Challenges for a global inclusive V_{cb} analysis

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Belle II Physics Week 2023

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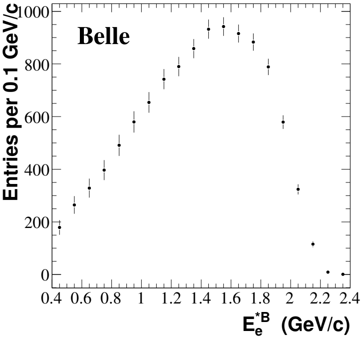
Global analysis

Are data and theory predictions aligned?

How do we combine the various datasets?

Traditional approach



[Gambino, Schwanda, '13]

	experiment	values of E_{cut}(GeV)	Ref.
R^*	BaBar	0.6, 1.2, 1.5	[26, 27]
ℓ_1	BaBar	0.6, 0.8, 1, 1.2, 1.5	[26, 27]
ℓ_2	BaBar	0.6, 1, 1.5	[26, 27]
ℓ_3	BaBar	0.8, 1.2	[26, 27]
h_1	BaBar	0.9, 1.1, 1.3, 1.5	[26]
h_2	BaBar	0.8, 1, 1.2, 1.4	[26]
h_3	BaBar	0.9, 1.3	[26]
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h_1	Belle	0.7, 1.1, 1.3, 1.5	[29]
h_2	Belle	0.7, 0.9, 1.3	[29]
$h_{1,2}$	CDF	0.7	[31]
$h_{1,2}$	CLEO	1, 1.5	[32]
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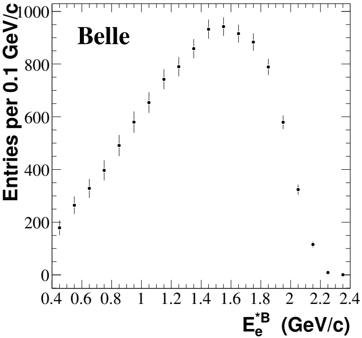
Traditional approach



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2007

New data are welcome

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	−0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b = (\overline{m}_b) = 4.198(12)\,\text{GeV}$ and $\overline{m}_c = (\overline{m}_c) = 0.988(7)\,\text{GeV}$
- No new experimental input wrt to the one in 1411.6560
- The central value of V_{cb} is stable
- Without constraints on m_b, we extract $\bar{m}_b(\bar{m}_b) = 4.210(22)\,\text{GeV}$

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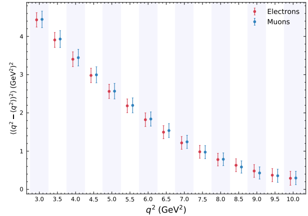
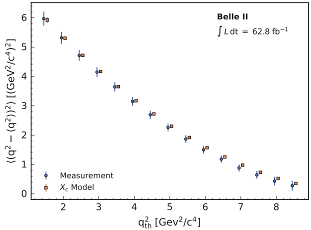
$$V_{cb} = 42.16(32)_{exp}(30)_{th}(25)_{\Gamma} \cdot 10^{-3}$$

New approach: q^2 moments

Idea: use q^2 spectrum to full use RPI relations in HQE





- First proposal

[Fael, Mannel, Vos, '18]

$$\langle (q^2)^n \rangle = \frac{\int_{q^2_{\rm min}} dq^2 (q^2)^n \frac{d\Gamma}{dq^2}}{\int_{q^2_{\rm min}} dq^2 \frac{d\Gamma}{dq^2}}$$

- V_{cb} extraction from Belle and Belle II data

[Bernlochner et al, '22]

$$V_{cb} = (41.79 \pm 0.57) \times 10^{-3}$$

- Value for ρ_D in tension with previous determinations

Where do we stand?

1. What else is needed on the theory side?

   $\Rightarrow$ Are QED corrections currently taken into account for the moments and the branching fractions?

   $\Rightarrow$ Are there any observables for which we need to compute higher order in α_s or $1/m$?

2. We have two methods that yield very compatible results for V_{cb}

   $\Rightarrow$ Can they be combined in a global fit?

