Inclusive $|V_{ub}|/|V_{cb}|$

Based on:
Belle Preprint 2023-17
KEK Preprint 2023-30
Submitting to arXiv and PRD today

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Belle II Physics Week
KEK
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Introduction

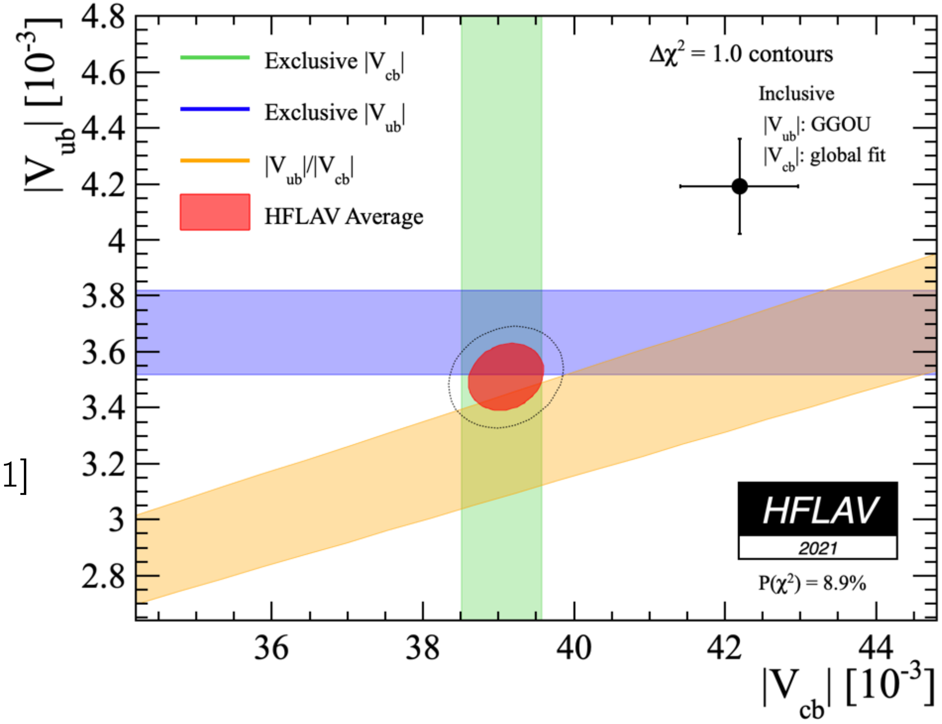
- Long standing discrepancy between inclusive and exclusive $|V_{ub}|$ and $|V_{cb}|$ determinations

- $|V_{ub}|/|V_{cb}|$ extracted from B decays in good agreement:

$$\begin{aligned} |V_{ub}|/|V_{cb}|_{inc} &= (9.8 \pm 0.6) \times 10^{-2} \\ |V_{ub}|/|V_{cb}|_{exc} &= (9.4 \pm 0.5) \times 10^{-2} \\ |V_{ub}|/|V_{cb}|_{\Lambda_b, B_s} &= (8.4 \pm 0.7) \times 10^{-2} \end{aligned}$$

[PDG 2021]

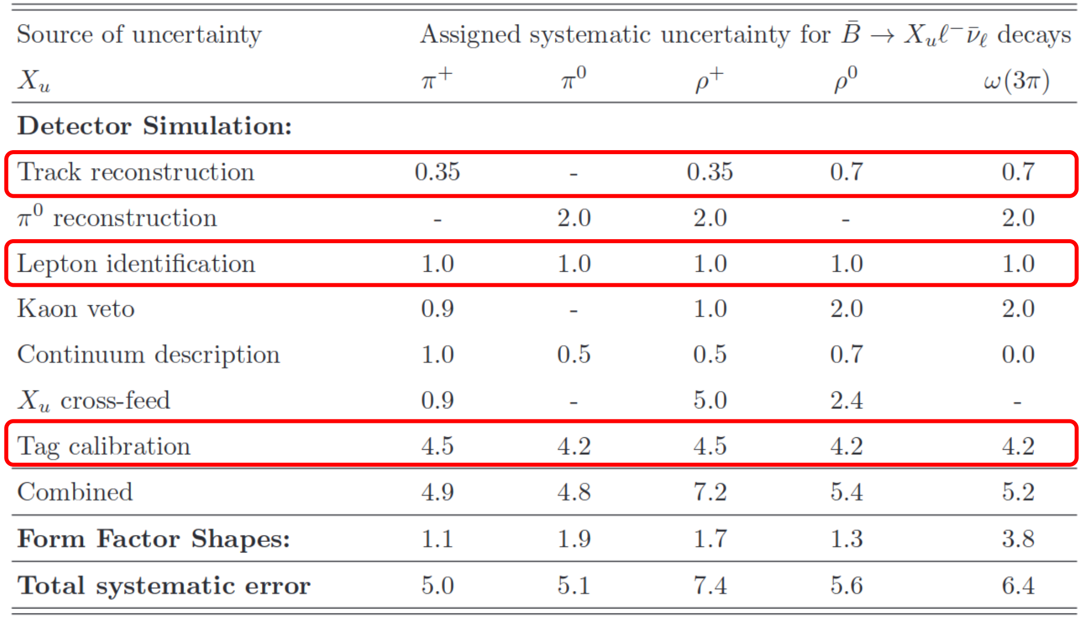
- Better precision and better understanding of possible biases needed



Experimental Uncertainties in $\overline{B} \to \pi/\rho/\omega\ell\overline{\nu}$

- 2013 Belle $\overline{B} \to \pi/\rho/\omega\ell\overline{\nu}$:

[*Phys.Rev.D* 88 (2013) 3, 032005]



Source of uncertainty	Assigned systematic uncertainty for $\bar{B} \to X_u \ell^- \bar{\nu}_\ell$ decays				
X_u	π^+	π^0	ρ^+	ρ^0	$\omega(3\pi)$
Detector Simulation:					
Track reconstruction	0.35	-	0.35	0.7	0.7
π^0 reconstruction	-	2.0	2.0	-	2.0
Lepton identification	1.0	1.0	1.0	1.0	1.0
Kaon veto	0.9	-	1.0	2.0	2.0
Continuum description	1.0	0.5	0.5	0.7	0.0
X_u cross-feed	0.9	-	5.0	2.4	-
Tag calibration	4.5	4.2	4.5	4.2	4.2
Combined	4.9	4.8	7.2	5.4	5.2
Form Factor Shapes:	1.1	1.9	1.7	1.3	3.8
Total systematic error	5.0	5.1	7.4	5.6	6.4
Stat Uncertainty:	6.0%	10%	5.5%	8.4%	15%

Calibrates to $\overline{B} \to D^{(*)}\ell\overline{\nu}$

Experimental Uncertainties in $\overline{B} \to \pi/\rho/\omega\ell\overline{\nu}$

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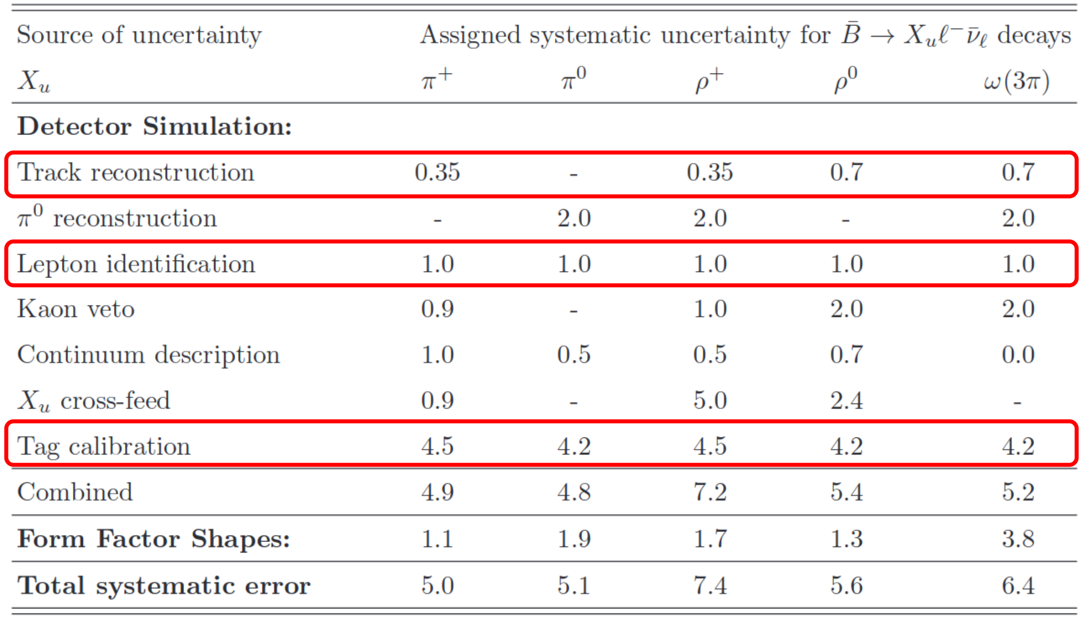

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Provocative comment:

Why bake in large normalization uncertainties when we can directly measure $\frac{\mathcal{B}(\overline{B}\to\pi\ell\overline{\nu})}{\mathcal{B}(\overline{B}\to D^{(*)}\ell\overline{\nu})}$ and $|V_{ub}|/|V_{cb}|$?

