V_{cb} and V_{ub} from Lattice

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Belle II Physics week
30 Oct - 3 Nov

Outline

I will discuss new and ongoing lattice predictions for

- $b \to u\ell\bar{\nu}$ semileptonic decays: $B \to \pi\ell\bar{\nu}$
- $b \to c\ell\bar{\nu}$ semileptonic decays: $B_{(s)} \to D^*_{(s)}\ell\bar{\nu}$
- Ongoing work on other relevant decays

$\to$ Focus will be mostly on new results for $B \to D^*$

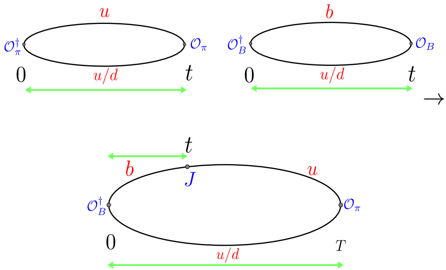
$B \to \pi$

In SM only f_+ needed to describe differential rate for light leptons $\ell = e, \mu$:

$$\frac{d\Gamma(B \to \pi \ell \bar{\nu})}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |\vec{p}_\pi(q^2)|^3 |f_+(q^2)|^2$$

$$\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu\right] + f_0(q^2)\frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

Form factors computed on the lattice from 2 and 3 point correlation functions, for $B \to \pi$ with current J:



$\to$

$$\sum_n |A_n^B|^2 e^{-E_n^B t}, \quad \sum_n |A_n^\pi|^2 e^{-E_n^\pi t}$$

$$\sum_{nm} A_n^\pi A_m^{B\dagger} V_{nm} e^{-tE_m^B - (T-t)E_n^\pi}$$

$$V_{nm} = \frac{\langle n, \pi|J|m, B\rangle}{\sqrt{2E_m^B 2E_n^\pi}}$$

$B \to \pi$

To simulate b-quarks on the lattice, we typically require $am_b < 1$. For fixed volume, this requires small lattice spacings $a < 1/m_b$ as well as a large number of lattice points. This makes calculations at the physical b very expensive.

Modern solution:

- Perform lattice calculations at multiple masses, m_h, below m_b, using the same relativistic action for all quarks.
- Fit results using some HQET-like form to disentangle am_h discretisation effects and physical m_h dependence.

This approach allows control of am_h discretisation effects while also obtaining precise results at the physical $m_h = m_b$ point.

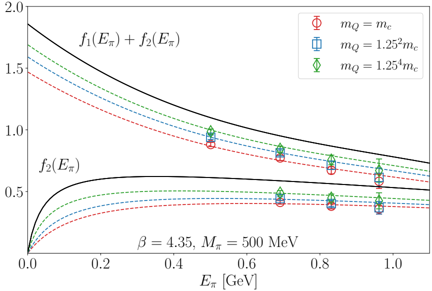
$B \to \pi$, JLQCD [2203.04938]

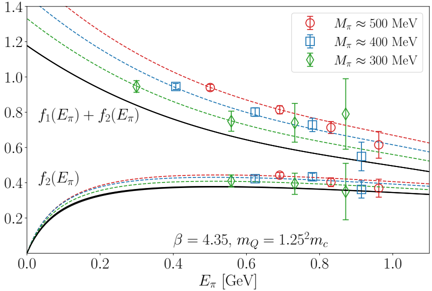
For $B \to \pi$, most recent calculation from JLQCD, using fully relativistic Möbius domain wall heavy quarks. Fit is done to form factors f_1 and f_2:

$$\langle \pi(p_\pi)|V^\mu|B(v = p_B/M_B)\rangle = 2\left[f_1(v \cdot p_\pi)v^\mu + f_2(v \cdot p_\pi)\frac{p_\pi^\mu}{v \cdot p_\pi}\right]$$

Lattice data for f_1 and f_2 are fit to a function describing chiral and $1/m_Q$ dependence, as well as discretisation and mistuning effects.

