# Theory challenges in exclusive $\left|V_{cb}\right|$

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#### Outline: I was told to talk about...

How to assess potential biases related to inputs from various models or sum rule based approaches?

How to decide in a systematic manner on the truncation order of form factor parametrizations?

Can we develop a benchmark test to compare different methods?

For HQET-based parametrizations of  $B \rightarrow D^{(*)}$  semileptonic decays, how do we assess the role of and incorporate second order power corrections?

(I'll focus on my opinions, prospects, open questions)





#### Some reasons $|V_{cb}|$ matters

- $|V_{cb}|$  important to assess if there is an  $\varepsilon_K$  tension, predict  $K \to \pi \nu \bar{\nu}, B \to (X) \ell \bar{\ell}$ SM predictions involve  $A^4$ , so 5% in  $|V_{cb}|$  yields 20%
- The  $b \to c \tau \bar{\nu}$  data should make  $|V_{cb}|$  much better understood are we there yet? To understand the  $\tau$  mode thoroughly, must understand the  $e, \mu$  modes better
- Recently:  $|V_{cb}|$  uncertainty limits future improvements in the sensitivity to NP in *B* and *B<sub>s</sub>* mixing

"Phase II" (LHCb upgrade 2 and Belle II upgrade) with / without  $|V_{cb}|$  uncertainty, maybe early 40s



[Charles, Descotes-Genon, ZL, Monteil, Papucci, Trabelsi, Vale Silva, 2006.04824]





Basics of  $B 
ightarrow D^{(*)} \ell ar{
u}$ 

• Heavy Quark Symmetry:  $v \to v'$  changes brown muck, but not  $m_b \to m_c$  or  $\vec{s_b} \to \vec{s_c}$ [Isgur & Wise]

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar{\nu})}{\mathrm{d}w} = (\dots) (w^2 - 1)^{3(1)/2} |V_{cb}|^2 \mathcal{F}^2_{(*)}(w)$$

$$\swarrow w \equiv v \cdot v' \qquad \text{Isgur-Wise function + corr.}$$

$$\mathcal{F}(1) = \mathbf{1}_{\text{Isgur-Wise}} + 0.02_{\alpha_s,\alpha_s^2} + \frac{(\text{compute})}{m_{c,b}} + \dots$$

$$\mathcal{F}_*(1) = \mathbf{1}_{\text{Isgur-Wise}} - 0.04_{\alpha_s,\alpha_s^2} + \frac{\mathbf{0}_{\text{Luke}}}{m_{c,b}} + \frac{(\text{compute})}{m_{c,b}^2} + \dots$$

(1) Lattice QCD:  $\mathcal{F}_*(1) = 0.906 \pm 0.012$ ,  $\mathcal{F}(1) = 1.054 \pm 0.009$ 

(2) Constraint on shape (slope vs. curvature) [Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert] (3) Some understanding of decays to higher mass  $X_c$  states (backgrounds)

Data:  $|V_{cb} \mathcal{F}_*(1)| = (34.77 \pm 0.36) \times 10^{-3}, |V_{cb} \mathcal{F}(1)| = (41.26 \pm 0.97) \times 10^{-3}$  [HFLAV]



HEORETICAL PHYSICS



[FLAG]

#### 2010s: hints of lepton universality violation

If established, likely impact  $|V_{cb}|$  and  $|V_{ub}|$ , cannot assume only  $\tau$  impacted



- Imply low scale NP, possible mediators:  $(bc)(\tau\nu)$  " $H^{\pm}$ ";  $(b\tau)(c\nu)$  "LQ";  $(b\nu)(c\tau)$  " $\tilde{b}$ "
- Rethink program, new & expanded searches: at high- $p_T$ , new channels, LFV
- Belle unfolded measurements reinvigorated the field (2017–) [1702.01521 & 1809.03290]
- Leads, at least, to more scrutiny and better understanding of  $|V_{cb}|$  determinations





