Belle II Physics Week

Form Factors interplay with polarisations and asymmetries in $B \to D^* \ell \nu$ decays

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based on arXiv:2305.15457 in collaboration with:

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Introduction to $b \to c$ anomalies

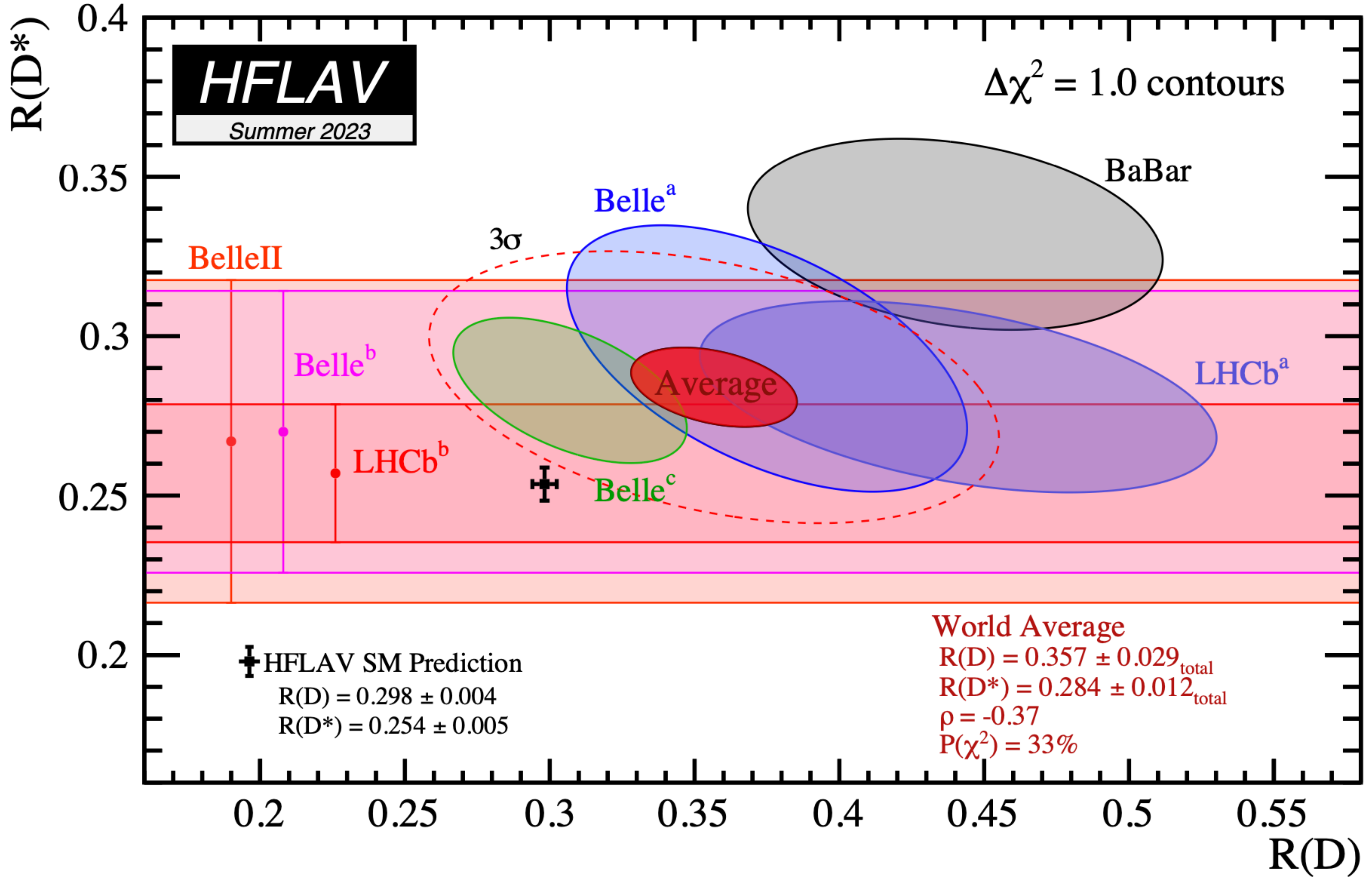
- Tree level, theoretically clean processes with large Br (~ few %)

- Sensitive to NP via LFUV tests

$$R(D^{(*)}) = \frac{\mathcal{B}(\overline{B} \to D^{(*)}\tau\overline{\nu}_\tau)}{\mathcal{B}(\overline{B} \to D^{(*)}\ell\overline{\nu}_\ell)}$$

$l = e, \mu, \tau$

$\ell = e, \mu$



Experimental average (HFLAV):

$$R(D) = 0.357 \pm 0.029$$

$$R(D^*) = 0.284 \pm 0.012$$

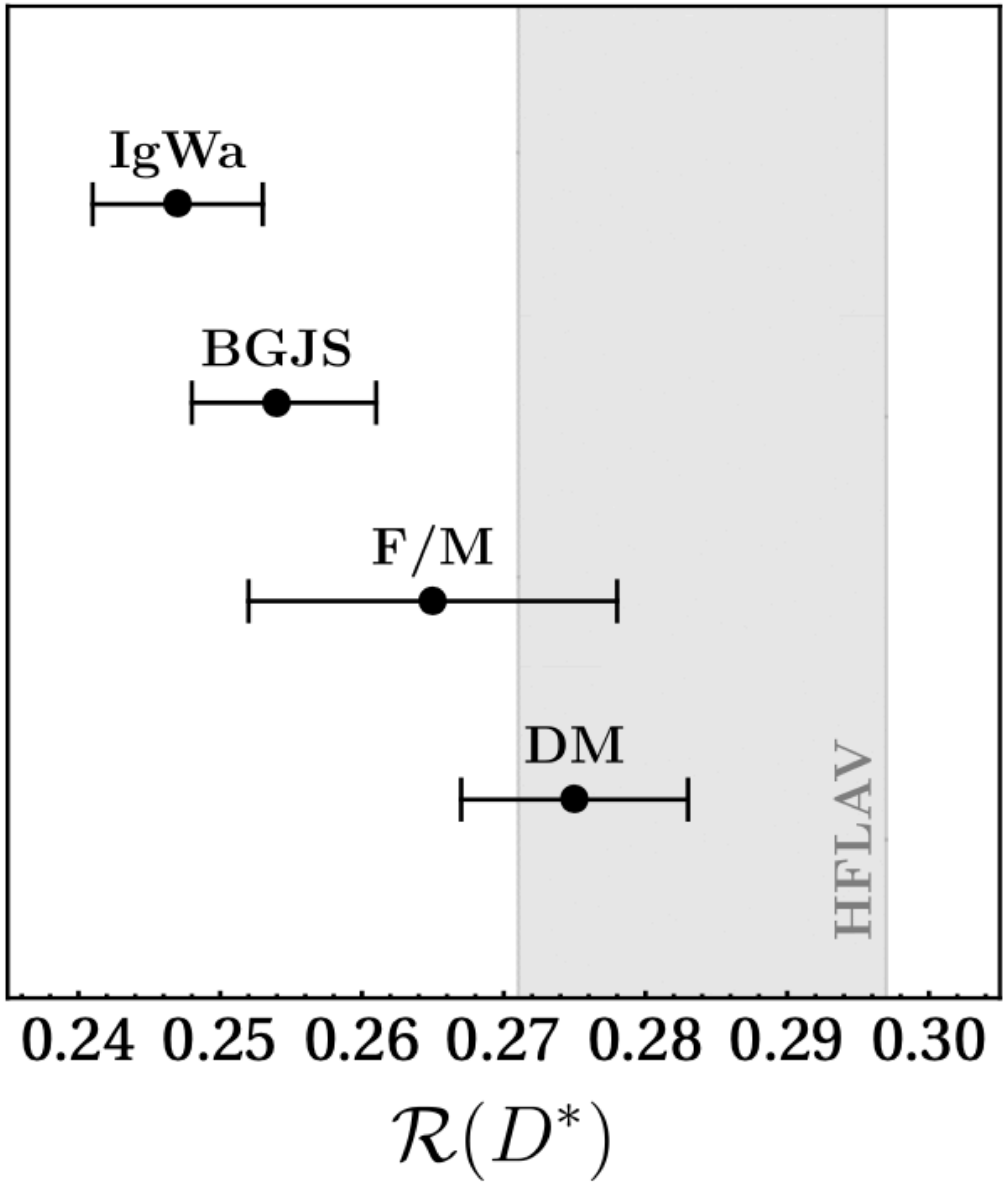
SM predictions:

$$R(D) = 0.298 \pm 0.004$$

$$R(D^*) = 0.254 \pm 0.005$$

Comb. discrepancy at ~3.3σ level hinting at τ over-abundance

What if it's a FF issue?



The SM prediction for $R(D^*)$ might not be as stable as originally thought!

Different Form Factors approaches have different predictions, with noticeable increase on the prediction for the latest determinations (and strongly correlated to $|V_{cb}^{excl}|$ determination)

Could the discrepancy actually arise from issues on the FF estimates?

The IgWa approach

Expand the FF $h_X(w) = \xi(w)\hat{h}_X(w)$, with $\xi(w)$ the leading Isgur-Wise function, in α_s and $1/m_{b,c}$

$$\hat{h}_X = \hat{h}_{X,0} + \frac{\alpha_s}{\pi}\delta\hat{h}_{X,\alpha_s} + \frac{\bar{\Lambda}}{2m_b}\delta\hat{h}_{X,m_b} + \frac{\bar{\Lambda}}{2m_c}\delta\hat{h}_{X,m_c} + \left(\frac{\bar{\Lambda}}{2m_c}\right)^2\delta\hat{h}_{X,m_c^2}$$

$\propto m_i$ $\propto$ sub-lead. I-W functs. $\xi_3(w), \chi_{2,3}(w)$ $\propto$ sub-lead. I-W functs. $\ell_{1-6}(w)$

Expand each of the 10 I-W functs. as a power of z, and fit to theory (LCSR and QCDSR) and experiment data up to a different order for each of the functions, selected by goodness-of-fit

$$f(w) = f^{(0)} + 8f^{(1)}z + 16\left(f^{(1)} + 2f^{(2)}\right)z^2 + \frac{8}{3}\left(9f^{(1)} + 48f^{(2)} + 32f^{(3)}\right)z^3 + \mathcal{O}(z^4)$$

2004.10208
Iguro, Watanabe

The BGJS approach

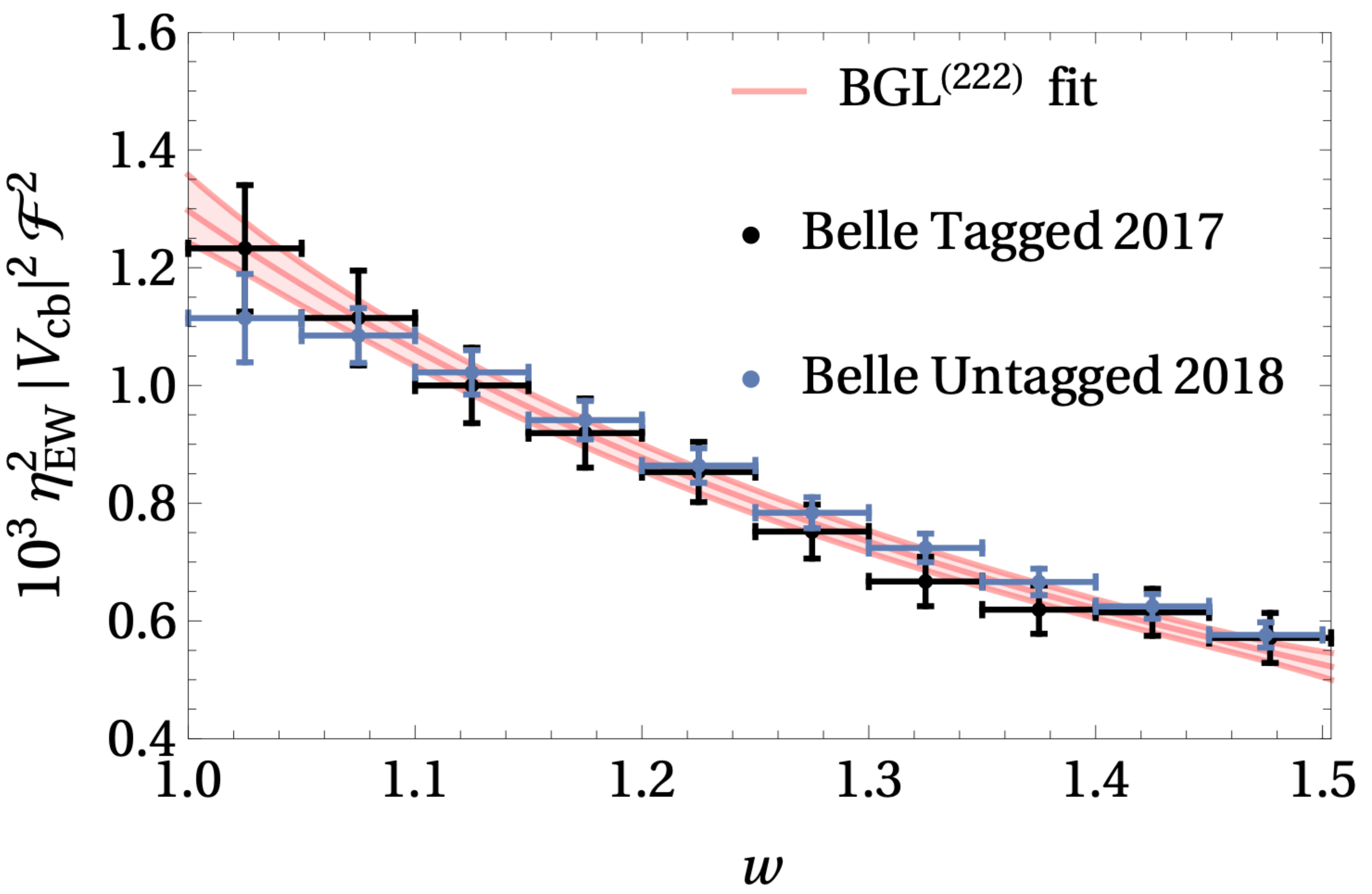
Expand the FF as a series in $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, where $w = (m_B^2 + m_{D*}^2 - q^2)/(2 m_B m_{D*})$

