

Experimental improvement of f_{+0}

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Outline (the pre-workshop slide)

- Why we care?
- The current state of the art:
<https://inspirehep.net/literature/2105326>
- How can we do better?
 - direct determination as nuisance parameter in a combined fit
 - isospin assumption
 - Double-semileptonic decay: <https://arxiv.org/pdf/hep-ex/0504001.pdf>
 - Other ideas: <https://inspirehep.net/literature/2667113>
 - \sqrt{s} dependent quantity – $\sigma(e^+e^- \rightarrow BB)$ lineshape
 -

Why we care? A systematic limitation

- Looking at our recent exclusive untagged V_{cb} paper [arXiv:2310.01170](https://arxiv.org/abs/2310.01170) (accepted PRD) we have this expression for the partial rate in each differential bin

$$\Delta\Gamma_x = \frac{\nu_x^{\text{unfolded}} \hbar}{\epsilon_x N_{B^0} \mathcal{B}(D^{*+} \rightarrow D^0 \pi^+) \mathcal{B}(D^0 \rightarrow K^- \pi^+) \tau_{B^0}}$$

- The number of B mesons is a common factor amongst all bins and its uncertainty propagates directly to $|V_{cb}|^2$ thus half of that to $|V_{cb}|$
- Two contributions to estimating it – the total no. of B-meson pairs (N_{BB}) and the ratio of $Y(4S)$ branching fractions to charged and neutral pairs (f_{+0})

$$N_{B^0} = 2N_{BB} (1 + f_{+0})^{-1}$$

Why we care? A systematic limitation

- Second largest uncertainty
 - Slow π (see D. Dorner's talk)
- So now we must address
 - why the current number is so large?
 - how we might do better?
- Aside: this affects all precision measurements of BFs too
 - Dominant systematic in recent $\text{BF}(B^0 \rightarrow h^+ h^-)$ [results](#)

	ρ^2	$R_1(1)$	$R_2(1)$	$ V_{cb} $
Statistical	3.0	4.1	2.8	0.7
Background subtraction	1.4	2.2	1.2	0.3
Size of simulated samples	1.2	1.7	1.1	0.3
Lepton ID efficiency	0.2	1.6	0.1	0.3
Slow pion efficiency	1.0	0.9	0.8	1.5
Tracking of K, π, ℓ				0.4
$N_{B\bar{B}}$				0.8
f_{+0}				1.3
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+)$				0.4
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$				0.4
B^0 lifetime				0.1
Signal modeling	2.6	2.6	2.0	0.5
Total	4.5	5.9	3.9	2.4

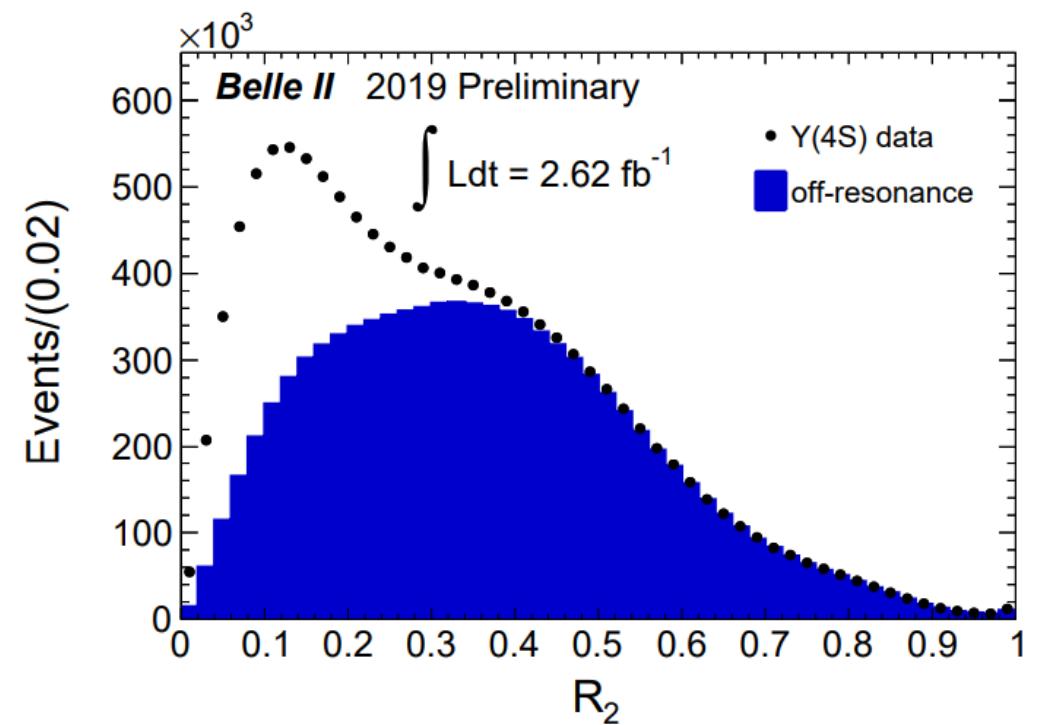
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N_{BB} uncertainty