Experimental Uncertainties in Inclusive $B \to X_u \ell \nu$

- Best sensitivity in hadronic tagged measurements – allows for measuring away from shape function regime.
- Usually dominated by $\overline{B} \to X_u \ell \overline{\nu}$ modelling.
- Expect to (partially) cancel some sub-leading tagging and lepton ID uncertainties by taking a ratio measurement.
- Tagged measurements are implicitly ratio measurements – why not make it explicit?

2021 Belle Inclusive |Vub|:

$\boldsymbol{M_X : q^2}$ Fit	
Data Statistics	4.4%
$B \to X_u \ell \nu$ Exclusive Modelling	2.5%
$B \to X_u \ell \nu$ Inclusive Modelling	8.8%
$B \to X_c \ell \nu$ Modelling	1.6%
Lepton Identification	1.9%
Tagging	3.6%
Other	2.6%

[*Phys.Rev.D* 104 (2021) 1, 012008]

Theory uncertainties in $\Gamma(B \to X_u \ell \nu)$

- Heavy-to-heavy and heavy-to-light OPEs in $\frac{1}{m_b}, \alpha_s$
  - shared dependence on m_b and non perturbative parameters $\mu_\pi, \mu_G, \rho_D, \rho_{LS}$
- Naïvely expect at least partial uncertainty cancellation in the ratio
- $|V_{ub}|$ world averages:

BLNP: $\pm\mathbf{5.7\%}$	
Statistics	1.6%
Detector Effects	1.7%
$B \to X_u \ell \nu$ Modelling	1.8%
$B \to X_c \ell \nu$ Modelling	1.0%
Heavy quark params.	**2.8%**
SF functional form	0.3%
Sub-leading SF	0.8%
Matching scale	3.8%
Weak Annihilation	−0.7%

GGOU: $\pm\mathbf{3.9\%}$	
Statistics	1.3%
Detector Effects	1.6%
$B \to X_u \ell \nu$ Modelling	1.7%
$B \to X_c \ell \nu$ Modelling	0.9%
Heavy quark params.	**1.8%**
Higher order corrections	1.5%
q^2 tail Modelling	1.3%
Func. form dist. function	0.1%
Weak Annihilation	−1.1%

[HFLAV, *Phys.Rev.D* 107 (2023) 5, 052008]

$$
\begin{aligned}
\mathit{\Gamma} =& |V_{cb}|^2 \frac{G_F^2 m_b^5(\mu)}{192\pi^3} |\eta_{\mathrm{EW}}|^2 \times \\
&\left[z_0^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_0^{(1)}(r) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 z_0^{(2)}(r) + \cdots \right. \\
&+ \frac{\mu_\pi^2}{m_b^2}\left(z_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_2^{(1)}(r) + \cdots \right) \\
&+ \frac{\mu_G^2}{m_b^2}\left(y_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_2^{(1)}(r) + \cdots \right) \\
&+ \frac{\rho_{\mathrm{D}}^3}{m_b^3}\left(z_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_3^{(1)}(r) + \cdots \right) \\
&\left. + \frac{\rho_{\mathrm{LS}}^3}{m_b^3}\left(y_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_3^{(1)}(r) + \cdots \right) + \ldots \right]
\end{aligned}
$$

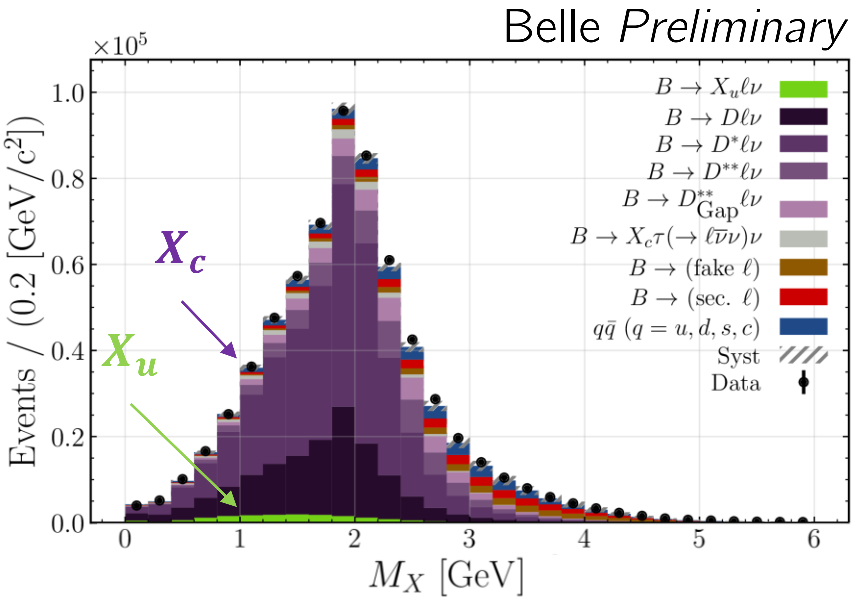
[PDG Review '21]

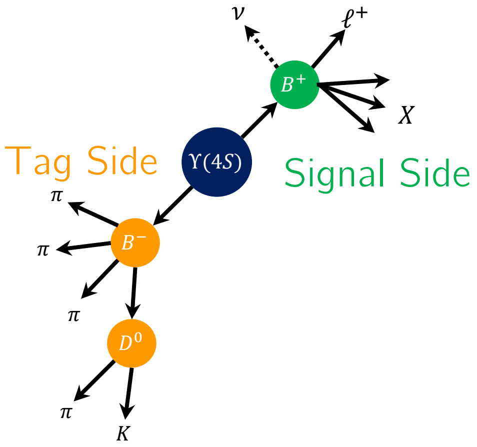
$$
\begin{aligned}
\Gamma_{tot} =& |V_{ub}|^2 G_F^2 m_b^5(\mu)/192\pi^3 \left(1 + 2\frac{\alpha}{\pi}\ln\frac{M_Z}{m_b}\right)\left[1 + \right. \\
& C_F \frac{\alpha_s}{\pi}\left(\frac{25}{8} - \frac{\pi^2}{2} + \frac{20}{3}\frac{\mu}{m_b} + 3\frac{\mu^2}{m_b^2} - \frac{77}{9}\frac{\mu^3}{m_b^3}\right) \\
& + C_F\ \beta_0 \left(\frac{\alpha_s}{\pi}\right)^2 \left(\frac{1009}{384} - \frac{77\pi^2}{288} - 2\zeta_3 \right. \\
& \left. + \frac{10(3\lambda+8)\mu}{9m_b} + \frac{(6\lambda+13)\mu^2}{4m_b^2} - \frac{77(\lambda+2)\mu^3}{18m_b^3}\right) \\
& - \frac{\mu_\pi^2(\mu)}{2m_b^2} - \frac{3\mu_G^2(\mu)}{2m_b^2} + \left(77 + 48\ln\frac{\mu_{\mathrm{WA}}^2}{m_b^2}\right)\frac{\rho_D^3(\mu)}{6m_b^3} \\
& \left. + \frac{3\rho_{LS}^3(\mu)}{2m_b^3} + C_{\mathrm{WA}}\, B_{\mathrm{WA}}(\mu_{\mathrm{WA}}) \right]
\end{aligned}
$$

[GGOU - *JHEP* 10 (2007) 058]

Measurement Strategy

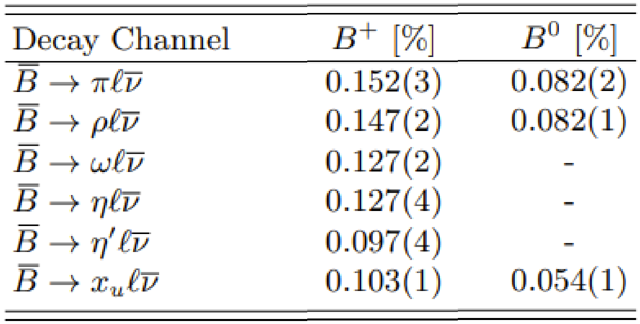
- Precise $B \to X_u \ell \nu$ extraction complicated by large $B \to X_c \ell \nu$ background.
  - $\frac{\mathcal{B}(B\to X_c\ell\nu)}{\mathcal{B}(B\to X_u\ell\nu)} \approx 50$ with similar experimental signature.
- Focus on reducing model dependence in $B \to X_u \ell \nu$ and $B \to X_c \ell \nu$.
- High inclusivity $B \to X_u \ell \nu$ extraction, $p_\ell^B > 1.0$ GeV/c (f_u~86%).
- Reconstruct $\overline{B} \to X \ell \overline{\nu}$ with hadronic tagging (FEI).
- The challenge is $B \to X_u \ell \nu$, simple treatment of $B \to X_c \ell \nu$.