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

Different expansion order for each FF (selected by goodness-of-fit)

Weak unitarity constraints imposed on series coefficients to ensure a rapid convergence of the series in the physical region, $0 < z < 0.056$

$$\sum_{k=0}^{n_g} (a_k^g)^2 < 1, \quad \sum_{i=0}^{n_f} (a_k^f)^2 + \sum_{k=0}^{n_{F_1}} (a_k^{\mathcal{F}_1})^2 < 1$$



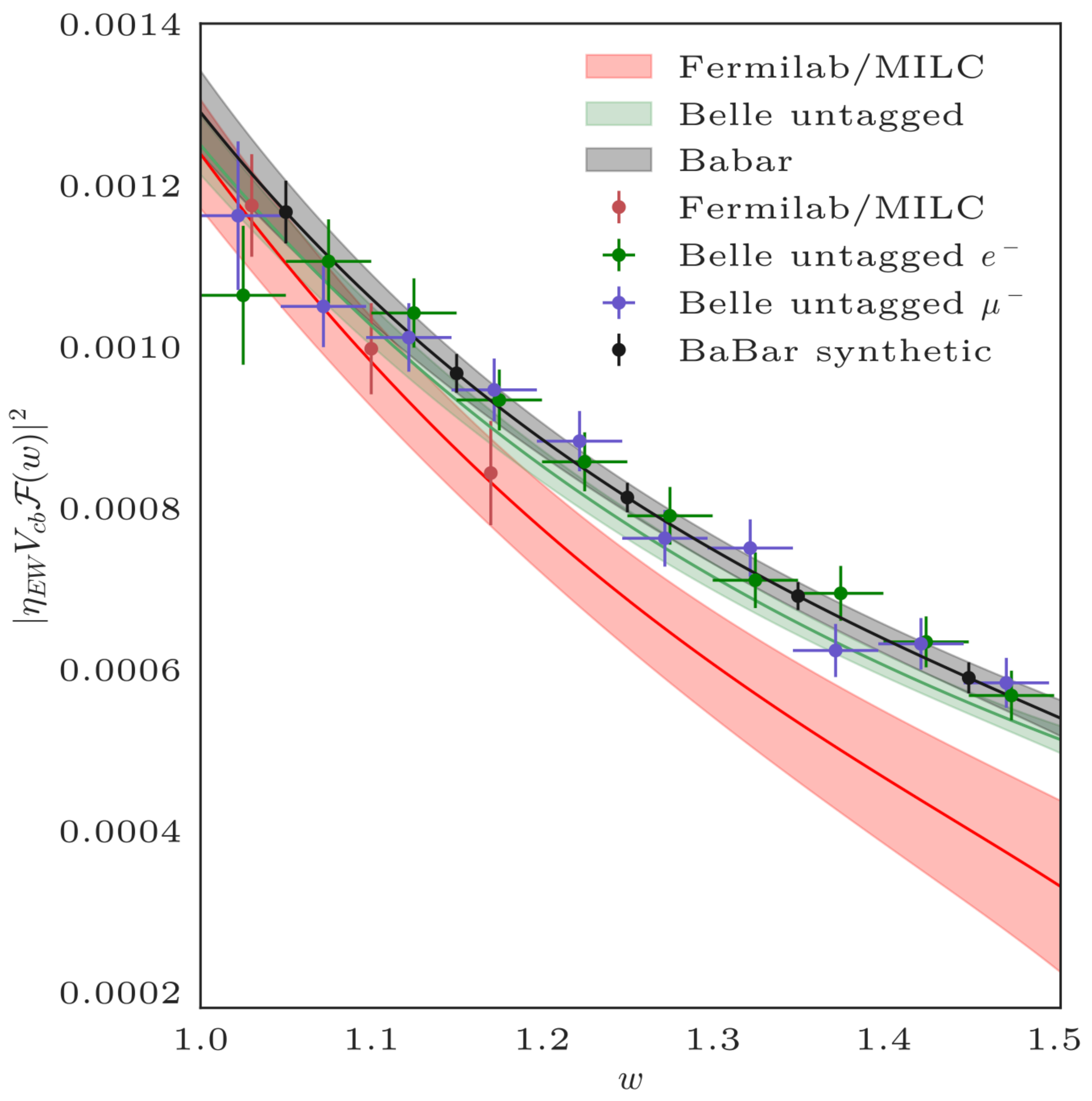
Additional input coming from HQET required for pseudoscalar FF

1905.08209
Gambino, Jung, Schacht

The Lattice approach

Employs the same parameterization as the BGL approach, first results beyond non-zero recoil have been recently obtained

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$



Result is however not fully compatible with exp. Problem with the slope?

2105.14019
Fermilab Lattice and MILC collaborations

(I will mainly focus on F/M here, only published result at time of pub.)

The Dispersive Matrix approach

Goal: determine a form factor f(t) starting from known values of $f(t_i)$, e.g. from Lattice

The starting point is the introduction of 2 ingredients: inner product and auxiliary function:

$$\langle g|h\rangle = \frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z}\bar{g}(z)h(z)$$

$$g_t(z) \equiv \frac{1}{1-\bar{z}(t)z}$$

$$\Rightarrow \quad \mathbf{M} \equiv \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_N}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_N}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_N}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_N}|\phi f\rangle & \langle g_{t_N}|g_t\rangle & \langle g_{t_N}|g_{t_1}\rangle & \cdots & \langle g_{t_N}|g_{t_N}\rangle \end{pmatrix}$$

Matrix built out of inner products, hence its determinant is by construction positive semidefinite