What about QED Effect?

Why do we care about QED Effects?

- We want to match the theory description with the experimental measurements that are always affected by photon emissions
- The MC PHOTOS accounts for QED effects, reporting results which can be compared with the non-radiative theory predictions
- PHOTOS knows only about real emission and obtains the virtual part by normalisation

$$\frac{d\Gamma}{dzdx} = \mathcal{F}^{(0)}(\omega_{\text{virtual}} + \omega_{\text{real}}) \Rightarrow \int dx(\omega_{\text{virtual}} + \omega_{\text{real}}) = 1$$

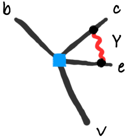
Are virtual corrections under control?

The inclusive case

- If wrt QCD the hadronic and leptonic system are separated, QED corrections mix them

  ⇒ Defining fully inclusive observables is harder

  ⇒ Analogy with experiments is essential



- The OPE is still valid for the total decay width

- At the differential level, this is generally not true

  ⇒ Large contributions factorise wrt to tree-level

  ⇒ Useful to go beyond NLO

Two calculation approaches

1. Splitting Functions

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi}\bar{L}_{b/e}\int_y^{1-\rho}\frac{dx}{x}P_{ee}^{(0)}\left(\frac{y}{x}\right)\left(\frac{d\Gamma}{dx}\right)^{(0)}$$

$\log(m_b^2/m_e^2)$ plus distribution

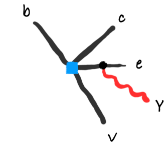
- Correction vanishes for the inclusive branching fraction
- Suitable for evaluating $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha/m_b^n)$ corrections

2. Full $\mathcal{O}(\alpha)$ corrections

- Access all corrections, not only the one that factorise
- Real corrections are computationally expensive
  - $\Rightarrow$ Cuba library employed to carry out the 4-body integration
  - $\Rightarrow$ Phase space splitting used to reduce the size of the integrands

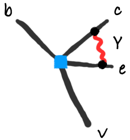
Leading contributions

1. **Collinear logs**: captured by splitting functions



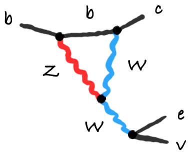
$$\sim \frac{\alpha_e}{\pi} \log^2 \left(\frac{m_b^2}{m_e^2} \right)$$

2. **Threshold effects** or Coulomb terms



$$\sim \frac{4\pi\alpha_e}{9}$$

3. **Wilson Coefficient**



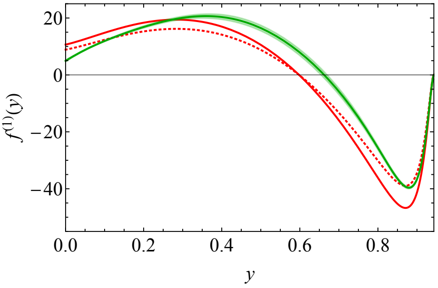
$$\sim \frac{\alpha_e}{\pi} \left[\log \left(\frac{M_Z^2}{\mu^2} - \frac{11}{6} \right) \right]$$

Lepton Energy spectrum

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- We compute bins in the lepton energy using the full $\mathcal{O}(\alpha)$ calculation
- We compare them to the results given by the splitting functions
- The difference the two calculations for the lepton energy spectrum and obtain a full analytic formula for the radiative corrections

  $\Rightarrow$ Relatively small, easy-to-use formula to obtain branching fractions, lepton energy moments w/o cuts



$$f^{(1)}(y) = \frac{\bar{L}_{b/e}}{2}\, f^{(1)}_{\mathrm{LL}}(y) + \Delta f^{(1)}(y)$$

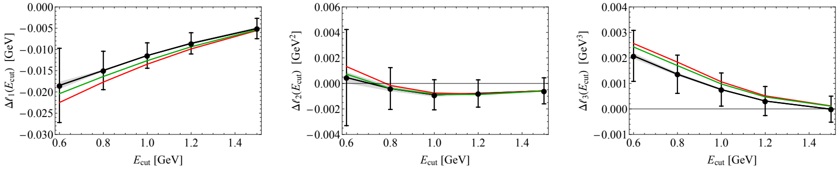
Comparison with data

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- Babar provides data with and without applying PHOTOS to subtract QED effects

  ⇒ Perfect ground to test our calculations

  ⇒ Not the same for Belle at the moment, could be possible for future analysis?