- Use event shape to distinguish BB from continuum
- Off-resonance data ($\sim 10\%$ of $Y(4S)$) used to subtract continuum
 - Largest uncertainties from matching conditions of off-resonance and on-resonance running
 - Luminosity for scaling
- On-resonance only technique being developed



This is for 2022 (not latest) but indicative

Source	systematics on $N_{B\bar{B}}$ (%)
missing off-resonance	1.14
luminosity measurement	0.9
selection efficiency	0.5
tracking efficiency	0.1
trigger efficiency	0.2
Total	1.5

Our current best estimate:

[PRD 107, L031102 \(2023\)](#) [Belle Collaboration]

- Uses $B \rightarrow J/\psi K$ events and isospin assumptions

$$N_{\text{sig}}^+ = 2N_{B\bar{B}} f^{+-} \epsilon^+ \mathcal{B}[B^+ \rightarrow J/\psi(\ell\ell)K^+]$$

$$N_{\text{sig}}^0 = 2N_{B\bar{B}} f^{00} \epsilon^0 \mathcal{B}[B^0 \rightarrow J/\psi(\ell\ell)K^0]$$

Note that
 $R^{+/0} \equiv f_{+0}$

$$\frac{N_{\text{sig}}^+/\epsilon^+}{N_{\text{sig}}^0/\epsilon^0} = R^{+/0} \frac{\mathcal{B}[B^+ \rightarrow J/\psi(\ell\ell)K^+]}{\mathcal{B}[B^0 \rightarrow J/\psi(\ell\ell)K^0]}$$

$$= R^{+/0} \frac{\Gamma[B^+ \rightarrow J/\psi(\ell\ell)K^+] \tau^+}{\Gamma[B^0 \rightarrow J/\psi(\ell\ell)K^0] \tau^0}$$

$$\Rightarrow R^{+/0} = \frac{N_{\text{sig}}^+ \epsilon^0 \tau_0}{N_{\text{sig}}^0 \epsilon^+ \tau_+}$$

But how good is the assumption
 $\Gamma(B^+ \rightarrow J/\psi K^+) = \Gamma(B^0 \rightarrow J/\psi K^0)$?

Our current best estimate:

[PRD 107, L031102 \(2023\)](#) [Belle Collaboration]

- The assumption is that it is of the order $\bar{\lambda}^3$ from isospin breaking rescattering [\[Fleischer and Mannel, 2001\]](#)
 - $\bar{\lambda} = 0.2$ is a generic expansion parameter the same order as $\sin \theta_c = 0.22$
- This results in

$$f_{+0} = 1.065 \pm 0.012 \text{ (stat)} \pm 0.019 \text{ (syst)} \pm \mathbf{0.043 \text{ (Isospin)}}$$

