Inclusive semileptonic decays

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Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established, precise framework
- Extract important CKM parameters V_{cb} and V_{ub}
- Extract power corrections from data
- Cross check of exclusive decays

Inclusive $B \to X_c$ decays: short intro

Inclusive Decays

Inclusive $B \to X_c \ell \nu$: Heavy Quark Expansion (HQE)

- b quark mass is large compared to $\Lambda_{\rm QCD}$
- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\to$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B | O_i^{(k)} | B \right\rangle$$

- $C_i^{(k)}$ perturbative Wilson coefficients
- $\langle B | \ldots | B \rangle$ non-perturbative matrix elements $\to$ string of iD
- operators contain chains of covariant derivatives

$\langle B | \mathcal{O}_i^{(n)} | B \rangle = \langle B | \bar{b}_v (iD_\mu) \ldots (iD_{\mu_n}) b_v | B \rangle$

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$$\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\dots(iD_{\mu_n})b_v|B\rangle$$

- HQE parameters extracted from lepton energy, hadronic mass and q^2 moments

Decay rate

Γ_i are power series in $\mathcal{O}(\alpha_s)$

$$\Gamma = \Gamma_0 + \frac{1}{m_b}\Gamma_1 + \frac{1}{m_b^2}\Gamma_2 + \frac{1}{m_b^3}\Gamma_3 \cdots$$

- Γ_0: decay of the free quark (partonic contributions), $\Gamma_1 = 0$
- Γ_2: μ_π^2 kinetic term and the μ_G^2 chromomagnetic moment

$$2M_B\mu_\pi^2 = -\left\langle B|\bar{b}_v iD_\mu iD^\mu b_v|B\right\rangle$$
$$2M_B\mu_G^2 = \left\langle B|\bar{b}_v(-i\sigma^{\mu\nu})iD_\mu iD_\nu b_v|B\right\rangle$$

- Γ_3: ρ_D^3 Darwin term and ρ_{LS}^3 spin-orbit term

$$2M_B\rho_D^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left[iD_\mu,[ivD, iD^\mu]\right]b_v|B\right\rangle$$
$$2M_B\rho_{LS}^3 = \frac{1}{2}\left\langle B|\bar{b}_v\left\{iD_\mu,\left[ivD, iD_\nu\right]\right\}(-i\sigma^{\mu\nu})b_v|B\right\rangle$$

- Γ_4: 9 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109
- Γ_5: 18 parameters Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005

Non-perturbative matrix elements obtained from moments of differential rate

Charged lepton energy

$$\langle E^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dE_\ell \, E_\ell^n \, \frac{d\Gamma}{dE_\ell}}{\int_{E_\ell > E_{\rm cut}} dE_\ell \, \frac{d\Gamma}{dE_\ell}}$$

Hadronic invariant mass

$$\langle (M_X^2)^n \rangle_{\rm cut} = \frac{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, (M_X^2)^n \, \frac{d\Gamma}{dM_X^2}}{\int_{E_\ell > E_{\rm cut}} dM_X^2 \, \frac{d\Gamma}{dM_X^2}}$$

Dilepton momentum

$$\langle (q^2) \rangle_{\rm cut} = \frac{\int_{q^2 > q^2_{\rm cut}} dq^2 (q^2)^n \, \frac{d\Gamma}{dq^2}}{\int_0 dq^2 \, \frac{d\Gamma}{dq^2}}$$

- Moments up to $n = 3, 4$ and with several cuts available
- Experimentally necessary to use some cut on the leptons