- The moments, since they are normalised, are not affected by the large threshold corrections
- The agreement with BaBar is very good

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\rm cut}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\rm cut}}}$$

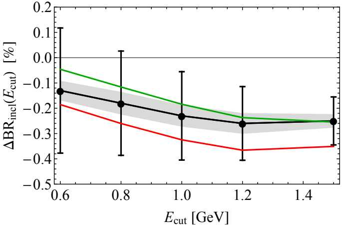
Branching ratio

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- The total branching ratio is not affected by large logs due to KLN theorem
- The large corrections are from the Wilson Coefficient and the threshold effects

$$\frac{\Gamma}{\Gamma^{(0)}g(\rho)} = 1 + \frac{\alpha}{\pi}\left[\ln\left(\frac{M_Z^2}{m_b^2}\right) - \frac{11}{6} + 5.516(14)\right]$$

$$= 1 + 1.43\% - 0.44\% + 1.32\% = 1 + 2.31\%$$



E_{cut}	$\delta\mathrm{BR}^{\mathrm{BaBar}}_{\mathrm{incl}}$	$\delta\mathrm{BR}^{\mathrm{LL}}_{\mathrm{incl}}$	$\delta\mathrm{BR}^{\mathrm{NLL}}_{\mathrm{incl}}$	$\delta\mathrm{BR}^{\alpha}_{\mathrm{incl}}$	$\delta\mathrm{BR}^{1/m_b^2}_{\mathrm{incl}}$	$\delta\mathrm{BR}_{\mathrm{incl}}$	σ
0.6	−1.26%	−1.92%	−1.95%	−0.54%	−0.50%	−0.45%	+0.34
0.8	−1.87%	−2.88%	−2.91%	−1.36%	−1.29%	−1.22%	+0.30
1.0	−2.66%	−4.03%	−4.04%	−2.38%	−2.26%	−2.15%	+0.25
1.2	−3.56%	−5.43%	−5.41%	−3.65%	−3.43%	−3.27%	+0.14
1.5	−5.22%	−8.41%	−8.26%	−6.37%	−5.73%	−5.39%	−0.09

QED for exclusive decays

- For $B^0 \to D^+ \ell \bar{\nu}$, the threshold effects were calculated and are $1 + \alpha\pi$

  [Ginsberg, '66, De Boer, Kitahara, Nisandzic, '18]

- For $B^0 \to D^{*+} \ell \bar{\nu}$, the threshold effects might have a different structure because the hadronic matrix element is different

  $\Rightarrow$ To verify explicitly

- Structure-dependent terms are unknown, but maybe something is doable in the HQE?

- How do we reconcile the threshold effects between the exclusive and the inclusive?

$$\mathcal{B}(B \to X_c \ell \nu) = \mathcal{B}(B \to D \ell \nu) + \mathcal{B}(B \to D^* \ell \nu) + \mathcal{B}(B \to D^{**} \ell \nu) + \ldots$$

Global fit

- The results for the the V_{cb} determination using lepton energy and hadronic mass moments, and the q^2 moments seem very compatible

- What would be the result of a combined fit?

  $\Rightarrow$ What's the combined value of V_{cb} and its uncertainty

  $\Rightarrow$ Relevant to extract the non-perturbative parameters

Main differences wrt Bernlochner et al:

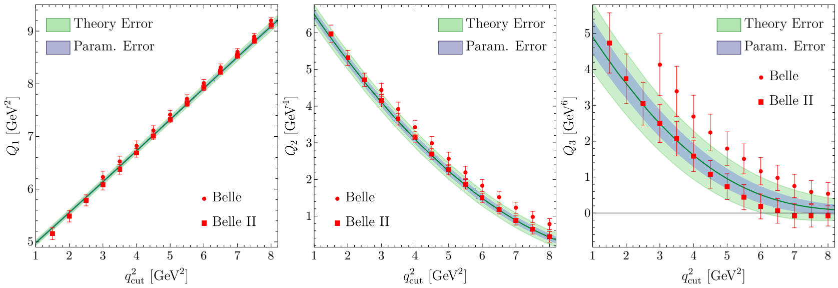
- Inclusion of the leading $\mathcal{O}(\alpha_s^2\beta_0)$ corrections

- Power corrections up to $1/m_b^3$

Global fit

[Finauri, Gambino, '23]

	$m_b^{\rm kin}$	$\overline{m}_c$	μ_π^2	μ_G^2	ρ_D^3	ρ_{LS}^3	$10^2{\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$	$\chi^2_{\rm min}$(/dof)
without	4.573	1.092	0.477	0.306	0.185	−0.130	10.66	42.16	22.3
q^2-moments	0.012	0.008	0.056	0.050	0.031	0.092	0.15	0.51	0.474
Belle II	4.573	1.092	0.460	0.303	0.175	−0.118	10.65	42.08	26.4
	0.012	0.008	0.044	0.049	0.020	0.090	0.15	0.48	0.425
Belle	4.572	1.092	0.434	0.302	0.157	−0.100	10.64	41.96	28.1
	0.012	0.008	0.043	0.048	0.020	0.089	0.15	0.48	0.476
Belle &	4.572	1.092	0.449	0.301	0.167	−0.109	10.65	42.02	41.3
Belle II	0.012	0.008	0.042	0.048	0.018	0.089	0.15	0.48	0.559



Global fit + QED

[Finauri, Gambino, '23]

- Small changes compared to the inputs in 2107.00604

  ⇒ New FLAG average for the heavy quark masses

- New computation of QED effects including threshold effects included by correcting the BaBar branching fraction

$$R_{\rm QCD}^{\rm new} = \zeta_{\rm QED} R_{\rm QCD}^{\rm Babar}$$

  ⇒ $\zeta_{\rm QED}$ accounts for the misalignment between the corrected BaBar results and the results from the full $\mathcal{O}(\alpha_e)$ computation

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4.573	1.090	0.453	0.288	0.176	−0.113	10.62	41.95
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

Can we apply the same procedure to Belle measurements?

Summary and Outlook

Summary

- Main message: the inclusive $|V_{cb}|$ determination is stable

  ⇒ There are different datasets and different methods that yield separated very compatible results and a first joint fit stresses their compatibility

  ⇒ Personal opinion: if there are no major changes in the data, it is unlikely that the central value for V_{cb} will change by a large amount

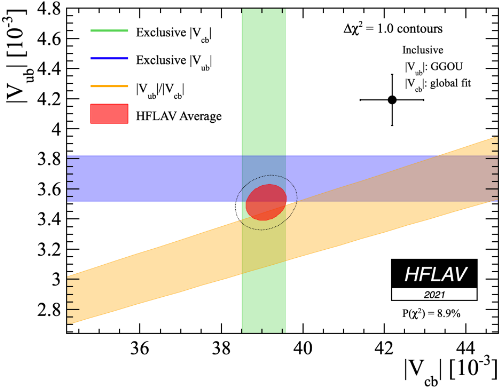
Outlook

- We need to understand how to correct for the QED effects not accounted for by PHOTOS

  ⇒ Can we rescale the branching fractions by Belle?

  ⇒ Would it be profitable to build an ad hoc MC based on a dedicated calculation for the inclusive decays?

Appendix



- Longstanding discrepancy between inclusive and exclusive determinations

- A lot of activity lately
  - ⇒ new experimental determinations
  - ⇒ new calculations of exclusive form factors

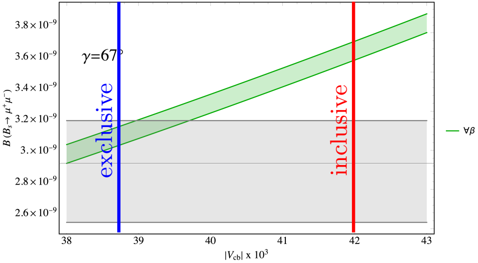
Why is V_{cb} important?