2105.07851, 2105.08674, 2109.15248
Martinelli, Simula, Vittorio

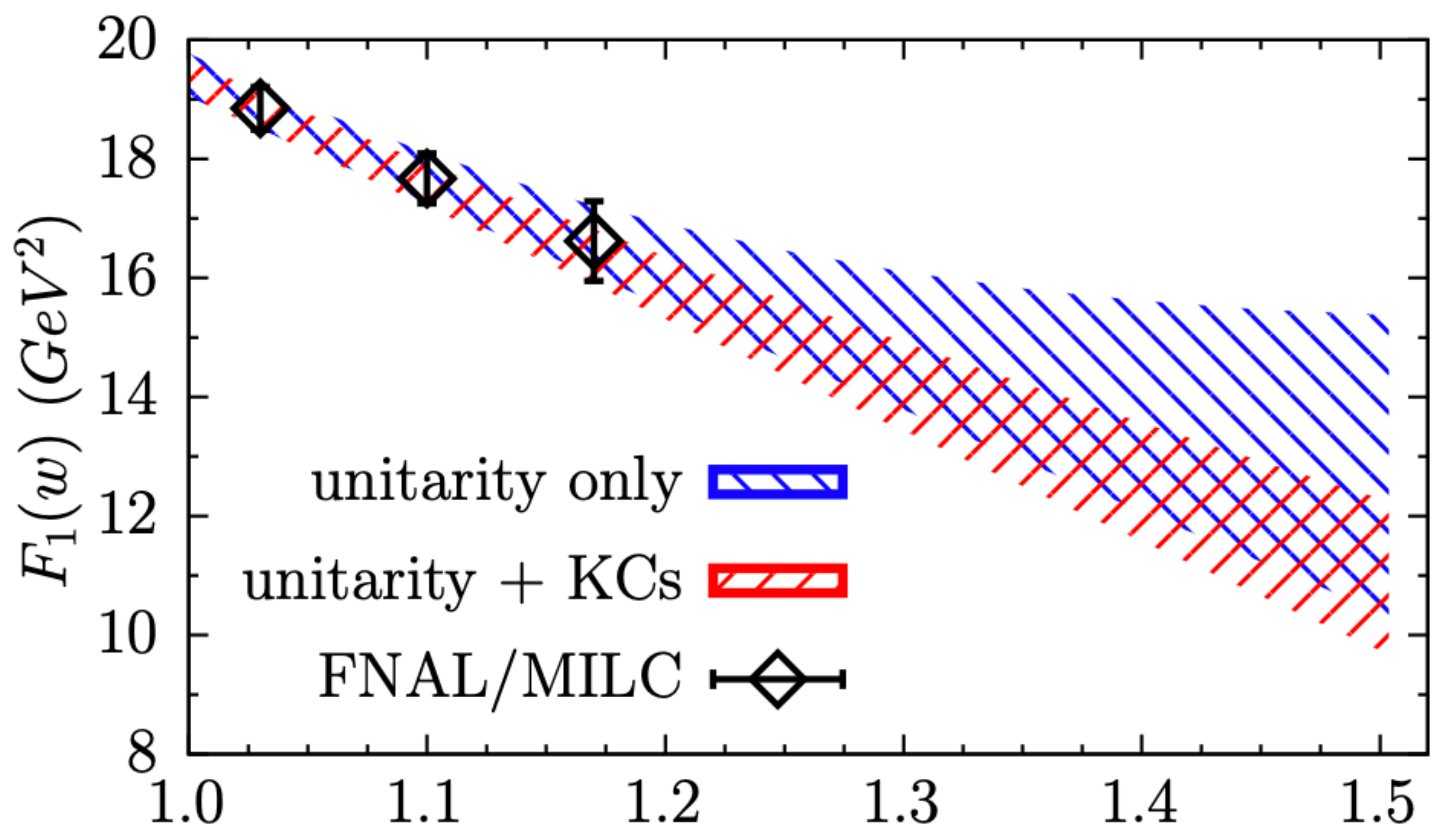
The Dispersive Matrix approach

M_{11} obeys to the dispersion relation

$$\frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z}|\phi(z)f(z)|^2 \leq \chi \longrightarrow 0 \leq \langle \phi f | \phi f \rangle \leq \chi$$

The Cauchy theorem allows to compute the remaining terms, and the semidefinite positiveness is not spoiled by replacing M_{11} by its upper limit

$$\Rightarrow \quad \mathbf{M}_\chi = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \dots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \dots & \frac{1}{1-z_1 z_N} \\ \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \dots & \frac{1}{1-z_N^2} \end{pmatrix}$$



Requiring the positiveness of the determinant allows to obtain a band for the FF, representing the envelope of the results of all possible (non) truncated z-expansions, like BGL ones

$$\beta(z) - \sqrt{\gamma(z)} \leq f(z) \leq \beta(z) + \sqrt{\gamma(z)}$$

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The Dispersive Matrix approach

$$\beta(z) - \sqrt{\gamma(z)} \leq f(z) \leq \beta(z) + \sqrt{\gamma(z)}$$

$$\beta(z) \equiv \frac{1}{\phi(z)d(z)} \sum_{j=1}^{N} \phi_j f_j d_j \frac{1 - z_j^2}{z - z_j} ,$$

$$\gamma(z) \equiv \frac{1}{1 - z^2} \frac{1}{\phi^2(z)d^2(z)} (\chi - \chi_{\rm DM}) ,$$

$$\chi_{\rm DM} \equiv \sum_{i,j=1}^{N} \phi_i f_i \phi_j f_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j} ,$$

$$d(z) \equiv \prod_{m=1}^{N} \frac{1 - z z_m}{z - z_m} ,$$

$$d_j \equiv \prod_{m \neq j = 1}^{N} \frac{1 - z_j z_m}{z_j - z_m} .$$

$\Rightarrow$

Unitarity requires $\gamma(z) \geq 0$, which implies $\chi \geq \chi_{\rm DM}$. Therefore, the FF at any given z is given by the convolution of $\gamma(z)$ and $\beta(z)$ with the distribution of input (lattice) data with $\chi > \chi_{\rm DM}$: input data is therefore filtered by unitarity!

Martinelli et al. employ only lattice as input data (and no exp.) because they want to have a fully theoretical prediction of FFs, without having to assume data to be SM-like