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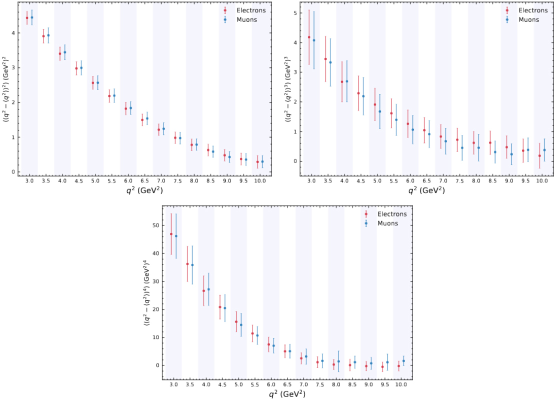
- Moments up to $n = 3, 4$ and with several cuts available
- Experimentally necessary to use some cut on the leptons
- [What is next?] New measurement (with q^2 and lepton energy cuts)!

q^2 moments: Belle and Belle II

Belle Collaboration [2109.01685, 2105.08001]



Centralized moments as function of $q^2_{\rm cut}$

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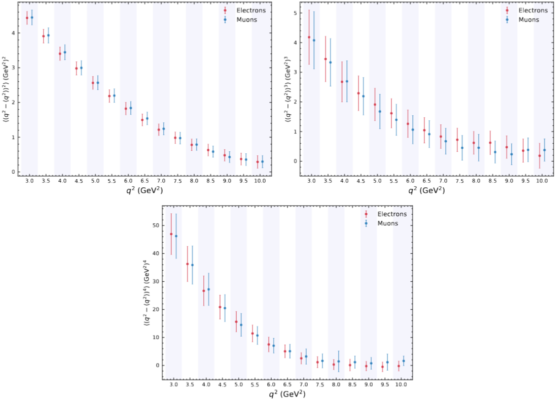
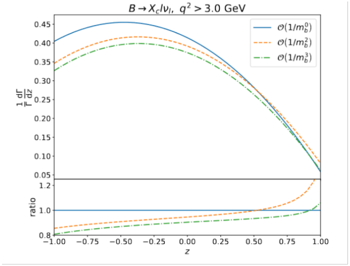


First measurements of inclusive moments since 2009!

Herren [2205.03427] (see also Turczyk [1602.02678])

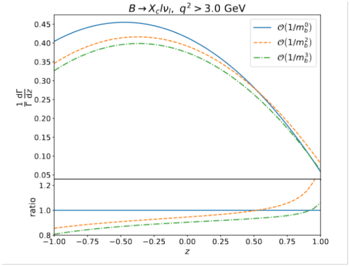


$$A_{FB} = \frac{\int_{-1}^{0} \frac{d\Gamma}{dz} - \int_{-1}^{0} \frac{d\Gamma}{dz}}{\int_{-1}^{0} \frac{d\Gamma}{dz} + \int_{-1}^{0} \frac{d\Gamma}{dz}} \qquad z = \cos\theta = \frac{v \cdot p_\nu - v \cdot p_\ell}{\sqrt{(v \cdot q)^2 - q^2}}$$

- θ is the angle between spacial momenta of the lepton and the B meson in the rest-frame of the dilepton pair
- more sensitive to μ_G^2

Forward-Backward Asymmetry

Herren [2205.03427] (see also Turczyk [1602.02678])



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- θ is the angle between spacial momenta of the lepton and the B meson in the rest-frame of the dilepton pair
- more sensitive to μ_G^2
- [What is next?] Measurements?

Determining V_{cb} and the HQE elements

$$\langle E_\ell^n \rangle, \langle (M_X^2)^n \rangle \qquad \langle (q^2)^n \rangle_{\rm cut}$$

$$\downarrow$$

$$m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, r_E, r_G, s_E, s_B, s_{qB}, + \cdots$$

$$\downarrow$$

$$\mathrm{Br}(\bar{B} \to X_c \ell \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \frac{s_{qB}^4}{m_b^4} \right]$$

$$\downarrow$$

$$V_{cb}$$

State-of-the-art in inclusive $b \to c$

Jezabek, Kuhn, NPB 314 (1989) 1; Melnikov, PLB 666 (2008) 336; Pak, Czarnecki, PRD 78 (2008) 114015; Becher, Boos, Lunghi, JHEP 0712 (2007) 062; Alberti, Gambino, Nandi, JHEP 1401 (2014) 147; Mannel, Pivovarov, Rosenthal, PLB 741 (2015) 290; Fael, Schonwald, Steinhauser, Phys Rev. D 104 (2021) 016003; Fael, Schonwald, Steinhauser, Phys Rev. Lett. 125 (2020) 052003; Fael, Schonwald, Steinhauser, Phys Rev. D 103 (2021) 014005,