- Key parameter in the prediction of flavour observables

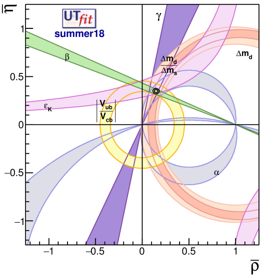
  $\Rightarrow \; \epsilon_K \sim |V_{cb}|^4$

  $\Rightarrow \; \mathcal{B}(\bar{B}_s \to \mu^+\mu^-) \sim |V_{tb}V_{ts}^*|^2 \sim |V_{cb}|^2[1+\mathcal{O}(\lambda^2)]$

[Buras, Venturini, '21]



- Tests the SM flavour structure



Theory framework

$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T\left\{\mathcal{H}_{\mathrm{eff}}^{\dagger}(x)\mathcal{H}_{\mathrm{eff}}(0)\right\} |B(p)\rangle$$

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↑

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

Theory framework

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↑

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative

  ⇒ They need to be determined with non-perturbative methods **[Hashimoto's talk]**

  ⇒ They can be extracted from data

  ⇒ With large n, large number of operators

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↑

loss of predictivity

Theory framework

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$
$$+\left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \dots\Big]$$

$$\mu_\pi^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v(i\vec{D})^2 b_v|B\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2m_B}\langle B|\bar{b}_v\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b_v|B\rangle_\mu$$

- Coefficients of the expansions are known
- Ellipses stands for higher orders

Theory framework

NEW

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[1 + a_1\left(\frac{\alpha_s}{\pi}\right) + a_2\left(\frac{\alpha_s}{\pi}\right)^2 + a_3\left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_\pi^2}{m_b^2}$$

$$+\left(g_0 + g_1\left(\frac{\alpha_s}{\pi}\right)\right)\frac{\mu_G^2(m_b)}{m_b^2} + d_0\frac{\rho_D^3}{m_b^3} - g_0\frac{\rho_{LS}^3}{m_b^3} + \dots\Big]$$

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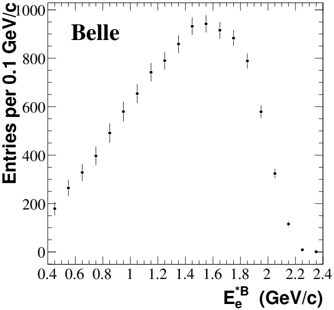
How do we constrain the OPE parameters?



- Lepton energy and hadronic invariant mass distributions can be used to extract non perturbative information
- The moments admit a Heavy Quark Expansion

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + M_i^{\mu_\pi^2} \frac{\mu_\pi^2}{m_b^2} + M_i^{\mu_G^2} \frac{\mu_G^2}{m_b^2} + M_i^{\rho_D^3} \frac{\rho_D^3}{m_b^3} + M_i^{\rho_{LS}^3} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

- q^2 moments can also be used [Fael, Mannel, Vos, '18]

Scheme conventions

- Pole mass scheme

  $\Rightarrow$ Renormalon ambiguity

  $\Rightarrow$ Perturbative series is factorially divergent

$$\Gamma_{sl} \sim \sum_k k! \left(\frac{\beta_0}{2} \frac{\alpha_s}{\pi} \right)^k$$

- We choose to use to b-quark mass and the non perturbative parameters in the kinetic scheme

[Bigi, Shifman,Uraltsev,Vainshtein]

  $\Rightarrow$ Wilsonian cutoff $\mu = 1\,\mathrm{GeV}$

- We express the charm mass in the $\overline{\mathrm{MS}}$ scheme

Inclusion of $\mathcal{O}(\alpha_s^3)$ results

[Fael, Schönwald, Steinhauser, '20]

b-quark mass:

$$m_b^{kin}(1\,\text{GeV}) = [4169 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3}]\,\text{MeV} = (4526 \pm 15)\,\text{MeV}$$