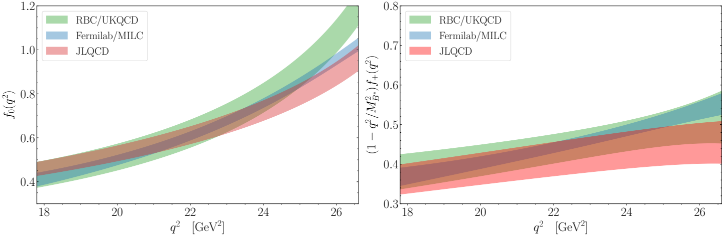
The extrapolation in m_Q and M_π looks very reasonable





$B \to \pi$, JLQCD [2203.04938]

Resulting form factors (in $f_{+/0}$ basis) can be compared to older calculations from RBC/UKQCD [1501.05373] and Fermilab/MILC [1503.07839]



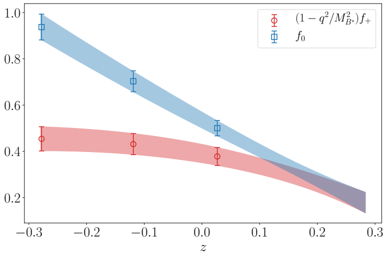
$\to$ good general agreement on f_+

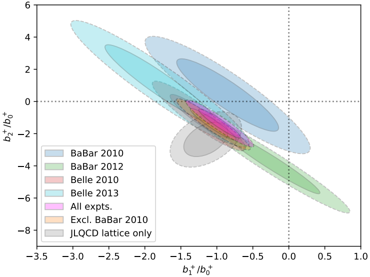
$B \to \pi$, JLQCD [2203.04938]

Continuum FFs commonly parameterised using Bourrely-Caprini-Lellouch (BCL) expansion [0807.2722]

$$f_+(q^2) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{k=0}^{N_z-1} b_k^+ \left[z^k - (-1)^{k-N_z} \frac{k}{N_z} z^{N_z} \right]$$

$$f_0(q^2) = \sum_{k=0}^{N_z-1} b_k^0 z^k, \quad z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ + t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ + t_0}}$$



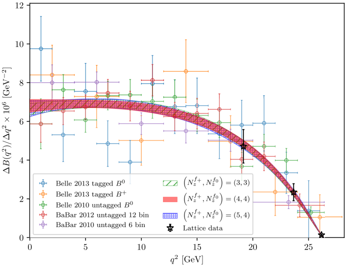


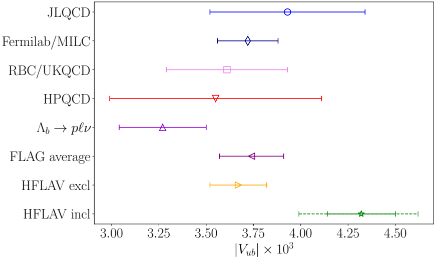
Good agreement between experimentally measured BCL parameters and JLQCD lattice-only results.

$B \to \pi$, JLQCD [2203.04938]

JLQCD fit their lattice FFs together with experimental data to find

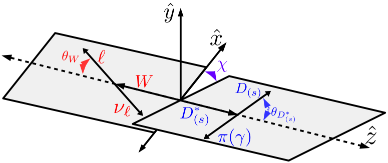
$$|V_{ub}| = 3.93 \pm 0.41 \times 10^{-3}$$





Large uncertainty on $|V_{ub}|$ from JLQCD - need more precise lattice results.

$B \to D^* \ell \bar{\nu}$



For $B \to D^* \ell \bar{\nu}$ we require 4 form factors to describe the decay in SM, three for the axial-vector current and 1 for the vector current:

$$\langle D^* | \bar{c} \gamma^\mu b | \overline{B} \rangle = i \sqrt{M_B M_{D^*}} \varepsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu v'_\alpha v_\beta h_V$$

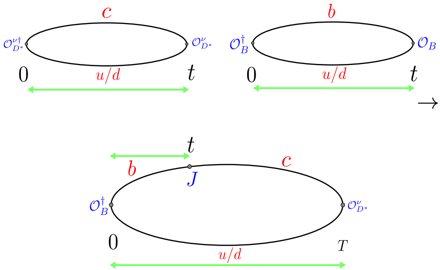
$$\begin{aligned} \langle D^* | \bar{c} \gamma^\mu \gamma^5 b | \overline{B} \rangle = \sqrt{M_B M_{D^*}} \big[& h_{A_1} (w+1) \epsilon^{*\mu} \\ & - h_{A_2} (\epsilon^* \cdot v) v^\mu - h_{A_3} (\epsilon^* \cdot v) v'^\mu \big] \end{aligned}$$

