameters: 7 (real/imag amps)

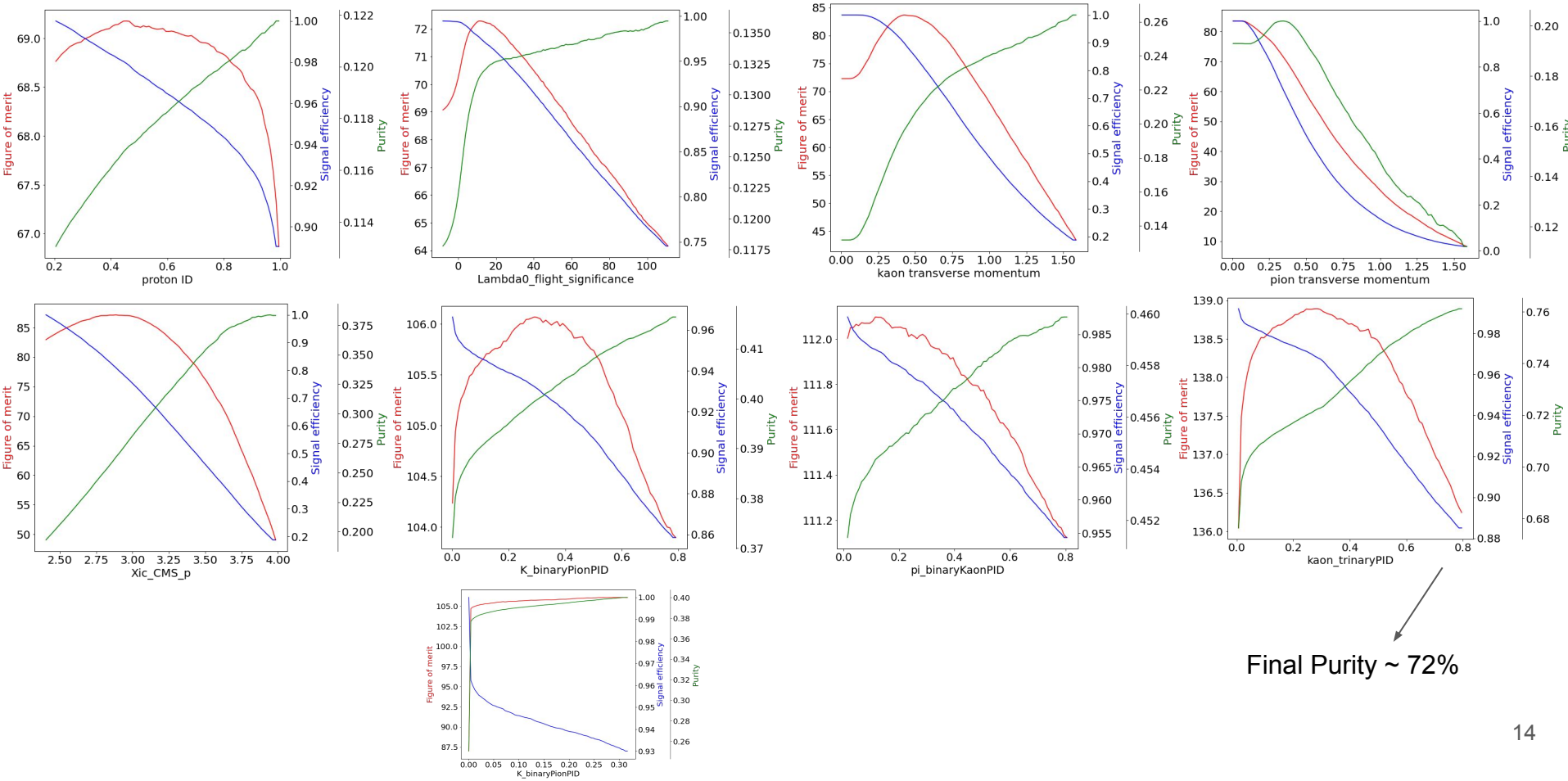


## Next steps and plans

- Ongoing: Amplitude analysis of  $\Lambda_c^+ \rightarrow p K \pi$  to check the fitting model (by comparing with the result from LHCb)
- Fine tune fitting method to find global minimum by randomizing the parameters
- Finalize check on uncertainties by bootstrapping from a larger sample
- Study the helicity angles of  $K^*$  distribution and finalize the model to be used for fitting.
- Before looking Data: Final steps of fitting method include adding and removing amplitudes and checking their significance
- Systematic studies