↑ 50% **reduction!**

Semileptonic width

$\Rightarrow$ $\mu = 1\,\text{GeV}$, $\mu_b = m_b^{kin}$, $\mu_c = 3\,\text{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1163_{\alpha_s} - 0.0349_{\alpha_s^2} - 0.0097_{\alpha_s^3}\Big]$$

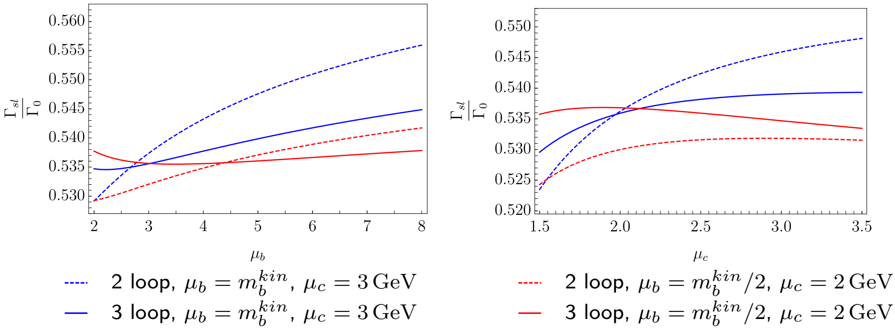
$\Rightarrow$ $\mu = 1\,\text{GeV}$, $\mu_b = m_b^{kin}/2$, $\mu_c = 2\,\text{GeV}$

$$\Gamma_{sl} = \Gamma_0 f(\rho)\Big[0.9257 - 0.1138_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0104_{\alpha_s^3}\Big]$$

residual uncertainty $\sim 0.5\%$

Residual uncertainty

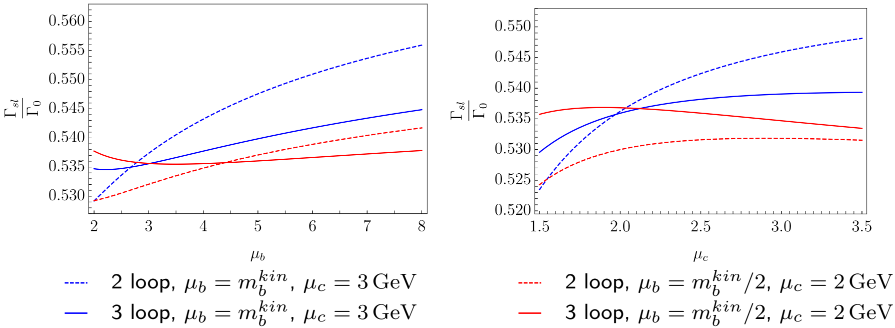
[MB, Capdevila, Gambino, '21]



- Theory uncertainties are essential for a good fit to data [Gambino, Schwanda, '14]
- Residual scale dependence
  - ⇒ Milder including $\mathcal{O}(\alpha_s^3)$
  - ⇒ We choose $\mu_c = 2\,\mathrm{GeV}$, $\mu_b = m_b^{kin}/2$ and $\mu = 1\,\mathrm{GeV}$ to minimize scale dependence
- Other sources of uncertainties e.g. higher power corrections are slightly smaller

Residual uncertainty

[MB, Capdevila, Gambino, '21]



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1.2% **residual uncertainty**

The semileptonic fit

[MB, Capdevila, Gambino, '21]

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_g(m_b)$	ρ_{LS}	$\text{BR}_{c\ell\nu}$	$10^3\vert V_{cb}\vert$
4.573	1.092	0.477	0.185	0.306	−0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

- Constraints from FLAG $N_f = 2+1+1$: $\overline{m}_b = (\overline{m}_b) = 4.198(12)\,\text{GeV}$ and $\overline{m}_c = (\overline{m}_c) = 0.988(7)\,\text{GeV}$
- No new experimental input wrt to the one in 1411.6560
- The central value of V_{cb} is stable
- Without constraints on m_b, we extract $\bar{m}_b(\bar{m}_b) = 4.210(22)\,\text{GeV}$