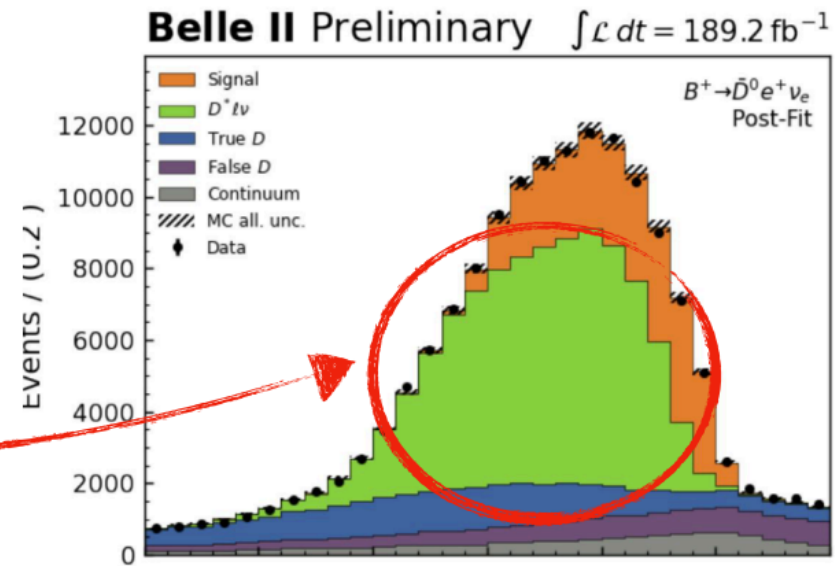
So, it is isospin breaking assumption that dominates the current estimate on V_{cb}

Note that [HFLAV](#) does not include the Belle measurement or isospin uncertainty resulting in 1.058 ± 0.024

A different isospin assumption – ongoing M. Dorigo and M. Mantovano

Simultaneous fit of $D^0 l \bar{\nu}$ and $D^- l \nu$

- Target an analysis of $B \rightarrow D l \nu X$ decays to measure $\mathcal{B}(B \rightarrow D l \nu)$, $\mathcal{B}(B \rightarrow D^* l \nu)$, form-factors (FFs) and $|V_{cb}|$ using both $D^0 l \nu X$ and $D^- l \nu X$.
The $B \rightarrow D^* l \nu$ are partially reconstructed.



- Same method used by BaBar in PRD [[arXiv:0809.0828](https://arxiv.org/abs/0809.0828)] to obtain the world's best result of $\mathcal{B}(B^+ \rightarrow D^{0*} l \nu)$.

Measurement of f_{+-}/f_{00}

- Assuming the SL decay-width equality between B^- and B^0 , we can also provide a measurement of f_{+-}/f_{00} .

- The signal yields can be expressed as:

$$N_{D^0\bar{l}X}(B \rightarrow D^*l\nu) = 2N_{BB}\epsilon\mathcal{B}(B^- \rightarrow D^{*0}l\nu)\left[\frac{f}{1+f} + \frac{1}{1+f}\frac{\tau_{B^0}}{\tau_{B^-}}\mathcal{B}(D^{*+} \rightarrow D^0X)\right]\mathcal{B}(D^0 \rightarrow k\pi)$$

$$N_{D^+lX}(B \rightarrow D^*l\nu) = 2N_{BB}\epsilon\mathcal{B}(B^- \rightarrow D^{*0}l\nu)\left[\frac{1}{1+f}\frac{\tau_{B^0}}{\tau_{B^-}}\mathcal{B}(D^{*+} \rightarrow D^+X)\right]\mathcal{B}(D^+ \rightarrow k\pi\pi)$$

$$N_{D^0\bar{l}X}(B \rightarrow D^0l\nu) = 2N_{BB}\epsilon\frac{f}{1+f}\mathcal{B}(B^- \rightarrow D^0l\nu)\mathcal{B}(D^0 \rightarrow k\pi)$$

$$N_{D^+lX}(B \rightarrow D^+l\nu) = 2N_{BB}\epsilon\frac{1}{1+f}\mathcal{B}(B^- \rightarrow D^0l\nu)\frac{\tau_{B^0}}{\tau_{B^-}}\mathcal{B}(D^+ \rightarrow k\pi\pi)$$

- With the SL decay-equality we will have 4 equations (yields) and only 2 fit parameters ($\mathcal{B}(B \rightarrow Dl\nu)$ and $\mathcal{B}(B \rightarrow D^*l\nu)$) so we can also measure f .