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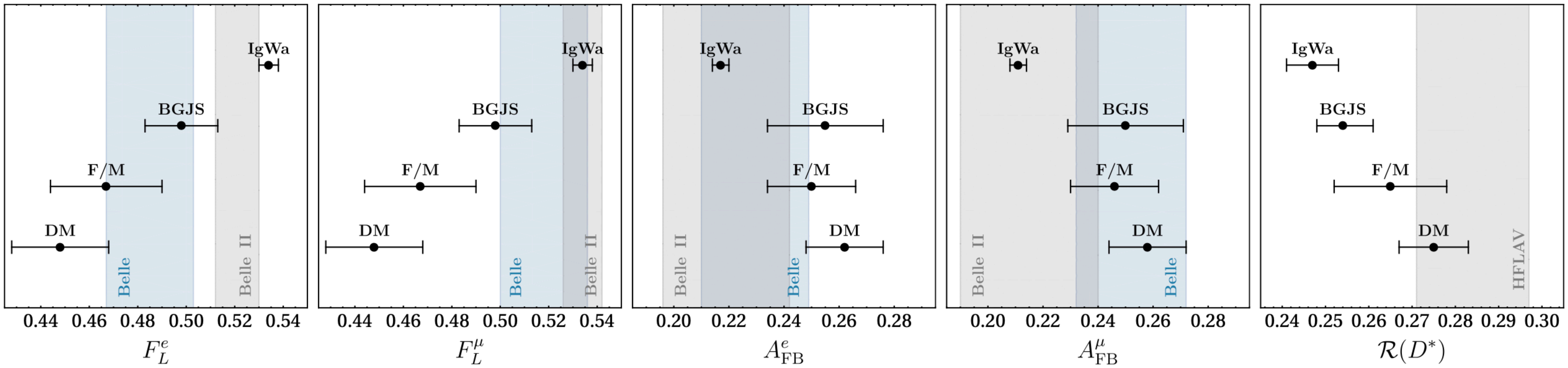
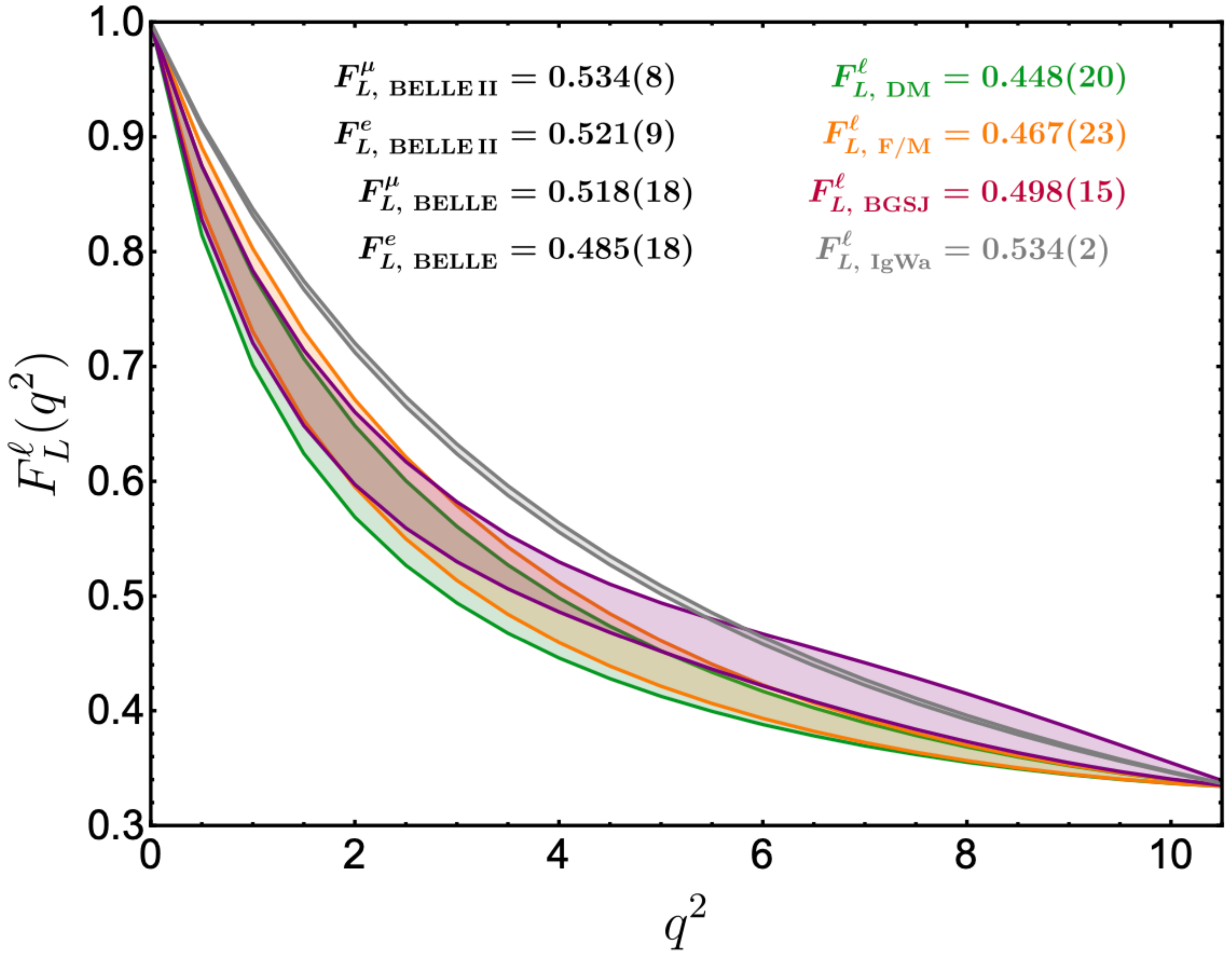
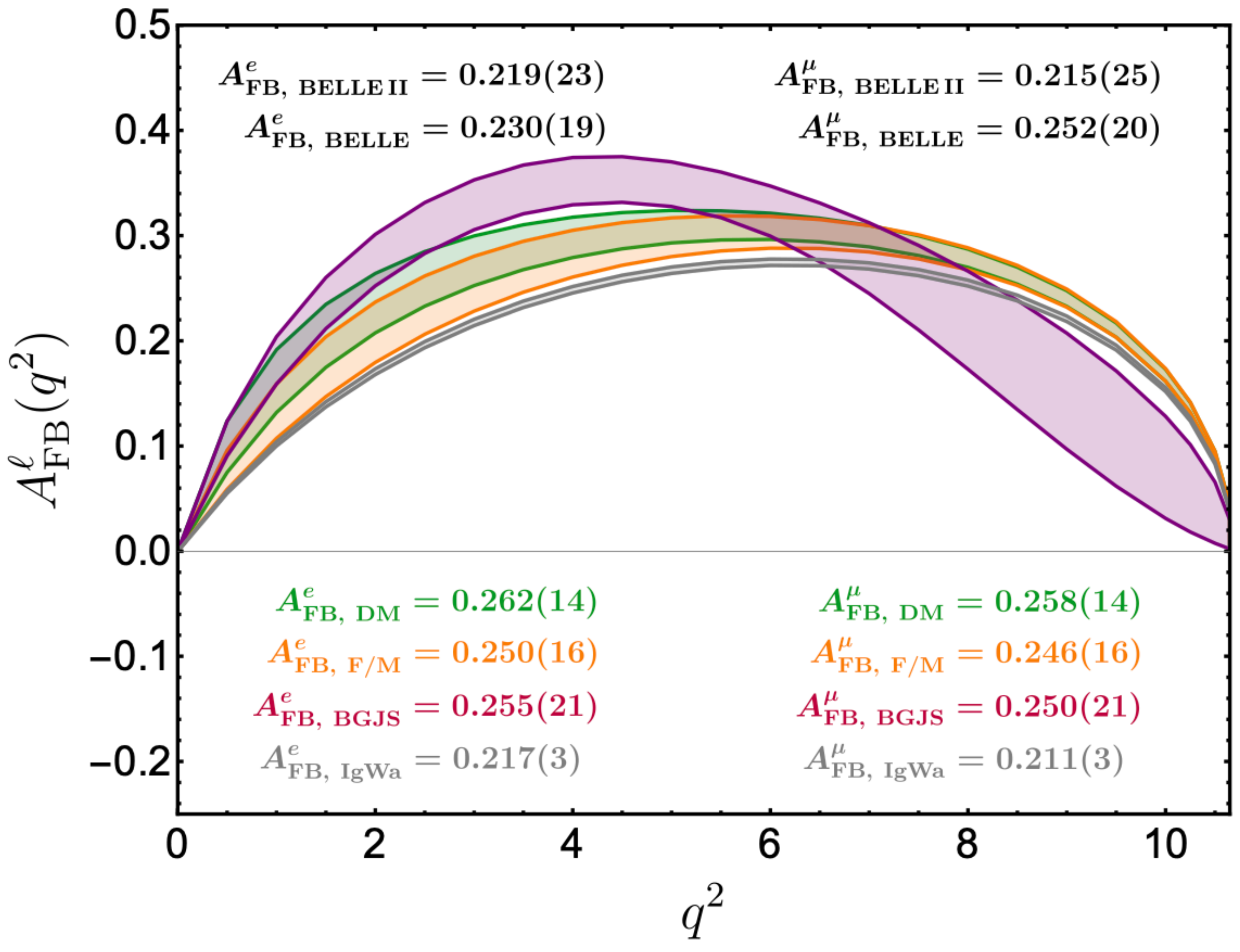
Martinelli, Simula, Vittorio

Not all that glitters is gold…



The DM FF approach is capable to address tension in $R(D^*)$ (and $|V_{cb}|$ incl. vs excl. discrepancy), but however in tension with new F_L^ℓ and A_{FB}^ℓ data!