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2}\left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) + \frac{\mu_G^2}{m_b^2}\left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3}(\Gamma^{(D,0)} + \Gamma_0^{(1)}\left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots\right)$$

- Include terms up to $1/m_b3^*$ see also Gambino, Healey, Turczyk [2016]
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- $\alpha_s \rho_D^3$ for total rate Mannel, Pivovarov [2020]
- Kinetic mass scheme 1411.6560,1107.3100; hep-ph/0401063

E_ℓ, M_X moments:

$$|V_{cb}|^{\mathrm{BCG}}_{\mathrm{incl}} = (42.16 \pm 0.51) \times 10^{-3}$$

q^2 moments*:

$$|V_{cb}|^{q^2}_{\mathrm{incl}} = (41.79 \pm 0.57) \times 10^{-3}$$

Gambino, Schwanda, PRD 89 (2014) 014022;
Alberti, Gambino et al, PRL 114 (2015) 061802;
Bordone, Capdevila, Gambino, Phys.Lett.B 822 (2021) 136679; Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

First combined fit [fresh of the ArXiV]

First combined fit of all measured moments Gambino, Finauri [2310.20324]

- Includes terms up to $1/m_b^3$
- α_s^3 to total rate and kinetic mass Fael, Schonwald, Steinhauser [2020, 2021]
- Calculation of BLM α_s^2 corrections to q^2 moments included
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- Includes QED corrections to the lepton moments

Combined E_ℓ, M_X, q^2 moments:

$$|V_{cb}|^{\rm GF}_{\rm incl,all} = (41.95 \pm 0.27|_{\rm exp} \pm 0.31|_{\rm th} \pm 0.25|_{\Gamma}) \times 10^{-3} = (41.95 \pm 0.48) \times 10^{-3}$$

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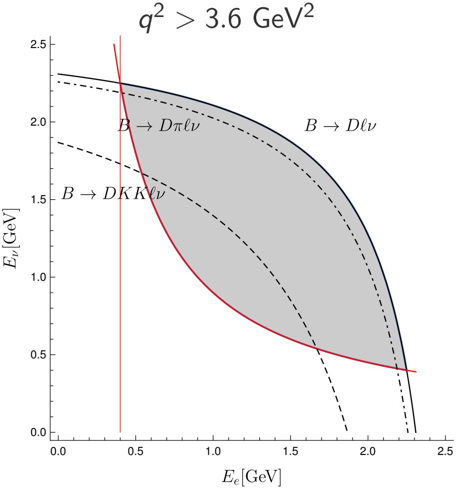
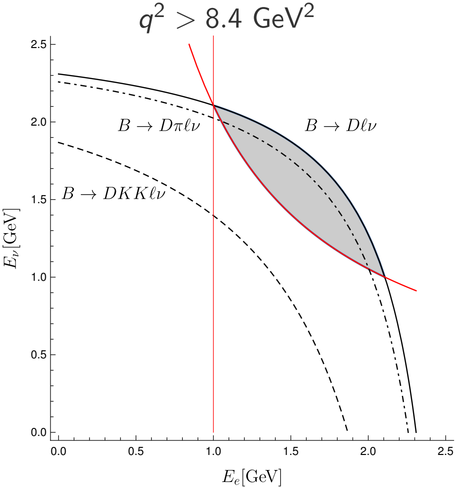
- Agrees with determination from three points at fixed cut
- [What is next?] New measurements of branchingratio (with q^2 cut)?

q^2 versus energy cut





Towards the ultimate precision in inclusive V_{cb}

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What is next?