#### Available for the first time in 2017



 $\cos \theta_{\ell}$  [Grinstein & Kobach, 1703.08170]  $\chi$ 



1.0



#### Using HQET at $\mathcal{O}(1/m)$ and $\mathcal{O}(1/m^2)$

- One leading and 3 subleading Isgur-Wise functions in  $B \to D^{(*)} \ell \bar{\nu}$ Can constrain all 4 from  $B \to D^{(*)} l \bar{\nu} \Rightarrow \mathcal{O}(\Lambda^2_{\text{QCD}}/m^2_{c,b}, \alpha^2_s)$  uncertainties  $(l = e, \mu)$ [Bernlochner, ZL, Papucci, Robinson, 1703.05330]
- Observables:  $B \to Dl\bar{\nu}$ :  $d\Gamma/dw$  $B \to D^*l\bar{\nu}$ :  $d\Gamma/dw$  and  $R_{1,2}(w)$  form factor ratios

(CLN fit prescription: QCD sum rules built in + linear slope vs. curvature relation)

• At  $\mathcal{O}(1/m_{c,b}^2)$  the number of "universal" functions for  $B \to D^{(*)} \ell \bar{\nu}$  proliferate

Proposal to include  $1/m_c^2$  corrections using LCSR [Bordone, Jung, van Dyk, 1908.09398]

We explored truncating the number of order  $1/m^2$ Isgur-Wise functions: vanishing chromomagnetic (VC) "limit" & residual chiral (RC) "expansion"

[Bernlochner, ZL, Papucci, Prim, Robinson, Xiong, "BLPR-XP", 2206.11281]

HQET		Isgur-Wise functions				
order	All	<b>RC</b> Expansion	VC Limit			
1	1	1	1			
$1/m_{c,b}$	3	3	2			
$1/m_{c}^{2}$	20	1	2			
$1/m_{c,b}^{2}$	32	3	3			





#### **Boyd-Grinstein-Lebed (BGL) constraints**

• Constrain form factor shapes, based on analyticity & unitarity; Taylor expansions:

$$rac{1}{P_i(z)\phi_i(z)}\sum a_n^i z^n \qquad \quad i=g,\,f,\,\mathcal{F}_1 ext{ (lin. comb.)}$$

z(w) is a conformal parameter, maps physical region 1 < w < 1.5 to 0 < z < 0.056 $P_i(z)$ ,  $\phi_i(z)$  are known functions  $c_0$  is fixed by  $b_0$ 

Some papers use notation:  $\{a_n, b_n, c_n\} \longleftrightarrow \{a_n^g, a_n^f, a_n^{\mathcal{F}_1}\}$ 

- Does not use constraints from heavy quark symmetry, but can be added
- Denote by  $BGL_{ijk}$  a BGL fit with parameters:  $\{a_{0,...,i-1}, b_{0,...,j-1}, c_{1,...,k}\}$

Literature contains various choices: N = i + j + k = 4, 5, 6, 8

• Must truncate expansions at some order — what is the optimal choice?





#### The CLN fits used 1997–2017

- CLN added QCD SR to BGL:  $R_{1,2}(w) = \underbrace{R_{1,2}(1)}_{\text{fit}} + \underbrace{R'_{1,2}(1)}_{\text{fixed}}(w-1) + \underbrace{R''_{1,2}(1)}_{\text{fixed}}(w-1)^2/2$ 
  - In HQET:  $R_{1,2}(1) = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$   $R_{1,2}^{(n)}(1) = 0 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)$

The  $\mathcal{O}(\Lambda_{
m QCD}/m_{c,b})$  terms are determined by 3 subleading Isgur-Wise functions

- Inconsistent fits: same param's determine  $R_{1,2}(1) 1$  (fit) and  $R_{1,2}^{(1,2)}(1)$  (QCDSR) Sometimes calculations using QCD sum rules are called the HQET predictions
- Devised fits to "interpolate" between BGL and CLN [Bernlochner, ZL, Robinson, Papucci, 1708.07134]

form factors	BGL	CLN	CLNnoR	noHQS
axial $\propto \epsilon_{\mu}^{*}$	$b_0,  b_1$	$h_{A_1}(1), \ \rho_{D^*}^2$	$h_{A_1}(1), \ \rho_{D^*}^2$	$h_{A_1}(1),\;\rho_{D^*}^2,\;c_{D^*}$
vector	$a_0,  a_1$	$\int R_1(1)$	$\int R_1(1), R'_1(1)$	$\int R_1(1), R'_1(1)$
axial $(\mathcal{F}_1)$	$c_1,  c_2$	$R_2(1)$	$R_2(1), R'_2(1)$	$R_2(1), R'_2(1)$