There are also 3 tensor form factors needed to include potential new physics:

$$\begin{aligned} \langle D^* | \bar{c} \sigma^{\mu\nu} b | \overline{B} \rangle = - \sqrt{M_B M_{D^*}} \varepsilon^{\mu\nu\alpha\beta} \big[& h_{T_1} \epsilon^*_\alpha (v + v')_\beta \\ & + h_{T_2} \epsilon^*_\alpha (v - v')_\beta + h_{T_3} (\epsilon^* \cdot v) v_\alpha v'_\beta \big] \end{aligned}$$

$B \to D^* \ell \bar{\nu}$

Calculation strategy on lattice is similar to $B \to \pi$ case. Extract form factors from 2 and 3 point correlation functions:



$$\sum_n |A_n^B|^2 e^{-E_n^B t}, \quad \sum_n |A_n^{D^*}|^2 e^{-E_n^{D^*} t}$$

$$\to \quad \sum_{nm} A_n^{D^*} A_m^{B\dagger} V_{nm} e^{-tE_m^B - (T-t)E_n^{D^*}}$$

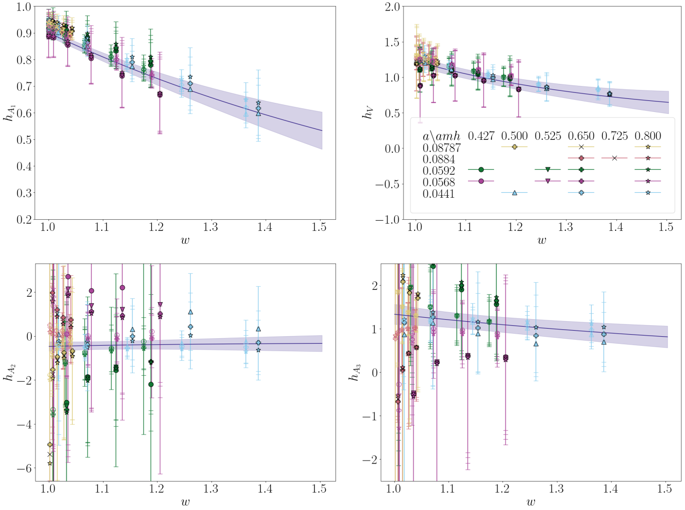
$$V_{nm} = \frac{\sum_\lambda \epsilon^\nu(\lambda, p') \langle n, D^*(\lambda, p')|J|m, B\rangle}{\sqrt{2E_m^B 2E_n^{D^*} (1 + \vec{p}_\nu'^2 / M_{D^*}^2)}}$$

D^* interpolating operator $\mathcal{O}^\nu_{D^*} = \bar{u}\gamma^\nu c$ comes with Lorentz index ν, chosen to pick out FFs.

$B \to D^* \ell \bar{\nu}$: HPQCD [2304.03137]

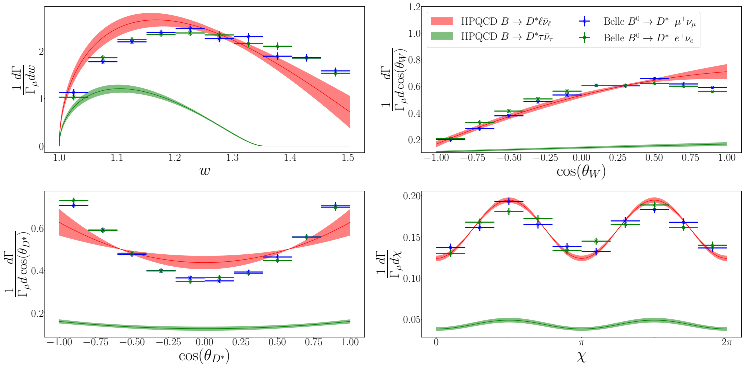
HPQCD calculation of SM+Tensor FFs, fully relativistic HISQ action for all quarks