- Include higher-order α_s corrections [Talk by Matteo Fael]
- Add higher-order non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

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- V_{cb} and HQE from B_s decays

Theory guidance to include power corrections

Lowest State Saturation Approximation (LSSA)

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

$$\rho_D^3 = \varepsilon\mu_\pi^2, \qquad \rho_{LS}^3 = -\varepsilon\mu_G^2, \qquad \varepsilon \sim 0.4\ \text{GeV}$$

Mannel, Turczyk, Uraltsev JHEP 1011 (2010) 109; Heinonen, Mannel, NPB 889 (2014) 46

- LSSA estimated as priors (60% gaussian uncertainty)
- $\mathcal{O}(1/m_b^4, 1/m_b^5)$ can then be included in fit Healey, Turczyk, Gambino, PLB 763 (2016) 60, Bordone, Capdevila, Gambino [2107.00604]
- Using Lepton and MX moments:

$$|V_{cb}|_{\text{incl}} = (42.00 \pm 0.64) \times 10^{-3}$$

- -0.25% shift due to power corrections

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

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Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\ v_\mu = \delta v_\mu \ \text{ and } \ \delta_{RP}\ iD_\mu = -m_b \delta v_\mu$$

- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)

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  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)

- q^2 moments enable (?) a full extraction up to $1/m_b^4$

Inputs into the global fits

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274], Bordone, Capdevila, Gambino [2107.00604], Gambino, Schwanda [2014]

- Quark masses precisely known Fael, Schonwald, Steinhauser [2020,2021]

$$m_b^{\rm kin}(1\ {\rm GeV}) = 4.565 \pm 0.015 \pm 0.013\ {\rm GeV} \qquad \overline{m}_c(2\ {\rm GeV}) = 1.093 \pm 0.008\ {\rm GeV}$$

- Mass difference of the B meson constraints:

$$\frac{3}{4}(m_{B^*}^2 - m_B^2) = C_{cm}(\mu)\mu_G^2(\mu) + \mathcal{O}(1/m_b^3) \sim (0.36 \pm 0.07){\rm GeV}^2$$

- q^2 insensitive to μ_π^2: requires constraint

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Theoretical uncertainties

- Vary scale α_s to account for missing higher order
- Vary ρ_D^3 by 30% to account for missing HQE parameters
- Vary μ_G^2 by 20% to account for missing $\alpha_s \times$ HQE parameters
- Account for theoretical correlations!

Current uncertainties on the inclusive

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using q^2 moments with $1/m_b^4$ terms
- Large uncertainties on HQE elements
- ρ_D^3 smaller than previous
- [What's next?] α_s^2 corrections to moments not (yet) included
- Agreement with BCG extraction (differs due to branching ratio inputs)

  Bordone,Capdevila, Gambino [2021]

q^2 moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\mathrm{incl}} = (41.69 \pm 0.63) \times 10^{-3}$$

- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\mathrm{GeV}^4$$

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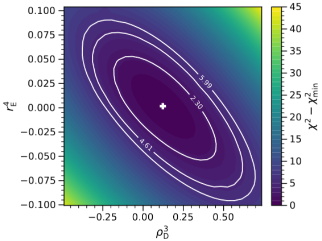
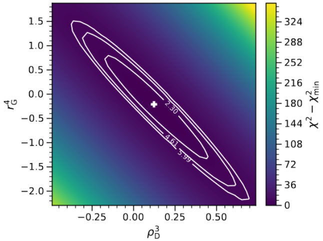


