## Theory challenges in exclusive $\left|V_{c b}\right|$

## Zoltan Ligeti

2023 Belle II Physics Week KEK, Oct. 30 - Nov. 3, 2023

## Outline: I was told to talk about...

How to assess potential biases related to inputs from various models or sum rule based approaches?

How to decide in a systematic manner on the truncation order of form factor parametrizations?

Can we develop a benchmark test to compare different methods?
For HQET-based parametrizations of $B \rightarrow D^{(*)}$ semileptonic decays, how do we assess the role of and incorporate second order power corrections?
(I'll focus on my opinions, prospects, open questions)

## Some reasons $\left|V_{c b}\right|$ matters

- $\left|V_{c b}\right|$ important to assess if there is an $\varepsilon_{K}$ tension, predict $K \rightarrow \pi \nu \bar{\nu}, B \rightarrow(X) \ell \bar{\ell}$ SM predictions involve $A^{4}$, so $5 \%$ in $\left|V_{c b}\right|$ yields $20 \%$
- The $b \rightarrow c \tau \bar{\nu}$ data should make $\left|V_{c b}\right|$ much better understood - are we there yet? To understand the $\tau$ mode thoroughly, must understand the $e, \mu$ modes better
- Recently: $\left|V_{c b}\right|$ uncertainty limits future improvements in the sensitivity to NP in $B$ and $B_{s}$ mixing "Phase Il" (LHCb upgrade 2 and Belle II upgrade) with / without $\left|V_{c b}\right|$ uncertainty, maybe early 40s


[Charles, Descotes-Genon, ZL, Monteil, Papucci, Trabelsi, Vale Silva, 2006.04824]

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z L-p .2
$$



## Basics of $B \rightarrow D^{(*)} \ell \bar{\nu}$

- Heavy Quark Symmetry: $v \rightarrow v^{\prime}$ changes brown muck, but not $m_{b} \rightarrow m_{c}$ or $\vec{s}_{b} \rightarrow \vec{s}_{c}$ [lsgur \& Wise]

$$
\begin{gathered}
\frac{\mathrm{d} \Gamma\left(B \rightarrow D^{(*)} \ell \bar{\nu}\right)}{\mathrm{d} w}=(\ldots)\left(w^{2}-1\right)^{3(1) / 2}\left|V_{c b}\right|^{2} \mathcal{F}_{(*)}^{2}(w) \\
\nwarrow_{\nearrow} \equiv v \cdot v^{\prime} \quad \text { Isgur-Wise function }+ \text { corr. } \\
\mathcal{F}(1)=1_{\text {Isgur-Wise }}+0.02_{\alpha_{s}, \alpha_{s}^{2}}+\frac{(\text { compute })}{m_{c, b}}+\ldots \\
\mathcal{F}_{*}(1)=1_{\text {Isgur-Wise }}-0.04_{\alpha_{s}, \alpha_{s}^{2}}+\frac{0_{\text {Luke }}}{m_{c, b}}+\frac{(\text { compute })}{m_{c, b}^{2}}+\ldots
\end{gathered}
$$


(1) Lattice QCD: $\mathcal{F}_{*}(1)=0.906 \pm 0.012, \mathcal{F}(1)=1.054 \pm 0.009$
(2) Constraint on shape (slope vs. curvature)
[Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert]
(3) Some understanding of decays to higher mass $X_{c}$ states (backgrounds)

- Data: $\left|V_{c b} \mathcal{F}_{*}(1)\right|=(34.77 \pm 0.36) \times 10^{-3}, \quad\left|V_{c b} \mathcal{F}(1)\right|=(41.26 \pm 0.97) \times 10^{-3} \quad \| F L A V \mid$

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Z L-p .3
$$

## 2010s: hints of lepton universality violation

- If established, likely impact $\left|V_{c b}\right|$ and $\left|V_{u b}\right|$, cannot assume only $\tau$ impacted