- The simultaneous fit provides a measurement of f with a statistical uncertainty of 1.2%. (competitive with the statistical uncertainty of the current measurement [\[link\]](#)).

- Isospin breaking uncertainty <1% in SL decays, to be compared with 4.5% in $B \rightarrow J/\psi K$ (dominated by factorisation uncertainty).

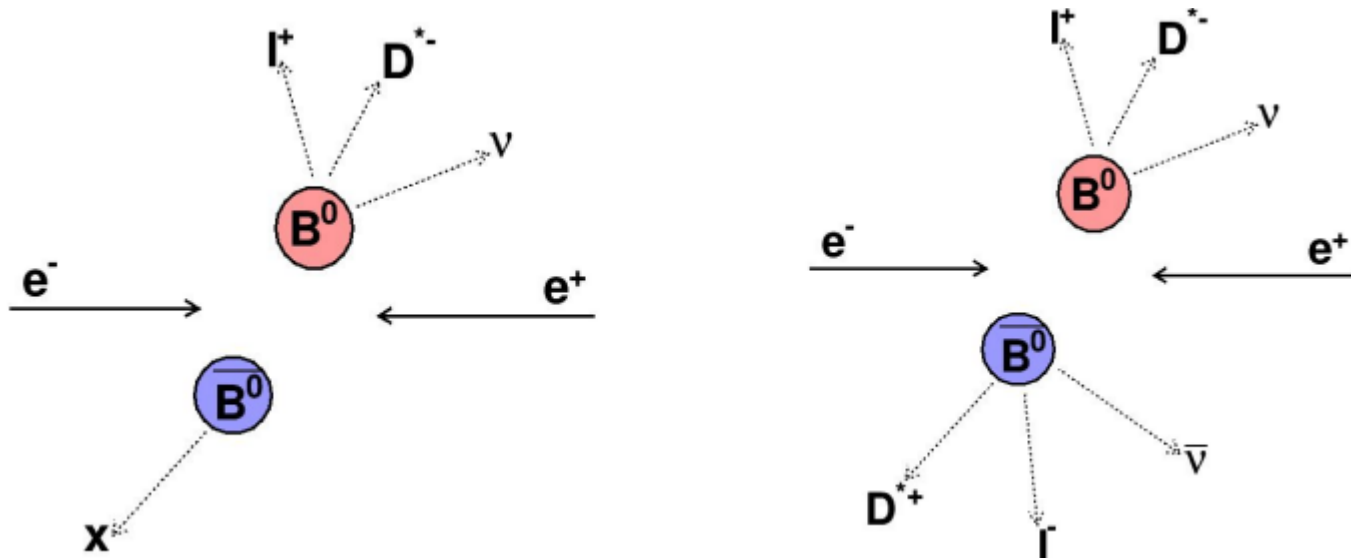
- No rescattering in semileptonic so anticipate isospin assumption is good < 1%
- Coulomb effect
 - Neglected in many publications
 - Estimated to be very small O(1%) in [arXiv:1503.07237](#)
- Other systematics, e.g., D^{**} ?