Extra slides

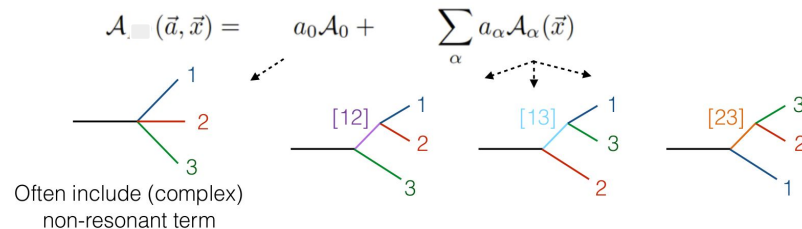
# FOM optimization



# Theoretical background

- amplitude for  $\Xi_c^0 \rightarrow \Lambda^0 K^- \pi^+$  :  $U^{M,\lambda_\Lambda}(\vec{x}) = \langle \Lambda K^- \pi^+ | H | \Xi_c^0 \rangle$
- can be parameterized as:  $U^{M,\lambda_\Lambda}(\vec{x}) = \sum_{j_X, \lambda_X} V_{j_X, \lambda_X} A_{j_X, \lambda_X}^{M, \lambda_\Lambda}(\vec{x})$

$V_{j_X, \lambda_X} = k_{j_X, \lambda_X} BW_{j_X, \lambda_X}$  describes the propagator of the intermediate state and its coupling to  $\Xi_c^0$  and  $A_{j_X, \lambda_X}^{M, \lambda_\Lambda}$  describes the angular distribution of final-state particles.

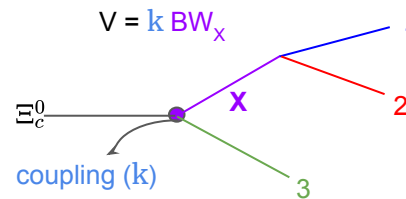


- The density of events at  $\vec{x}$  is given by the intensity:  $I(\vec{x}) = \sum_{M, \lambda_\Lambda} |U^{M, \lambda_\Lambda}(\vec{x})|^2$
- To extract the couplings,  $k_{j_X, \lambda_X}$ , an unbinned maximum likelihood fit is performed on  $\Lambda^0 K^- \pi^+$  invariant mass.
- The probability to find an event in the detector at some location in phase space,  $\vec{x}$ , is given by

$$f(\vec{x}_i | \vec{\zeta}) = \frac{\eta(\vec{x}) I(\vec{x} | \vec{\zeta})}{\int \eta(\vec{x}) I(\vec{x} | \vec{\zeta}) d\vec{x}}, \quad \eta(\vec{x}) \text{ is the efficiency of the detector to find an event at } \vec{x}$$

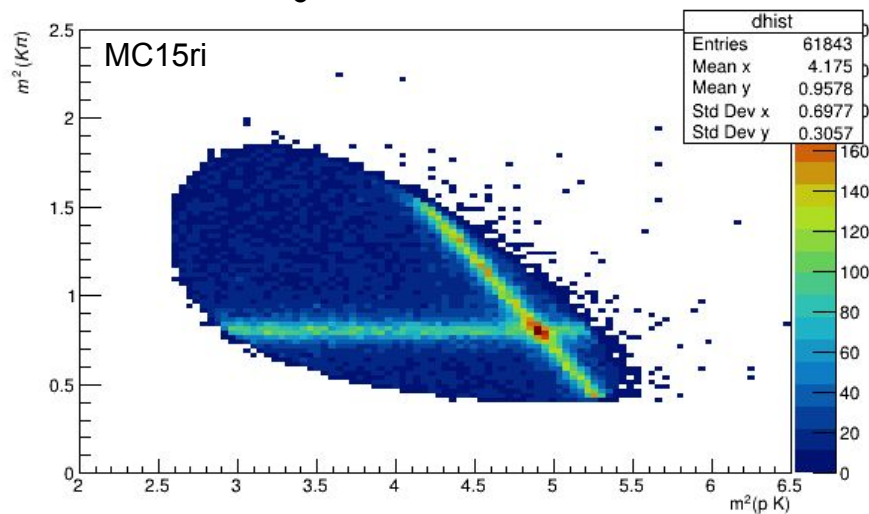
- The propagator is described by a Breit-Wigner function and the coupling is set as a free parameter in an unbinned, maximum likelihood fit to the data. The number and type of intermediate states is varied until an optimal solution is found. Any remaining backgrounds are accounted using background samples added to the data set with negative weights.

$$L(\vec{\zeta}) = \prod_{i=1}^{N_{data}} f(\vec{x}_i | \vec{\zeta}) \prod_{j=1}^{N_{MC}^{bkg}} f(\vec{x}_j | \vec{\zeta})^{-w_j}$$