- 2+1+1 second generation MILC HISQ ensembles with charm in the sea
- non-perturbative current renormalisation
- small am_h discretisation effects in HISQ $\to$ good coverage of w-range



$B \to D^* \ell \bar{\nu}$: HPQCD [2304.03137]

HPQCD normalised differential decay rates do not agree well with Belle data [1809.03290]

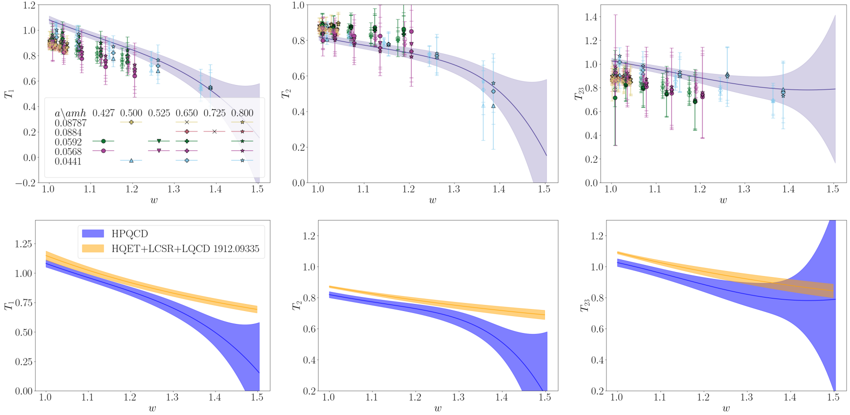


Simultaneous fit to HPQCD FFs, Belle data and LHCb $B_s \to D_s^* \ell \bar{\nu}$ data gives

$$|V_{cb}| = 39.36(54)_{\rm exp}(61)_{\rm latt} \times 10^{-3}$$

$B \to D^* \ell \bar{\nu}$: HPQCD [2304.03137]

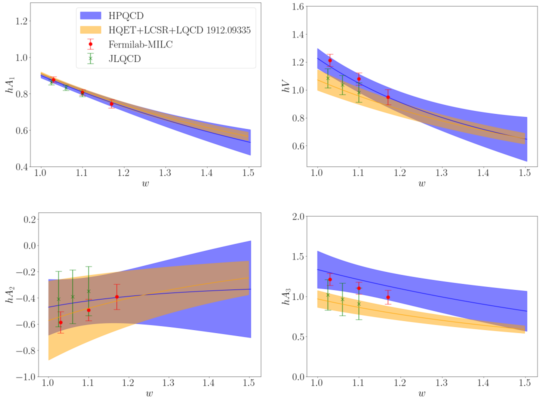
HPQCD calculation of SM+Tensor FFs, fully relativistic HISQ action for all quarks



Tensor FFs do not agree well with heavy quark expansion (HQE) fits to light-cone sum-rules and older $h_{A_1}(w = 1)$ and $B \to D$ lattice results [1912.09335]

$B \to D^*\ell\bar{\nu}$

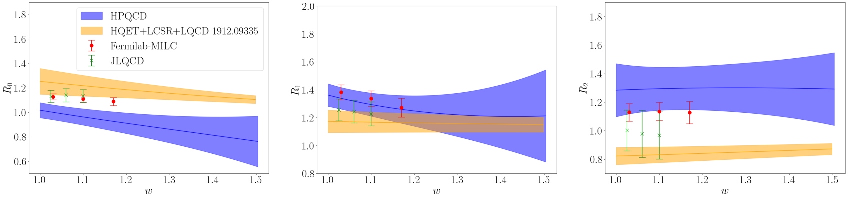
Two other recent lattice calculations of vector and axial-vector FFs: Fermilab/MILC SM FFs [2105.14019], JLQCD SM FFs [2306.05657].
h_{A_1} and h_V agree reasonably with heavy quark expansion (HQE) fits to light-cone sum-rules and older $h_{A_1}(w=1)$ and $B \to D$ lattice results [1912.09335].