Relaxing constraints on  $R'_{1,2}(1)$ , fit results similar to BGL





#### Abstract of recent Belle paper [2310.01170]

We determine the CKM matrix-element magnitude  $|V_{cb}|$  using  $\overline{B}^0 \to D^{*+} \ell^- \bar{\nu}_{\ell}$  decays reconstructed in 189 fb<sup>-1</sup> of collision data collected by the Belle II experiment, located at the SuperKEKB  $e^+e^-$  collider. Partial decay rates are reported as functions of the recoil parameter w and three decay angles separately for electron and muon final states. We obtain  $|V_{cb}|$  using the Boyd-Grinstein-Lebed and Caprini-Lellouch-Neubert parametrizations, and find  $|V_{cb}|_{BGL} = (40.57 \pm 0.31 \pm 0.95 \pm 0.58) \times 10^{-3}$  and  $|V_{cb}|_{CLN} = (40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$  with the uncertainties denoting statistical components, systematic components, and components from the lattice QCD input, respectively. The branching fraction is measured to be  $\mathcal{B}(\overline{B}^0 \to D^{*+}\ell^- \bar{\nu}_{\ell}) = (4.922 \pm 0.023 \pm 0.220)\%$ . The ratio of branching fractions for electron and muon final states is found to be  $0.998 \pm 0.009 \pm 0.020$ . In addition, we determine the forward-backward angular asymmetry and the  $D^{*+}$  longitudinal polarization fractions. All results are compatible with lepton-flavor universality in the Standard Model.

- If CLN fit is quoted (maybe to compare with past results?) on equal footing with BGL, readers will assume that the Collaboration views them equally meaningful
- While CLN gave a simple recipe (that is not self-consistent), using BGL some choices must be made (truncation order, additional input from HQET, LQCD, unitarity?)





#### The how-to-fit saga



"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk." (John von Neumann)

Overfitting? Truncation orders? Additional inputs/constraints from HQET, LQCD, unitarity?

#### Nested hypothesis tests

Optir	mal B	GL fit p	aramet	ers? (ı	pper: $\chi^2$ ,	lower: $ V_c $	$_b  \times 10^3$ )	[From	1902.09553	8, using 1702.	
	$n_a$ $n_c$	1	2	3	1	2	3	1	2	3	
-	1	33.2	31.6	31.2	33.0	29.1	28.9	30.4	29.1	28.9	
	1	$38.6 \pm 1.0$	$38.6 \pm 1.0$	$38.6 \pm 1.0$	$39.0\pm1.5$	$40.7\pm1.6$	$40.7 \pm 1.6$	$40.7\pm1.7$	$40.6 \pm 1.8$	$40.6 \pm 1.8$	
	2	32.9	31.3	31.1	32.7	27.7	27.7	29.2	27.7	27.7	
	2	$38.8 \pm 1.1$	$38.7 \pm 1.1$	$38.8 \pm 1.0$	$39.5 \pm 1.7$	$41.7 \pm 1.8$	$41.6 \pm 1.8$	$41.8\pm2.0$	$41.8 \pm 2.0$	$41.7\pm2.0$	
	9	31.7	31.3	31.0	29.1	27.7	27.6	29.2	27.6	23.2	
	3	$39.0 \pm 1.1$	$38.6 \pm 1.2$	$38.6 \pm 1.1$	$41.9\pm2.0$	$41.8 \pm 2.0$	$41.7 \pm 2.0$	$41.8 \pm 2.0$	$41.7 \pm 1.9$	$41.4\pm2.0$	
-			$n_b = 1$			$n_b = 2$		$a_{nb} = 3$			