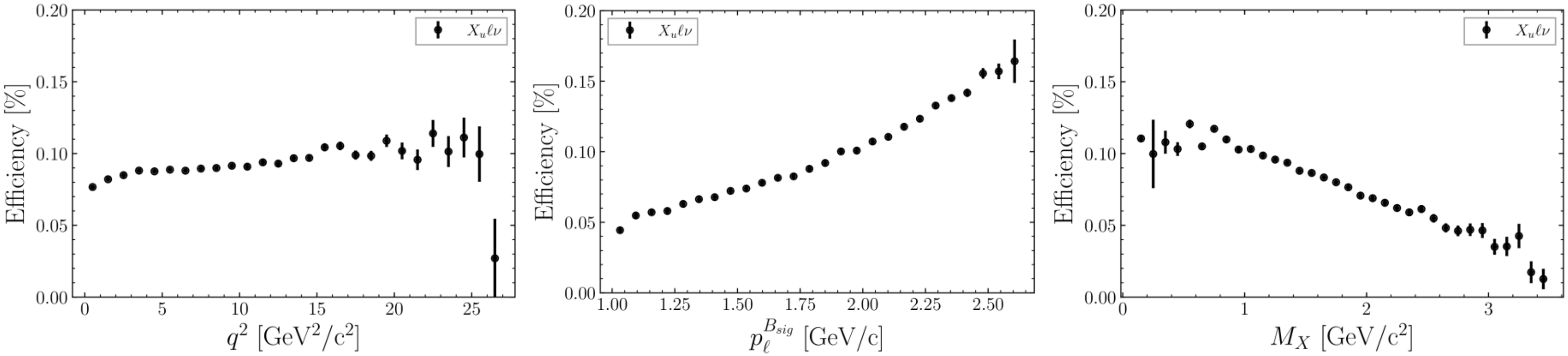
$B \to X_u \ell \nu$ Selection

- Simple cut-based selection to suppress $B \to X_c \ell \nu$ background
  - loose to reduce bias towards resonances and $B \to X_u \ell \nu$ sculpting
    - $|m_\nu^2| \approx |m_{Miss}^2| < 0.43\ GeV^2$
    - Charged slow pion veto.
    - Kaon veto: even $N_{K^\pm} + N_{K_S^0}$



Decay Channel	B^+ [%]	B^0 [%]
$\overline{B} \to \pi \ell \overline{\nu}$	0.152(3)	0.082(2)
$\overline{B} \to \rho \ell \overline{\nu}$	0.147(2)	0.082(1)
$\overline{B} \to \omega \ell \overline{\nu}$	0.127(2)	-
$\overline{B} \to \eta \ell \overline{\nu}$	0.127(4)	-
$\overline{B} \to \eta' \ell \overline{\nu}$	0.097(4)	-
$\overline{B} \to x_u \ell \overline{\nu}$	0.103(1)	0.054(1)

- $B \to X_u \ell \nu$ Efficiency:

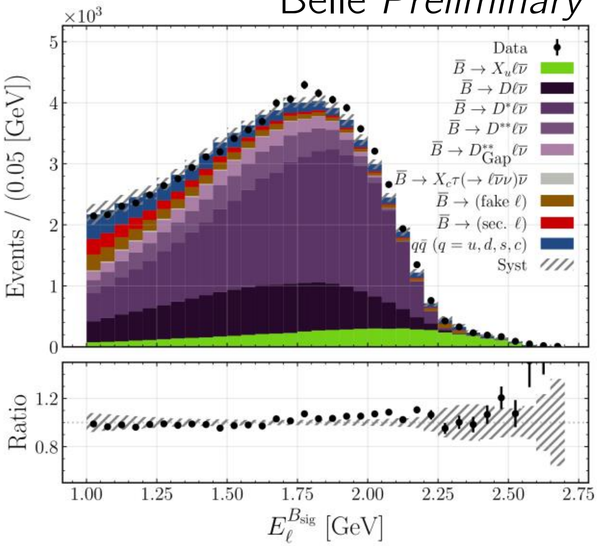


- Still a large dependence on E_ℓ^B - can improve on this in Belle II.

$B \to X_u \ell \nu$ Sample

- Data excess at high p_ℓ^B, q^2.
- Repeated indications seen by Belle, BaBar, and $B \to X_c \ell \nu$ moments analysis.
  Bernlocher, et al. 2014 [Eur.Phys.J.C 74 6, 2914],
  BaBar 2012 [PRD 86, 032004]
  Belle 2021 [PRD 104, 012008]
  Belle 2021 [PRD 104, 112011]
- Reason unclear, $B \to D^{**} \ell \nu$ modelling?
- Mismodelling might cause bias in inclusive $|V_{ub}|$ determinations.

Belle *Preliminary*



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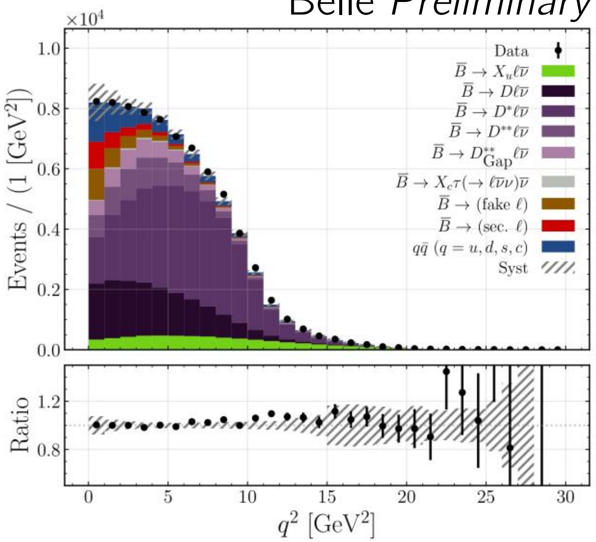

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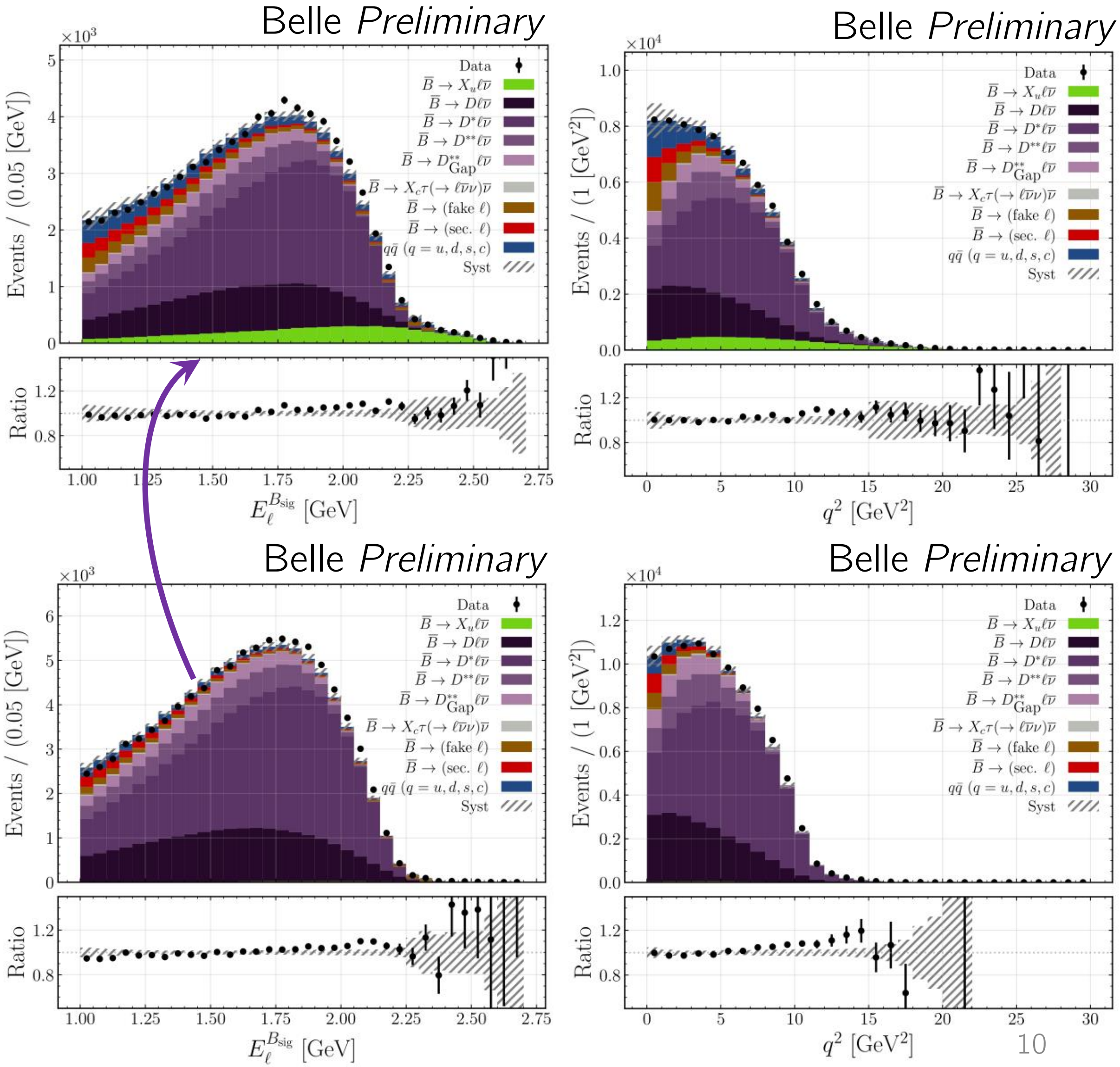
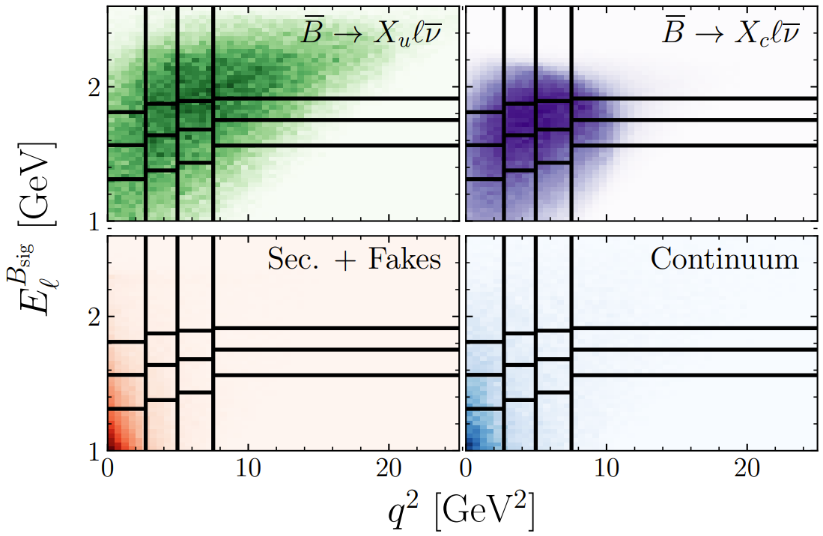
- Reason unclear, $B \to D^{**} \ell \nu$ modelling?
- Mismodelling might cause bias in inclusive $|V_{ub}|$ determinations.
- Kaon vetoed sample →
- Consistent Data-MC disagreement
  - Expected if issue is semi-leptonic decay modelling.
  - Use for modelling corrections.