$B \to D^*\ell\bar{\nu}$

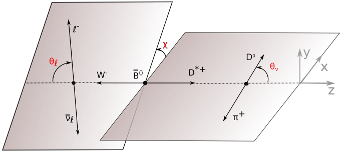
However, ratios do not seem to agree so well $\to$ correlations?

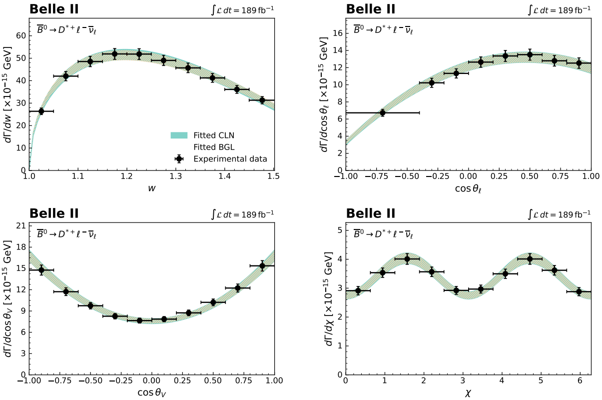
$$R_0 = \frac{1}{1+r}\left(w + 1 + w\frac{rh_{A_2} - h_{A_3}}{h_{A_1}} - \frac{h_{A_2} - rh_{A_3}}{h_{A_1}}\right), \quad R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{rh_{A_2} + h_{A_3}}{h_{A_1}}$$



$B \to D^*\ell\bar{\nu}$: Belle II

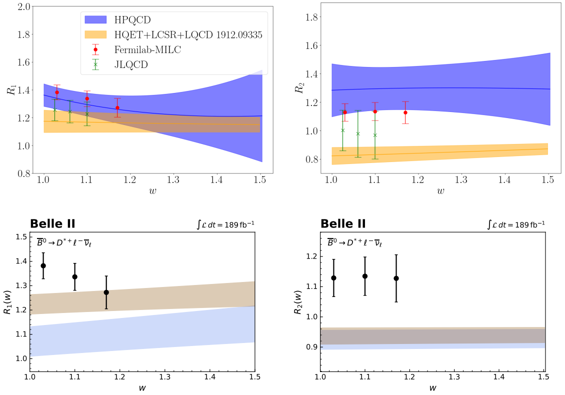
New results from Belle II [2310.01170]!





$B \to D^* \ell \bar{\nu}$: Belle II

New results from Belle II [2310.01170] seem to agree with expectations from HQE, particularly for R_2



- Including only Fermilab/MILC h_{A_1} (blue band): $|V_{cb}| = 40.3 \pm 1.2 \times 10^{-3}$, $p - \text{value} = 21\%$
- Including h_{A_1}, R_1 and R_2 (brown band):$|V_{cb}| = 38.3 \pm 1.1 \times 10^{-3}$, $p - \text{value} = 0.04\%$

$B \to D^*\ell\bar{\nu}$: Things to look into

Issue seems to be with h_{A2}, h_{A3} - need to look for possible biases in chiral-continuum fit.

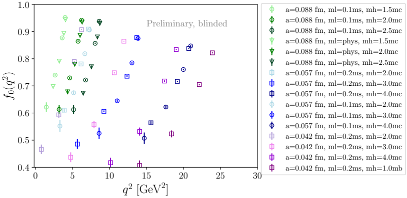
- Are we including enough kinematic terms/estimating truncation errors correctly/using broad enough priors?
- Are we including discretisation effects consistently? e.g. do different parameterisations allow for $(ap)^2$ effects?

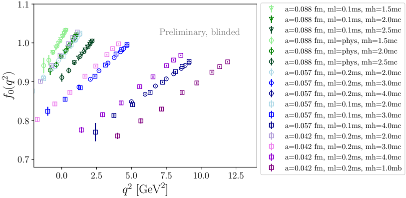
Other things we can improve:

- HPQCD and JLQCD have been conservative with lattice heavy quark mass - can get up to the B in future updates.
- HPQCD can use time reversed 3pt functions together with lattice rotations of current setup, to improve statistics.
- Fermilab heavy quark + HISQ calculation in not too distant future.
- Work to incoorporate HQE information into HPQCD chiral continuum extrapolation.

