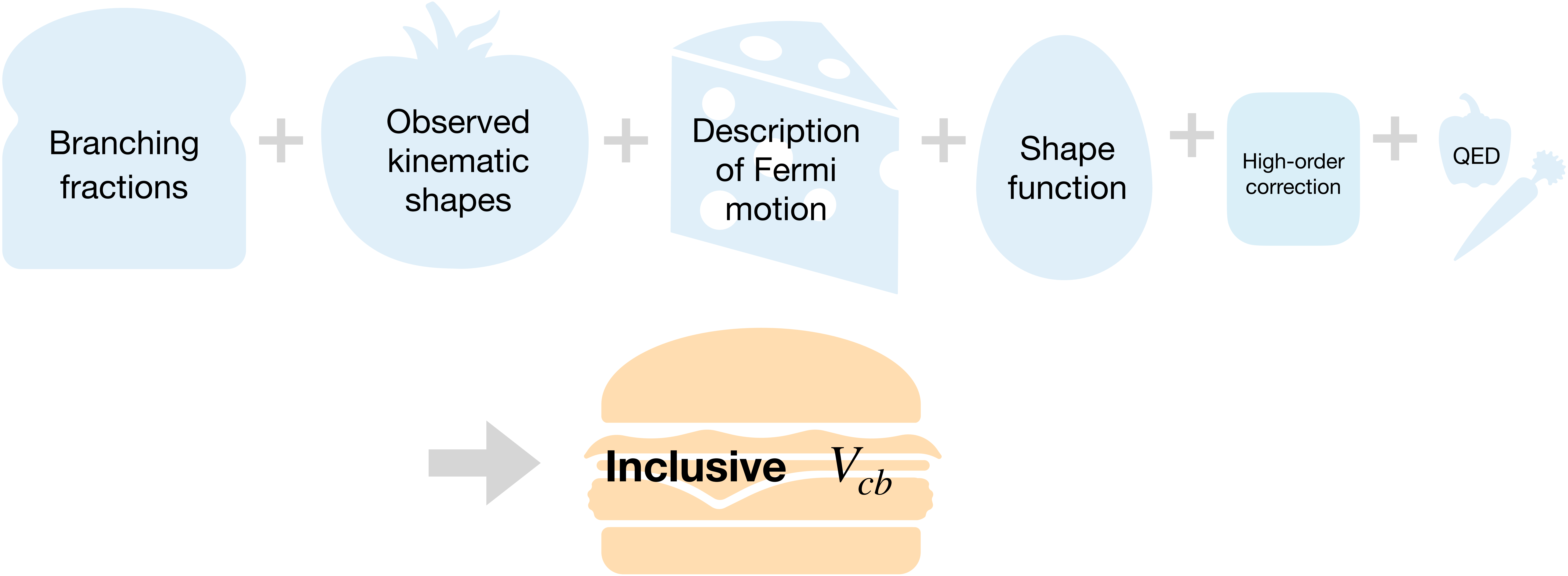
Experimental Developments of Inclusive Inputs

Lu Cao

(for the Belle & Belle II Collaboration)

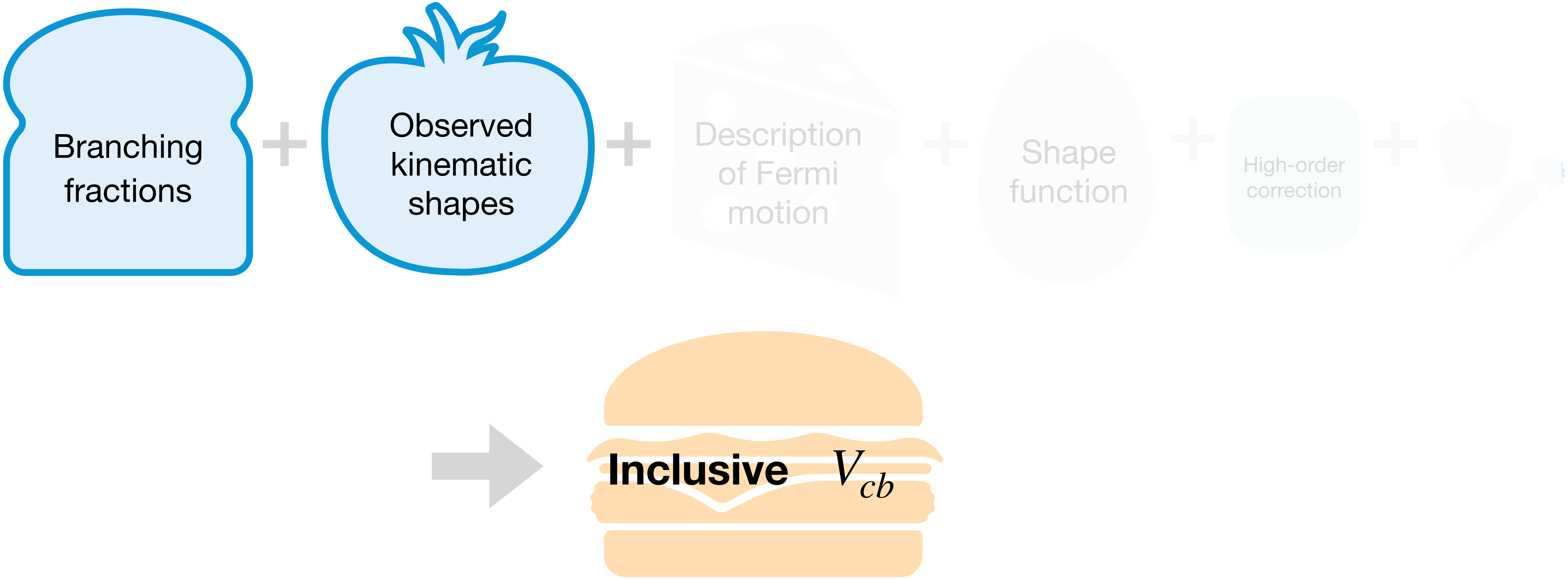
Belle II Physics Week - Vcb Workshop

Outline


Branching fractions
+
Observed kinematic shapes
+
Description of Fermi motion
+
Shape function
+
High-order correction
+
QED
Inclusive V_{cb}

Outline

Experimental ingredients



Branching Fraction Measurements

- As a normalisation factor, the total branching fraction is important for $|V_{cb}|$ extraction
- Experimental inputs are $\mathscr{B}(B \to X_c \ell \nu)$,

  or indirectly via $\mathscr{B}(B \to X_c \ell \nu) = \mathscr{B}(B \to X \ell \nu) - \mathscr{B}(B \to X_u \ell \nu)$

  if measured as partial $\Delta\mathscr{B}$, need rescale to full phase space $\mathscr{B} = \Delta\mathscr{B}/\epsilon_\Delta$ $\quad \epsilon_\Delta = \frac{\Delta\Gamma(\mathrm{e.g.}\ E_\ell^B > 0.6\,\mathrm{GeV})}{\Gamma}$

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- PDG status

$\mathscr{B}(B \to X \ell \nu)$

VALUE (%)		DOCUMENT ID		TECN
10.84 ± 0.16	OUR EVALUATION See the ideogram below. [HFLAV]			
10.49 ± 0.20	OUR AVERAGE Error includes scale factor of 1.3. See the ideogram below.			
10.34 ±0.04 ±0.26		[1] LEES	2017B	BABR
10.28 ±0.18 ±0.24		[2] URQUIJO	2007	BELL
10.91 ±0.09 ±0.24		[3] MAHMOOD	2004	CLEO
9.7 ±0.5 ±0.4		[4] ALBRECHT	1993H	ARG

$\mathscr{B}(B \to X_c \ell \nu)$

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10.65 ± 0.16	OUR EVALUATION [HFLAV]			
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10.18 ±0.03 ±0.24		[1] LEES	2017B	BABR
10.44 ±0.19 ±0.22		[2] URQUIJO	2007	BELL

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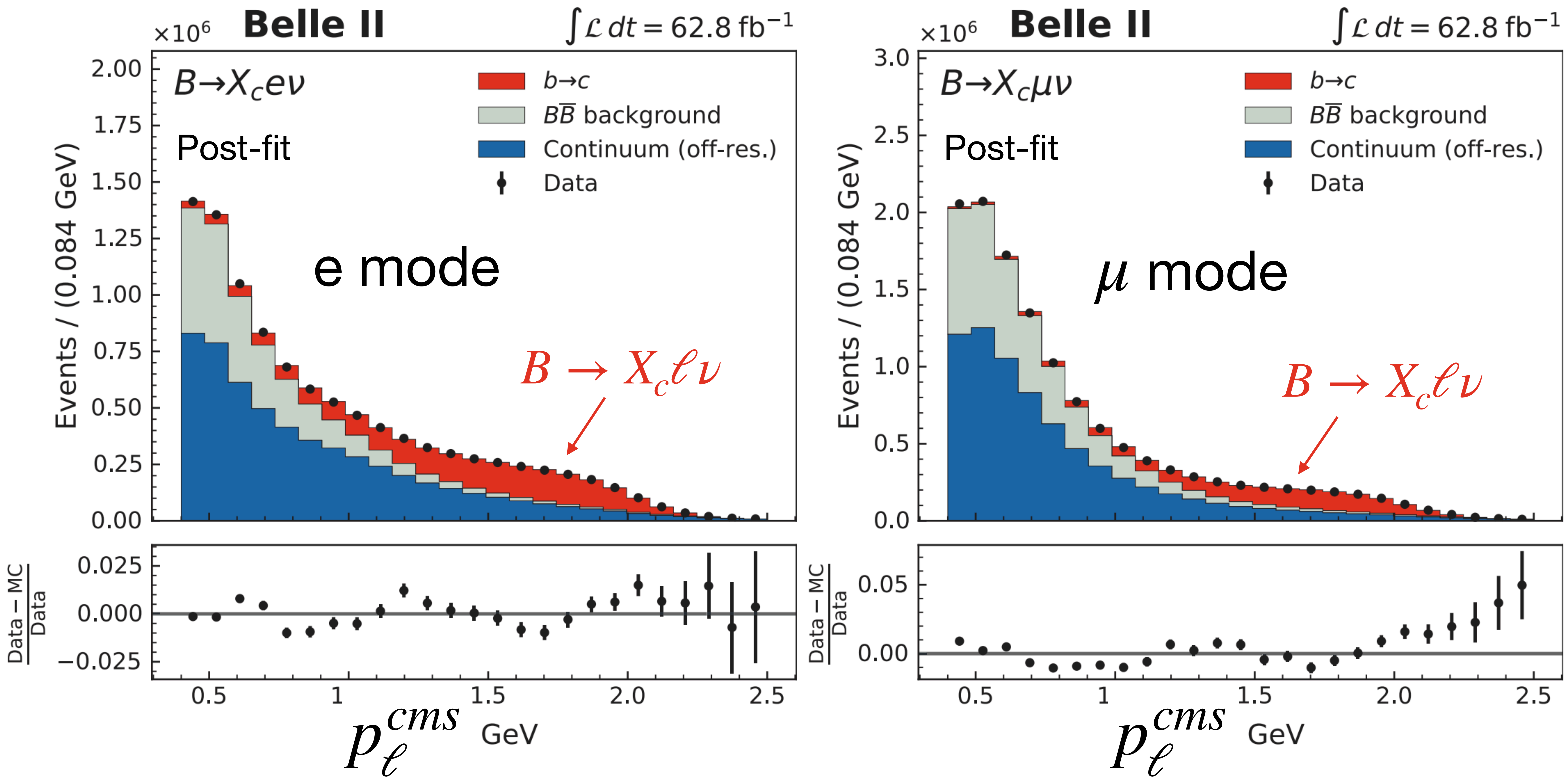
- Tension exits among various measurements/conversions [see details in JHEP 10 068 (2022)]

Branching Fraction Measurement at Belle II

- Belle II provided preliminary result on $\mathscr{B}(B \to X_c \ell \nu)$ with 62.8 fb^{-1} dataset [2111.09405]



- **Untagged** strategy
- Select energetic lepton within [0.4, 2.5] GeV in centre-of-mass frame
- Addition selections applied to enhance signal (e.g. M^2_{miss}, θ_{miss}, and total event charge)

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μ mode: $\mathcal{B}(B \to X_c \mu \nu_\mu) = (9.47 \pm 0.05(\text{stat}) \pm 0.45(\text{sys}))\%$

combined: $\mathcal{B}(B \to X_c \ell \nu_\ell) = (9.75 \pm 0.03(\text{stat}) \pm 0.47(\text{sys}))\%$

weighted mean assuming fully correlated syst. unc., quote larger error from μ mode for the average

Contribution	Relative uncertainty [%]	
	Electron mode	Muon mode
Tracking	0.69	0.69
$N_{B\bar{B}}$	1.1	1.1
Lepton ID corrections	1.64	2.33
f_0/f_+, B lifetime	1.2	1.2
$B \to X_c \ell \nu_\ell$ branching fractions	2.65	2.15
$B \to X_c \ell \nu_\ell$ form factors	1.11	1.11
$B\bar{B}$ background model	0.24	0.34
Off-resonance data model	0.34	2.91
Sum	3.77	4.79

Results are not stat. limited but some important syst. uncertainties are due to only limited data were used for calibrations

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- **This untagged measurement** will be **updated to larger dataset** and incorporating new knowledge for modelling, e.g. the recent Belle measurement of $B \to D^{(*)}\pi(\pi)\ell\nu$ [PRD 107, 092003 (2023)] **The lepton energy spectrum/moments could be also provided** (very limited resolutions for M_X, q^2)

- We will also measure branching fractions (total & differential) for **semi-inclusive** $B \to DX\ell\nu$

Measurements of Moments in $B \to X_c \ell \nu$

- Beyond normalisation $\mathscr{B}$, detailed shapes of key kinematic variables are needed to derive HQE parameters, shape function and $|V_{cb}|$

- $< E_\ell^n >$: **BaBar**(2004, 2010), **Belle**(2007), **CLEO**(2001), **DELPHI**

- $< M_x^{2n} >$: **BaBar**(2010), **Belle**(2007), **CDF**(2005), **CLEO**(2004), **DELPHI**(2006)

- $< q^{2n} >$: **Belle** (2021), **Belle II** (2023)

$B \to X_c \ell \nu$ HADRONIC MASS MOMENTS	
$\langle M_X^2 - \overline{M}_D^2 \rangle$ (First Moments)	0.36 ± 0.08 GeV^2 (S = 1.8)
$\langle M_X^2 \rangle$ (First Moments)	4.156 ± 0.029 GeV^2
$\langle (M_X^2 - \overline{M}_X^2)^2 \rangle$ (Second Moments)	0.55 ± 0.08 GeV^4
$\langle (M_X^2 - \overline{M}_D^2)^2 \rangle$ (Second Moments)	0.64 ± 0.19 GeV^4
$B \to X_c \ell \nu$ LEPTON MOMENTUM MOMENTS	
R_0 ($\Gamma_{E_l > 1.7 GeV}$ / $\Gamma_{E_l > 1.5 GeV}$)	0.6187 ± 0.0021
R_1 ($\langle E_l \rangle_{E_l > 1.5 GeV}$)	1.7797 ± 0.0018 (S = 1.8)
R_2 ($\langle E_l^2 - \bar{E}_l^2 \rangle_{E_l > 1.5 GeV}$)	0.0308 ± 0.0008 GeV^2
R_3 ($\langle E_l^3 - \bar{E}_l^3 \rangle_{E_l > 1.5 GeV}$)	0.0021 ± 0.0005 GeV^3