No isospin assumption

[PRL 95, 042001 \(2005\)](#) BABAR Collaboration

- A measurement of $f_{00} = B(\Upsilon(4S) \rightarrow B^0 \bar{B}^0)$
- Either assume zero width of $\Upsilon(4S)$ to non-B-meson pairs and derive f_{+0} or use an independent measure of f_{+0} to bound the non-B width.

$$N_s = 2N_{B\bar{B}}f_{00}\varepsilon_s\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell), \quad N_d = N_{B\bar{B}}f_{00}\varepsilon_d[\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\ell^-\bar{\nu}_\ell)]^2$$



$$f_{00} = \frac{\varepsilon_d}{\varepsilon_s^2} \frac{N_s^2}{4N_{BB}N_d}$$

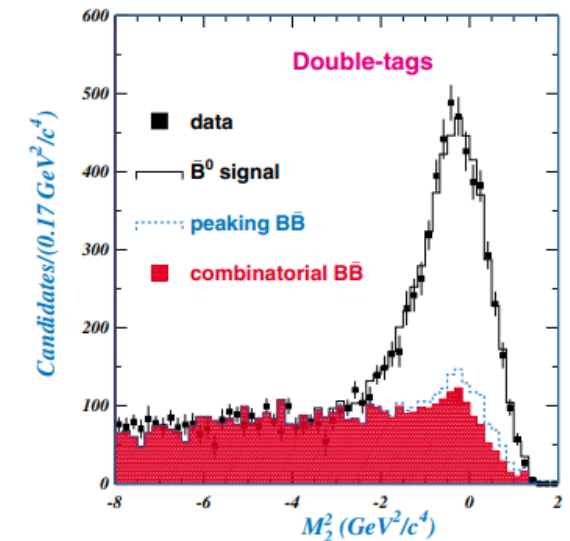
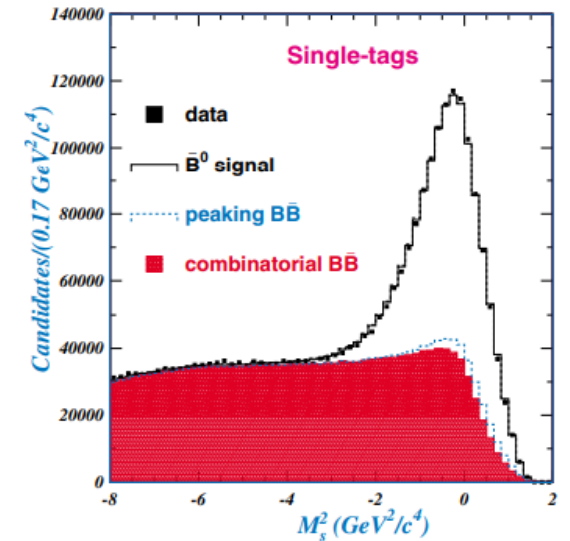
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PRL 95, 042001 (2005) BABAR Collaboration

- Inclusive reconstruction just using lepton and soft pion
- Then D^* momentum inferred from the slow pion given it is at rest in the D^* frame
- Then calculate missing mass with $\mathbf{p}_B=0$ assumption
- Off-resonance and same charge samples used for combinatorial background subtraction
- Peaking background D^{**} etc is one of the largest sources of systematics along with N_{BB}

$$f_{00} = 0.487 \pm 0.010(\text{stat}) \pm 0.008(\text{syst})$$

Only 82 fb^{-1} data sample



New ideas

[2306.04686](#) [hep-ph] Florian et al.

Part I – reappraising the current uncertainties

- Emphasise the non-B width is poorly known
- Lower bound of 0.24% from the sum of Y transitions
- Measurement from [CLEO](#) $f_B = (-0.11 \pm 1.43 \pm 1.07) \%$
- They reappraise the isospin violation uncertainty reported by Belle in $B \rightarrow J/\psi K$ to be 0.030 rather than 0.043 based on testing isospin violation in these decays with independent values of f_{0+}
- Adding non-B width information, updated Belle and Belle II measurements of $B \rightarrow J/\psi K$ and A_1 in $b \rightarrow s\gamma$ update the HFLAV average to get $f_{+0} = 1.057 \pm 0.023$
- Investigate impact of the non-B width and show the value is consistent with the phase space and Coulomb effects in B pair production

New ideas

[2306.04686](#) [hep-ph] Florian et al.