BaBar, Belle,

- Imply low scale NP, possible mediators: $(b c)(\tau \nu)$ " $H^{ \pm " ;}(b \tau)(c \nu)$ "LQ"; $(b \nu)(c \tau)$ " $\tilde{b}$ "
- Rethink program, new \& expanded searches: at high- $p_{T}$, new channels, LFV
- Belle unfolded measurements reinvigorated the field (2017-) $\sqrt{1702.01521 / ~ \& ~} 1809.03290$
- Leads, at least, to more scrutiny and better understanding of $\left|V_{c b}\right|$ determinations

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Z L-p .4
$$

## Available for the first time in 2017

- Belle published unfolded $B \rightarrow D^{*} l \bar{\nu}$ ( $l=e, \mu$ ) distributions [1702.01521]

- Input on the fitted shapes:

BGL: Boyd, Grinstein, Lebed, '95-97 CLN: Caprini, Lellouch, Neubert, '97 1997-2017: all measurements used CLN

- Can perform different fits to data







## Using HQET at $\mathcal{O}(1 / m)$ and $\mathcal{O}\left(1 / m^{2}\right)$

- One leading and 3 subleading Isgur-Wise functions in $B \rightarrow D^{(*)} \ell \bar{\nu}$ Can constrain all 4 from $B \rightarrow D^{(*)} l \bar{\nu} \Rightarrow \mathcal{O}\left(\Lambda_{Q C D}^{2} / m_{c, b}^{2}, \alpha_{s}^{2}\right)$ uncertainties $(l=e, \mu)$
[Bernlochner, ZL, Papucci, Robinson, 1703.05330]
- Observables: $B \rightarrow D l \bar{\nu}: \mathrm{d} \Gamma / \mathrm{d} w$

$$
B \rightarrow D^{*} l \bar{\nu}: \mathrm{d} \Gamma / \mathrm{d} w \text { and } R_{1,2}(w) \text { form factor ratios }
$$

(CLN fit prescription: QCD sum rules built in + linear slope vs. curvature relation)

- At $\mathcal{O}\left(1 / m_{c, b}^{2}\right)$ the number of "universal" functions for $B \rightarrow D^{(*)} \ell \bar{\nu}$ proliferate

Proposal to include $1 / m_{c}^{2}$ corrections using LCSR
[Bordone, Jung, van Dyk, 1908.09398]
We explored truncating the number of order $1 / m^{2}$ Isgur-Wise functions: vanishing chromomagnetic (VC) "limit" \& residual chiral (RC) "expansion"
[Bernlochner, ZL, Papucci, Prim, Robinson, Xiong, "BLPR-XP", 2206.11281]

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Z L-p .6
$$



## Boyd-Grinstein-Lebed (BGL) constraints

- Constrain form factor shapes, based on analyticity \& unitarity; Taylor expansions:

$$
\frac{1}{P_{i}(z) \phi_{i}(z)} \sum a_{n}^{i} z^{n} \quad i=g, f, \mathcal{F}_{1} \text { (lin. comb.) }
$$

$z(w)$ is a conformal parameter, maps physical region $1<w<1.5$ to $0<z<0.056$ $P_{i}(z), \phi_{i}(z)$ are known functions
$c_{0}$ is fixed by $b_{0}$
Some papers use notation: $\left\{a_{n}, b_{n}, c_{n}\right\} \longleftrightarrow\left\{a_{n}^{g}, a_{n}^{f}, a_{n}^{\mathcal{F}_{1}}\right\}$

- Does not use constraints from heavy quark symmetry, but can be added
- Denote by $\mathrm{BGL}_{i j k}$ a BGL fit with parameters: $\left\{a_{0, \ldots, i-1}, b_{0, \ldots, j-1}, c_{1, \ldots, k}\right\}$

Literature contains various choices: $N=i+j+k=4,5,6,8$

- Must truncate expansions at some order - what is the optimal choice?