$B \to X_u \ell \nu$ Extraction

- Continuum MC calibrated to off-resonance sample.
- Secondary and fake lepton MC calibrated to high M_X, low p_ℓ^B control region.



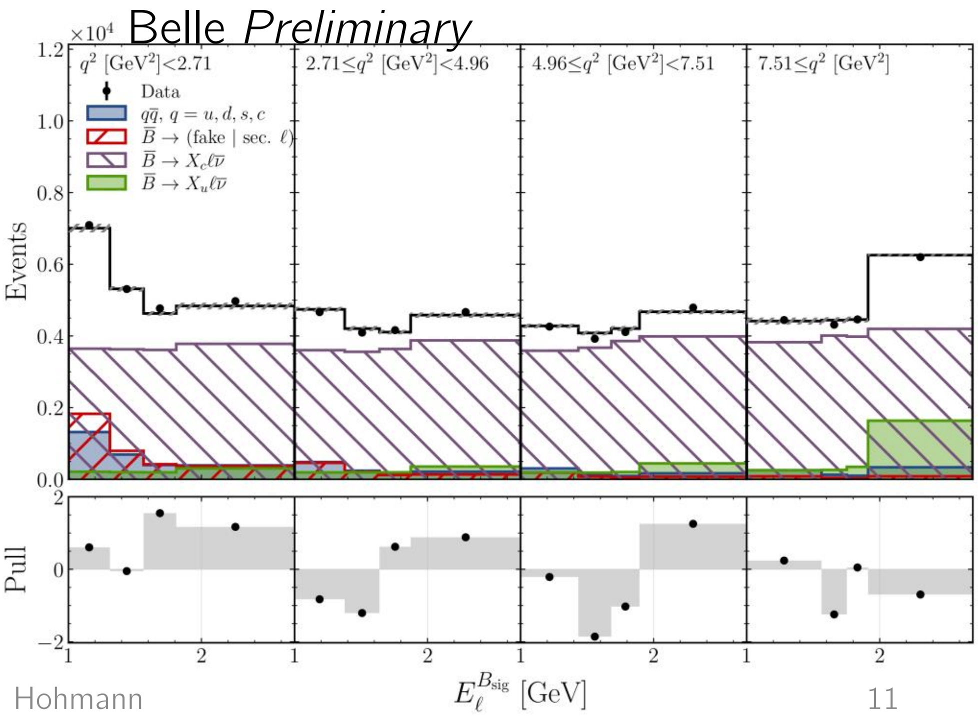
- Extract $B \to X_u \ell \nu$ yield from 2D fit to $q^2 : p_\ell^B$.
  - Equal frequency $B \to X_c \ell \nu$.
  - Final broad bins average over shape function region - reduce dependence on B $\to X_u \ell \nu$ modelling.
- Reduce dependence on $B \to X_c \ell \nu$ modeling - take template from kaon vetoed sample as:

Data and MC yields in kaon vetoed sample

$$T_i = \tau_i(N^{Data}_{i,K} - a\eta^{B\to X_u\ell\nu}_{i,K} - \eta^{q\bar{q}}_{i,K} - \eta^{Sec.Fakes}_{i,K})$$

Transfer factor taken from MC

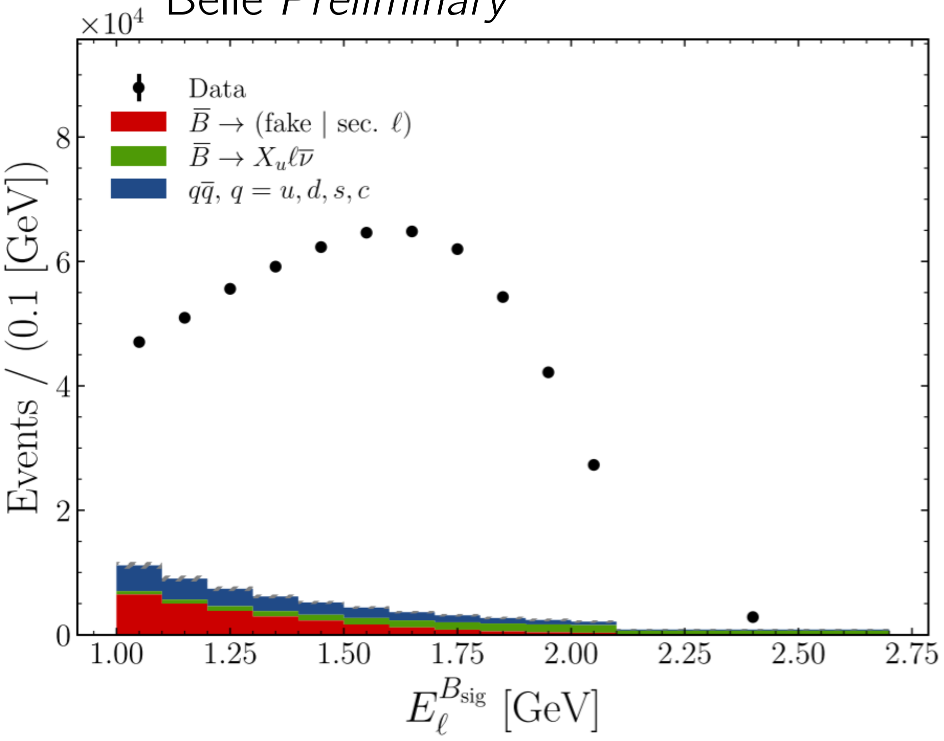
- Maximum likelihood fit floating proportion of $B \to X_u \ell \nu / B \to X_c \ell \nu$.
  - $\chi^2/dof = 14.6/15$



$B \to X_c \ell \nu$ Extraction

- Extract $B \to X_c \ell \nu$ yield via simple background subtraction in total $\mathrm{B} \to \mathrm{X}\ell\nu$ sample.
- Normalize $B \to X_u \ell \nu$ by fit result.
- Continuum scaled by calibration in off-resonance sample.
- Secondary and fake lepton component fixed after calibration to high M_X, low p_ℓ^B control region.