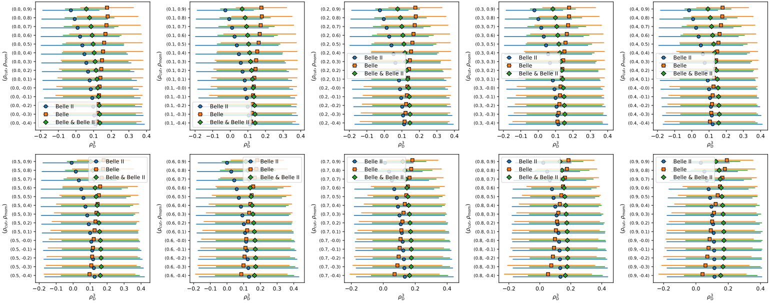


Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Flexible correlations between moments $\rho_{\rm mom}$ and different cuts $\rho_{\rm cut}$

$$\rho_n[q_n(q_A^2) - q_n(q_B^2)] = \rho_{\rm cut}^x \qquad x = \frac{|q_A^2 - q_B^2|}{0.5\mathrm{GeV}^2}$$

- Included by adding a penalty term to the χ^2
- Scan over large range of values + add as nuisance parameters in fit
- V_{cb} stable w.r.t. theory correlations



What about theory correlations?

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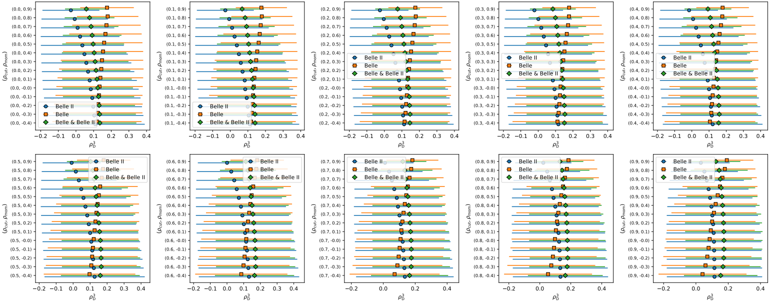

What is next?

Even higher corrections?

Mannel, Mulatin, KKV [2311.xxxx]

- HQE set up with $m_c/m_b \sim \mathcal{O}(1)$
- IR sensitive terms for $m_c \to 0$ Bigi, Mannel, Turczyk, Uraltsev [0911.3322]
  - at dim-6: $1/m_b^3 \ln m_c^2$
  - at dim-8: $1/m_b^5 m_b^2/m_c^2 \sim 1/m_b^3 1/m_c^2$
- Numerically: $m_c^2 \sim m_b \Lambda_{\mathrm{QCD}}$
- What is next? Calculation and estimate of these effects
- One parameter to parametrize these **intrinic charm effects**

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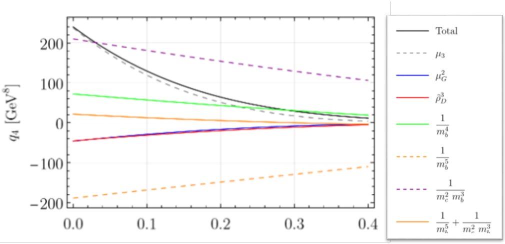


Ratio of inclusive V_{ub}/V_{cb}

See Gambino , Giordano [0805.0271]

- New measurements by Belle See talk by Marcel
- New! α_s^3 corrections for $b \to u$ Fael, Usovitsch [2310.03685]

Theoretical description of $B \to X_u \ell \nu$

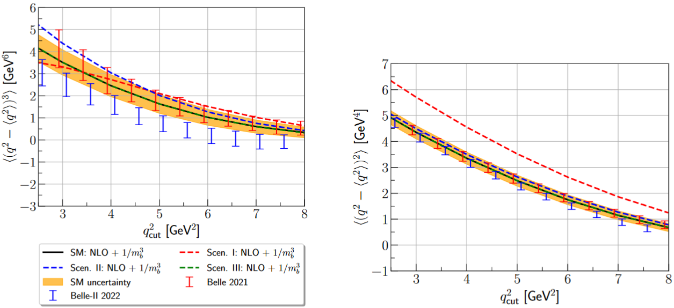
- Breakdown of local OPE due to experimental cuts
- Sensitivity to non-perturbative shape function (GGOU, BLNP, DFE, AFD)
- What is next? Direct calculation of the ratio

$$C \equiv \left| \frac{V_{cb}}{V_{ub}} \right|^2 \frac{\mathcal{B}(B \to X_u \ell \nu)}{\mathcal{B}(B \to X_c \ell \nu)}$$

- Either in shapefunction region or in local OPE (see also Mannel, Rahimi, KKV [2105.02163])
- We can also predict the $B \to X \ell nu$ rate in local OPE!