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Z L-p .7
$$



## The CLN fits used 1997-2017

- CLN added QCD SR to BGL: $R_{1,2}(w)=\underbrace{R_{1,2}(1)}_{\text {fit }}+\underbrace{R_{1,2}^{\prime}(1)}_{\text {fixed }}(w-1)+\underbrace{R_{1,2}^{\prime \prime}(1)}_{\text {fixed }}(w-1)^{2} / 2$ In HQET:

$$
R_{1,2}(1)=1+\mathcal{O}\left(\Lambda_{\mathrm{QCD}} / m_{c, b}, \alpha_{s}\right) \quad R_{1,2}^{(n)}(1)=0+\mathcal{O}\left(\Lambda_{\mathrm{QCD}} / m_{c, b}, \alpha_{s}\right)
$$

The $\mathcal{O}\left(\Lambda_{\mathrm{QCD}} / m_{c, b}\right)$ terms are determined by 3 subleading Isgur-Wise functions

- Inconsistent fits: same param's determine $R_{1,2}(1)-1$ (fit) and $R_{1,2}^{(1,2)}(1)$ (QCDSR)

Sometimes calculations using QCD sum rules are called the HQET predictions

- Devised fits to "interpolate" between BGL and CLN [Berlochner, zL, Robinson, Papuci, [1708.07134]

| form factors | BGL | CLN | CLNnoR | noHQS |
| :---: | :---: | :---: | :---: | :---: |
| axial $\propto \epsilon_{\mu}^{*}$ | $b_{0}, b_{1}$ | $h_{A_{1}}(1), \rho_{D^{*}}^{2}$ | $h_{A_{1}}(1), \rho_{D^{*}}^{2}$ | $h_{A_{1}}(1), \rho_{D^{*}}^{2}, c_{D^{*}}$ |
| vector | $a_{0}, a_{1}$ |  |  |  |
| axial $\left(\mathcal{F}_{1}\right)$ | $c_{1}, c_{2}$ |  |  |  |\(\quad\left\{\begin{array}{l}R_{1}(1) <br>

R_{2}(1)\end{array} \quad\left\{$$
\begin{array}{l}R_{1}(1), R_{1}^{\prime}(1) \\
R_{2}(1), R_{2}^{\prime}(1)\end{array}
$$ \quad\left\{$$
\begin{array}{l}R_{1}(1), R_{1}^{\prime}(1) \\
R_{2}(1), R_{2}^{\prime}(1)\end{array}
$$\right.\right.\right.\)

Relaxing constraints on $R_{1,2}^{\prime}(1)$, fit results similar to BGL

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Z L-p .8
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## Can one move past CLN?

- Abstract of recent Belle paper [2310.01170]

We determine the CKM matrix-element magnitude $\left|V_{c b}\right|$ using $\bar{B}^{0} \rightarrow D^{*+} \ell^{-} \bar{\nu}_{\ell}$ decays reconstructed in $189 \mathrm{fb}^{-1}$ of collision data collected by the Belle II experiment, located at the SuperKEKB $e^{+} e^{-}$collider. Partial decay rates are reported as functions of the recoil parameter $w$ and three decay angles separately for electron and muon final states. We obtain $\left|V_{c b}\right|$ using the Boyd-Grinstein-Lebed and Caprini-Lellouch-Neubert parametrizations, and find $\left|V_{c b}\right|_{\text {BGL }}=(40.57 \pm 0.31 \pm 0.95 \pm 0.58) \times$ $10^{-3}$ and $\left|V_{c b}\right|_{\text {CLN }}=(40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$ with the uncertainties denoting statistical components, systematic components, and components from the lattice QCD input, respectively. The branching fraction is measured to be $\mathcal{B}\left(\bar{B}^{0} \rightarrow D^{*+} \ell^{-} \bar{\nu}_{\ell}\right)=(4.922 \pm 0.023 \pm 0.220) \%$. The ratio of branching fractions for electron and muon final states is found to be $0.998 \pm 0.009 \pm 0.020$. In addition, we determine the forward-backward angular asymmetry and the $D^{*+}$ longitudinal polarization fractions. All results are compatible with lepton-flavor universality in the Standard Model.

- If CLN fit is quoted (maybe to compare with past results?) on equal footing with BGL, readers will assume that the Collaboration views them equally meaningful
- While CLN gave a simple recipe (that is not self-consistent), using BGL some choices must be made (truncation order, additional input from HQET, LQCD, unitarity?)