Belle *Preliminary*



- Split sample into electron and muon modes, subtract background and extrapolate to full phase-space:

$$\frac{\mathcal{B}(\overline{B} \to X_c e\, \overline{\nu})}{\mathcal{B}(\overline{B} \to X_c \mu\, \overline{\nu})} = 1.003\ (1\ \pm 0.5\%_{stat} \pm 2.4\%_{syst})$$

Ratio of Partial Branching Fractions

- Take ratio as:

1.98 ± 0.04

$5390 \pm 450 \pm 350$

$$\frac{\Delta\mathcal{B}\left(B \to X_u \ell\nu\!:\ p_\ell^B > 1.0\ GeV/c\right)}{\Delta\mathcal{B}\left(B \to X_c \ell\nu\!:\ p_\ell^B > 1.0\ GeV/c\right)} = \frac{\epsilon^{B \to X_c \ell\nu}}{\epsilon^{B \to X_u \ell\nu}} \frac{N^{B \to X_u \ell\nu}}{N^{B \to X_c \ell\nu}}$$

$545000 \pm 1400 \pm 2300$

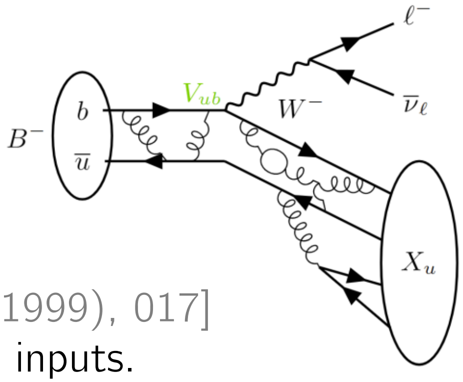
Belle *Preliminary*

$$\boxed{\frac{\Delta\mathcal{B}\left(B \to X_u \ell\nu\!:\ p_\ell^B > 1.0\ GeV/c\right)}{\Delta\mathcal{B}\left(B \to X_c \ell\nu\!:\ p_\ell^B > 1.0\ GeV/c\right)} = 1.96\left(1 \pm 8.4\%_{stat} \pm 7.9\%_{syst}\right) \times 10^{-2}}$$

- Final Step: Extract $\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(B\to X_u\ell\nu)}{\Delta\mathcal{B}(B\to X_c\ell\nu)} \boxed{\frac{\Delta\Gamma(B\to X_c\ell\nu)}{\Delta\Gamma(B\to X_u\ell\nu)}}}$

No public predictions of ratio of partial rates

Systematics - $B \to X_u \ell \nu$ Modelling



Belle *Preliminary*

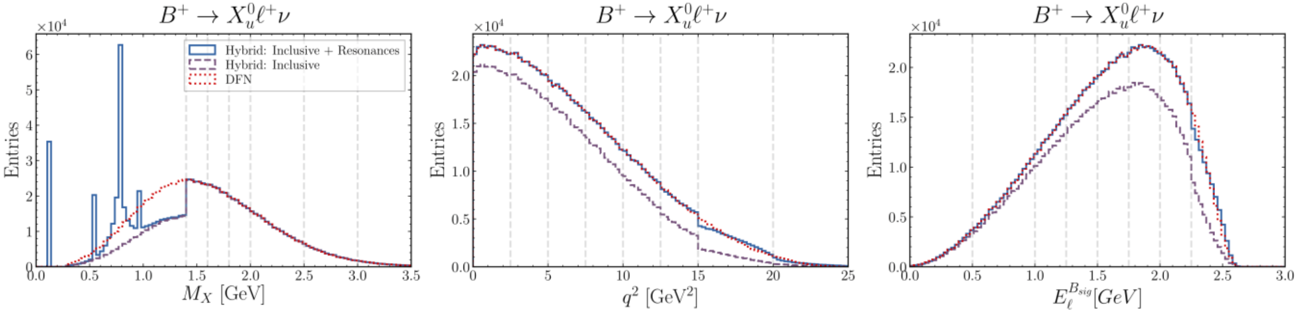
	%
Stat. Error (Data)	8.4
$\mathcal{B}(\overline{B} \to \pi/\eta/\rho/\omega/\eta'\ell\overline{\nu})$	0.2
$\mathcal{FF}(\overline{B} \to \pi/\eta/\rho/\omega/\eta'\ell\overline{\nu})$	0.3
$BF(B \to x_u \ell \overline{\nu})$	0.6
Hybrid Model (BLNP)	0.5
DFN $(m_b^{\mathrm{KN}}, a^{\mathrm{KN}})$	5.0
$N_{g \to s\overline{s}}$	1.3
$\mathcal{B}(\overline{B} \to D\ell\overline{\nu})$	0.1
$\mathcal{B}(\overline{B} \to D^*\ell\overline{\nu})$	0.8
$\mathcal{B}(\overline{B} \to D^{**}\ell\overline{\nu})$	0.3
$\mathcal{B}(\overline{B} \to D^{(*)}\eta\ell\overline{\nu})$	0.2
$\mathcal{B}(\overline{B} \to D^{(*)}\pi\pi\ell\overline{\nu})$	0.2
$\mathcal{FF}(\overline{B} \to D\ell\overline{\nu})$	0.2
$\mathcal{FF}(\overline{B} \to D^*\ell\overline{\nu})$	0.9
$\mathcal{FF}(\overline{B} \to D^{**}\ell\overline{\nu})$	0.4
Sec.Fakes. Composition	3.8
In-situ q^2 Calibration	2.8
ℓID Efficiency	0.1
ℓID Fake Rate	0.3
$K\pi$ID Efficiency	1.1
$K\pi$ID Fake Rate	0.7
K_S^0 Efficiency	0.2
π_{slow} Efficiency	< 0.1
Tracking	0.1
Continuum Calibration	0.4
N_{BB}	< 0.1
$f_{+/0}$	< 0.1
Stat. Error (MC)	2.8
Total Syst.	7.9

- Nominal $B \to X_u \ell \nu$ MC: DFN [JHEP 06(1999), 017] NLO calculation + non-perturbative QCD inputs.
  - Hadronised with JETSET/Pythia $(m_X > 2m_\pi)$

- Resonances $(B \to (\pi, \rho, \omega, \eta, \eta')\ell\nu)$ added ad-hoc via hybrid approach [PRD 41, 1496].

  PDG 2020 branching fractions and up-to-date models.

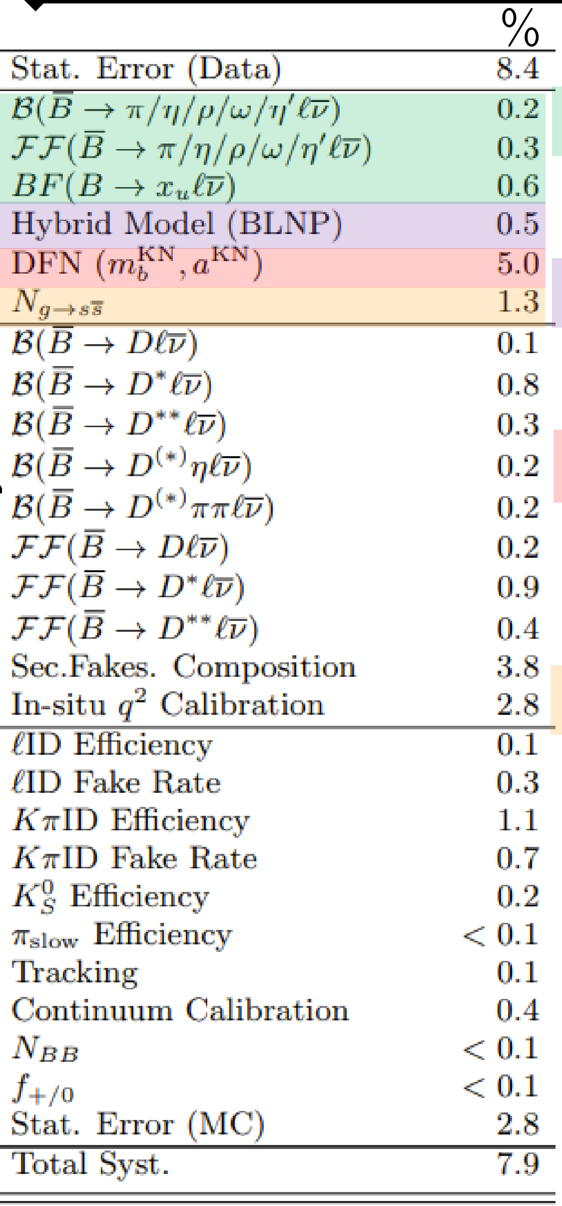
  $\Delta\mathcal{B}_{ijk}^{inc.} = \Delta\mathcal{B}_{ijk}^{exc.} + \mathrm{w_{ijk}} \cdot \Delta\mathcal{B}_{ijk}^{inc.}$



Decay Channel	B^+ $[\times 10^{-3}]$	B^0 $[\times 10^{-3}]$
$\overline{B} \to X_u \ell \overline{\nu}$	2.21 ± 0.31	2.05 ± 0.29
$\overline{B} \to \pi \ell \overline{\nu}$	0.078 ± 0.003	0.150 ± 0.006
$\overline{B} \to \rho \ell \overline{\nu}$	0.158 ± 0.011	0.294 ± 0.021
$\overline{B} \to \omega \ell \overline{\nu}$	0.119 ± 0.009	-
$\overline{B} \to \eta \ell \overline{\nu}$	0.039 ± 0.005	-
$\overline{B} \to \eta' \ell \overline{\nu}$	0.023 ± 0.008	-
$\overline{B} \to x_u \ell \overline{\nu}$	1.79 ± 0.32	1.60 ± 0.30