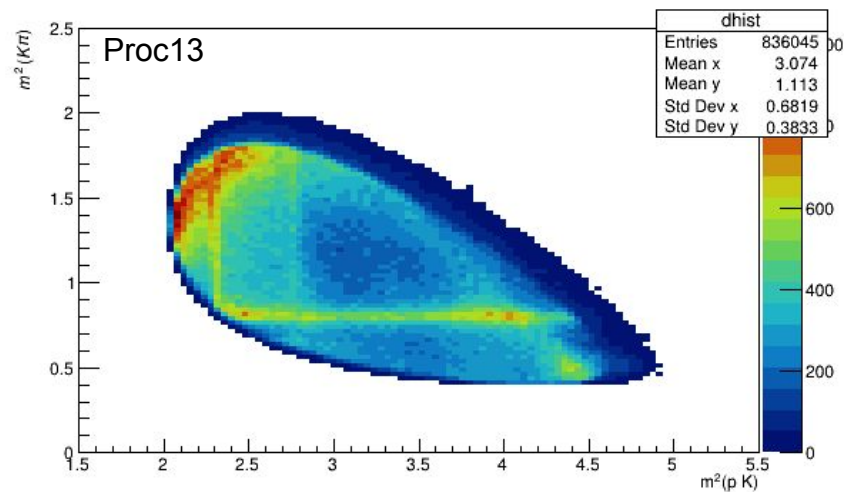


# Dalitz plots

Dalitz plot:  $\Xi_c^0 \rightarrow \Lambda K \pi$



Dalitz plots:  $\Lambda_c^+ \rightarrow p K \pi$





# Helicity angles of $K^*$

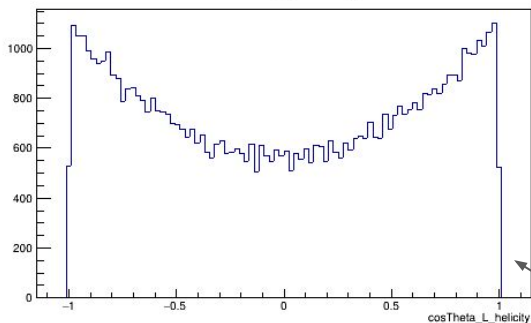
$K^*$  being a vector,  $J = 1$

Toy sample by AmpTools

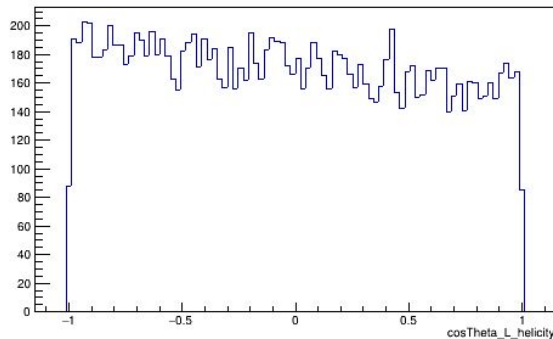
Signal MC by basf2

Generic MC by basf2

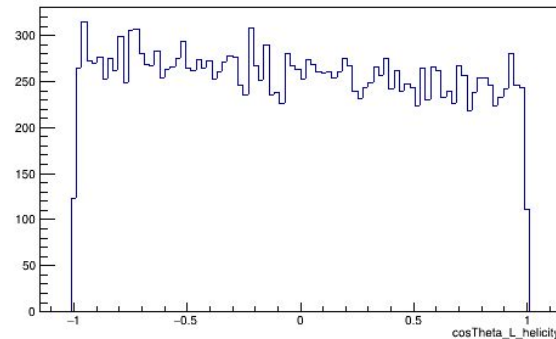
$J = 1$  cosTheta\_L\_helicity



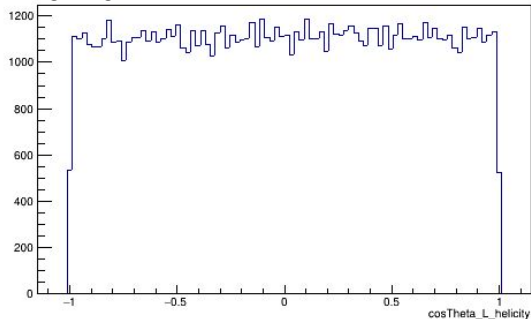
$J = 1$  cosTheta\_L\_helicity



$J = 1$  cosTheta\_L\_helicity



$J = 0$  cosTheta\_L\_helicity



- Expected curved helicity angle
- basf2 generated sample have almost flat distribution (study going on: might be helpful to address fitting fractions)