2305.15457
MF, Blanke, Crivellin, Iguro, Nierste, Simula, Vittorio

Where is this coming from?

In order to understand the origin of the FF behaviours, it's instrumental to take a look at the helicity amp.

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{m_B^2 + m_{D^*}^2 - 2 m_B m_{D^*} w}} \qquad H_\pm(w) = f(w) \mp m_B m_{D^*} \sqrt{w^2 - 1}\, g(w)$$

which are used to build

$$\frac{1}{|V_{cb}|^2}\frac{d\Gamma^\ell}{dw} \propto |H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2$$

$$F_L^\ell(w) = \frac{|H_0(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

$$A_{\rm FB}^\ell(w) = \frac{|H_-(w)|^2 - |H_+(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

$\Rightarrow$

$\mathcal{F}_1(w)$	$\frac{1}{|V_{cb}|^2}\frac{d\Gamma^\ell}{dw}$	$|V_{cb}|$	$\mathcal{R}(D^*)$	$A_{\rm FB}^\ell$	F_L^ℓ
↗	↗	↘	↘	↘	↗
↘	↘	↗	↗	↗	↘

A change in the shape of $\mathscr{F}_1(w)$ has a direct proportional impact on $R(D^*)$, $|V_{cb}|$, A_{FB}^ℓ and F_L^ℓ

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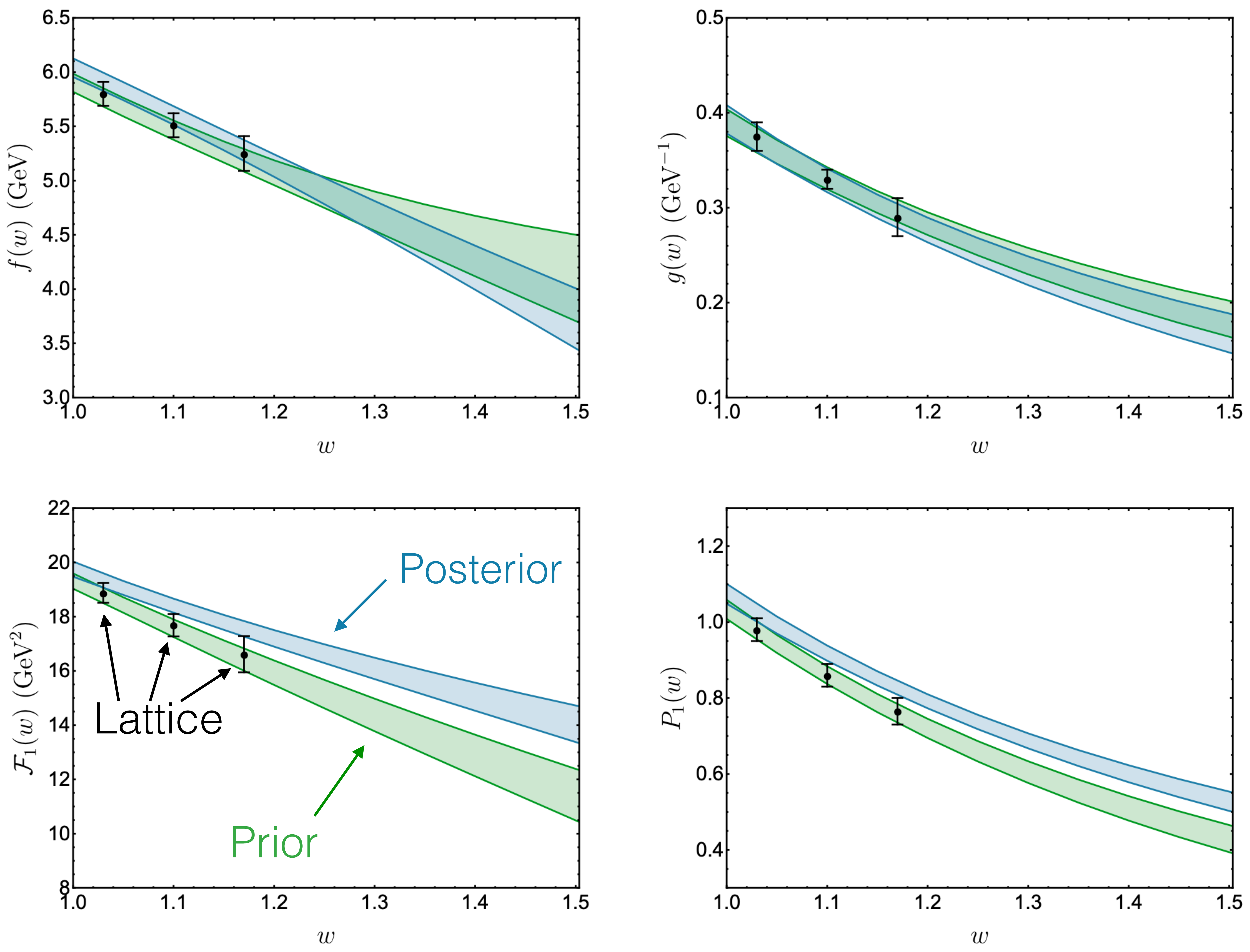
What if we try to perform a fit to this data?

Goal: perform a fit to A_{FB}^{ℓ} and F_L^{ℓ} using DM results for the FF as priors

$$\begin{aligned}\mathcal{R}(D^*)_{\rm fit} &= 0.265 \pm 0.005 \\ F^{\ell}_{L,\,\rm fit} &= 0.515 \pm 0.005 \\ A^{e}_{\rm FB,\,fit} &= 0.227 \pm 0.007 \\ A^{\mu}_{\rm FB,\,fit} &= 0.222 \pm 0.007\end{aligned}$$

Re-emergence of $R(D^*)$ anomaly, disappearance of F_L^{ℓ} and $A_{\rm FB}^{\ell}$ ones, change of $F_1(w)$ slope



Strong discrepancy between prior and posterior values, lattice results not even reproduced anymore!

2305.15457
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Can we reproduce everything introducing NP in light leptons?

The DM FF offer the unique possibility to employ NP in light leptons to address anomalies (forbidden in other scenarios due to CKM limits)

Could this fix the issue?

Only evidence found for g_{V_L}; however F_L^ℓ and $A_{\rm FB}^\ell$ are ratios, hence insensitive to it!