- Fit w/ 1 param added / removed:  $BGL_{(n_a \pm 1)n_b n_c}$ ,  $BGL_{n_a(n_b \pm 1)n_c}$ ,  $BGL_{n_a n_b(n_c \pm 1)}$
- Accept descendant (parent) if  $\Delta\chi^2$  is above (below) a boundary, say,  $\Delta\chi^2 = 1$
- Repeat until "stationary" fit is found, preferred over its parents and descendants
- If multiple stationary fits, choose smallest N, then smallest  $\chi^2$  (333 is an overfit!) Start from small N, to avoid overfitting e.g.:  $\begin{cases} 111 \rightarrow 211 \rightarrow 221 \rightarrow 222 \\ 121 \rightarrow 131 \rightarrow 231 \rightarrow 232 \rightarrow 222 \end{cases}$





#### Toy studies: check $|V_{cb}|$ is unbiased

• Set  $\{\tilde{a}_{0,1}, \tilde{b}_{0,1}, \tilde{c}_{1,2}\} = BGL_{222}$  fit result, and  $\{\tilde{a}_2, \tilde{b}_2, \tilde{c}_3\} = (1 \text{ or } 10) \times \{\tilde{a}_1, \tilde{b}_1, \tilde{c}_2\}$ Generate MC data using experimental covariance, fit each set w/ our prescription



Frequency of the selected hypotheses, with two scenarios for higher order terms:

	$BGL_{122}$	$BGL_{212}$	$\mathrm{BGL}_{221}$	$\mathrm{BGL}_{222}$	$\mathrm{BGL}_{223}$	$\mathrm{BGL}_{232}$	$\mathrm{BGL}_{322}$	$\mathrm{BGL}_{233}$	$\mathrm{BGL}_{323}$	$\mathrm{BGL}_{332}$	$\mathrm{BGL}_{333}$
'1-times'	6%	0%	37%	27%	6%	6%	11%	0%	2%	4%	0.4%
'10-times'	0%	0%	8%	38%	14%	8%	16%	3%	4%	8%	1%





#### **Akaike information criterion**

- There is no frequentist approach to deciding on number of fit parameters (Each choice is effectively a different theory)
- How to pick number of BGL fit parameters? (depends on data, multiple proposals) Just looking at goodness of fit is not enough ["us" 1902.09553; Gambino, Jung, Schacht 1905.08209, etc.]
- Akaike information criterion (AIC): Started by Bhattacharya, Nandi, Patra, 1611.04605, 1805.08222, 1908.04835 for BSM fits, recently revisited by Simons *et al.* 2304.13045 for |V<sub>cb</sub>|

"AIC sometimes selects a much better model than BIC even when the "true model" is in the candidate set" [Wikipedia]

• Vast literature: desirable to use a procedure that is somewhat "standard", easy to communicate, and not to reinvent the wheel (especially if resuts in tension w/ SM)