Experimental Strategy

Binned

1. Extract signal yields for each kinematic region (e.g. E_ℓ thresholds)
2. **Unfold** binned spectra with **migration matrix**, correct eff. & acc.
3. Calculate moments via **summing up bins**

Unbinned

1. Signal extraction based on **event-wise** probability
2. Calculate moments with weighted events and corrections
3. **Calibrate moments** for distortion, acceptance and bias

Also discussed by F. Bernlochner at Barolo

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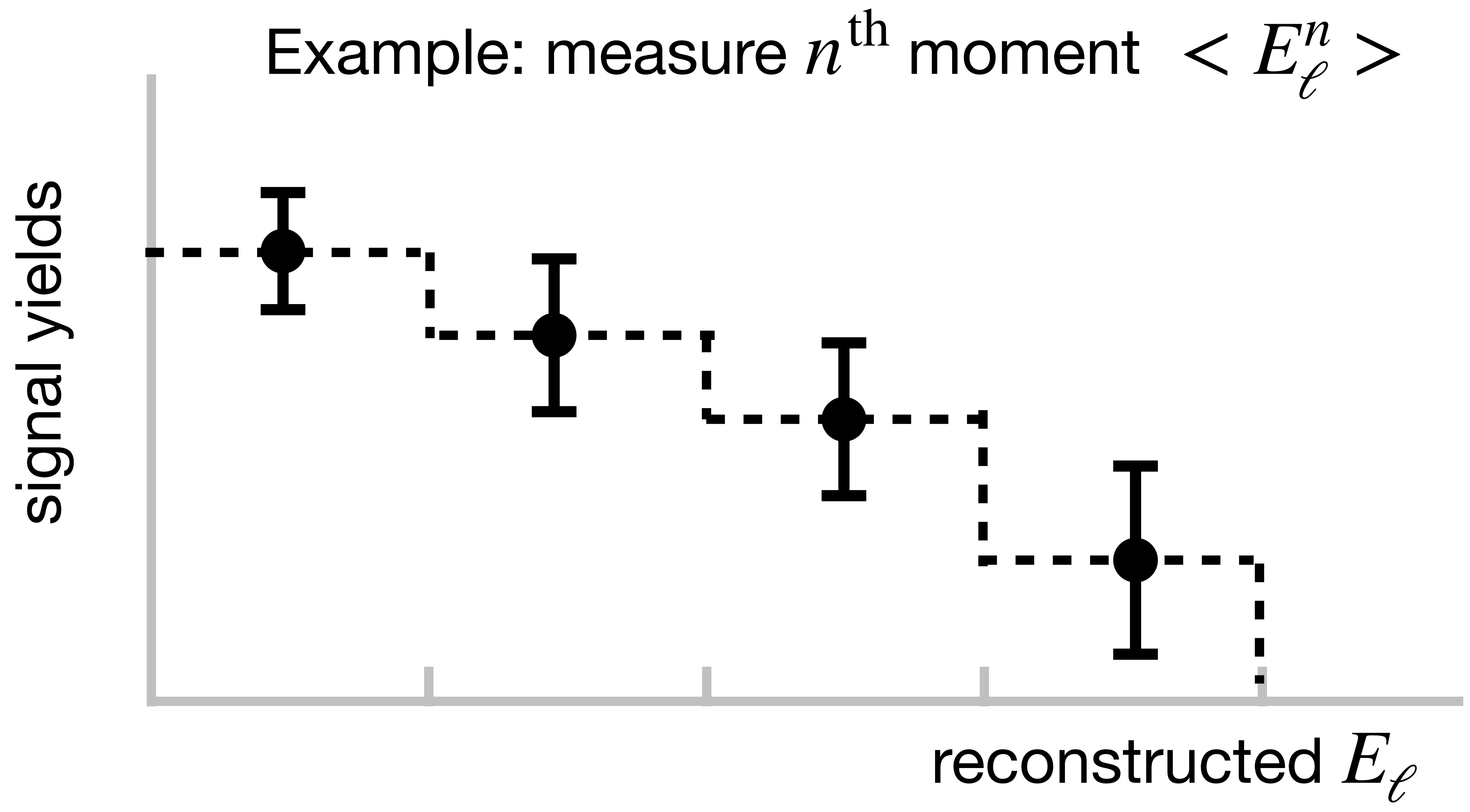

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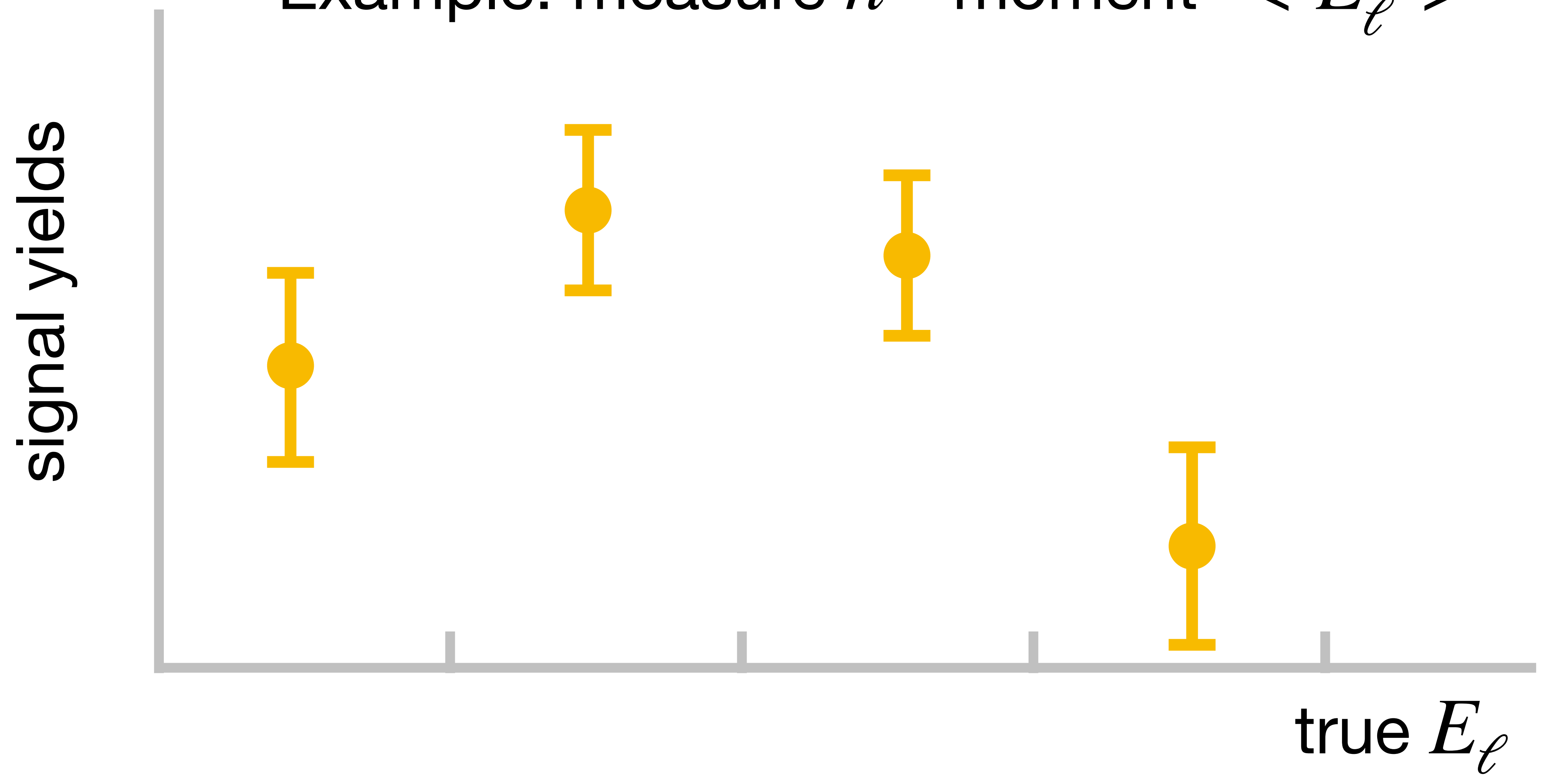

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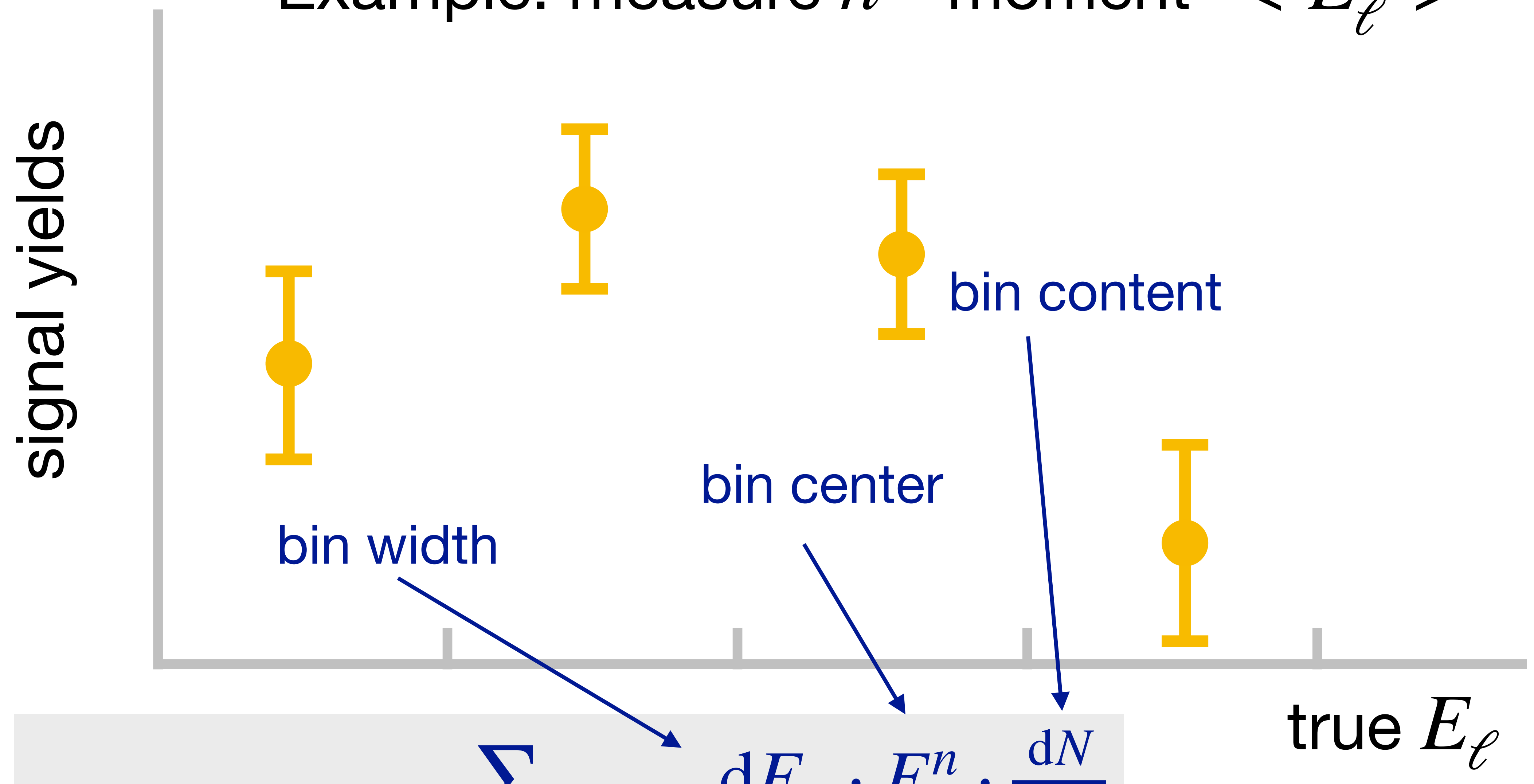

$$\left\langle E_\ell^n \right\rangle_{E_\ell > E_\ell^{\text{cut}}} = \frac{\sum_{E_\ell > E_\ell^{\text{cut}}} \mathrm{d}E_\ell \cdot E_\ell^n \cdot \frac{\mathrm{d}N}{\mathrm{d}E_\ell}}{\sum_{E_\ell > E_\ell^{\text{cut}}} \mathrm{d}E_\ell \frac{\mathrm{d}N}{\mathrm{d}E_\ell}}$$

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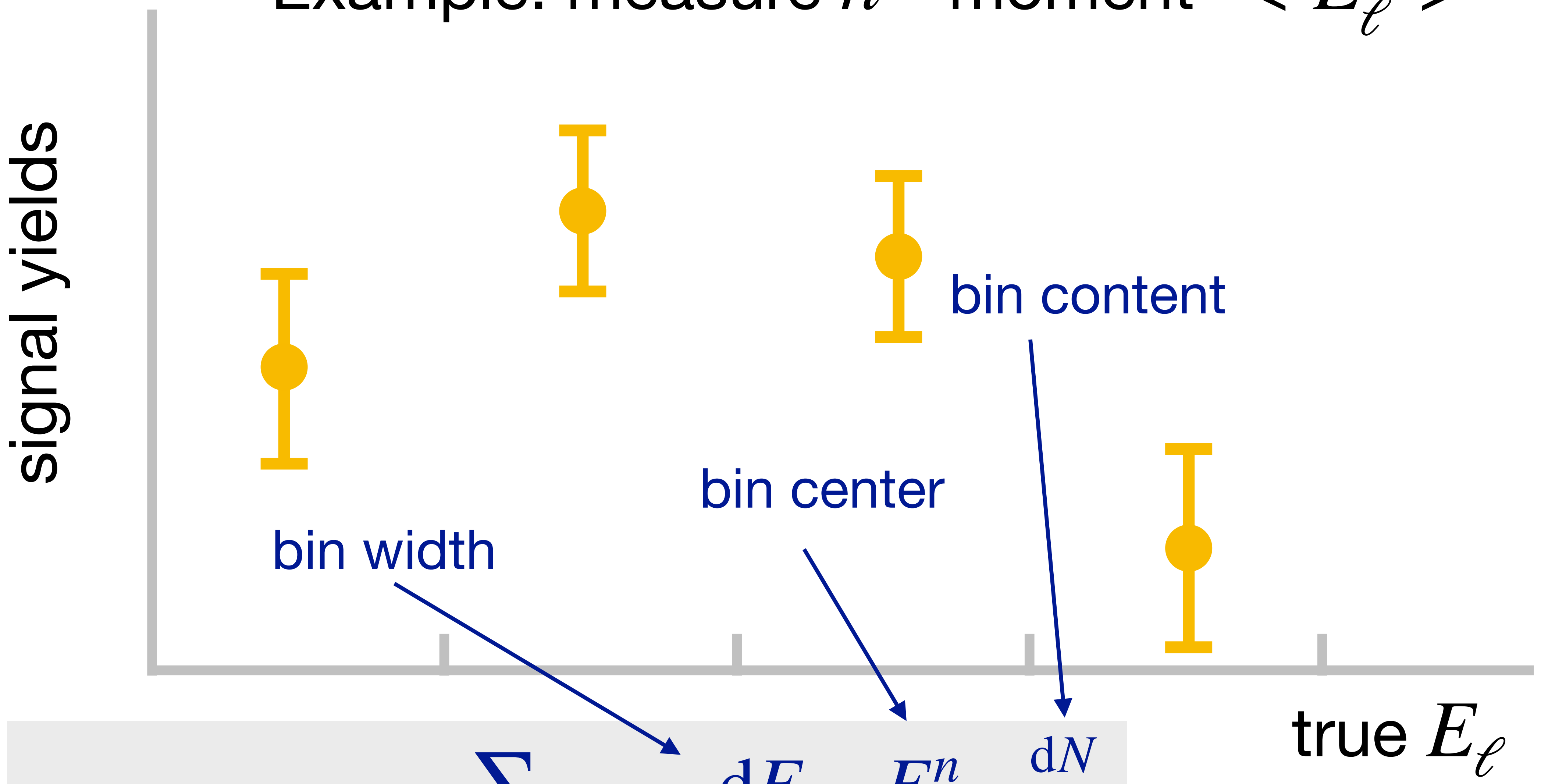