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[MB, Capdevila, Gambino, '21]

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$$V_{cb} = 42.16(32)_{exp}(30)_{th}(25)_{\Gamma} \cdot 10^{-3}$$

Higher power corrections

- At $\mathcal{O}(1/m^4)$ the number of operators become large
  - $\Rightarrow$ 9 at dim 7
  - $\Rightarrow$ 18 at dim 8

Lowest Lying State Saturation Approximation: [Mannel, Turczyk, Uraltsev, '11]

$$\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle = \sum_n \langle B|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|B\rangle$$

↑ complete set of states

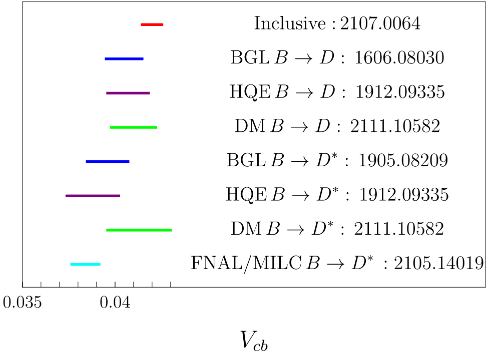
At dimension 6 the LLSA works well:

$$\rho_D^3 = \epsilon\mu_\pi^2 \qquad \rho_{LS}^3 = -\epsilon\mu_G^2 \qquad \epsilon \sim 0.4\,\mathrm{GeV}$$

- Large corrections to the LLSA are possible [Gambino, Mannel, Uraltsev, '12]
- 60% gaussian uncertainty on higher order parameters

$$V_{cb} = 42.00(53) \times 10^{-3}$$

Inclusive vs. Exclusive



- There is a **spread** between inclusive and exclusive determinations of V_{cb}
- The tension between inclusive and FNAL/MILC accounts to almost $4\,\sigma$!
- Determination from q^2 moments

[see Keri's talk]

The inclusive B_s width

[MB, Gambino, '21]

- B_d and B_s widths are linked through violation of $SU(3)_F$

$$\frac{\delta_{\mu_\pi^2}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -0.9(1)\% \qquad \frac{\delta_{\mu_G^2}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -3.2(5)\%$$
$$\frac{\delta_{\rho_D^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -3.2(5)\% \qquad \frac{\delta_{\rho_{LS}^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -0.3(2)\%$$

- Previous studies used sum rules and HQ relations [Bigi, Mannel, Uraltsev, '11]

- We update those estimates

  ⇒ Preliminary lattice estimates [Gambino, Melis, Simula, '17]

  ⇒ Most recent semileptonic fit

$$\frac{\Gamma_{\rm sl}(B_s)}{\Gamma_{\rm sl}(B_d)} - 1 = -(1.8 \pm 0.8)\% \,.$$

The inclusive Λ_b width

[MB, Gambino, '21]

- Same arguments as before
- μ_G^2 and ρ_{LS}^3 terms vanish for ground state baryons

$$\frac{\delta_{\mu_G^2}\Gamma_{\rm sl}(B) + \delta_{\rho_{LS}^3}\Gamma_{\rm sl}(B)}{\Gamma_{\rm sl}(B)} = -(3.5 \pm 0.6)\%$$

$\Rightarrow$ Biggest difference comes from these terms

$\Rightarrow$ No big numerical changes from previous determinations

$$\frac{\Gamma_{\rm sl}(\Lambda_b)}{\Gamma_{\rm sl}(B_d)} - 1 = (4.1 \pm 1.6)\%\,,$$

Summary and Prospects

Summary:

- Tension between inclusive and exclusive determination of V_{cb} is not resolved
- New $\mathcal{O}(\alpha_s^3)$ contributions to Γ_{sl} show that
  - $\Rightarrow$ perturbative effects are under control
  - $\Rightarrow$ reduction of the final uncertainty of 1/3
  - $\Rightarrow$ the central value of V_{cb} is stable

Prospects:

- α_s corrections for the hadronic parameters in the moments
- Lattice calculations for the B_s width are ongoing
- Moments measurements for B_s and Λ_b modes