#### **Notorius BGL: truncation orders matter**

Belle (711/fb) [2301.07529] and Belle II (189/fb) [2310.01170]	)]					
		$ V_{\rm cb} $	$\chi^2$	d.o.f.	Ν	$ \rho_{\rm max} $
papers impose a constraint that eliminates models	BGL <sub>111</sub>	$40.4\pm0.8$	45.6	34	3	0.70
	BGL <sub>112</sub>	$40.9\pm0.9$	43.4	33	4	0.98
with highly correlated fit parameters	<b>BGL</b> <sub>121</sub>	$40.7\pm0.9$	45.2	33	4	0.60
	BGL <sub>122</sub>	$41.5 \pm 1.1$	42.3	32	5	0.98
	BGL <sub>131</sub>	$38.1 \pm 1.7$	41.7	32	5	0.98
Belle paper choose BGL 101 while both AIC and	BGL <sub>132</sub>	$39.0 \pm 1.6$	37.5	31	6	0.98
$\sim$ Defice paper checke $Dar_{121}$ , while beth fire and	$BGL_{211}$	$39.7 \pm 1.0$	42.7	33	4	0.99
NHT $r_{\rm rest}$ and $r_{\rm rest}$ would pick the BCI $r_{\rm rest}$ fit $\rightarrow$	BGL <sub>212</sub>	$40.4 \pm 1.0$	39.3	32	5	0.99
NTT [as in 1902.09553] WOULD PICK THE DUL $221$ III $\rightarrow$	BGL <sub>221</sub>	$37.1 \pm 1.2$	37.7	32	5	1.00
	BGL <sub>222</sub>	$37.9 \pm 2.0$ $37.2 \pm 1.8$	37.5	31	6	0.00
Difference not negligible: $A   \mathbf{I} \mathbf{I}   = 0.0 \pm 10^{-3}$	BGL 231	$37.2 \pm 1.8$ $38.8 \pm 1.7$	37.7	30	7	0.99
Difference not negligible: $\Delta  V_{cb}  = 3.6 \times 10^{-9}$	BGL <sub>232</sub>	$38.5 \pm 0.9$	40.1	32	5	0.95
	BGL <sub>311</sub>	$39.9 \pm 1.1$	36.9	31	6	0.98
	BGL <sub>321</sub>	$37.3 \pm 1.2$	37.3	31	6	0.97
Detailed validation of model selection with tov MC	BGL <sub>322</sub>	$38.9\pm2.1$	36.5	30	7	0.99
	BGL <sub>331</sub>	$39.6\pm2.3$	36.3	30	7	0.99
seems to be essential	BGL <sub>332</sub>	$40.1 \pm 2.3$	35.9	29	8	0.99

(Maybe more from Florian or Markus during discussions)

See also: Juttner's talk at LHCb implications, last week [2303.11285]

[2301.07529, Table XVI from PRD]





## **Higher orders**

Order  $\mathcal{O}(1/m_{c,b}^2)$  terms

- Baryons: much fewer form factors, more tractable [Bernlochner, ZL, Robinson, Sutcliffe, 1808.09464]
- At  $\mathcal{O}(1/m_{c,b}^2)$  the number of "universal" functions for  $B \to D^{(*)} \ell \bar{\nu}$  proliferate

Proposal to include  $1/m_c^2$  corrections using LCSR [Bordone, Jung, van Dyk, 1908.09398]

We explored truncating the number of order  $1/m^2$ Isgur-Wise functions: vanishing chromomagnetic (VC) "limit" & residual chiral (RC) "expansion"

[Bernlochner, ZL, Papucci, Prim, Robinson, Xiong, "BLPR-XP", 2206.11281]

HQET		Isgur-Wise functions					
order	All	RC Expansion	VC Limit				
1	1	1	1				
$1/m_{c,b}$	3	3	2				
$1/m_{c}^{2}$	20	1	2				
$1/m_{c,b}^2$	32	3	3				

I am not convinced that it's optimal or necessary to account for all  $\mathcal{O}(1/m_{c,b}^2)$  terms

• Toys to estimate ability to constrain many terms (5 or 50/ab?) would be interesting (Try to identify what's important and what's not, etc.)





#### Main differences in our 2022 vs. 2017 paper

- BLPR [1703.05330] preceded Belle'19 [1809.03290], which is in tension w/ Belle'17 [1702.01521] Changes are not due to (partly) including  $1/m^2$ ; enough freedom at 1/m at current precision
- CLN approximated the constraint on the slope vs. curvature plane by a linear relationship
   The precision of experimental and LQCD data are high enough that this no longer applies

• BLPR (no SR): 
$$R(D) = 0.298(3), R(D^*) = 0.261(4)$$
  
 $\downarrow \downarrow$   
BLPR-XP:  $R(D) = 0.288(4), R(D^*) = 0.249(3)$ 

(Includes a scale factor for the  $D^*$  prediction to account for tension)



• Slight increase in tension between SM prediction and  $R(D^{(*)})$  measurements