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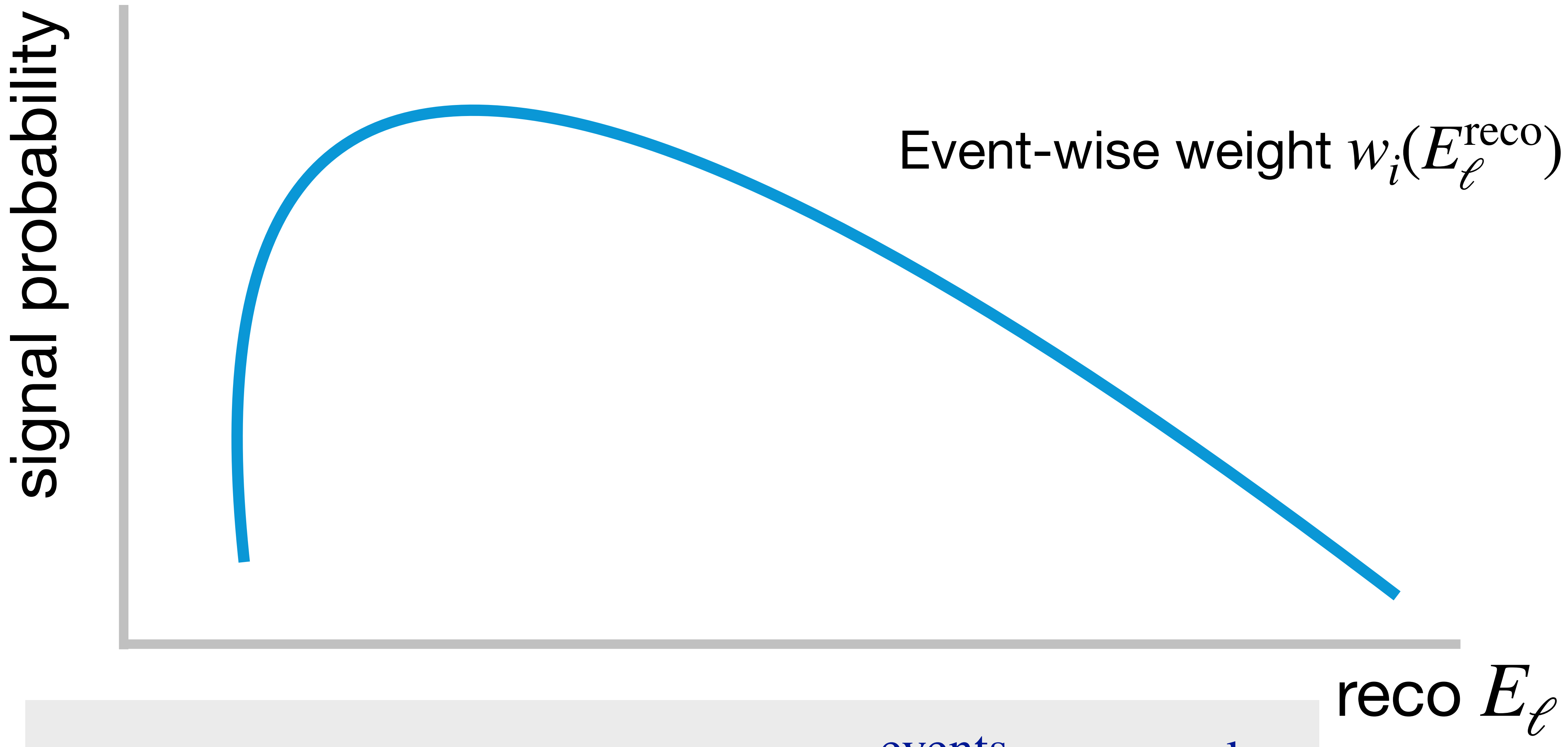


$$\left\langle E_\ell^n \right\rangle_{E_\ell > E_\ell^{\text{cut}}} = \frac{C_{\text{cal.}} \cdot C_{\text{acc.}} \cdot \sum_i^{\text{events}} w_i \cdot E_\ell^{\text{cal,n}}}{\sum_i^{\text{events}} w_i}$$

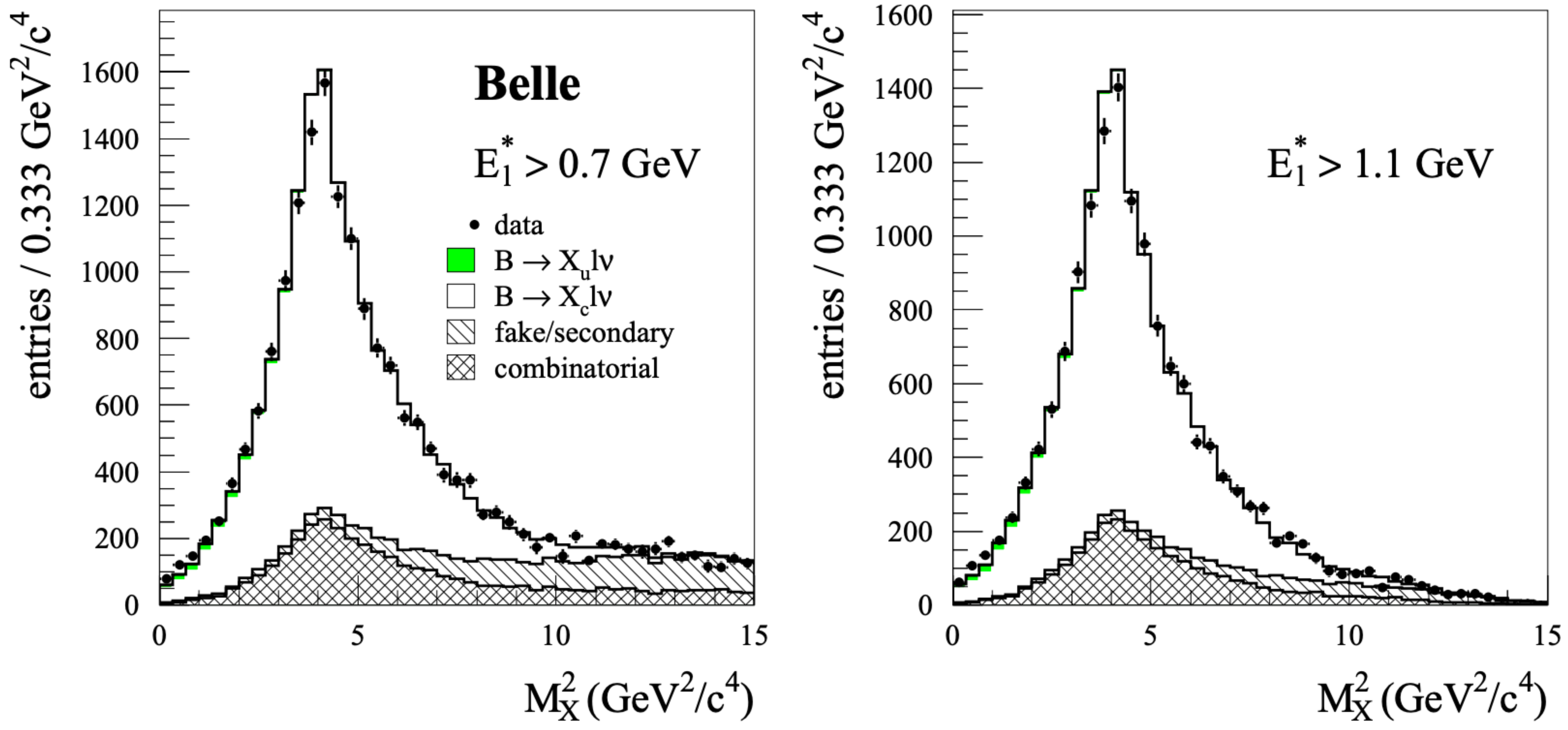
Hadronic Mass Moments in $B \to X_c \ell \nu$

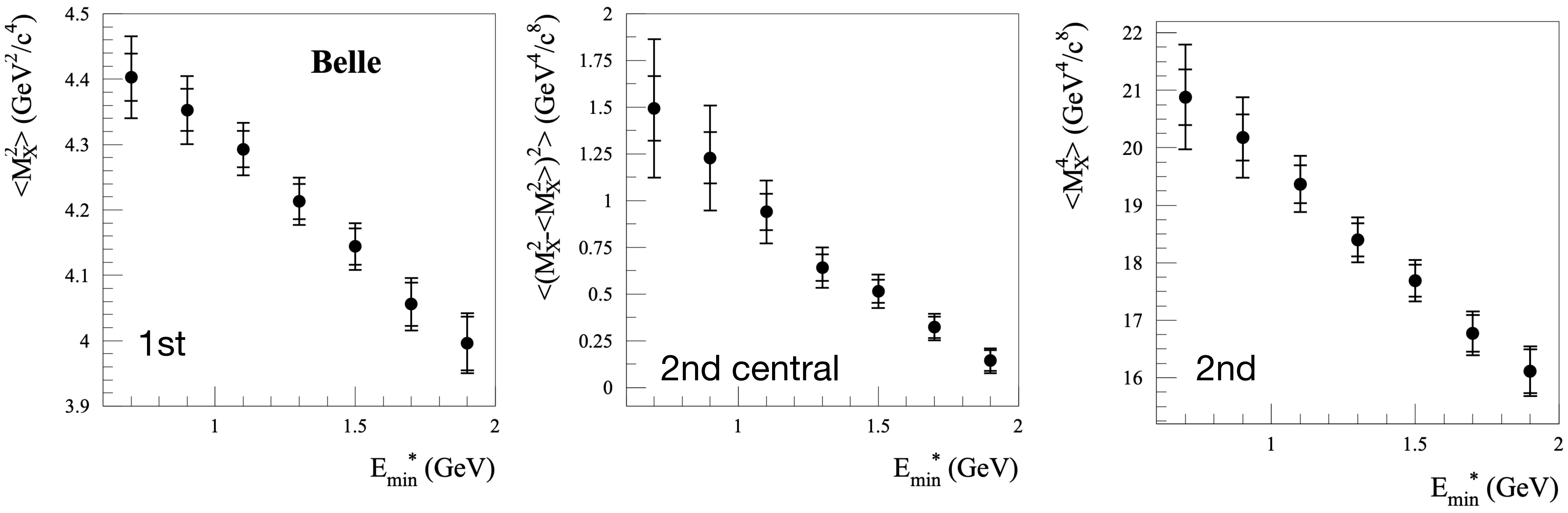
Belle: PRD 75, 032005 (2007)

- Binned strategy applied on 140 fb^{-1} of Belle data
- 1st, 2nd central moments and 2nd raw moments measured with E_ℓ thresholds ranging in [0.7, 1.9] GeV