New Physics?

Fael, Rahimi, KKV [2208.04282]



NP Scenarios	C_{V_L}	C_{V_R}	C_{S_R}	C_{S_L}	C_T
I	0	0	1	1	0
II	0	0	0	-1	0.5
III	-1	0.5	0	0	0

- NP would also influence the moments of the spectrum
- What's next? Requires a simultaneous fit of hadronic parameters and NP

Tests of Lepton Flavor Universality

KKV, Rahimi [2207.03432]; Ligeti, Tackmann [1406.7013];Bernlocner, Sevilla, Robinson, Wormser [2101.08326]

$$R_{e/\mu}(X) \equiv \frac{\Gamma(B \to X_c e\bar{\nu}_e)}{\Gamma(B \to X_c \mu\bar{\nu}_\mu)}$$

- Belle II result: $R_{e/\mu}(X) = 1.033 \pm 0.022$ PRL131 [2023] [2301.08266]
- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ

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- In agreement with new SM predictions: 1.006 ± 0.001 at 1.2σ
- New! Belle II result: $R_{\tau/\ell}(X) = 0.228 \pm 0.016 \pm 0.036$ @EPS
- In agreement with SM prediction:

$$R_{\tau/\ell}(X) = 0.221 \pm 0.004$$

What is next?

Inclusive V_{cb} in the high-precision era

- New measurement with q^2 and lepton energy cuts!
- Forward-backward asymmetry Herren [2205.03427]
- Branching ratio measurements (with q^2 cut)

- Include full α_s^2 in global fit
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Thank you for this initiative!

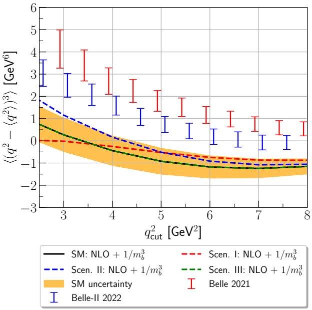
Backup

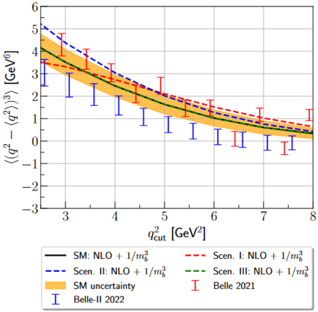
What about ρ_D^3?

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

- Large uncertainties on HQE elements
- Important: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included

Rahimi, Fael, Vos [2208.04282]





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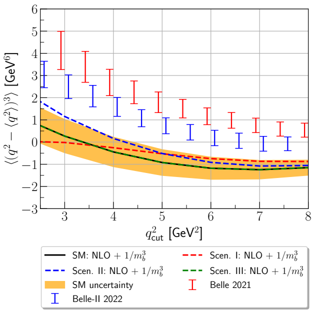
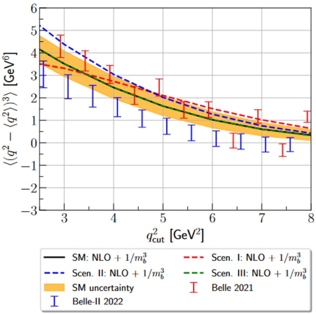
- Important: ρ_D^3 much smaller than previous!
- α_s^2 corrections to moments not yet included
- Leading power corrections are negative Steinhauser, Fael, Schoenwald [2205.03410]
- Full analysis including all data is necessary! Bernlochner, Fael, Prim, KKV [in progress]

Rahimi, Fael, Vos [2208.04282]