### $B ightarrow D^{(*)}$ : some open questions

- Won't talk about fits to unfolded vs. folded data what I think is important is that different approaches and new ideas can be tested ⇒ See Markus' talk
- Can we have a more systematic approach to  $1/m_{c,b}^2$ , with continuum methods?
- How to pick number of BGL fit parameters? (depends on data, multiple proposals) Just looking at goodness of fits is not the full story [1708.07134, 1905.08209]
- Can lattice QCD determine form factors as precisely as  $F_{(*)}(1)$ ? Some tension between FNAL/MILC results and data (less so for JLQCD)  $R(D^*)_{\text{Lat}} = 0.265 \pm 0.013, \quad R(D^*)_{\text{Lat}+\text{Exp}(e,\mu)} = 0.2484 \pm 0.0013$  [FNAL & MILC, 2105.14019]  $B_s \rightarrow D_s^* \ell \bar{\nu}: R(D_s^*) = 0.249 \pm 0.007$  [Harrison & Davies, 2105.11433]
- Need (a lot) more data to resolve all outstanding issues





Importance known since the early 1990s ratios of form factors defined to be unity in HQS limit at all w

Lattice calculations not as consistent as one would like:



**Form factor ratios** 

w

Belle II

 $\overline{B}^0 \rightarrow D^{*+} \ell^- \overline{\nu}_\ell$ 

1.5

1.4 1.3

(M)<sup>1</sup> W 1.2

1.1

1.0 1.0

- Observing a large violation of HQS would have significant consequences
- Need both more precise measurements and lattice QCD calculations

FY CENTER FOR THEORETICAL PHYSICS



 $\int \mathcal{L} dt = 189 \, \text{fb}^{-1}$ 

 $\int \mathcal{L} dt = 189 \, \text{fb}^{-1}$ 

1.5

1.4

<sup>[</sup>Belle II, 2310.01170]

#### $B \to D^{**}$ : many open questions

- Significant backgrounds, masses in some tension w/ quark model predictions Some nonleptonic  $D^{**}\pi$  rates far from facotrization prediction
- The "1/2 vs. 3/2 puzzle" remains... puzzling [Le Yaouanc, Leroy, Roudeau, 2102.11608]
- How well can  $B \rightarrow D^{**}$  and nonresonant rates be measured at Belle II?
- Four  $D_s^{**}$  states much narrower than non-strange counterparts nice for LHCb
- $D_{s0}^*(2317)$ : orbitally excited state or "molecule"? Might make HQET inapplicable

If  $D_{s0}^*$  is excited  $c\bar{s}$  state, predict  $\mathcal{B}(D_{s0}^* \to D_s^*\gamma)/\mathcal{B}(D_{s0}^* \to D_s\pi)$  above CLEO bound, < 0.059 [Mehen & Springer, hep-ph/0407181; Colangelo & De Fazio, hep-ph/0305140; Godfrey, hep-ph/0305122] CLEO used 13.5/fb, the Belle bound < 0.18 used 87/fb, the BaBar bound < 0.16 used 232/fb

• Understanding Inclusive =  $\sum$  Exclusive may be necessary to resolve issues





## Conclusions

- Independent of hints of NP, reducing the uncertainty of  $|V_{cb}|$  is important
- $B \to D^* \ell \bar{\nu}$ : Need (much) more data and consistent LQCD results
- What are the largest useful data sets? No one has seriously explored it! (Recall, Sanda, 2003: the question is not 10<sup>35</sup> or 10<sup>36</sup>...)

With  $\infty$  statistics, would  $B \to D^* e \bar{\nu}$  in the bin  $q^2 > 9 \,\text{GeV}^2$  give the cleanest  $|V_{cb}|$ ? (No  $D^{**}$  backgrounds, no  $D^* \to D$  down-feed)

Of course, in reality, there is always a tradeoff to minimize the overall uncertainty...