Systematics - $B \to X_u \ell \nu$ Modelling

Belle *Preliminary*



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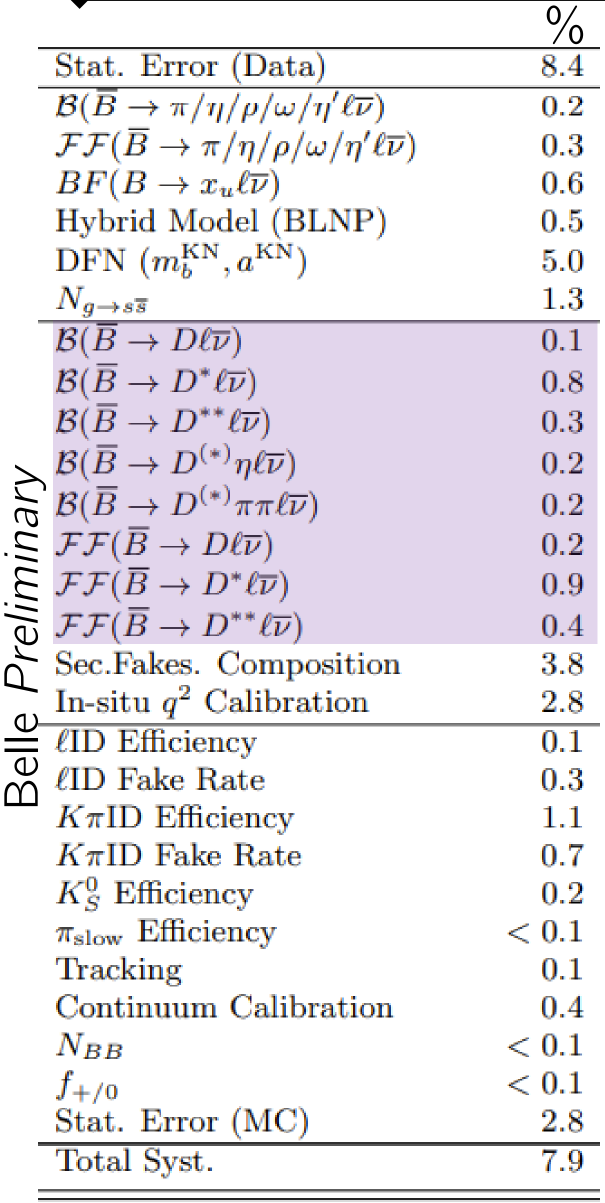
- 🟩 Small exposure to resonances in hybrid modelling composition.
- 🟪 Dominant systematic due to $m_b^{KN} = 4.66 \pm 0.04$ GeV , $a^{KN} = 1.3 \pm 0.5$ uncertainty [PRD 73:073008].
- 🟥 Switch DFN -> BLNP [PRD 72, 073006] for inclusive. Shape difference mostly at endpoint – averaged over in broad bins!
- 🟧 K production in X_u via gluon splitting – vary relative contribution $\pm 25\%$.

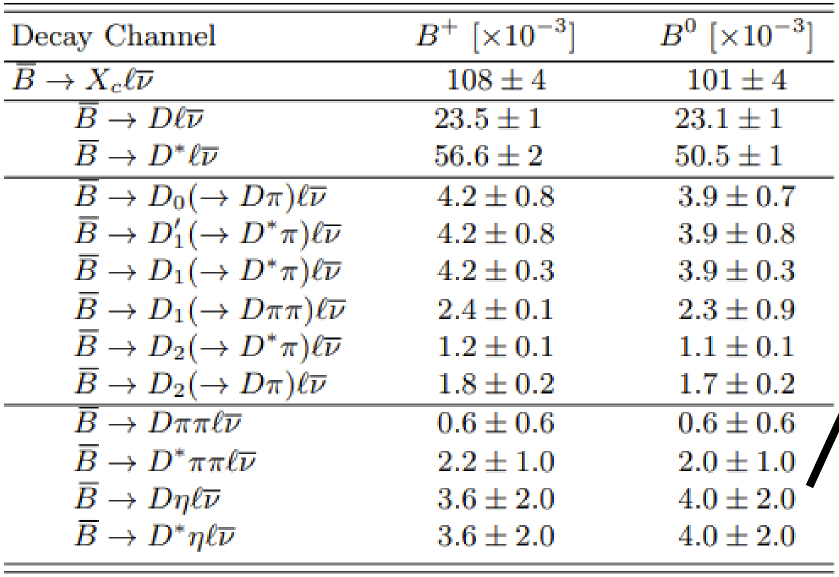
Systematics - $B \rightarrow X_c \ell \nu$ Modelling

Belle *Preliminary*



	%
Stat. Error (Data)	8.4
$\mathcal{B}(\bar{B} \to \pi/\eta/\rho/\omega/\eta'\ell\bar{\nu})$	0.2
$\mathcal{FF}(\bar{B} \to \pi/\eta/\rho/\omega/\eta'\ell\bar{\nu})$	0.3
$BF(B \to x_u \ell\bar{\nu})$	0.6
Hybrid Model (BLNP)	0.5
DFN $(m_b^{\mathrm{KN}}, a^{\mathrm{KN}})$	5.0
$N_{g\to s\bar{s}}$	1.3
$\mathcal{B}(\bar{B} \to D\ell\bar{\nu})$	0.1
$\mathcal{B}(\bar{B} \to D^*\ell\bar{\nu})$	0.8
$\mathcal{B}(\bar{B} \to D^{**}\ell\bar{\nu})$	0.3
$\mathcal{B}(\bar{B} \to D^{(*)}\eta\ell\bar{\nu})$	0.2
$\mathcal{B}(\bar{B} \to D^{(*)}\pi\pi\ell\bar{\nu})$	0.2
$\mathcal{FF}(\bar{B} \to D\ell\bar{\nu})$	0.2
$\mathcal{FF}(\bar{B} \to D^*\ell\bar{\nu})$	0.9
$\mathcal{FF}(\bar{B} \to D^{**}\ell\bar{\nu})$	0.4
Sec.Fakes. Composition	3.8
In-situ q^2 Calibration	2.8
ℓID Efficiency	0.1
ℓID Fake Rate	0.3
$K\pi$ID Efficiency	1.1
$K\pi$ID Fake Rate	0.7
K_S^0 Efficiency	0.2
π_{slow} Efficiency	< 0.1
Tracking	0.1
Continuum Calibration	0.4
N_{BB}	< 0.1
$f_{+/0}$	< 0.1
Stat. Error (MC)	2.8
Total Syst.	7.9

- $B \rightarrow D^{(*)}\ell\nu$: latest BGL form factors [PRD 93, 0.32006, PRD 103, 073005] and PDG2020 branching fractions
- $B \rightarrow D^{**}\ell\nu,\ D^{**} \in (D_1, D_2, D_0, D_1')$: D^{**} masses and widths updated to PDG2020. LLSW for form factors.
- $\sim$10% gap between inclusive and sum of exclusive measurements.
  - Filled with "best guess" $B \rightarrow D^{(*)}\eta\ell\nu, B \rightarrow D^{(*)}\pi\pi\ell\nu$