Sensitivity of q^2 moments

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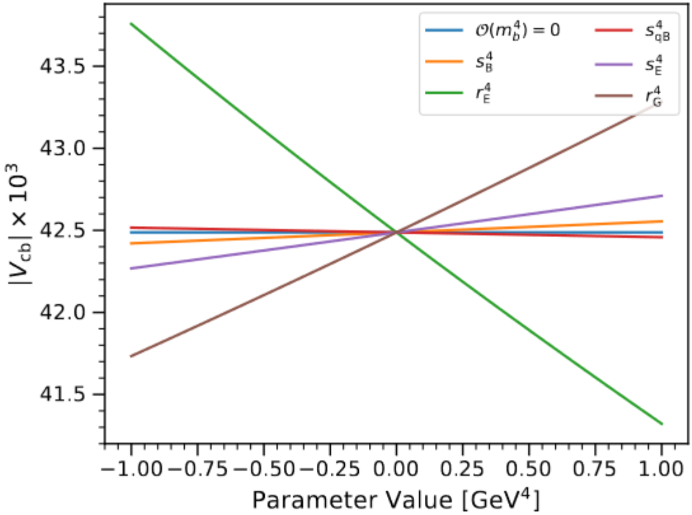
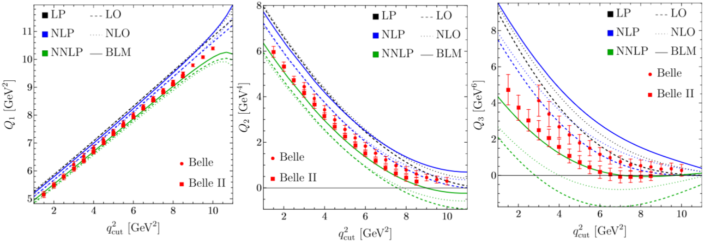


Sensitivity of q^2 moments to α_s^2

Gambino, Finauri [2310.20324]



More information on the q^2 moments

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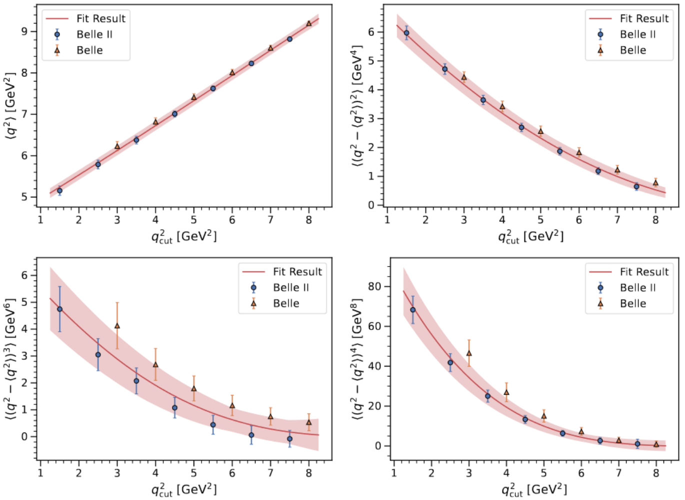

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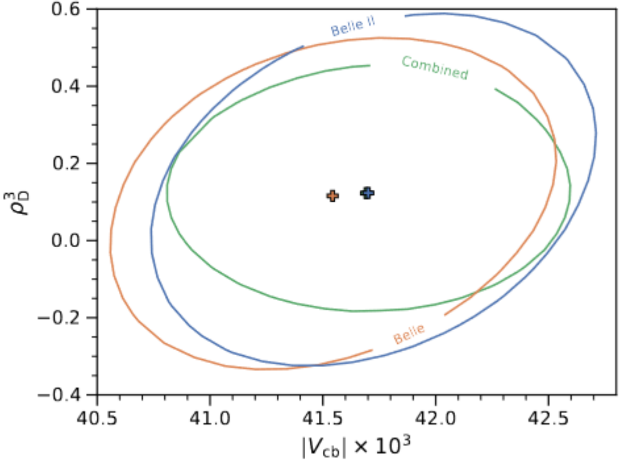

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