- Their conclusions
 1. Currently level of precision not good enough for future V_{cb}
 2. Isospin assumptions hard to reduce further
 3. Only the double-tag independent of isospin – a lot of data requiredCan't argue with this but the combined fit is a new approach to add

New ideas proposed

Use $\Upsilon(5S)$ information:

[2306.04686](#) [hep-ph] Florian et al.

- Phase space factor reduced so assume
- Then measure double-ratio

$$R_{5S}^{\pm 0} = \frac{\Gamma(\Upsilon(5S) \rightarrow B^{(*)+} B^{(*)-})}{\Gamma(\Upsilon(5S) \rightarrow B^{(*)0} \bar{B}^{(*)0})} \simeq 1.$$

$$r(f, f') = \left[\frac{N(B^+ \rightarrow f)}{N(B^0 \rightarrow f')} \right]_{\Upsilon(4S)} / \left[\frac{N(B^+ \rightarrow f)}{N(B^0 \rightarrow f')} \right]_{\Upsilon(5S)}$$

	Belle	Belle II partial	Belle II full
$\mathcal{L}_{\Upsilon(5S)} / \mathcal{L}_{\Upsilon(4S)}$ [ab ⁻¹ /ab ⁻¹]	0.12 / 0.71	0.5 / 5	5 / 50
$N_{B^{(*)}B^{(*)}}^{\Upsilon(5S)} / N_{BB}^{\Upsilon(4S)}$	$2.74 \times 10^7 / 7.72 \times 10^8$	$1.13 \times 10^8 / 5.55 \times 10^9$	$1.13 \times 10^9 / 5.55 \times 10^{10}$
f, f'	$\Delta r(f, f')/r(f, f')$		
$J/\psi K^+, J/\psi K^0$	7.1%	3.5%	1.1%
$\bar{D}^0 \pi^+, D^- \pi^+$	2.4%	1.2%	0.4%
$\bar{D}^{*0} \ell^+ \nu, D^{*-} \ell^+ \nu$	4.5%	2.2%	0.7%
$\bar{D}^0 \pi^+, D^{*-} \ell^+ \nu$	1.8%	0.9%	0.3%

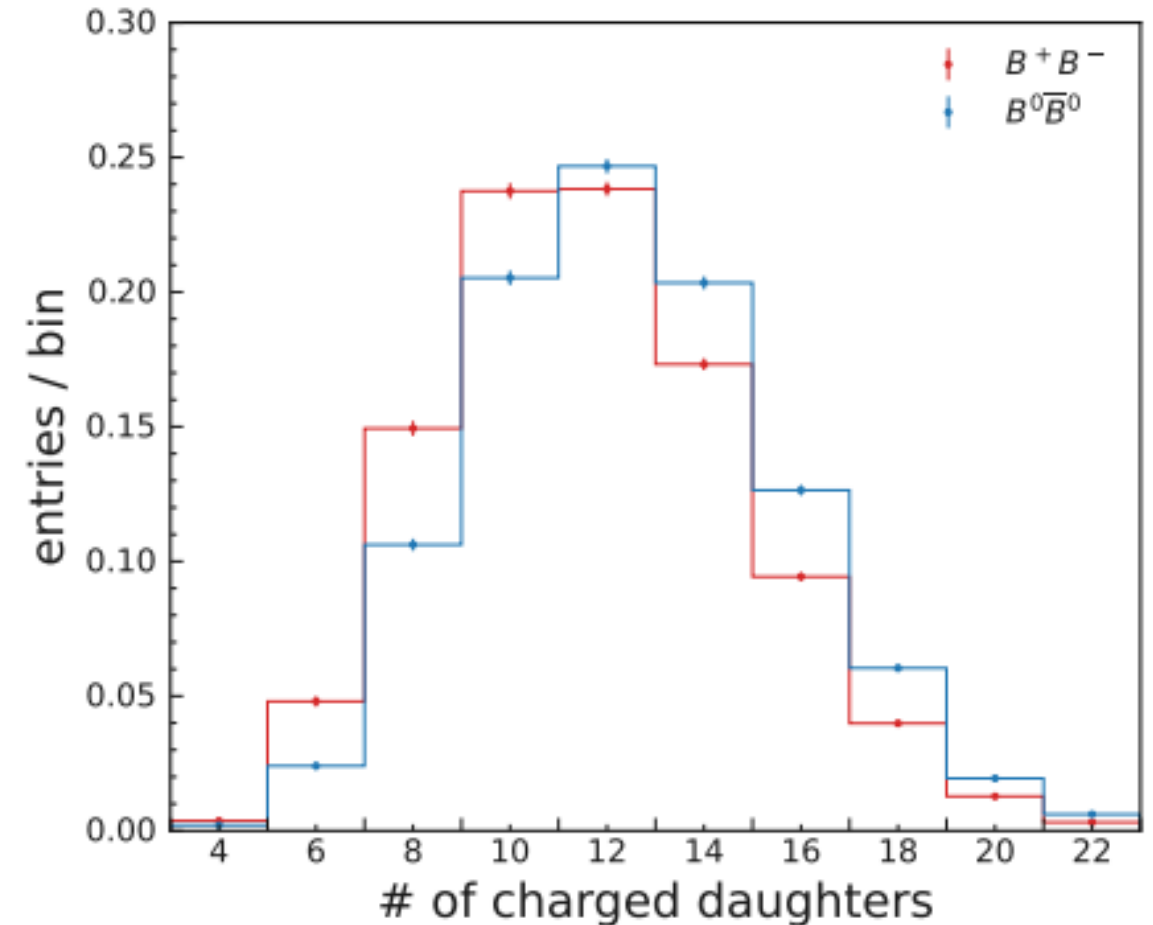
Sub-percent precision possible but large $\Upsilon(5S)$ data sample unlikely soon