## The how-to-fit saga


"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

Overfitting? Truncation orders? Additional inputs/constraints from HQET, LQCD, unitarity?

## Nested hypothesis tests

- Optimal BGL fit parameters? (upper: $\chi^{2}$, lower: $\left|V_{c b}\right| \times 10^{3}$ )
[From 1902.09553, using 1702.01521]

| $n_{c}$ | 1 | 2 | 3 | 1 | 2 | 3 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 33.2 | 31.6 | 31.2 | 33.0 | 29.1 | 28.9 | 30.4 | 29.1 | 28.9 |
|  | $38.6 \pm 1.0$ | $38.6 \pm 1.0$ | $38.6 \pm 1.0$ | $39.0 \pm 1.5$ | $40.7 \pm 1.6$ | $40.7 \pm 1.6$ | $40.7 \pm 1.7$ | $40.6 \pm 1.8$ | $40.6 \pm 1.8$ |
| 2 | 32.9 | 31.3 | 31.1 | 32.7 | 27.7 | 27.7 | 29.2 | 27.7 | 27.7 |
|  | $38.8 \pm 1.1$ | $38.7 \pm 1.1$ | $38.8 \pm 1.0$ | $39.5 \pm 1.7$ | $41.7 \pm 1.8$ | $41.6 \pm 1.8$ | $41.8 \pm 2.0$ | $41.8 \pm 2.0$ | $41.7 \pm 2.0$ |
| 3 | 31.7 | 31.3 | 31.0 | 29.1 | 27.7 | 27.6 | 29.2 | 27.6 | 23.2 |
|  | $39.0 \pm 1.1$ | $38.6 \pm 1.2$ | $38.6 \pm 1.1$ | $41.9 \pm 2.0$ | $41.8 \pm 2.0$ | $41.7 \pm 2.0$ | $41.8 \pm 2.0$ | $41.7 \pm 1.9$ | $41.4 \pm 2.0$ |
|  | $n_{b}=1$ |  |  | $n_{b}=2$ |  |  | Q $n_{b}=3$ |  |  |

- Fit w/ 1 param added / removed: $\mathrm{BGL}_{\left(n_{a} \pm 1\right) n_{b} n_{c}}, \mathrm{BGL}_{n_{a}\left(n_{b} \pm 1\right) n_{c}}, \mathrm{BGL}_{n_{a} n_{b}\left(n_{c} \pm 1\right)}$
- Accept descendant (parent) if $\Delta \chi^{2}$ is above (below) a boundary, say, $\Delta \chi^{2}=1$
- Repeat until "stationary" fit is found, preferred over its parents and descendants
- If multiple stationary fits, choose smallest $N$, then smallest $\chi^{2} \quad$ (333 is an overfit!) Start from small $N$, to avoid overfitting e.g.: $\left\{\begin{array}{l}111 \rightarrow 211 \rightarrow 221 \rightarrow 222 \\ 121 \rightarrow 131 \rightarrow 231 \rightarrow 232 \rightarrow 222\end{array}\right.$

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Z L-p .10
$$

## Toy studies: check $\left|V_{c b}\right|$ is unbiased

- Set $\left\{\tilde{a}_{0,1}, \tilde{b}_{0,1}, \tilde{c}_{1,2}\right\}=\operatorname{BGL}_{222}$ fit result, and $\left\{\tilde{a}_{2}, \tilde{b}_{2}, \tilde{c}_{3}\right\}=(1$ or 10$) \times\left\{\tilde{a}_{1}, \tilde{b}_{1}, \tilde{c}_{2}\right\}$ Generate MC data using experimental covariance, fit each set w/ our prescription


- Frequency of the selected hypotheses, with two scenarios for higher order terms:

|  | BGL $_{122}$ | BGL $_{212}$ | BGL $_{221}$ | BGL $_{222}$ | BGL $_{223}$ | BGL $_{232}$ | BGL $_{322}$ | BGL $_{233}$ | BGL $_{323}$ | BGL $_{332}$ | BGL $_{333}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| '1-times' | $6 \%$ | $0 \%$ | $37 \%$ | $27 \%$ | $6 \%$ | $6 \%$ | $11 \%$ | $0 \%$ | $2 \%$ | $4 \%$ | $0.4 \%$ |
| '10-times' | $0 \%$ | $0 \%$ | $8 \%$ | $38 \%$ | $14 \%$ | $8 \%$ | $16 \%$ | $3 \%$ | $4 \%$ | $8 \%$ | $1 \%$ |