- "Best" case: new physics, new directions
   "Worst" case: better SM tests, better CKM determinations, and NP sensitivity
- Good reasons to want to collect the largest possible  $\Upsilon(4S)$  data sets







## Extra slides

#### **Factor of 2 improvements can matter!**

ANNALS OF PHYSICS: 5, 156-181 (1958)

#### VOLUME 6, NUMBER 10 PHYSICAL REVIEW LETTERS MAY 15, 1961 Long-lived Neutral K Mesons\* DECAY PROPERTIES OF K.º MESONS\* D. Neagu, E. O. Okonov, N. I. Petrov, A. M. Rosanova, and V. A. Rusakov M. BARDON, K. LANDE, AND L. M. LEDERMAN Joint Institute of Nuclear Research, Moscow, U.S.S.R. (Received April 20, 1961) Columbia University, New York, New York, and Brookhaven National Laboratories, Upton, New York Combining our data with those obtained in refer-AND ence 7, we set an upper limit of 0.3% for the relative probability of the decay $K_2^0 \rightarrow \pi^- + \pi^+$ . Our WILLIAM CHINOWSKY Brookhaven National Laboratories. Upton. New York set an upper limit < 0.6% on the reactions $K_2^0 \rightarrow \begin{cases} \mu^{\pm} + e^{\mp} \\ e^+ + e^- \end{cases}$ "At that stage the search was terminated by administration of the Lab." [Okun, hep-ph/0112031]

and on  $K_2^0 \to \pi^+ + \pi^-$ .

VOLUME 13, NUMBER 4 PHYSICAL REVIEW LETTERS 27 JULY 1964

#### EVIDENCE FOR THE $2\pi$ DECAY OF THE $K_2^{0}$ MESON\*<sup>†</sup>

J. H. Christenson, J. W. Cronin,<sup>‡</sup> V. L. Fitch,<sup>‡</sup> and R. Turlay<sup>§</sup> Princeton University, Princeton, New Jersey (Received 10 July 1964) We would conclude therefore that  $K_2^0$  decays to two pions with a branching ratio  $R = (K_2 - \pi^+ + \pi^-)/(K_2^0 - \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$  where the error is the standard deviation. As empha-

## Speculations on SU(3) in $B_{(s)} o D_{(s)}^{(*)} \ell ar{ u}$

Considerations that suggest possibly sizable effects:

Bjorken and Voloshin sum rules relate the behavior of  $B_{(s)} \rightarrow D_{(s)}^{(*)}$  ground state transition to the decays to excited states; e.g., Voloshin sum rule [PRD 46 (1992) 3062] "Also the sum rule shows that the slope parameter should be a growing function of the mass of the spectator quark."

$$\rho^{2} = -\frac{\mathrm{d}}{\mathrm{d}w} \frac{\mathrm{d}\Gamma}{\mathrm{d}w} \Big|_{w=1} < \frac{1}{4} + \frac{m_{M} - m_{Q}}{2(m_{M_{1}} - m_{M})} + \dots$$

where  $m_{M_1} - m_M$  is the gap to the first excited meson state above  $D_{(s)}^{(*)}$ 

Expect: slope parameter increases, if larger rates to excited states (not  $D_{(s)}^{(*)}$ ) if  $m_{M_1} - m_M$  smaller ("gap" above  $D_{(s)}^{(*)}$ )

Discovered in 2003:  $m_{D_{s0}^{*\pm}} - m_{D_s^{\pm}} \approx 206 \text{ MeV}$ , but  $m_{D_0^{*\pm}} - m_{D^{\pm}} \approx 484 \text{ MeV}$ 

• Interesting if these arguments for larger slope hold, or compensated by something Recently:  $\rho_{D_s^*}^2 = 1.16 \pm 0.09$  [LHCb, 2003.08453] vs. HFLAV:  $\rho_{D^*}^2 = 1.121 \pm 0.024$  (use CLN) LQCD: "no significant SU(3) symmetry breaking" [Harrison & Davies, 2105.11433]





#### Inclusive vs. exclusive: $P_{ au}$ in $B o X au ar{ u}$

- Inclusive  $= \sum$  exclusive type sum rules can give new information  $\tau$  polarization probes NP complementary to 3-body ( $X, \tau, \nu$ ) distributions (Could have calculated it when I was a grad student – no one would have cared...)
- Past calculations: τ polarization axis = directions of the 3-momenta of (i) the B (past inclusive calculations)
   (ii) the ν̄ (most exclusive decays & the only measurement)
   (iii) the transverse direction, x, violates CP







 $\vec{q} = \vec{p}_{\bar{\nu}}$