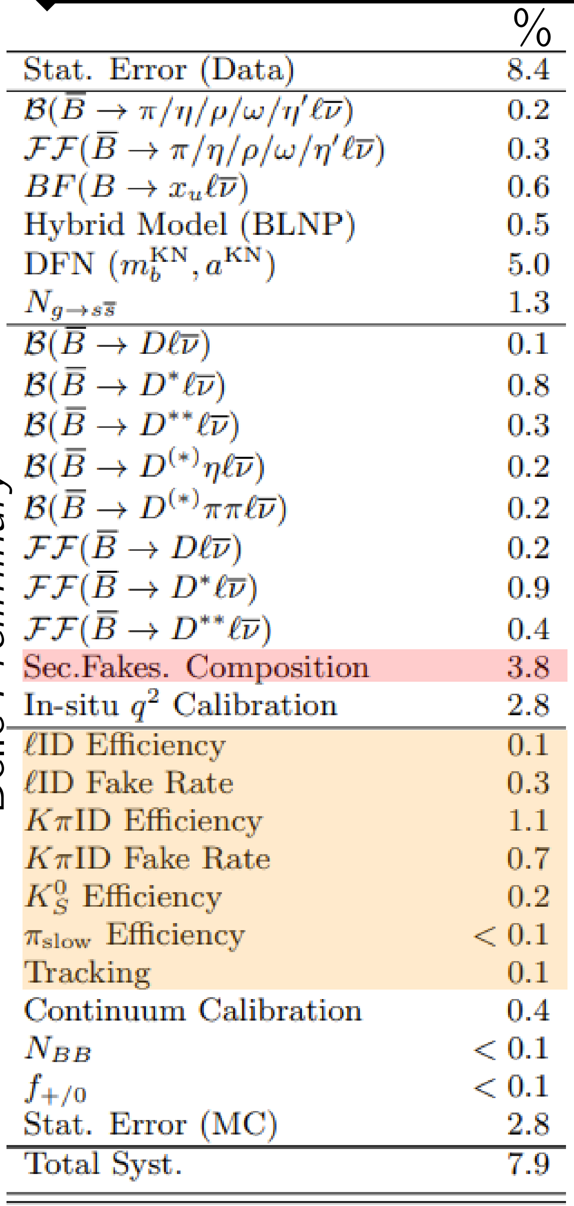


Decay Channel	B^+ $[\times 10^{-3}]$	B^0 $[\times 10^{-3}]$
$\bar{B} \to X_c\ell\bar{\nu}$	108 ± 4	101 ± 4
$\bar{B} \to D\ell\bar{\nu}$	23.5 ± 1	23.1 ± 1
$\bar{B} \to D^*\ell\bar{\nu}$	56.6 ± 2	50.5 ± 1
$\bar{B} \to D_0(\to D\pi)\ell\bar{\nu}$	4.2 ± 0.8	3.9 ± 0.7
$\bar{B} \to D_1'(\to D^*\pi)\ell\bar{\nu}$	4.2 ± 0.8	3.9 ± 0.8
$\bar{B} \to D_1(\to D^*\pi)\ell\bar{\nu}$	4.2 ± 0.3	3.9 ± 0.3
$\bar{B} \to D_1(\to D\pi\pi)\ell\bar{\nu}$	2.4 ± 0.1	2.3 ± 0.9
$\bar{B} \to D_2(\to D^*\pi)\ell\bar{\nu}$	1.2 ± 0.1	1.1 ± 0.1
$\bar{B} \to D_2(\to D\pi)\ell\bar{\nu}$	1.8 ± 0.2	1.7 ± 0.2
$\bar{B} \to D\pi\pi\ell\bar{\nu}$	0.6 ± 0.6	0.6 ± 0.6
$\bar{B} \to D^*\pi\pi\ell\bar{\nu}$	2.2 ± 1.0	2.0 ± 1.0
$\bar{B} \to D\eta\ell\bar{\nu}$	3.6 ± 2.0	4.0 ± 2.0
$\bar{B} \to D^*\eta\ell\bar{\nu}$	3.6 ± 2.0	4.0 ± 2.0

Via intermediate broad D^{**} → Better description than 4/5 body phase-space model.

Systematics - Other

Belle *Preliminary*



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Total Syst.	7.9

- Vary secondary lepton, fake lepton, and $B \to X\tau(\to \ell\nu\overline{\nu})\nu$ relative contributions
  - Combined normalisation constrained by fits to high mass, low lepton momentum control regions.

- Detector effects and particle identification well understood from control mode samples.

Naïve $|\boldsymbol{V_{ub}}|/|\boldsymbol{V_{cb}}|$Extraction

- Missing consistent theory evaluations of the ratio of partial rates.
- Naïve conversion:

$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell\nu)}{\Delta\mathcal{B}(B \to X_c \ell\nu)} \frac{\Delta\Gamma(B \to X_c \ell\nu)}{\Delta\Gamma(B \to X_u \ell\nu)}}$$

$1.96(1 \pm 8.4\% \pm 7.9\%) \times 10^{-2}$

$\Delta\Gamma(B \to X_u \ell\nu)$: $58.5^{+2.7}_{-2.3}\ ps^{-1}$ (GGOU)
$\Delta\Gamma(B \to X_c \ell\nu)$: $29.9 \pm 1.2\ ps^{-1}$ (Kin + phase space correction factor from MC)

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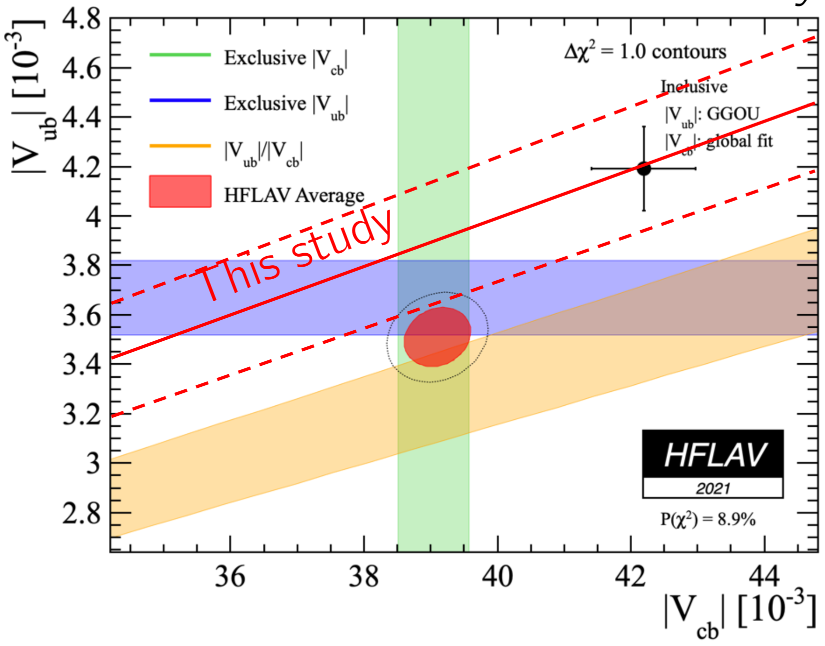
$$\frac{|V_{ub}|}{|V_{cb}|} = 9.96 \begin{pmatrix} 1 \\ \pm 4.2\%_{\text{stat}} \\ \pm 3.9\%_{\text{syst}} \\ \pm 3.1\%_{\Delta\Gamma(\overline{B} \to X_u \ell \overline{\nu})} \\ \pm 2.0\%_{\Delta\Gamma(\overline{B} \to X_c \ell \overline{\nu})} \end{pmatrix} \times 10^{-2}$$

Belle *Preliminary*

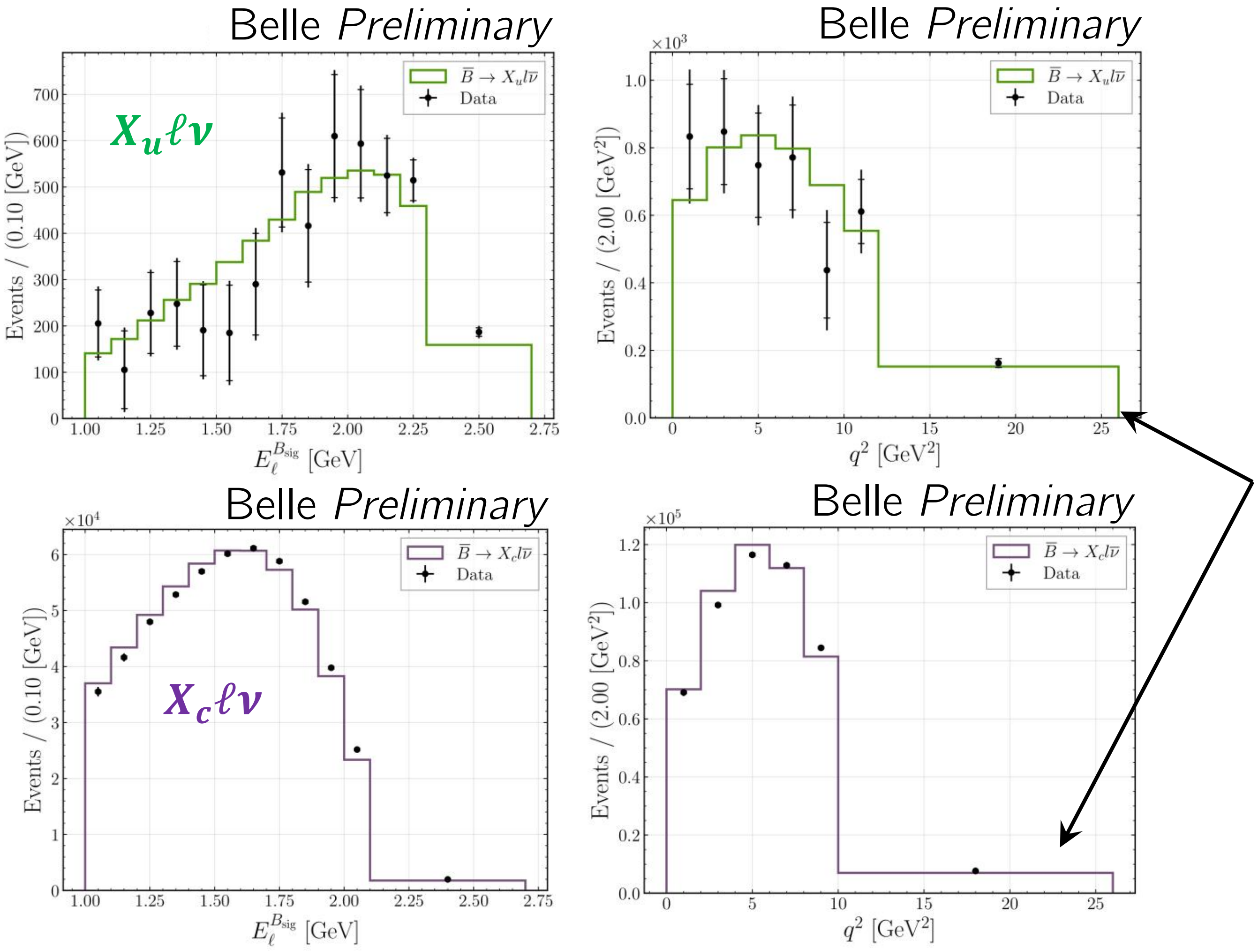
- Excellent agreement with inclusive and exclusive averages