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Z L-p .11
$$



## Akaike information criterion

- There is no frequentist approach to deciding on number of fit parameters (Each choice is effectively a different theory)
- How to pick number of BGL fit parameters? (depends on data, multiple proposals) Just looking at goodness of fit is not enough ["us" |1002.09553; Gambino, Jung, Schacht [1905.08209, etc.]
- Akaike information criterion (AIC): Started by Bhattacharya, Nandi, Patra, 1611.04605, [805.08222] 108.00835 for BSM fits, recently revisited by Simons et al. 2304.13045 for $\left|V_{c b}\right|$
AIC $=2 n+\chi^{2} \quad \sim$ demand $\chi^{2}$ decreases $\geq 2$, to include a new fit parameter
Variations: AIC $=$ AIC $+\frac{2 n^{2}+2 n}{k-n-1}$

$$
\mathrm{BIC}=n \ln k+\chi^{2}
$$

"AIC sometimes selects a much better model than BIC even when the "true model" is in the candidate set" Wikipedia]

- Vast literature: desirable to use a procedure that is somewhat "standard", easy to communicate, and not to reinvent the wheel (especially if resuts in tension w/ SM)

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Z L-p .12
$$

## Notorius BGL: truncation orders matter

- Belle (711/fb) [2301.05529] and Belle II (189/fb) [2310.01170 papers impose a constraint that eliminates models with highly correlated fit parameters
- Belle paper choose $\mathrm{BGL}_{121}$, while both AIC and NHT [as in 1902.09553 ] would pick the $\mathrm{BGL}_{221}$ fit Difference not negligible: $\Delta\left|V_{c b}\right|=3.6 \times 10^{-3}$
- Detailed validation of model selection with toy MC seems to be essential

| BGL $_{111}$ | $40.4 \pm 0.8$ | 45.6 | 34 | 3 | 0.70 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| BGL $_{112}$ | $40.9 \pm 0.9$ | 43.4 | 33 | 4 | 0.98 |
| BGL $_{121}$ | $40.7 \pm 0.9$ | 45.2 | 33 | 4 | 0.60 |
| BGL $_{122}$ | $41.5 \pm 1.1$ | 42.3 | 32 | 5 | 0.98 |
| BGL $_{131}$ | $38.1 \pm 1.7$ | 41.7 | 32 | 5 | 0.98 |
| BGL $_{132}$ | $39.0 \pm 1.6$ | 37.5 | 31 | 6 | 0.98 |
| BGL $_{211}$ | $39.7 \pm 1.0$ | 42.7 | 33 | 4 | 0.99 |
| BGL $_{212}$ | $40.4 \pm 1.0$ | 39.3 | 32 | 5 | 0.99 |
| $\Rightarrow$ BGL $_{221}$ | $37.1 \pm 1.2$ | 37.7 | 32 | 5 | 0.99 |
| BGL $_{222}$ | $37.9 \pm 2.0$ | 37.5 | 31 | 6 | 1.00 |
| BGL $_{231}$ | $37.2 \pm 1.8$ | 37.7 | 31 | 6 | 0.99 |
| BGL $_{232}$ | $38.8 \pm 1.7$ | 37.2 | 30 | 7 | 0.98 |
| BGL $_{311}$ | $38.5 \pm 0.9$ | 40.1 | 32 | 5 | 0.95 |
| BGL $_{312}$ | $39.9 \pm 1.1$ | 36.9 | 31 | 6 | 0.98 |
| BGL $_{321}$ | $37.3 \pm 1.2$ | 37.3 | 31 | 6 | 0.97 |
| BGL $_{322}$ | $38.9 \pm 2.1$ | 36.5 | 30 | 7 | 0.99 |
| BGL $_{331}$ | $39.6 \pm 2.3$ | 36.3 | 30 | 7 | 0.99 |
| BGL $_{332}$ | $40.1 \pm 2.3$ | 35.9 | 29 | 8 | 0.99 |