$$\langle M_X^2 \rangle = \frac{\sum_i (M_X^2)_i x_i'}{\sum_i x_i'}$$

Unfolded M_X^2 spectrum



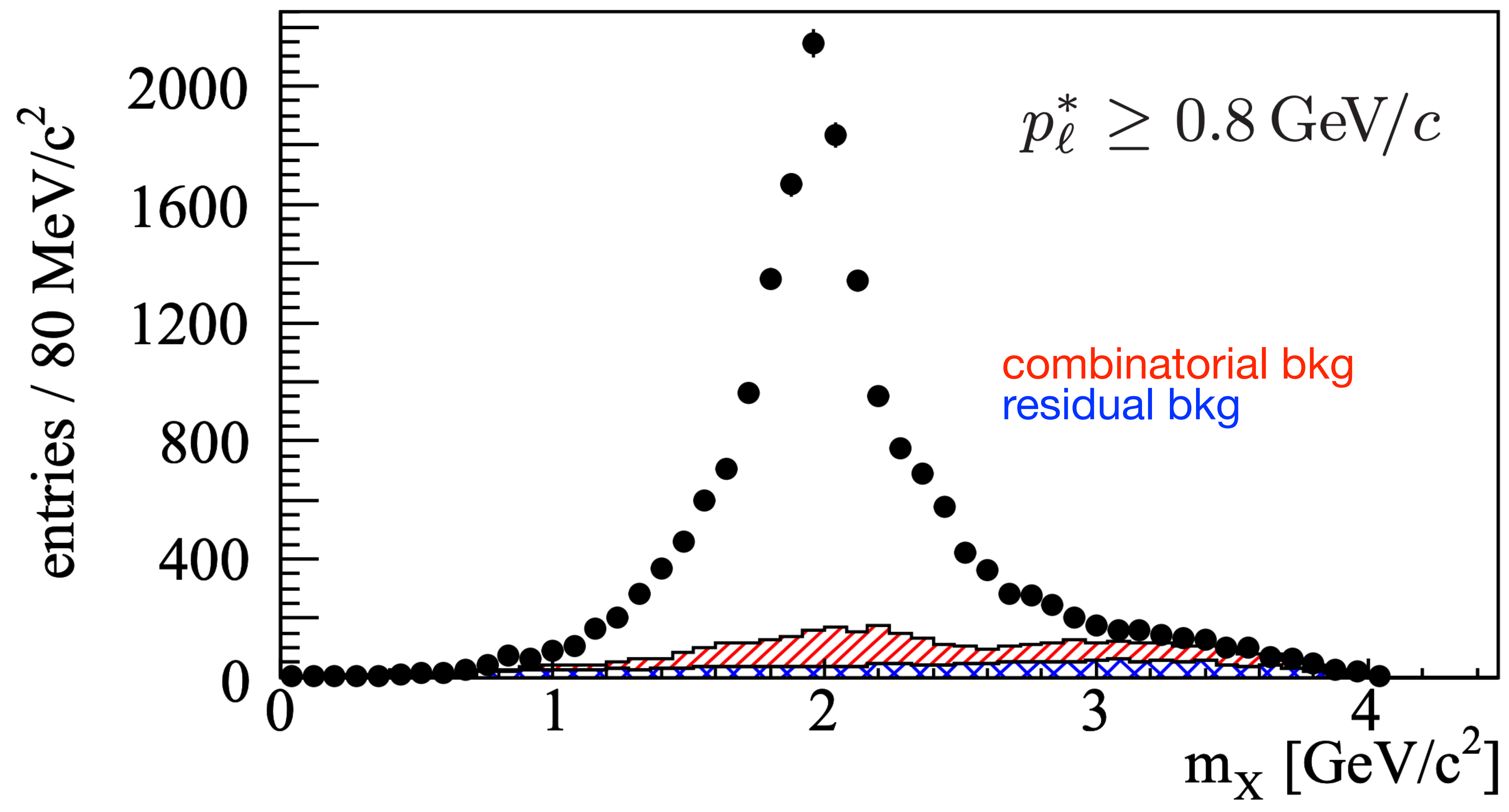


Hadronic Mass Moments in $B \to X_c \ell \nu$

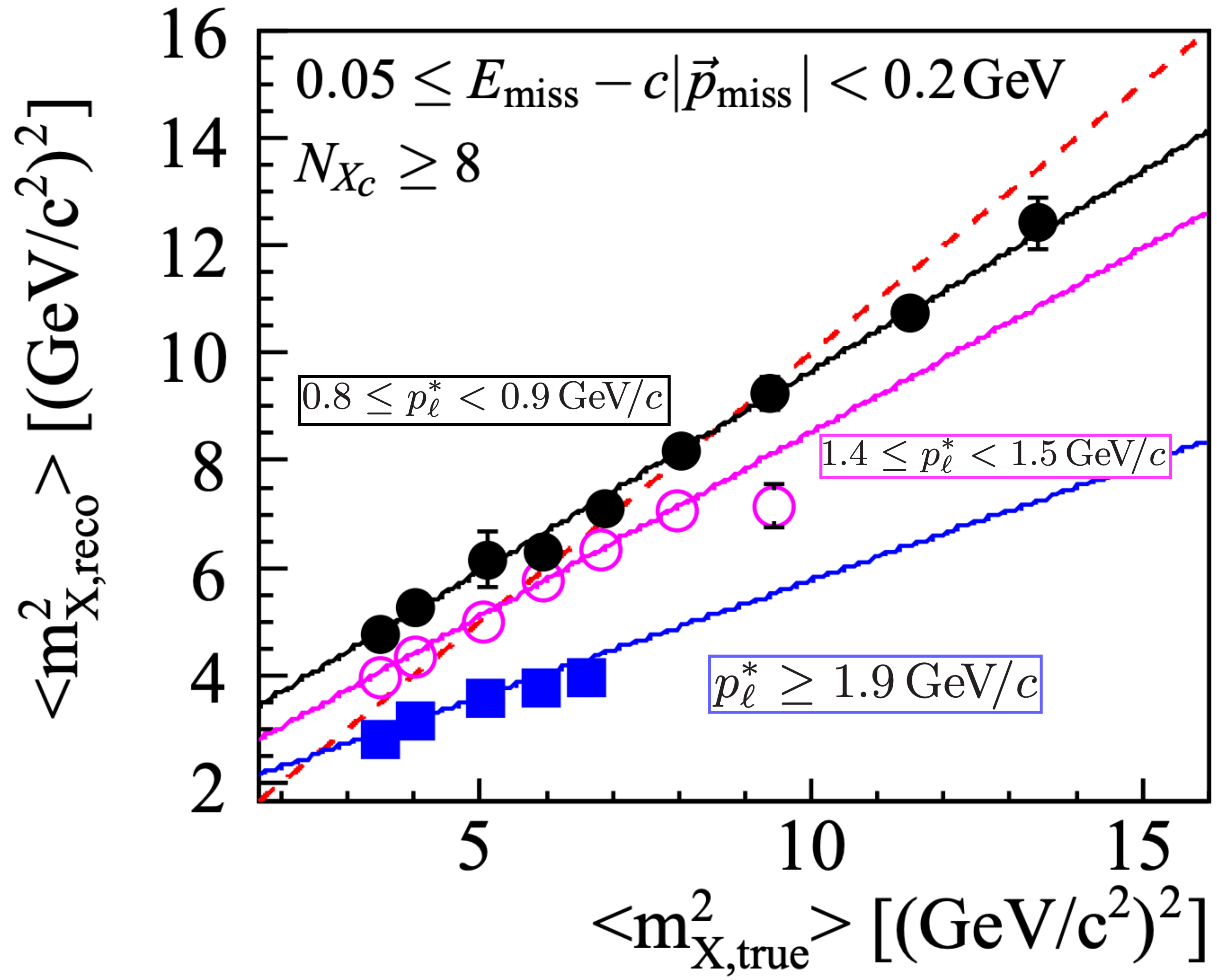
BaBar: PRD 81, 032003 (2010)

- Unbinned strategy applied for 210 fb^{-1} data of BaBar
- 1st, 2nd central moments and 2nd raw moments measured with E_ℓ thresholds ranging in [0.8, 1.9] GeV

$$\langle m_X^k \rangle = \frac{\sum_{i=1}^{N_{ev}} w_i(m_X)\, m^k_{X,calib,i}}{\sum_i^{N_{ev}} w_i} \times \mathcal{C}_{cal}(p_\ell^*, k) \times \mathcal{C}_{true}(p_\ell^*, k)$$



Calibration factors



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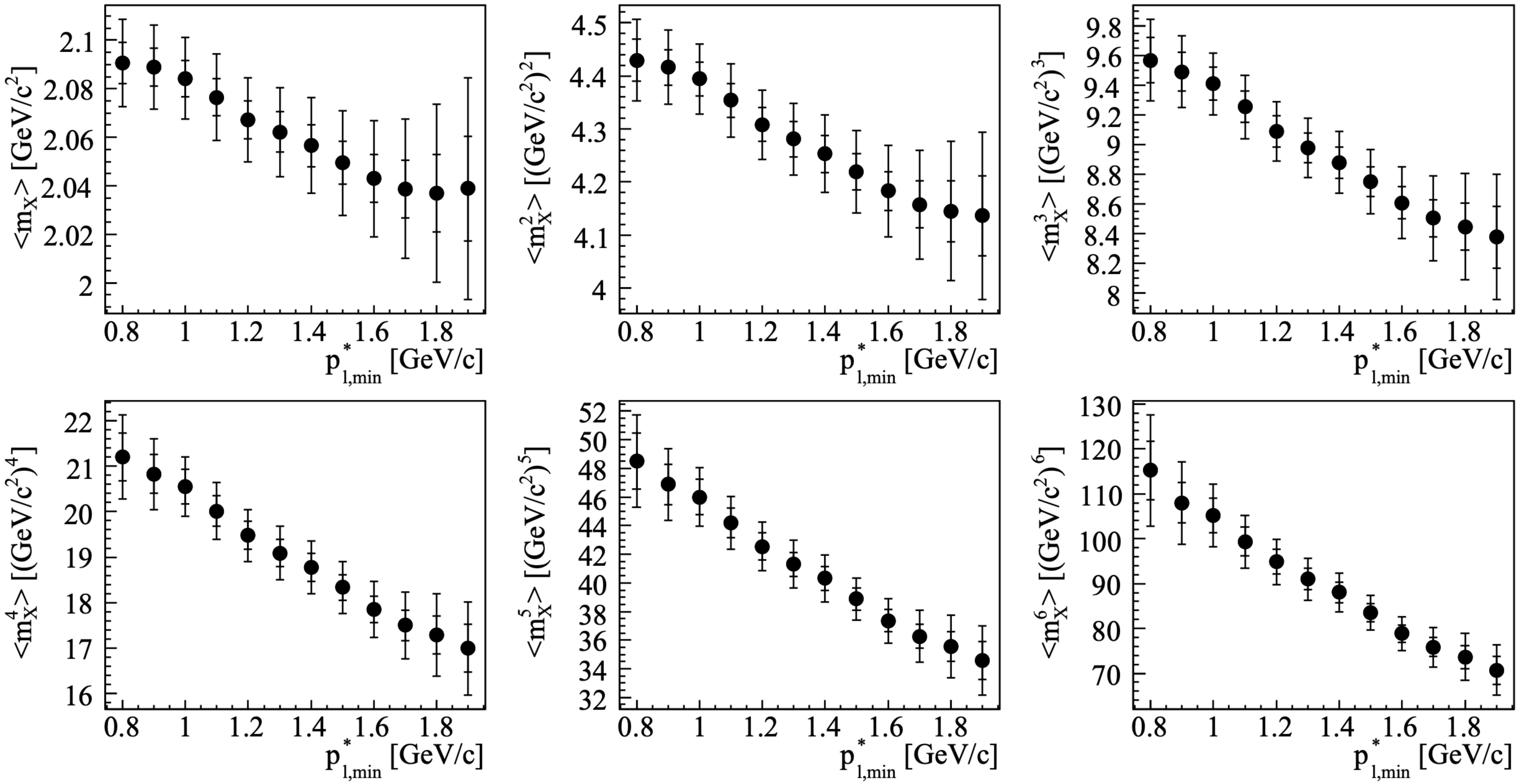
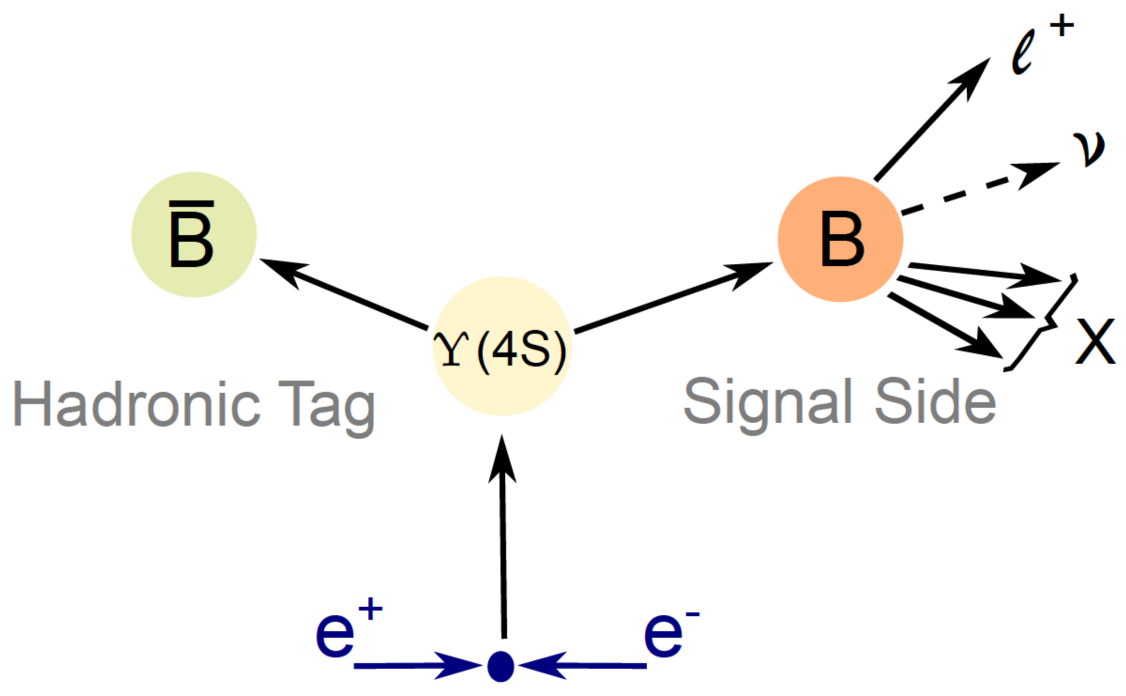


Measurement of Moments q^2 at Belle II

PRD 107, 072002 (2023)

- Measurement of q^2 moments with Belle II dataset of **62.8 fb^{-1}** , $\ell = e, \mu$
- Hadronic tagging to reconstruct B_{tag}
- Kinematic fit constraining missing system improves resolution
- Includes the experimentally challenging q^2 region of [1.5, 2.5] GeV^2, ~77% of phase space



Can fully assign each final state particle to either the tag or signal side

→ **Allows to reconstruct *X***

- Hadronic system X:

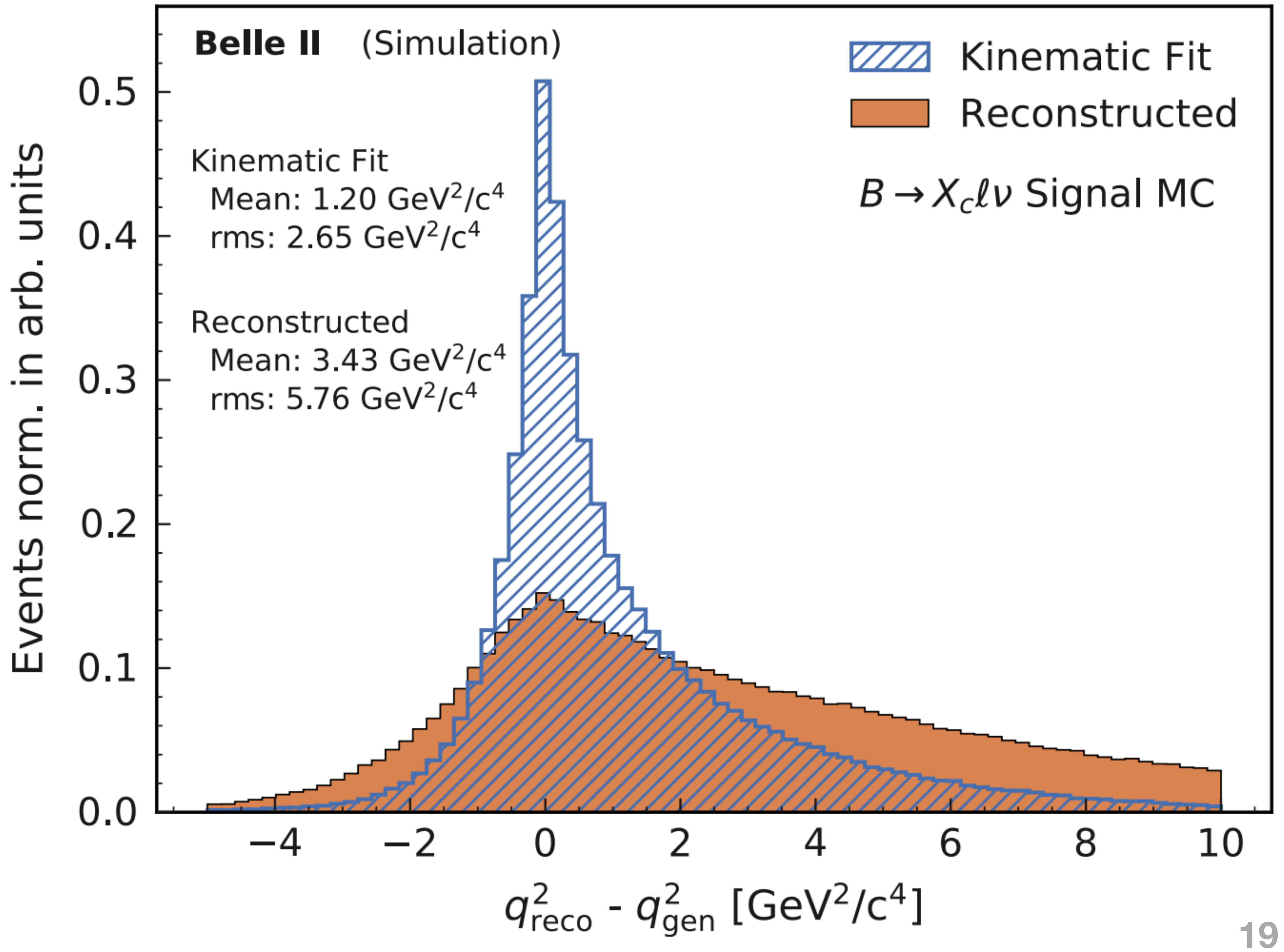
$$p_X = \sum_i\left(\sqrt{m_\pi^2 + |\mathbf{p_i}|^2}, \mathbf{p_i}\right) + \sum_i(E_i, \mathbf{k_i})$$