$b \to u\ell\bar{\nu}$: Ongoing exclusive lattice QCD calculations

- JLQCD working on $B \to \pi$ update with increased statistics and heavier masses.
- HPQCD currently working on $B_{(s)} \to \pi(K)$.
- RBC/UKQCD working on $B \to \pi$, $B_s \to D_s$ with new results this year for $B_s \to K$ ($|V_{ub}| = 3.8(6) \times 10^{-3}$) [2303.11280]
- Fermilab/MILC working on updates to $B \to D^*$ (see talk by Alejandro Vaquero)
- Fermilab/MILC also working on $B \to \pi$ and related decays $B_s \to K$ and $B_s \to D_s$ [2301.09229], from which the ratio $|V_{ub}/V_{cb}|$ can be computed:



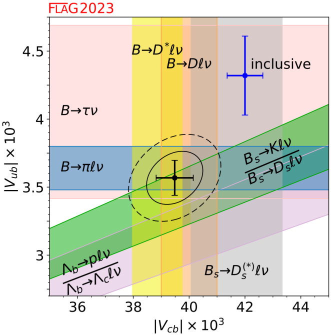


left: $B_s \to K$, right: $B_s \to D_s$

Inclusive V_{cb} and V_{ub}

Inclusive determinations still use operator product expansion (OPE), find (HFLAV)

$$|V_{ub}^{\text{inc}}| = 4.19 \pm 0.17 \times 10^{-3}$$
$$|V_{cb}^{\text{inc}}| = 42.19 \pm 0.78 \times 10^{-3}$$



However, new lattice methods (Alessandro Barone, Shoji Hashimoto, Andreas Jüttner, Takashi Kaneko, Ryan Kellermann) allow for fully non-perturbative calculation of inclusive observables - these will provide a check of existing OPE results.

Inclusive on the Lattice [2305.14092]

The inclusive rate depends on the hadronic tensor, e.g. for $B_s \to X_c \ell \nu$

$$\Gamma = \frac{G_F^2 |V_{cb}|^2}{24\pi^3} \int_0^{q^2_{\max}} dq^2 \sqrt{q^2}\,\overline{X}(q^2)$$

where

$$\overline{X}(q^2) = \sum_{l=0}^{2} \overline{X}^{(l)}(q^2) = \sum_{l=0}^{2} \int_{\omega_{\min}}^{\omega_{\max}} k_{\mu\nu}^{(l)}(q,\omega) W^{\mu\nu}(q,\omega)$$

and

$$W^{\mu\nu}(q) = \frac{1}{2M_{B_s}} \int dx^4 e^{iq\cdot x} \langle B_s | J^{\mu\dagger}(x) J^{\nu}(0) | B_s \rangle.$$

Lattice correlation functions can give

$$C_{\mu\nu}(q,t) = \int_0^{\infty} d\omega W_{\mu\nu}(q,\omega) e^{-\omega t}$$

Laplace transform cannot be inverted. Luckily, only need to get $\overline{X}$

Inclusive on the Lattice [2305.14092]

Approximate kernel (with smoothed sigmoid in place of step) as polynomial in $e^{-\omega}$

$$k^{(l)}_{\mu\nu}(q,\omega)\theta_\sigma(\omega_{\mathrm{max}}-\omega) \approx c^{(l)}_{0,\mu\nu}(q,\sigma) + c^{(l)}_{1,\mu\nu}(q,\sigma)e^{-\omega} + \ldots + c^{(l)}_{N,\mu\nu}(q,\sigma)e^{-N\omega}$$

then

$$\overline{X}^{(l)}(q^2) = \sum_{k=0}^{N} c^{(l)}_{k,\mu\nu} C^{\mu\nu}(q, k+2t_0)$$

This is not completely sufficient due to issues with noise - Chebyshev and Backus-Gilbert methods used to obtain accurate results.

- More details in [2305.14092], see [2005.13730] for details of pilot study on $B_s \to X_c \ell\nu$.