[2301.07529, Table XVI from PRD]
(Maybe more from Florian or Markus during discussions)
See also: Juttner's talk at LHCb implications, last week [2303.11285]

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Z L-p .13
$$




## Order $\mathcal{O}\left(1 / m_{c, b}^{2}\right)$ terms

- Baryons: much fewer form factors, more tractable [Berlochner, ZL, Robinson, Sutcliffe, [808.09464]
- At $\mathcal{O}\left(1 / m_{c, b}^{2}\right)$ the number of "universal" functions for $B \rightarrow D^{(*)} \ell \bar{\nu}$ proliferate Proposal to include $1 / m_{c}^{2}$ corrections using LCSR [Bordone, Jung, van Dyk, 1908.09398]

We explored truncating the number of order $1 / m^{2}$ Isgur-Wise functions: vanishing chromomagnetic (VC) "limit" \& residual chiral (RC) "expansion"
[Bernlochner, ZL, Papucci, Prim, Robinson, Xiong, "BLPR-XP", 2206.11281]

| HQET |  | Isgur-Wise functions |  |
| :---: | :---: | :---: | :---: |
| order | All | RC Expansion | VC Limit |
| 1 | 1 | 1 | 1 |
| $1 / m_{c, b}$ | 3 | 3 | 2 |
| $1 / m_{c}^{2}$ | 20 | 1 | 2 |
| $1 / m_{c, b}^{2}$ | 32 | 3 | 3 |

- I am not convinced that it's optimal or necessary to account for all $\mathcal{O}\left(1 / m_{c, b}^{2}\right)$ terms
- Toys to estimate ability to constrain many terms ( 5 or $50 / \mathrm{ab}$ ?) would be interesting (Try to identify what's important and what's not, etc.)

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Z L-p .14
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## Main differences in our 2022 vs. 2017 paper

- BLPR [1703.05330] preceded Belle'19 [1809.03200], which is in tension w/ Belle'17 [1702.01521] Changes are not due to (partly) including $1 / m^{2}$; enough freedom at $1 / m$ at current precision
- CLN approximated the constraint on the slope vs. curvature plane by a linear relationship The precision of experimental and LQCD data are high enough that this no longer applies
- $\mathrm{BLPR}_{(\text {no SR) })}: R(D)=0.298(3), R\left(D^{*}\right)=0.261(4)$ $\Downarrow$ BLPR-XP: $\quad R(D)=0.288(4), R\left(D^{*}\right)=0.249(3)$ (Includes a scale factor for the $D^{*}$ prediction to account for tension)

- Slight increase in tension between SM prediction and $R\left(D^{(*)}\right)$ measurements

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Z L-p .15
$$



## $B \rightarrow D^{(*)}$ : some open questions

- Won't talk about fits to unfolded vs. folded data - what I think is important is that different approaches and new ideas can be tested
- Can we have a more systematic approach to $1 / m_{c, b}^{2}$, with continuum methods?
- How to pick number of BGL fit parameters? (depends on data, multiple proposals) Just looking at goodness of fits is not the full story [1708.07134, 1905.08209]
- Can lattice QCD determine form factors as precisely as $F_{(*)}(1)$ ?

Some tension between FNAL/MILC results and data (less so for JLQCD)
$R\left(D^{*}\right)_{\text {Lat }}=0.265 \pm 0.013, \quad R\left(D^{*}\right)_{\text {Lat }+\operatorname{Exp}(e, \mu)}=0.2484 \pm 0.0013$ [FNAL\& \& MLC, 2105.14019]
$B_{s} \rightarrow D_{s}^{*} \ell \bar{\nu}: R\left(D_{s}^{*}\right)=0.249 \pm 0.007$
[Harrison \& Davies, 2105.11433]

- Need (a lot) more data to resolve all outstanding issues

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Z L-p .16
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## Form factor ratios