- Missing mass squared:

$$MM^2 = \left(P_{\Upsilon(4S)} - P_{\text{tag}} - P_X - P_\ell\right)^2$$

- Leptonic system:

$$q^2 = (P_B - P_X)^2 = (P_\ell + P_\nu)^2$$



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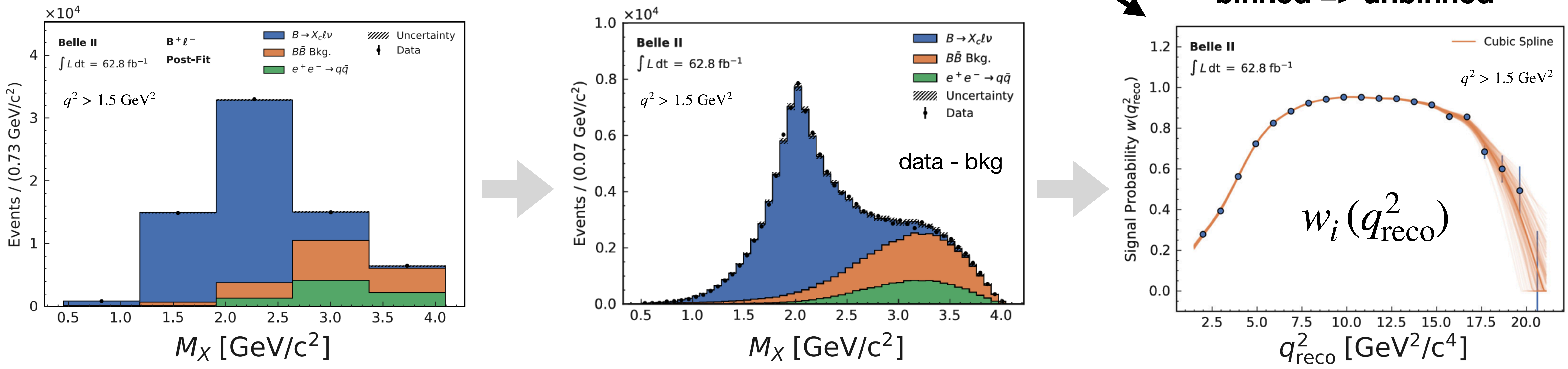
PRD 107, 072002 (2023)

- Background suppressed in hadronic mass M_X and converted to signal prob. on q^2
- **First to fourth moments (m=1~4)** & three central moments measured at a progression of cuts on q^2
- Spectra corrected for linear distortions, eff. & acc. & residual bias

$$\langle q^{2m} \rangle = \frac{C_{\mathrm{cal}} \cdot C_{\mathrm{acc}}}{\sum_i^{\mathrm{events}} w(q_i^2)} \times \sum_i^{\mathrm{events}} w(q_i^2) \cdot q_{\mathrm{cal}\, i}^{2m}$$

binned => unbinned



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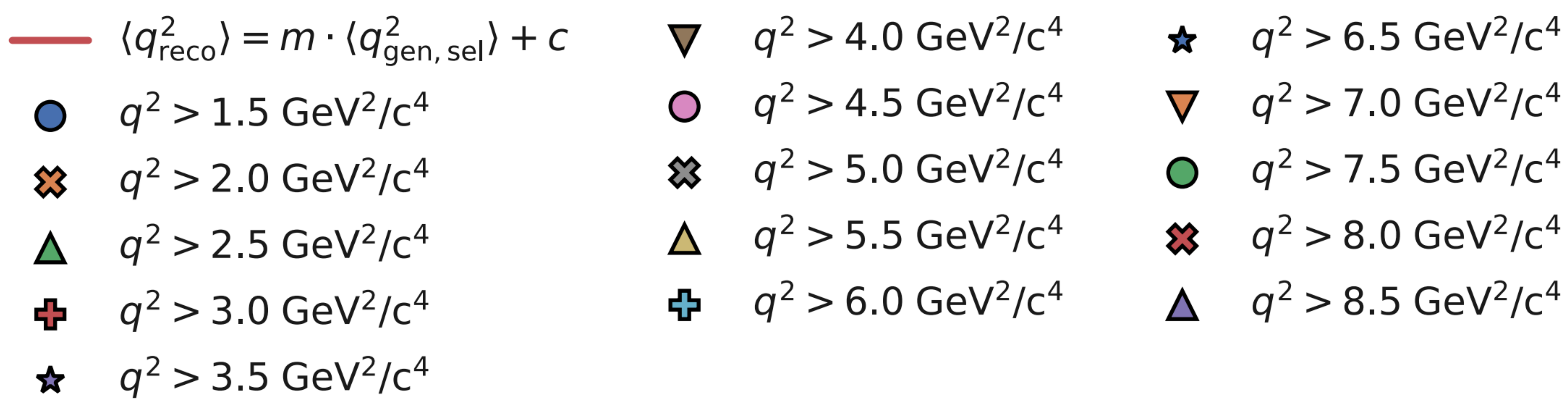

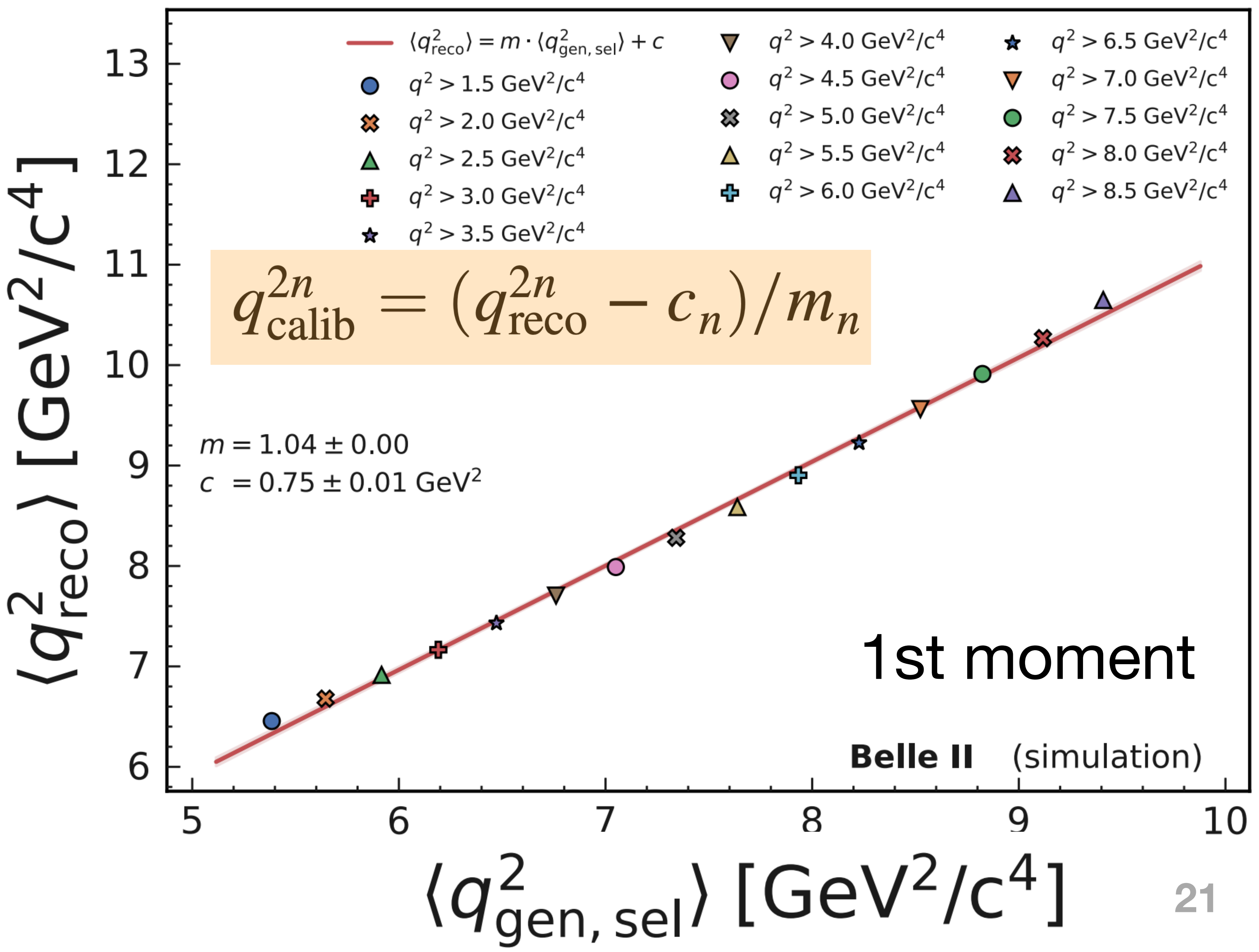

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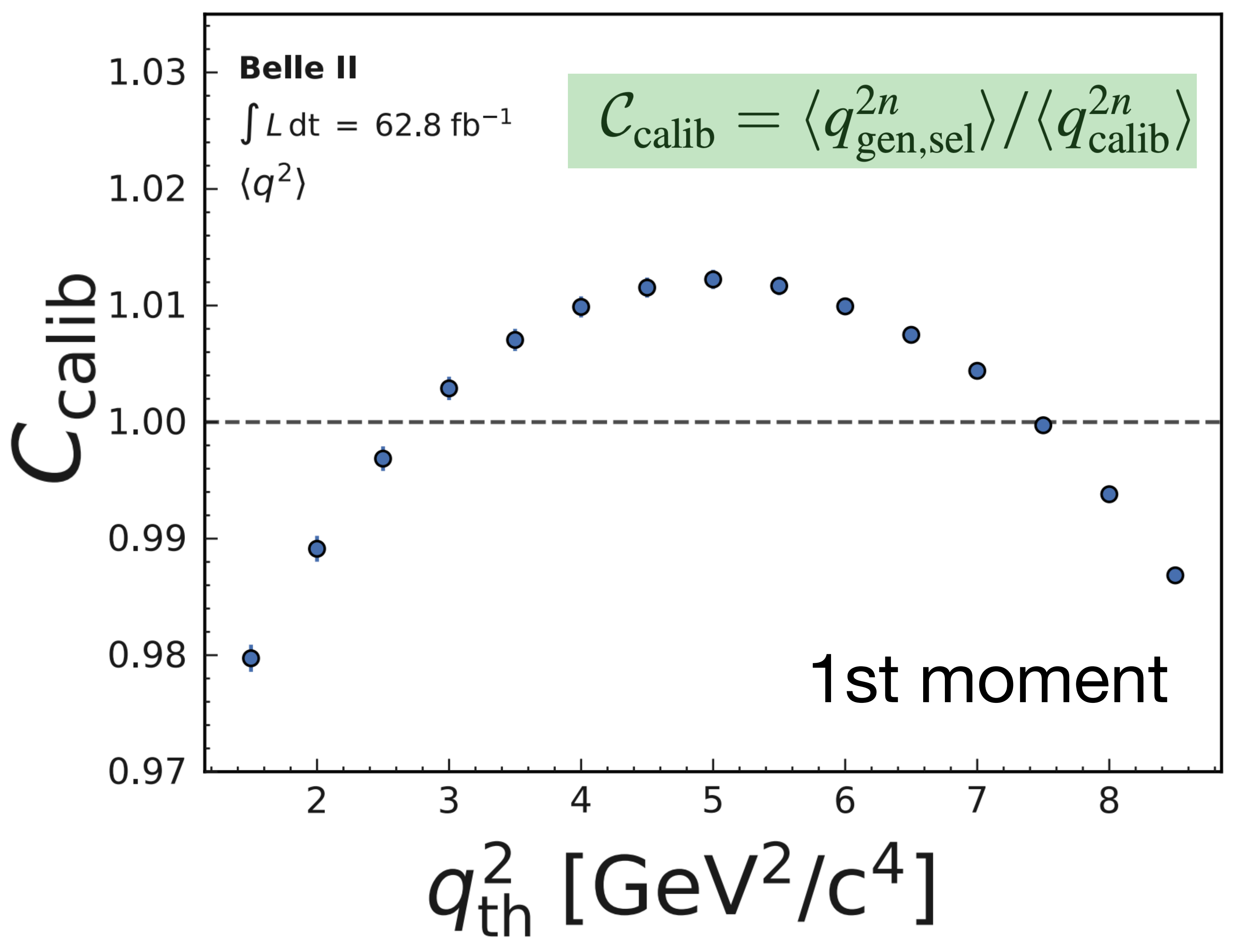