TABLE III. Estimated sensitivity to $r(f, f')$ in Eq. (18), with available Belle data and anticipated partial and full Belle II data.

Inclusive multiplicity measurements: [2306.04686](#) [hep-ph] Florian et al.

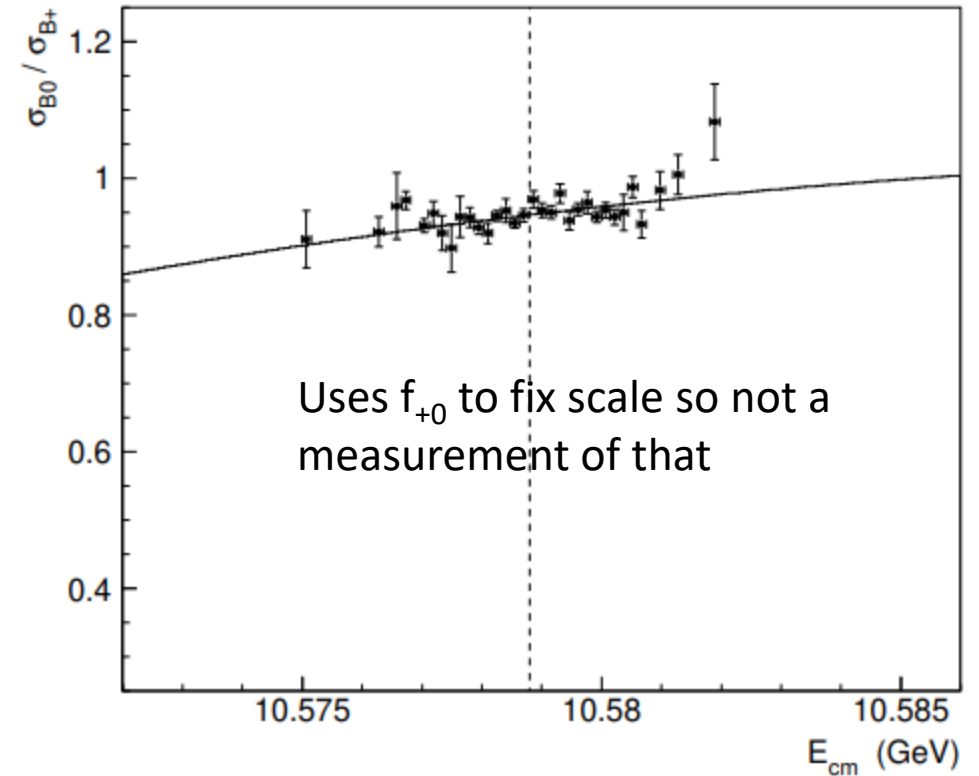
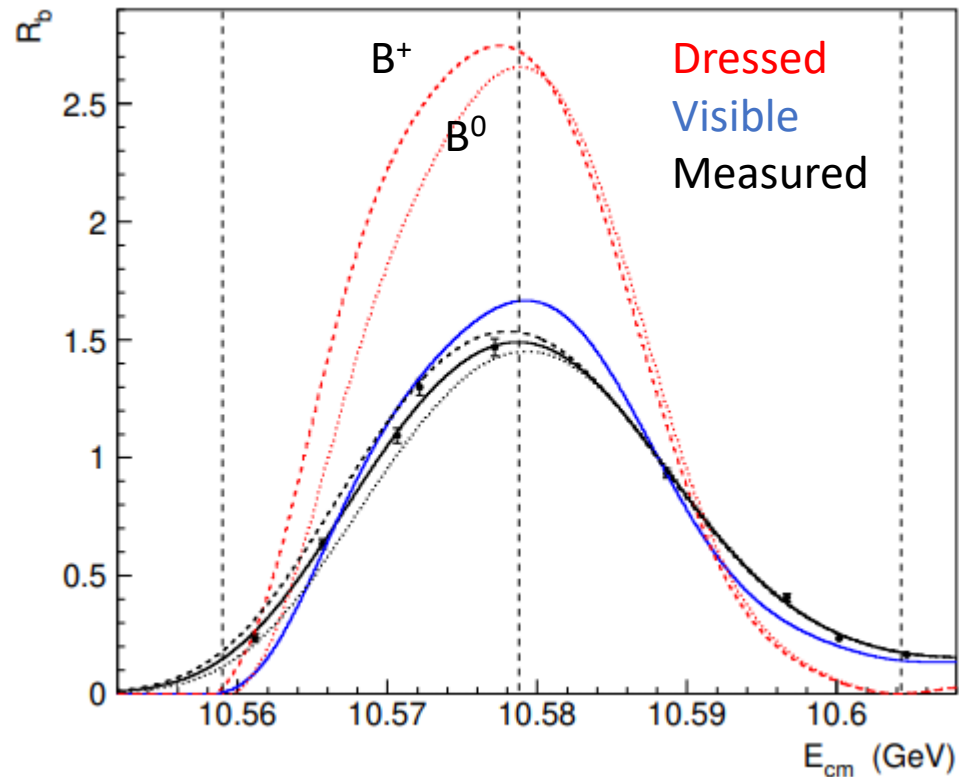
- Exploit the difference in charged particles for a fully inclusive measurement
- Calibration of templates due to reconstruction and simulation would require careful study of the recoil of hadronically tagged events

	Belle	Belle II partial	Belle II full
$\mathcal{L}_{\Upsilon(4S)}$ [ab^{-1}]	0.71	5	50
$\Delta(R^{\pm 0})/R^{\pm 0}$	2.2%	0.9%	0.3%



f_{+0} is a data-set dependent number

- Belle internal for now (Bondar and Mizuk) – B-production x-sec
- **We should have a value of f_{+0} that matches the data set**



To discuss

- Current techniques relying on isospin assumptions will not get us to the sub-percent level
- SL double-tag robust against isospin – certainly should be updated and pursued in the short term
- New ideas
 - Simultaneous $B \rightarrow D(^*)l\nu$ fit
 - $Y(5S)$ information
 - Inclusive technique
 -
- We should have a measurement that matches the data set used