- Importance known since the early 1990s ratios of form factors defined to be unity in HQS limit at all $w$
- Lattice calculations not as consistent as one would like:


[Harrison @ at LHCb implications last week]


(error bars: FNAL/MILC)
[Belle II, 2310.01170]
- Observing a large violation of HQS would have significant consequences
- Need both more precise measurements and lattice QCD calculations

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Z L-p .17
$$

## $B \rightarrow D^{* *}$ : many open questions

- Significant backgrounds, masses in some tension w/ quark model predictions Some nonleptonic $D^{* *} \pi$ rates far from facotrization prediction
- The " $1 / 2$ vs. $3 / 2$ puzzle" remains... puzzling [Le Yauanc, Leroy, Roudeau, 2102.11608]
- How well can $B \rightarrow D^{* *}$ and nonresonant rates be measured at Belle II?
- Four $D_{s}^{* *}$ states much narrower than non-strange counterparts - nice for LHCb
- $D_{s 0}^{*}(2317)$ : orbitally excited state or "molecule"? Might make HQET inapplicable If $D_{s 0}^{*}$ is excited $c \bar{s}$ state, predict $\mathcal{B}\left(D_{s 0}^{*} \rightarrow D_{s}^{*} \gamma\right) / \mathcal{B}\left(D_{s 0}^{*} \rightarrow D_{s} \pi\right)$ above CLEO bound, $<0.059$ [Mehen \& Springer, hep-ph/0407181]; Colangelo \& De Fazio, hep-phl0305140; Gootriey, hep-phl0305122] CLEO used 13.5/fb, the Belle bound $<0.18$ used $87 / \mathrm{fb}$, the BaBar bound $<0.16$ used $232 / \mathrm{fb}$
- Understanding Inclusive = Exclusive may be necessary to resolve issues

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Z L-p .18
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## Conclusions

- Independent of hints of NP, reducing the uncertainty of $\left|V_{c b}\right|$ is important
- $B \rightarrow D^{*} \ell \bar{\nu}$ : Need (much) more data and consistent LQCD results
- What are the largest useful data sets? No one has seriously explored it! (Recall, Sanda, 2003: the question is not $10^{35}$ or $10^{36} \ldots$...)

With $\infty$ statistics, would $B \rightarrow D^{*} e \bar{\nu}$ in the bin $q^{2}>9 \mathrm{GeV}^{2}$ give the cleanest $\left|V_{c b}\right|$ ? (No $D^{* *}$ backgrounds, no $D^{*} \rightarrow D$ down-feed)

Of course, in reality, there is always a tradeoff to minimize the overall uncertainty...

- "Best" case: new physics, new directions
"Worst" case: better SM tests, better CKM determinations, and NP sensitivity
- Good reasons to want to collect the largest possible $\Upsilon(4 S)$ data sets

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Z L-p .19
$$



## Extra slides

## Factor of 2 improvements can matter!

## Long-lived Neutral K Mesons*

M. Bardon, K. Lande, and L. M. Lederman

Columbia University, New York, New York, and Brookhaven National Laboratories, Upton, New York
A.vD

William Chinowsky
Brookhaven National Laboratories, Upton, New York:
set an upper limit $<0.6 \%$ on the reactions

$$
K_{2}^{0} \rightarrow\left\{\begin{array}{l}
\mu^{ \pm}+e^{\mp} \\
e^{+}+e^{-} \\
\mu^{+}+\mu^{-}
\end{array}\right.
$$

and on $K_{2}^{0} \rightarrow \pi^{+}+\pi^{-}$.