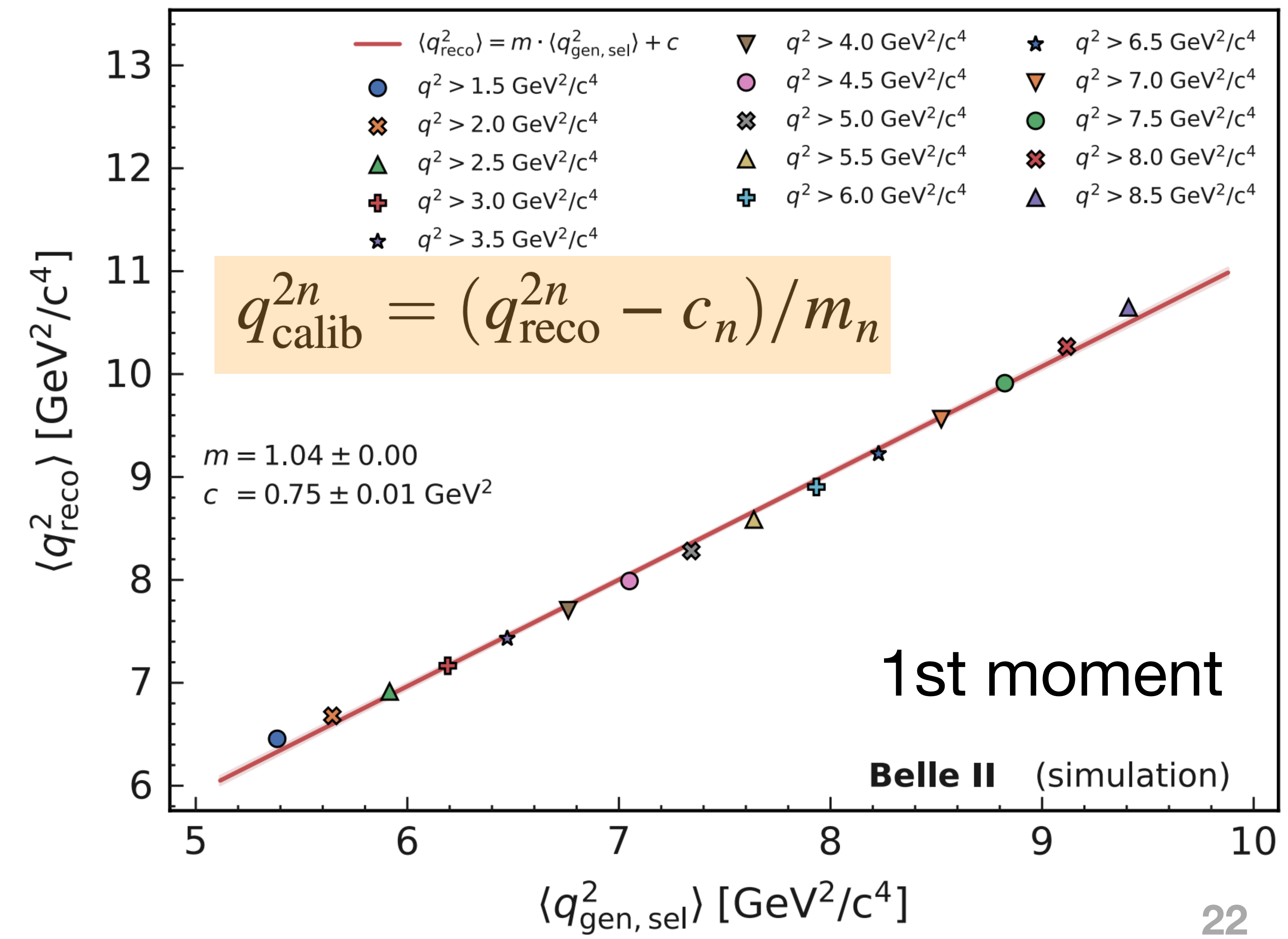

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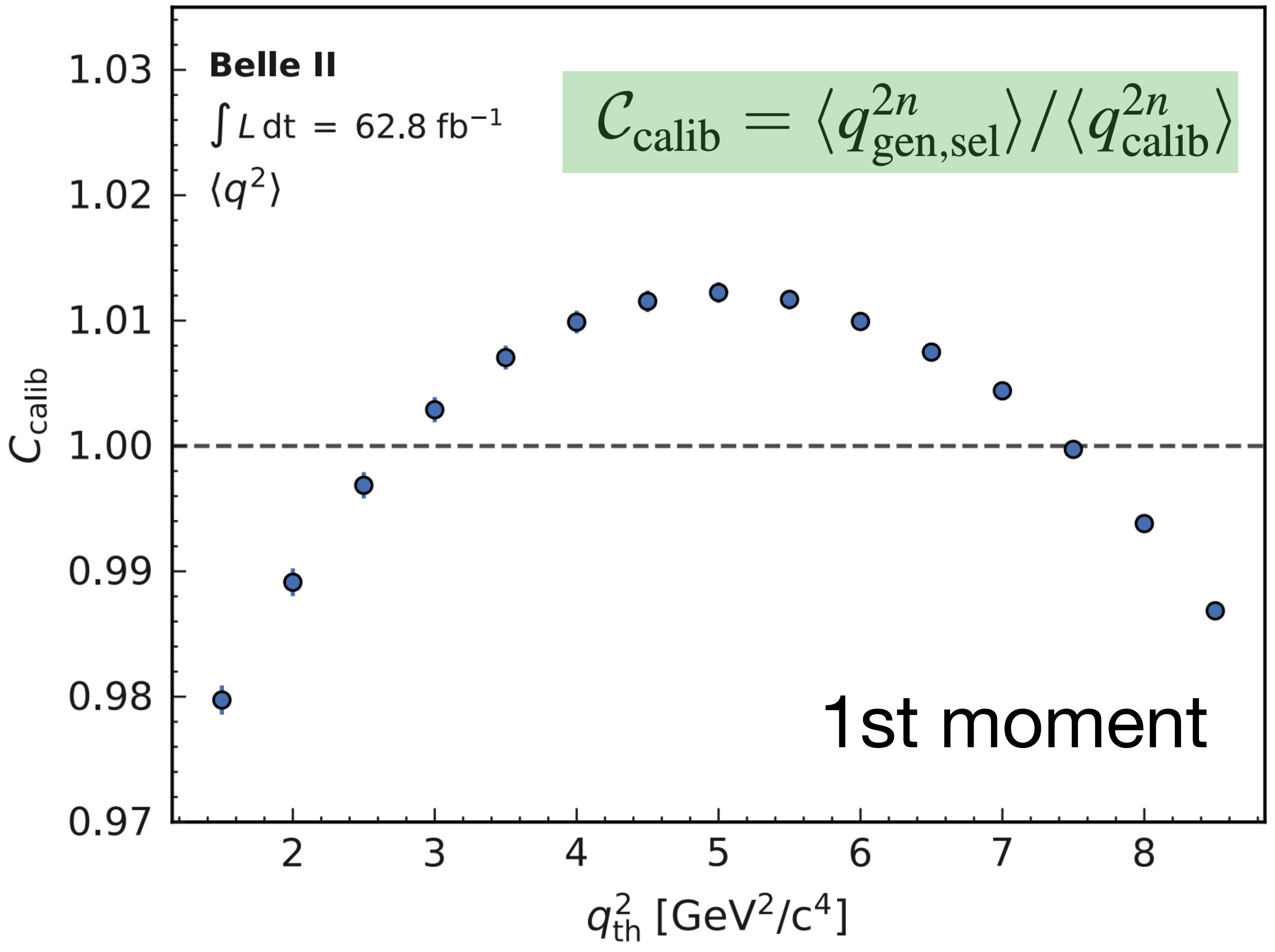

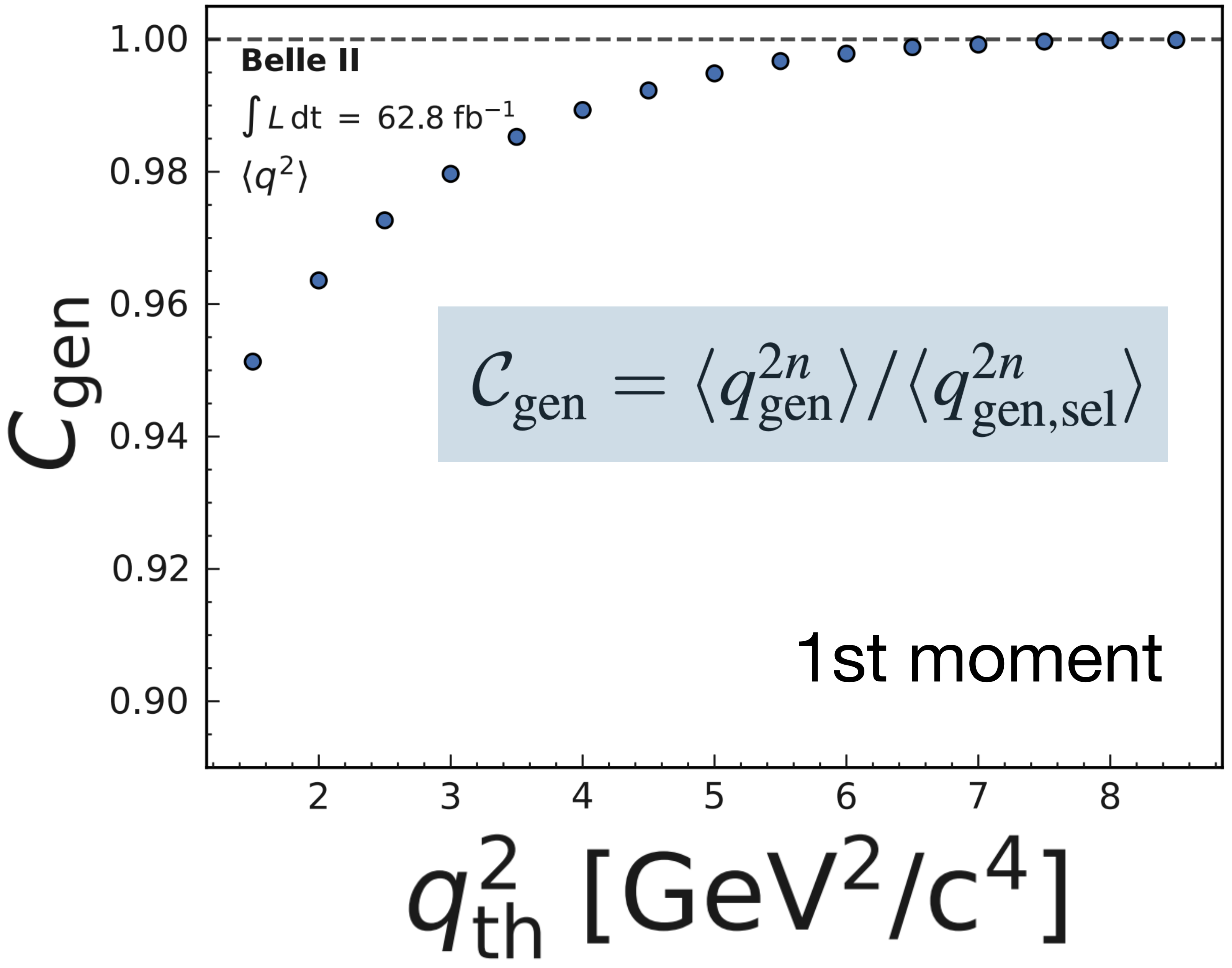

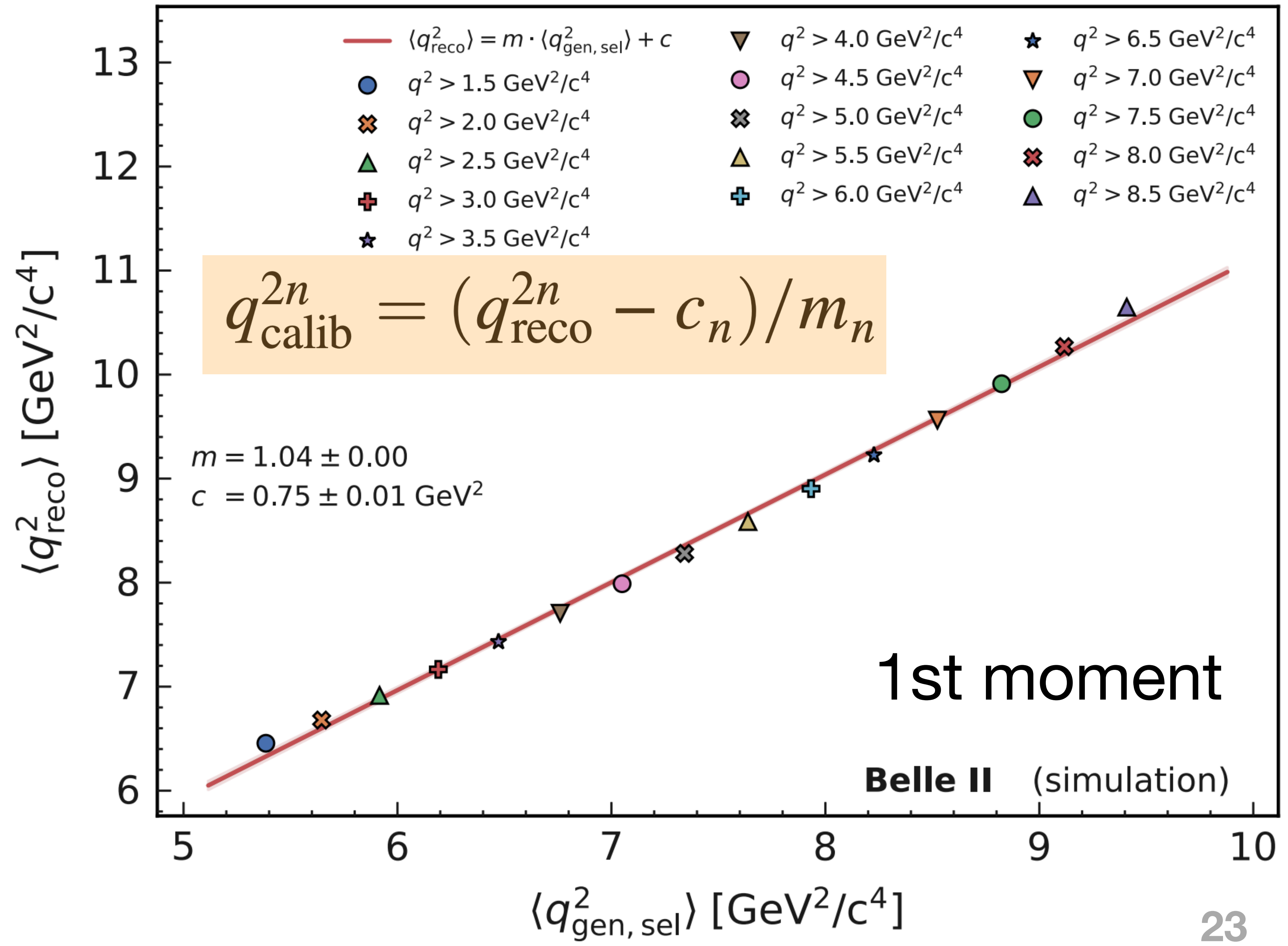

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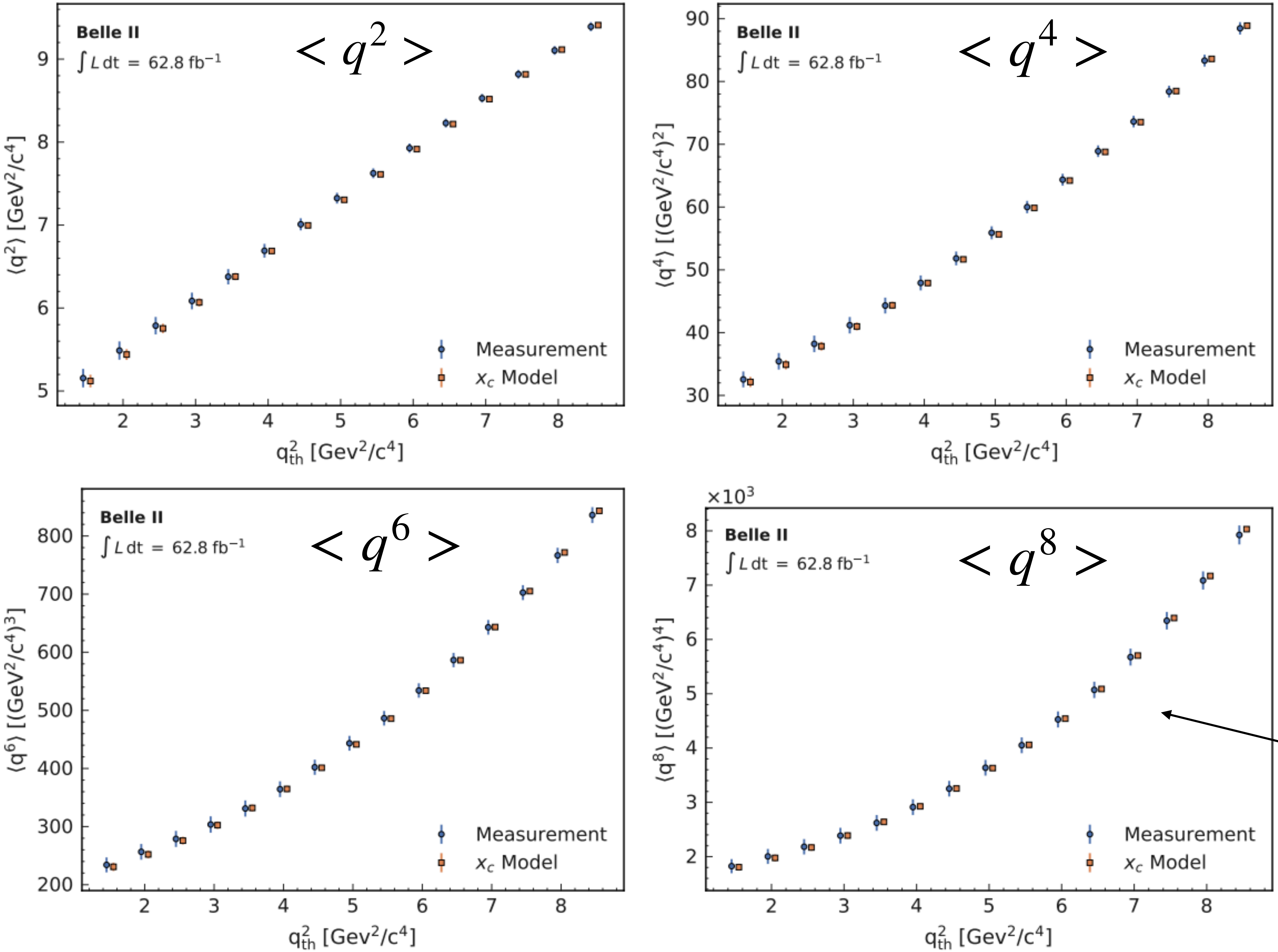