The absence of an hint for scalar/tensor WCs is due to more precise measurements in light lepton channel, together with m_ℓ suppression in interference terms with SM

$$g_{V_L} = -0.054 \pm 0.015$$
$$g_{V_R} \in [-0.04, 0.01]$$
$$g_{S_L} \in [-0.07, 0.02]$$
$$g_{S_R} \in [-0.05, 0.03]$$
$$g_T \in [-0.01, 0.02]$$

$\Rightarrow$ If the FF prediction for F_L^ℓ and $A_{\rm FB}^\ell$ does not reproduce data, this cannot be fixed by introducing NP effects in light leptons as could be done for $R(D^*)$!

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What about the DM results applied to other FFs?

Lattice FFs	$R(D^*)$	$P_\tau(D^*)$	$F_{L,\tau}$	$F_{L,\ell}$	$A_{FB,\ell}$
FNAL/MILC [14]	0.275(8)	-0.529(7)	0.418(9)	0.450(19)	0.261(14)
HPQCD [15]	0.276(8)	-0.558(13)	0.448(16)	0.426(30)	0.272(21)
JLQCD [16]	0.248(8)	-0.508(11)	0.398(16)	0.561(29)	0.220(21)
Average [14]-[16]	0.266(9)	-0.529(11)	0.420(11)	0.471(36)	0.254(14)
(PDG scale factor)	(2.0)	(2.1)	(1.6)	(2.6)	(1.3)
Combined [14]-[16]	0.262(5)	-0.525(5)	0.436(8)	0.468(14)	0.253(10)
Experimental value	0.284(12) [32]	$-0.38 \pm 0.51^{+0.21}_{-0.16}$ [37]	0.49(8) [34, 35]	0.523(8) [13, 36]	0.231(17) [13, 36]

We have an analogous pattern: either we reproduce $R(D^*)$ but observe a tension with new F_L^ℓ and $A_{\rm FB}^\ell$ data (HPQCD) or viceversa (JLQCD)!