Volume 6, number 10
PHYSICAL REVIEW LETTERS
May 15, 1961

## DECAY PROPERTIES OF $K_{2}{ }^{\circ}$ MESONS ${ }^{*}$

D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov Joint Institute of Nuclear Research, Moscow, U.S.S.R. (Received April 20, 1961)

Combining our data with those obtained in reference 7, we set an upper limit of $0.3 \%$ for the relative probability of the decay $K_{2}{ }^{0} \rightarrow \pi^{-}+\pi^{+}$. Our
"At that stage the search was terminated by administration of the Lab."
[Okun, hep-ph/0112031]

EVIDENCE FOR THE $2 \pi$ DECAY OF THE $K_{2}{ }^{\circ}$ MESON* $\dagger$
J. H. Christenson, J. W. Cronin ${ }^{\ddagger}$ V. L. Fitch, ${ }^{\ddagger}$ and R. Turlay ${ }^{\S}$

Princeton University, Princeton, New Jersey
(Received 10 July 1964)

We would conclude therefore that $K_{2}{ }^{0}$ decays to two pions with a branching ratio $R=\left(K_{2} \rightarrow \pi^{+}+\pi^{-}\right) /$ $\left(K_{2}{ }^{0} \rightarrow\right.$ all charged modes $)=(2.0 \pm 0.4) \times 10^{-3}$ where the error is the standard deviation. As empha-

## Speculations on $S U(3)$ in $B_{(s)} \rightarrow D_{(s)}^{(*)} \ell \bar{\nu}$

- Considerations that suggest possibly sizable effects:

Bjorken and Voloshin sum rules relate the behavior of $B_{(s)} \rightarrow D_{(s)}^{(*)}$ ground state transition to the decays to excited states; e.g., Voloshin sum rule [PRD 46 (1992) 3062]
"Also the sum rule shows that the slope parameter should be a growing function of the mass of the spectator quark."

$$
\rho^{2}=-\left.\frac{\mathrm{d}}{\mathrm{~d} w} \frac{\mathrm{~d} \Gamma}{\mathrm{~d} w}\right|_{w=1}<\frac{1}{4}+\frac{m_{M}-m_{Q}}{2\left(m_{M_{1}}-m_{M}\right)}+\ldots
$$

where $m_{M_{1}}-m_{M}$ is the gap to the first excited meson state above $D_{(s)}^{(*)}$

- Expect: slope parameter increases, if larger rates to excited states (not $D_{(s)}^{(*)}$ ) if $m_{M_{1}}-m_{M}$ smaller ("gap" above $\left.D_{(s)}^{(*)}\right)$
Discovered in 2003: $m_{D_{s 0}^{* \pm}}-m_{D_{s}^{ \pm}} \approx 206 \mathrm{MeV}$, but $m_{D_{0}^{* \pm}}-m_{D^{ \pm}} \approx 484 \mathrm{MeV}$
- Interesting if these arguments for larger slope hold, or compensated by something Recently: $\rho_{D_{s}^{*}}^{2}=1.16 \pm 0.09$ [นсь, 2003.08453] vs. HFLAV: $\rho_{D^{*}}^{2}=1.121 \pm 0.024$ (use CLN) LQCD: "no significant $S U(3)$ symmetry breaking" Harison \& Davies, ${ }^{2105.11433]}$


## Inclusive vs. exclusive: $P_{\tau}$ in $B \rightarrow X \tau \bar{\nu}$

- Inclusive $=\sum$ exclusive type sum rules can give new information $\tau$ polarization probes NP complementary to 3-body ( $X, \tau, \nu$ ) distributions (Could have calculated it when I was a grad student - no one would have cared...)
- Past calculations: $\tau$ polarization axis $=$ directions of the 3-momenta of (i) the $B$ (past inclusive calculations)
(ii) the $\bar{\nu}$ (most exclusive decays \& the only measurement)
(iii) the transverse direction, $x$, violates CP

- Could not compare inclusive and exclusive

Results: $P_{\tau}\left(X_{c}\right) \approx-0.24, \quad P_{\tau}\left(X_{u}\right) \approx-0.36$ (Compared with -0.71 and -0.77 for $\vec{p}_{B}$ direction) Sum rule: $P_{\tau}\left(X_{c}\right)=\sum_{H_{c}} \frac{\mathcal{B}\left(B \rightarrow H_{c} \tau \nu\right) P_{\tau}\left(H_{c}\right)}{\mathcal{B}\left(B \rightarrow X_{c} \tau \nu\right)}$ [Bernlochner, ZL, Papucci, Robinson, 2302.04764]


$$
Z L-p . i i
$$