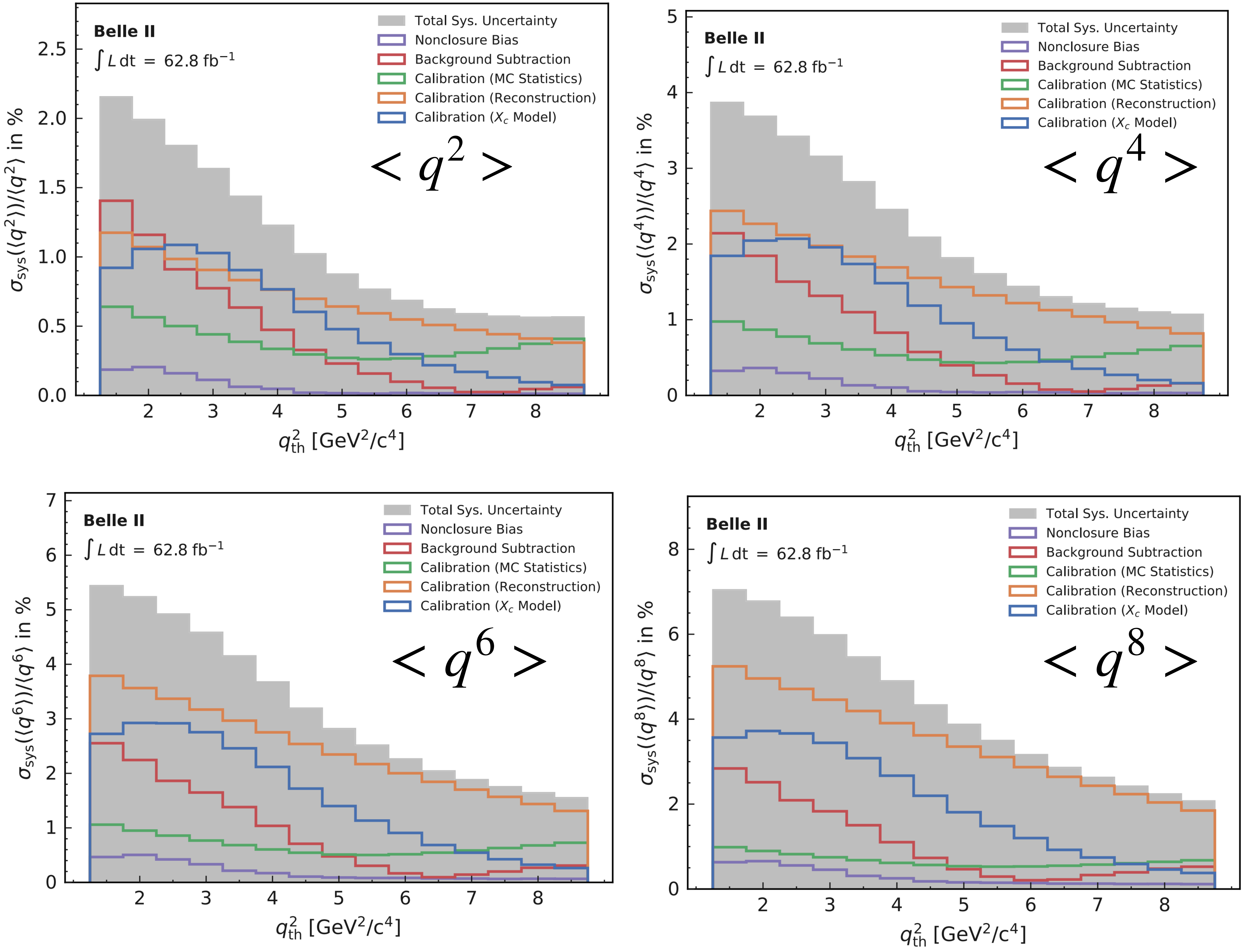
PRD 107, 072002 (2023)

q^2 thresholds at (**1.5, 2.0, 2.5**) GeV² are not measured in Belle (2021)

measured & simulation are compared

Split Relative Systematic Uncertainties

PRD 107, 072002 (2023)



Calibration (MC Statistics)	Calibration Curve (Statistical Uncertainty) Bias Correction (Statistical Uncertainty)
Calibration (X_c Model)	$\mathcal{B}(B \to D\ell\nu)$ $\mathcal{B}(B \to D^*\ell\nu)$ $\mathcal{B}(B \to D^{**}\ell\nu)$ Non-resonant X_c Dropped Non-resonant X_c Replaced w/ D_1', D_0^* $B \to D\ell\nu$ Form Factor $B \to D^*\ell\nu$ Form Factor
Calibration (Reconstruction)	PID Uncertainty N_γ Reweighted N_{tracks} Reweighted $E_{\mathrm{miss}} - p_{\mathrm{miss}}$ Reweighted Tracking Efficiency
Background Subtraction	Spline Smooth Factor Background Yield and Shape
Other	Nonclosure Bias

What's next for moments?

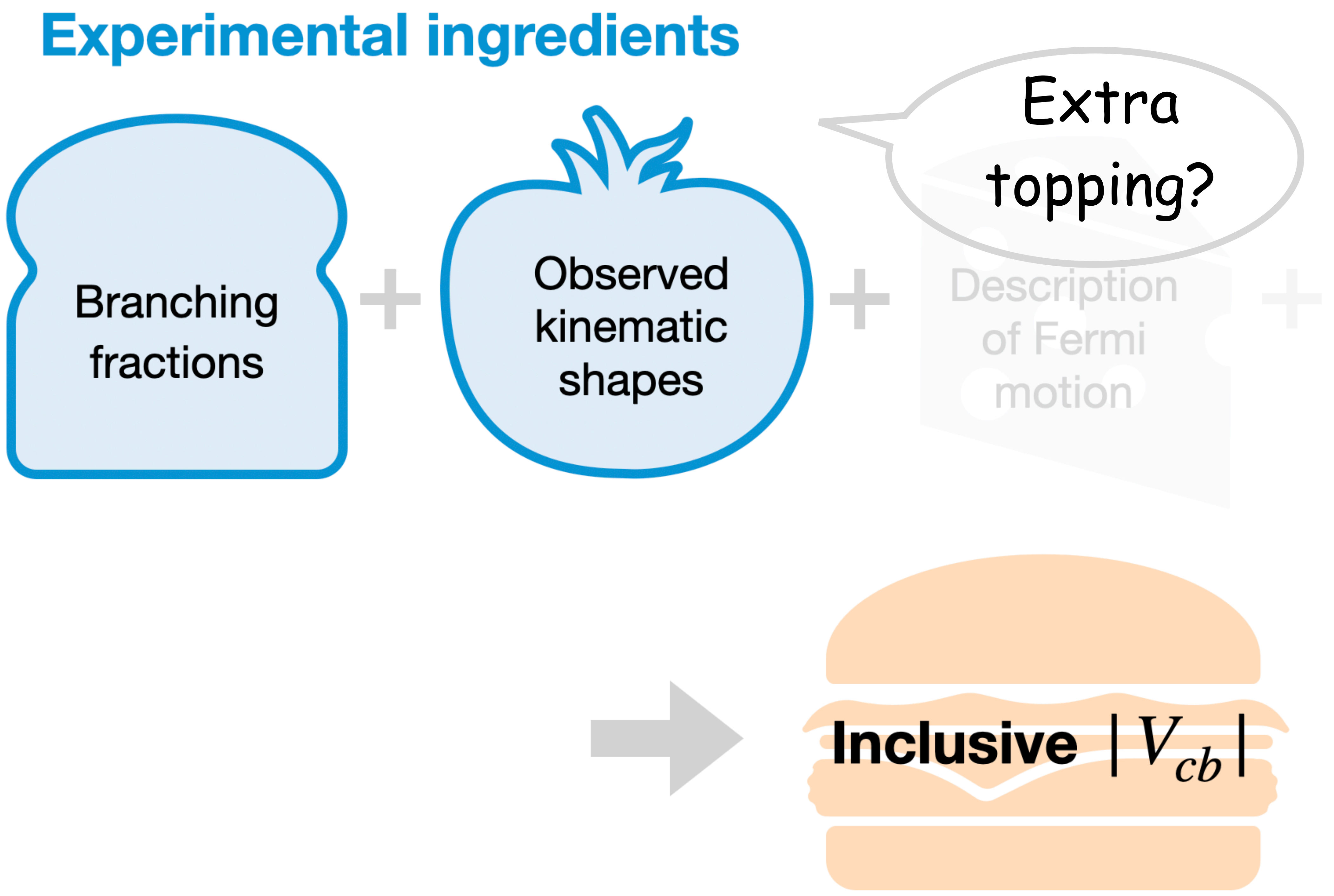
- Measure all kin. moments simultaneously as a function of q^2 (E_l^B) thresholds in

$B \to X\ell\nu$: $q^2, E_l^B, M_X, \cos\theta_\ell$, combined variables $n_X^2\,(M_X^2, E_X)$, $P_X^{\pm}\,(M_X, E_X)$

- Full experimental correlations will be derived => important for global analysis
- Only shape observation (drop tagging eff. calibration, separate from $\mathcal{B}$ measurement)

Summary

- Belle II will provide new results on **branching fractions** and **moments of all kinematic variables**
- New knowledge and techniques will be incorporated for future analyses (e.g. X_c modelling)
- Anything else? (e.g. sensitive regions, steps of thresholds,…)

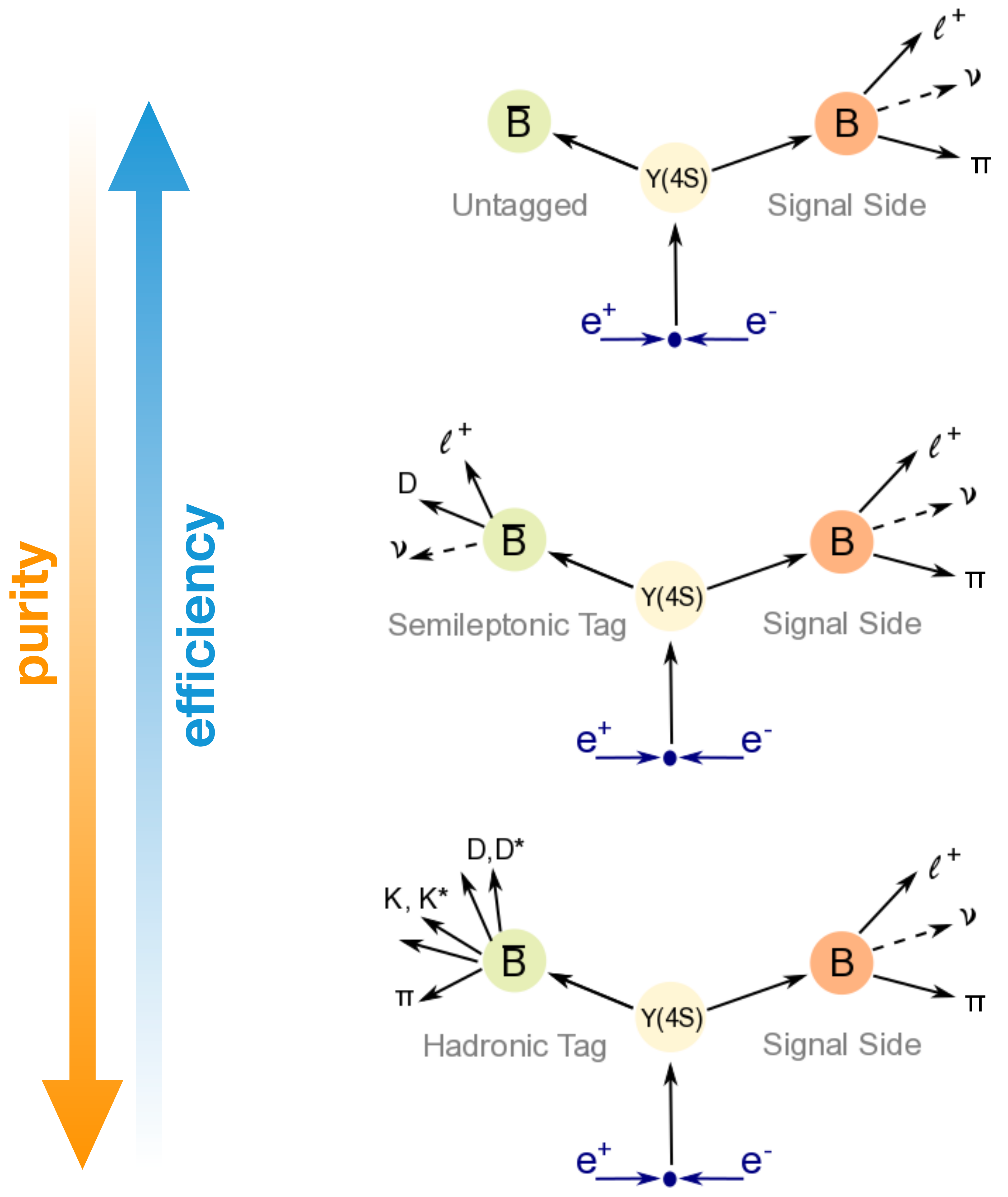


THANK YOU

THANK YOU

hmm, why do we have so many different values of V_{cb} ?