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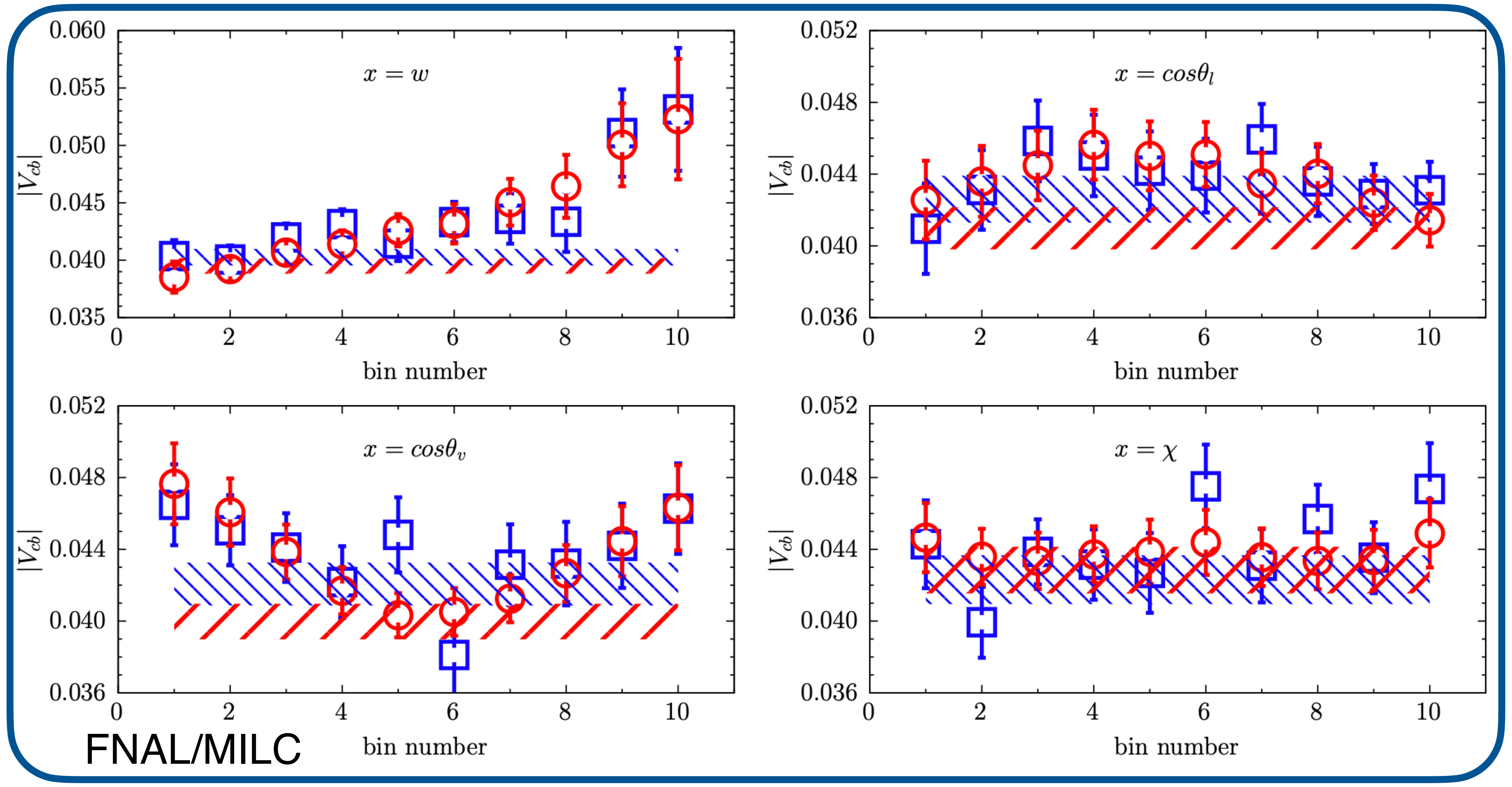
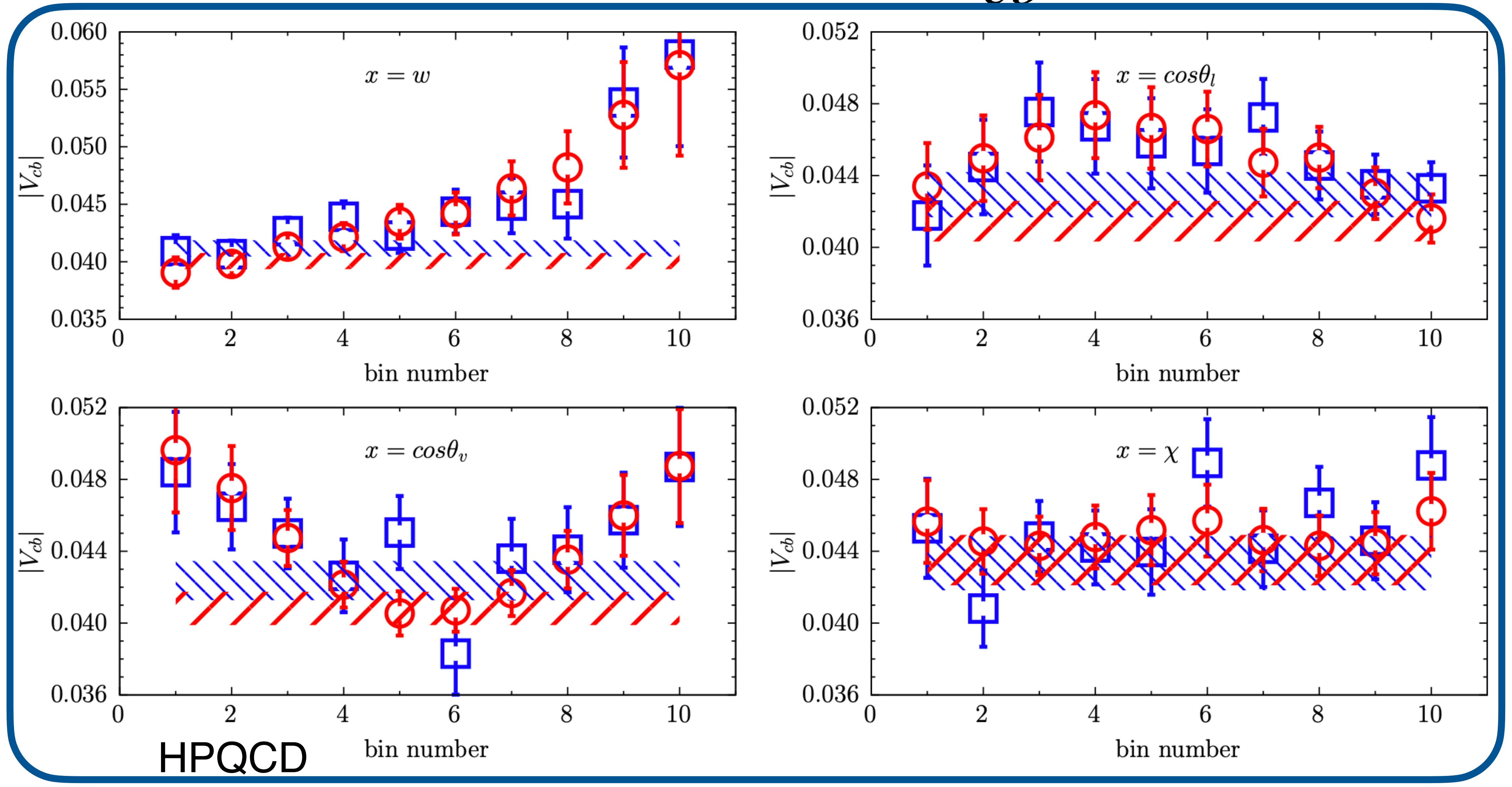
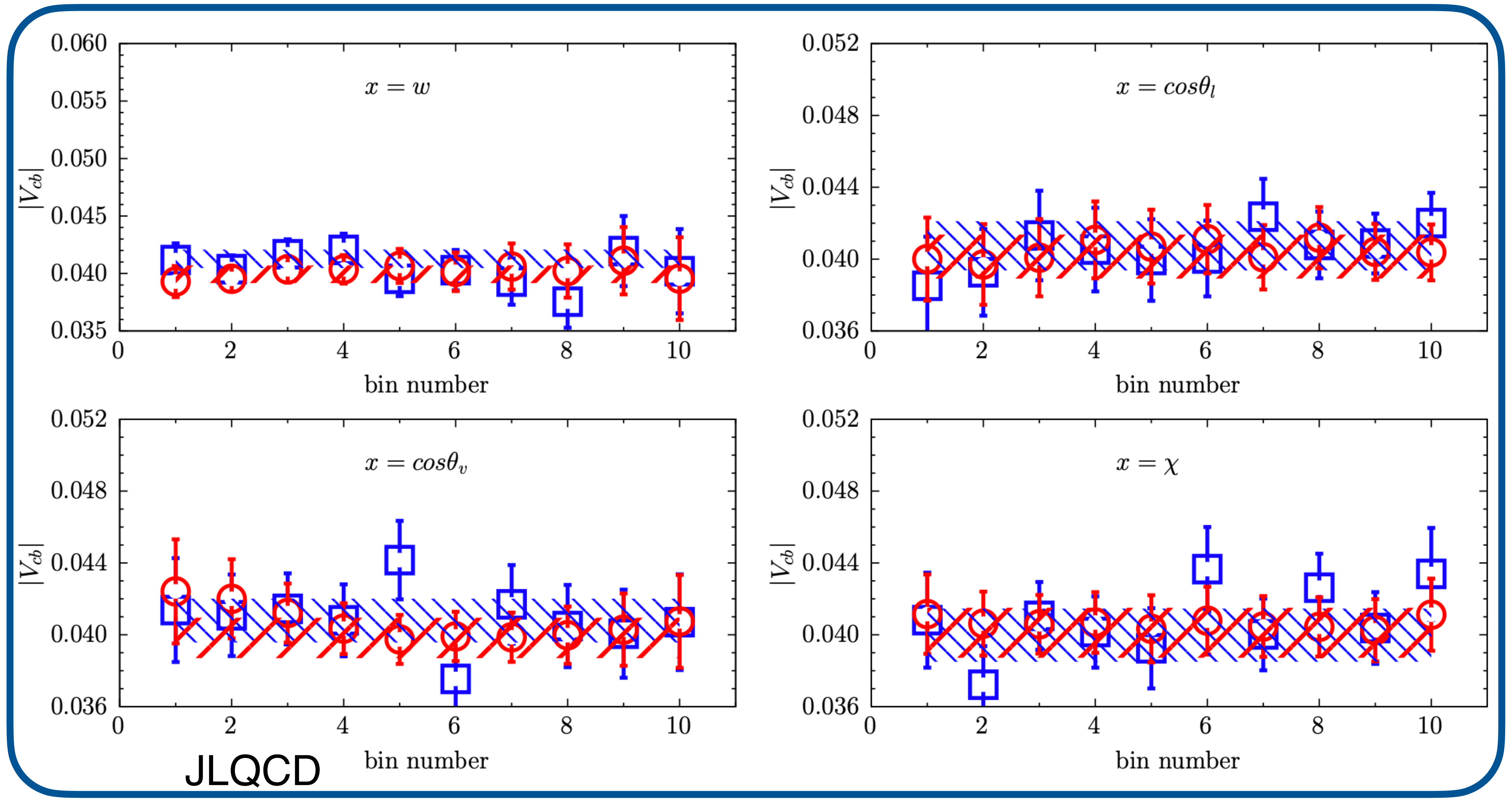
Implications to V_{cb} determinations

Due to not including differential data as an input, bin-by-bin extraction of V_{cb} is possible







Differences among these distributions reflect the differences among the different theoretical FFs results!

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Conclusions

- Recent determination of A_{FB}^{ℓ} and F_L^{ℓ} have become available from Belle and Belle II, already with great precision!

- Theory prediction of A_{FB}^{ℓ} and F_L^{ℓ} strongly correlated to the one of $R(D^*)$; while the latter can be modified by NP effects, the former are strongly NP-insensitive…

- Theory determinations of FF should therefore take in great attention their implications of the predictions for A_{FB}^{ℓ} and F_L^{ℓ}, and the consequent impact on the extraction of $|V_{cb}^{excl}|$!