Belle Preliminary



Background Subtracted Spectra

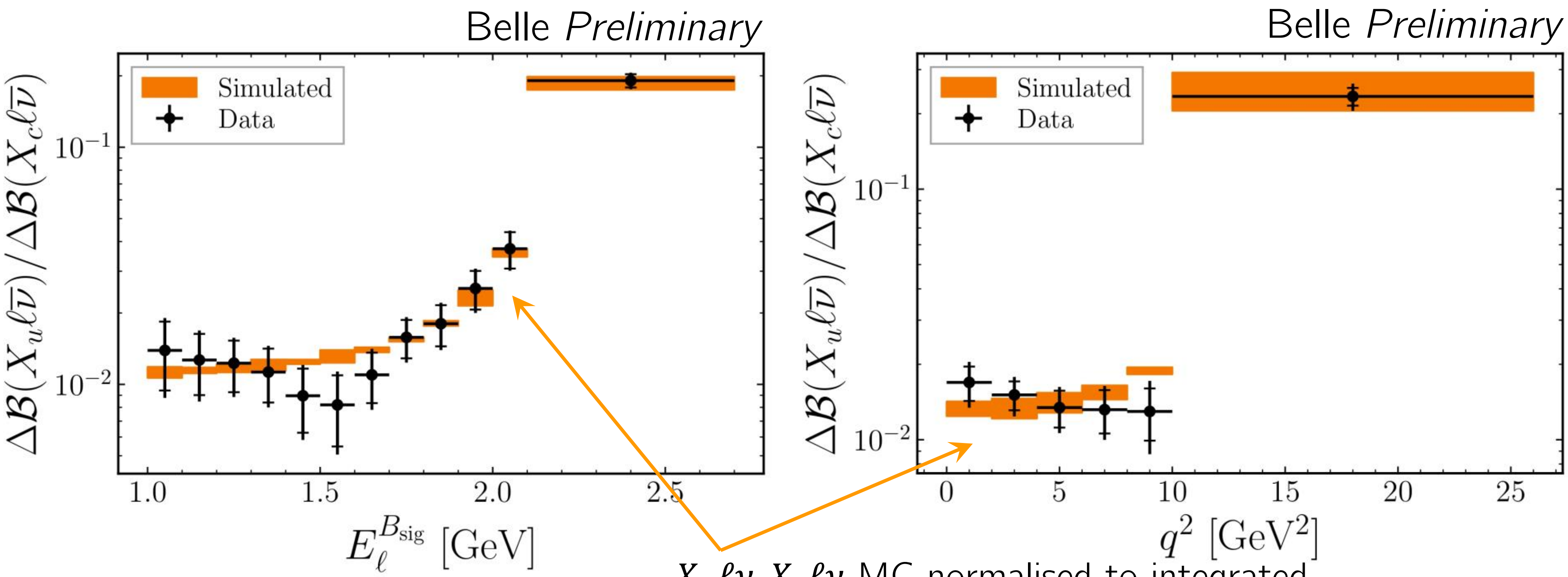
- Normalise to nominal fit results and project onto p_ℓ^B, q^2 - no additional selections.
- Subtract backgrounds, correct $B \to X_c \ell \nu$ shape from kaon vetoed sample.



Nominal MC normalised to background subtracted yields.

Unfolded Ratio

- Unfold $B \to X_u \ell \nu, B \to X_c \ell \nu$ yields via Singular Value Decomposition method of [NIMA 372:469(1996)]
- Tune regularisation parameter to minimise model bias.
- Take ratio and correct for efficiency to form differential ratios.



$X_u \ell \nu, X_c \ell \nu$ MC normalised to integrated efficiency corrected data yields.

Future – Belle II $|\boldsymbol{V_{ub}}|/|\boldsymbol{V_{cb}}|$

- We extracted $\mathbf{0}^{th}$ order ratio, can a simultaneous analysis of higher moments of $\bar{B} \rightarrow X_u \ell \bar{\nu}$ and $\bar{B} \rightarrow X_c \ell \bar{\nu}$ analysis be useful?
- How useful/feasible is it to include such ratio of moments in global fits to $\bar{B} \rightarrow X_c \ell \bar{\nu}$, and $\bar{B} \rightarrow X_s \gamma$ (and $\bar{B} \rightarrow X_u \ell \bar{\nu}$) spectra?

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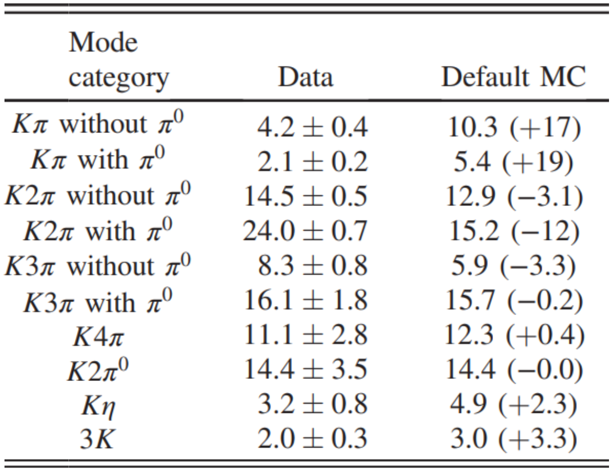
- Requirements for improved inclusive $|V_{ub}|$ determinations:
- Reduce $\overline{B} \to X_u \ell \overline{\nu}$ model dependence
    - Improve flatness of selections in kinematic variables - ongoing Belle II effort using machine learning with flatness penalty terms
- Kaon vetoes are extremely powerful – need to be able to reliably veto K_{L}
- Constrain 'weak annihilation' contribution
- Improve/tune fragmentation of X_u system in PYTHIA

Belle $\overline{B} \to X_s \gamma$ [PRD.91.052004]



Mode category	Data	Default MC
$K\pi$ without π^0	4.2 ± 0.4	10.3 (+17)
$K\pi$ with π^0	2.1 ± 0.2	5.4 (+19)
$K2\pi$ without π^0	14.5 ± 0.5	12.9 (−3.1)
$K2\pi$ with π^0	24.0 ± 0.7	15.2 (−12)
$K3\pi$ without π^0	8.3 ± 0.8	5.9 (−3.3)
$K3\pi$ with π^0	16.1 ± 1.8	15.7 (−0.2)
$K4\pi$	11.1 ± 2.8	12.3 (+0.4)
$K2\pi^0$	14.4 ± 3.5	14.4 (−0.0)
$K\eta$	3.2 ± 0.8	4.9 (+2.3)
$3K$	2.0 ± 0.3	3.0 (+3.3)

Excl. $(3.78 \pm 0.23_{\mathrm{stat}} \pm 0.16_{\mathrm{syst}} \pm 0.14_{\mathrm{theo}}) \times 10^{-3}$

Incl. $(3.88 \pm 0.20_{\mathrm{stat}} \pm 0.31_{\mathrm{syst}} \pm 0.09_{\mathrm{theo}}) \times 10^{-3}$

Ratio 0.97 ± 0.12 $(\rho = 0.11)$ compatible with the world average within 1.2σ

See also first simultaneous determination of inclusive and exclusive $|V_{ub}|$ [arXiv:2303.17309]

Summary

- Ratio measurements cancel some experimental systematic uncertainties – expect also some cancellation on theory side
- Data-driven $X_c \ell \nu$ modelling corrections can bypass issues with gap modelling
- Tagged measurements are implicitly ratio measurements – why not make it explicit?

- Preliminary result on $\frac{\Delta\mathcal{B}(B\to X_u \ell \nu)}{\Delta\mathcal{B}(B\to X_c \ell \nu)}$ at Belle. Theory predictions of ratio of partial rates needed to extract $|V_{ub}|/|V_{cb}|$.

- Naïve $|V_{ub}|/|V_{cb}|$ extraction in good agreement with world averages.

- Submitting to arXiv and PRD today

Belle Preliminary