Backup: Tagging vs. Untagging



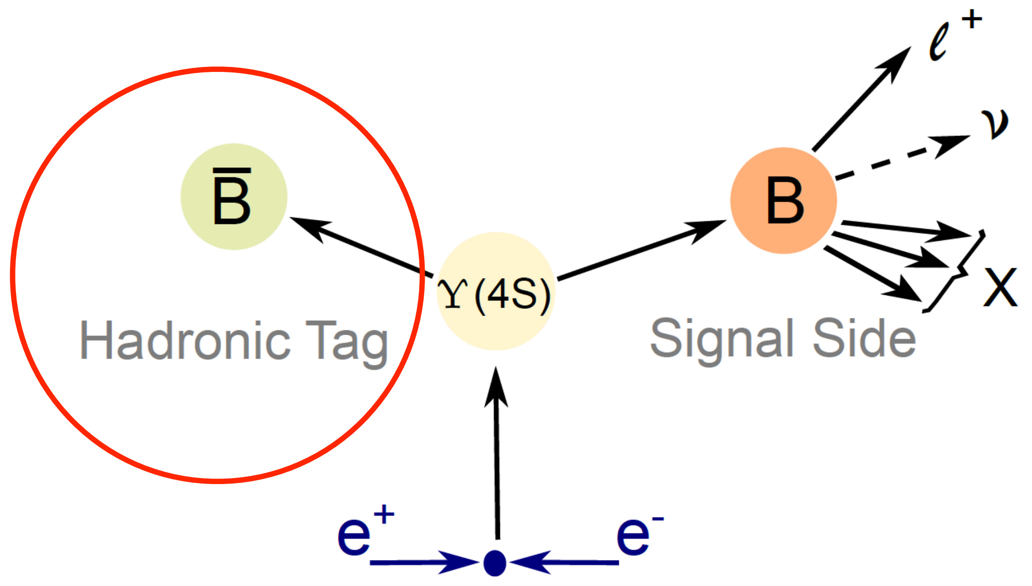
- Untagged
  - Loose constraints on signal
  - Very large statistics, but also very large background
  - Efficiency $\epsilon \approx \mathcal{O}(100\%)$

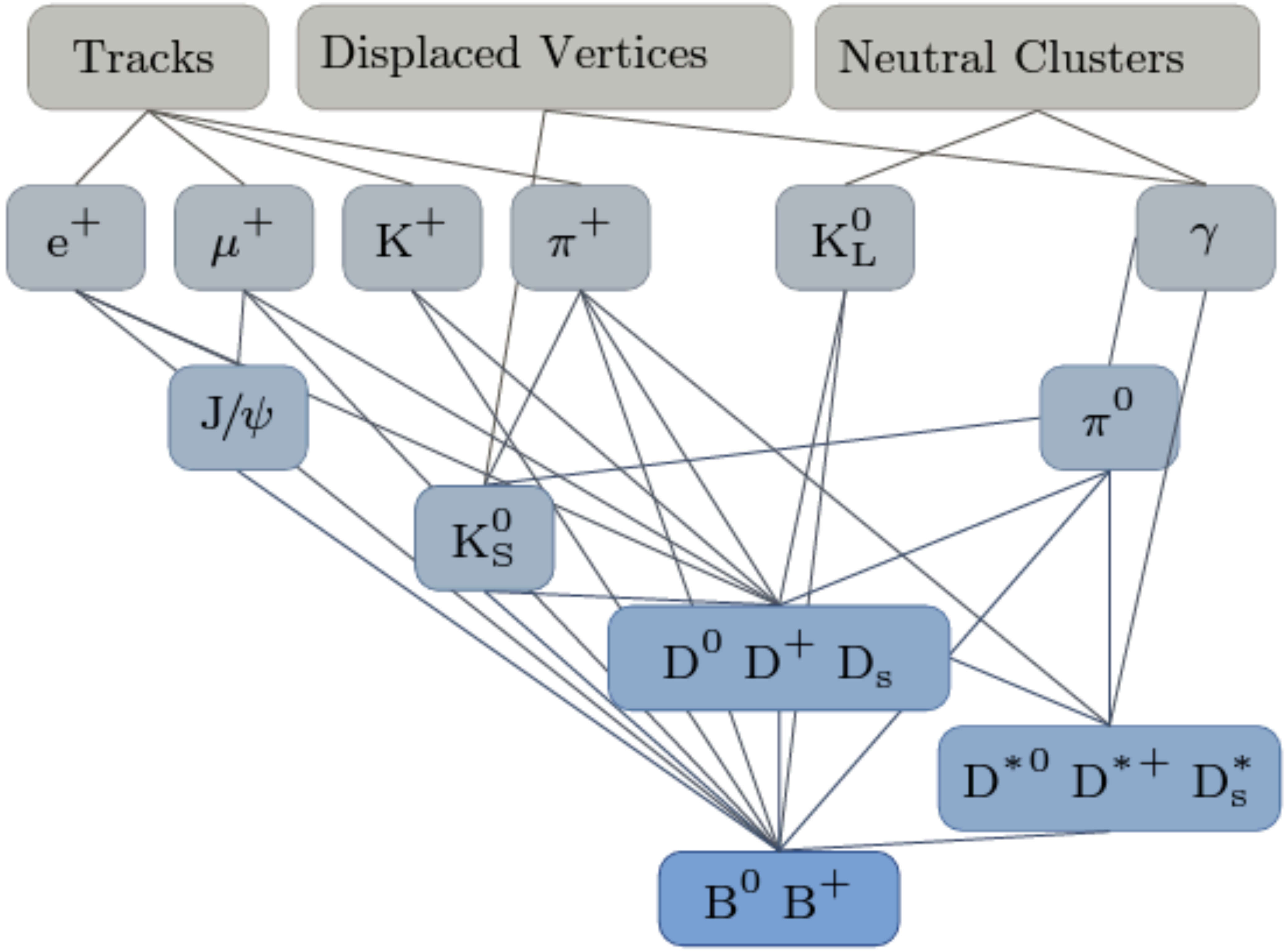
- Semileptonic tag
  - Mid-range reconstruction efficiency
  - Due to multiple neutrinos, less information about B_{tag}

- Hadronic tag
  - Cleaner sample
  - Knowledge of $p(B_{sig})$
  - Low tag-side efficiency $\epsilon \approx \mathcal{O}(0.1\%)$

Hadronic Tagging at Belle II

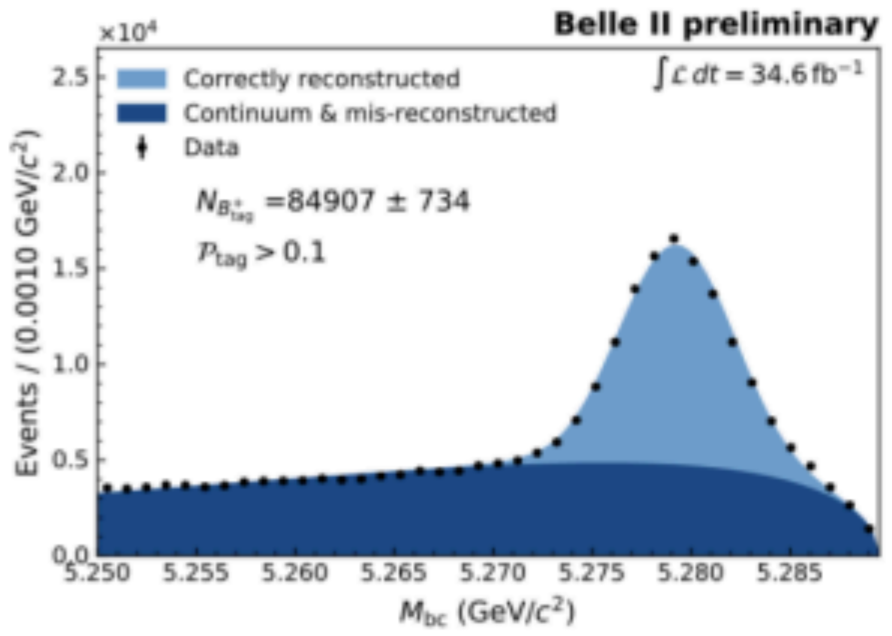
- Hadronic tagging with **Full Event Interpretation** algorithm [Comput Softw Big Sci 3, 6(2019)] to reconstruct B_{tag}
  - Reconstruct *B* candidate with all combination of daughters
  - Calculate signal probability with multivariate classifiers

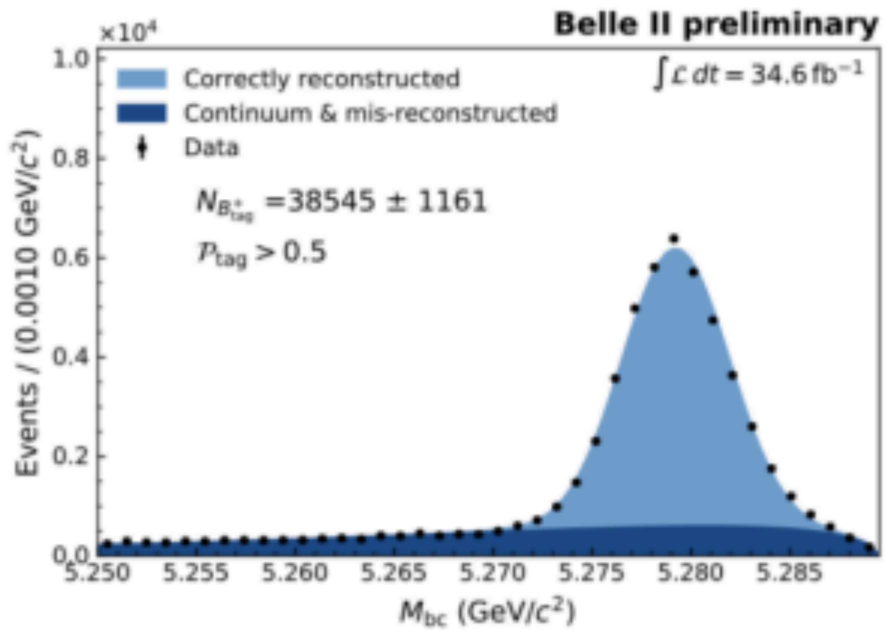




Hadronic FEI

- Over 200 BDTs to reconstruct $\mathcal{O}(10000)$ distinct decay chains
- Efficiency $\epsilon_{B^+} \approx 0.5\,\%$, $\epsilon_{B^0} \approx 0.3\,\%$ at ~$15\,\%$ purity

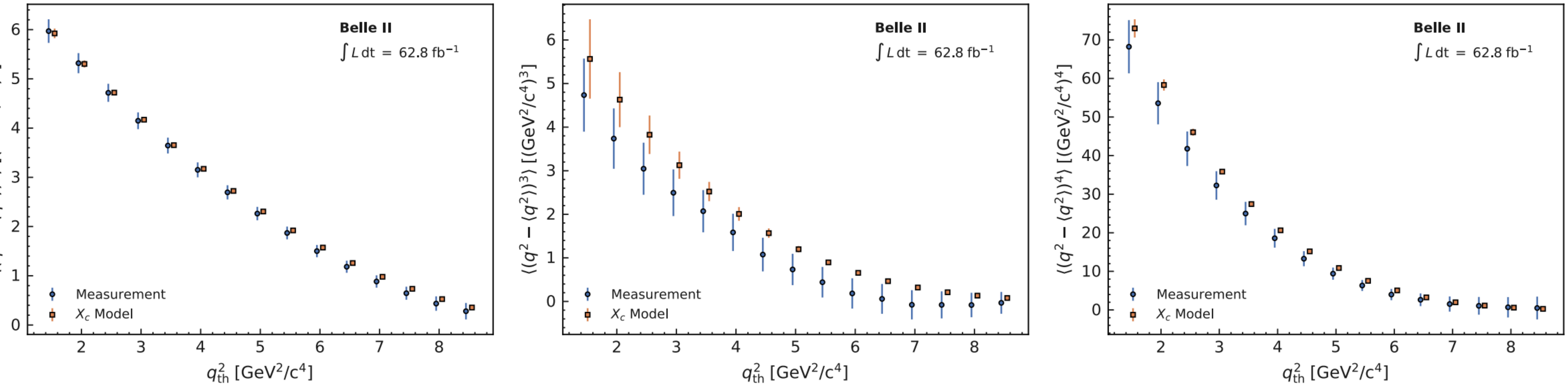




$$M_{bc} = \sqrt{E^2_{beam}/4 - (p^{cm}_{B_{tag}})^2} > 5.27\ \mathrm{GeV}/c^2$$

Measurement of Moments q^2 at Belle II

- Central moments



Hadronic Mass Moments in $B \to X_c \ell \nu$

BaBar: PRD 81, 032003 (2010)

k	$p^*_{\ell,\min}$ [GeV/c]	$\langle m_X^k \rangle$	σ_{stat}	σ_{sys}	MC statistics	simulation related	extraction method	back-groud	signal model
1	0.8	2.0906	±0.0063	±0.0166	0.0058	0.0099	0.0096	0.0047	0.0031
	0.9	2.0890	±0.0062	±0.0158	0.0048	0.0088	0.0103	0.0045	0.0028
	1.0	2.0843	±0.0061	±0.0153	0.0044	0.0076	0.0109	0.0044	0.0027
	1.1	2.0765	±0.0063	±0.0165	0.0044	0.0072	0.0127	0.0047	0.0026
	1.2	2.0671	±0.0064	±0.0160	0.0046	0.0073	0.0120	0.0045	0.0025
	1.3	2.0622	±0.0068	±0.0168	0.0048	0.0073	0.0131	0.0050	0.0023
	1.4	2.0566	±0.0073	±0.0183	0.0047	0.0069	0.0150	0.0054	0.0021
	1.5	2.0494	±0.0081	±0.0198	0.0036	0.0074	0.0168	0.0061	0.0019
	1.6	2.0430	±0.0092	±0.0221	0.0038	0.0082	0.0187	0.0070	0.0018
	1.7	2.0387	±0.0109	±0.0265	0.0047	0.0081	0.0232	0.0083	0.0015
	1.8	2.0370	±0.0143	±0.0337	0.0069	0.0097	0.0299	0.0098	0.0013
	1.9	2.0388	±0.0198	±0.0413	0.0082	0.0123	0.0355	0.0150	0.0008
2	0.8	4.429	±0.029	±0.070	0.027	0.047	0.030	0.018	0.008
	0.9	4.416	±0.027	±0.063	0.020	0.041	0.033	0.016	0.008
	1.0	4.394	±0.026	±0.058	0.020	0.033	0.035	0.015	0.008
	1.1	4.354	±0.026	±0.063	0.019	0.031	0.043	0.016	0.008
	1.2	4.308	±0.026	±0.058	0.019	0.030	0.039	0.015	0.007
	1.3	4.281	±0.027	±0.061	0.020	0.029	0.044	0.016	0.007
	1.4	4.253	±0.028	±0.066	0.021	0.028	0.051	0.018	0.006
	1.5	4.220	±0.031	±0.070	0.015	0.029	0.058	0.019	0.006
	1.6	4.183	±0.034	±0.078	0.015	0.032	0.065	0.022	0.005
	1.7	4.158	±0.040	±0.094	0.019	0.032	0.082	0.026	0.004
	1.8	4.145	±0.051	±0.120	0.026	0.036	0.107	0.031	0.004
	1.9	4.136	±0.069	±0.142	0.031	0.046	0.122	0.048	0.002

Belle: PRD 75, 032005 (2007)

	$\Delta\langle M_X^2 \rangle$ (GeV$^2/c^4$)				
$E^*_{\min}$ (GeV)	0.7	0.9	1.1	1.3	1.5
secondary/fake leptons	0.033	0.023	0.013	0.007	0.004
combinatorial background	0.006	0.004	0.003	0.002	0.002
continuum	0.000	0.000	0.000	0.000	0.000
$B \to X_u \ell \nu$ background	0.004	0.004	0.004	0.004	0.006
$\mathcal{B}(D^{(*)}\ell\nu)$	0.008	0.007	0.007	0.007	0.006
$\mathcal{B}(D^{**}\ell\nu)$	0.022	0.014	0.006	0.000	0.000
$\mathcal{B}((D^{(*)}\pi)_{\mathrm{non-res.}}\ell\nu)$	0.024	0.017	0.007	0.004	0.004
$D^{(*)}\ell\nu$ form factors	0.013	0.013	0.012	0.011	0.010
$D^{**}\ell\nu$ form factors	0.003	0.002	0.002	0.001	0.001
unfolding	0.015	0.015	0.015	0.015	0.015
binning	0.001	0.001	0.001	0.001	0.001
efficiency	0.008	0.011	0.012	0.009	0.008
total	0.052	0.041	0.029	0.024	0.022