Summary

- New exclusive determination of $|V_{ub}| = 3.93 \pm 0.41 \times 10^{-3}$ using JLQCD $B \to \pi$ FFs, consistent with existing inclusive and exclusive determinations. Large uncertainty, but update in progress and work by other collaborations in progress.
- New results for $B \to D^* \ell \bar{\nu}$ from Belle II inconsistent with Fermilab/MILC R_1 and R_2, give $|V_{cb}| = 40.3 \pm 1.2 \times 10^{-3}$, $p - \mathrm{value} = 21\%$ only including h_{A1} - closer to inclusive picture.
- New HPQCD $B \to D^* \ell \bar{\nu}$ FFs show similar discrepancy in R_1 and R_2 with HQE $\to$ need to understand the origin of this effect.
- New JLQCD $B \to D^* \ell \bar{\nu}$ FFs are in better agreement, but larger uncertainties.
- Application of non-perturbative LQCD methods to inclusive decays progressing nicely.

Backup Slides

$B \to \pi$, JLQCD [2203.04938] - fit function

For $B \to \pi$, most recent calculation from JLQCD, using fully relativistic Möbius domain wall heavy quarks. Fit is done to form factors f_1 and f_2:

$$\langle \pi(p_\pi) | V^\mu | B(v = p_B/M_B) \rangle = 2\left[f_1(v \cdot p_\pi) v^\mu + f_2(v \cdot p_\pi) \frac{p_\pi^\mu}{v \cdot p_\pi} \right]$$

Lattice data for f_1 and f_2 are fit using the functions

$$\begin{aligned}
f_1(v\cdot p_\pi) + f_2(v \cdot p_\pi) =& C_0 \left(1 + \sum_{n=1}^{3} C_{E^n} N_E^n E_\pi^n \right) (1 + C_{\chi\log} \delta f^{B\to\pi} + C_{M_\pi^2} N_{M_\pi^2} M_\pi^2) \\
& \times \left(1 + \frac{C_{M_Q} N_{M_Q}}{m_Q}\right) (1 + C_{m_{s\bar{s}^2}} \delta m_{s\bar{s}^2}{}^2) \\
& \times \left(1 + C_{a^2} (a\Lambda_{\mathrm{QCD}})^2 + C_{(am_Q)^2} (am_Q)^2 \right) \\
f_2(v \cdot p_\pi) =& D_0 \left[\frac{E_\pi}{E_\pi + \Delta_B} (1 + D_E N_E E_\pi) \right] (1 + D_{\chi\log} \delta f^{B\to\pi} + D_{M_\pi^2} N_{M_\pi^2} M_\pi^2) \\
& \times \left(1 + \frac{D_{M_Q} N_{M_Q}}{m_Q}\right) (1 + D_{m_{s\bar{s}^2}} \delta m_{s\bar{s}^2}{}^2) \\
& \times \left(1 + D_{a^2} (a\Lambda_{\mathrm{QCD}})^2 + D_{(am_Q)^2} (am_Q)^2 \right)
\end{aligned}$$

with C and D fit parameters.

$B \to D^*$, HPQCD [2304.03137] - fit function

For $B \to D^*$ FFs, extrapolation is done using power series in $(w-1)$

$$F^{Y^{(s)}}(w) = \sum_{n=0}^{10} a_n^{Y^{(s)}} (w-1)^n \mathcal{N}_n^{Y^{(s)}} + \frac{g^2_{D^* D\pi}}{16\pi^2 f_\pi^2} \left(\mathrm{logs}^{Y^{(s)}}_{SU(3)} - \mathrm{logs}^{Y}_{SU(3)\mathrm{phys}} \right)$$

The coefficients, a_n^Y, for each form factor take the form

$$\begin{aligned} a_n^{Y^{(s)}} =& \alpha_n^Y \\ &\times \left[1 + \sum_{j,k,l \neq 0}^{3} b_n^{Y,jkl} \Delta_h^{(j)} \left(\frac{am_c^{\mathrm{val}}}{\pi} \right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi} \right)^{2l} \right. \\ &\left. + \delta_\chi^{(s)} \sum_{j,k,l=0}^{3} \tilde{b}_n^{Y,jkl} \Delta_h^{(j)} \left(\frac{am_c^{\mathrm{val}}}{\pi} \right)^{2k} \left(\frac{am_h^{\mathrm{val}}}{\pi} \right)^{2l} \right] \end{aligned}$$