Helicity Amplitude Module
for Matrix Element Reweighting

HAMMER & EFFORT

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Caltech

BURKE INSTITUTE

NEED FOR REWEIGHING

- Large integrated luminosity requires large MC datasets and time consuming simulations
- Exclusive $b \to c\ell\nu$: need to vary shapes for
  - Fitting form factors
  - Constraining new physics
  - Propagate information from e,μ modes to τ
  - Get the "theory updates" on FFs
  - …
- This has to be done (consistently) among different modes D, D*, D**, non-resonant …

Reweighing tools are needed → HAMMER & eFFORT

EFFORT V2

(M. Prim) https://github.com/MarkusPrim/eFFORT2

- eFFORT: Form Factor Reweighting Tool
- A tool to reweigh exclusive $b \to c\ell\nu$, $b \to u\ell\nu$ semileptonic decays
- Lightweight, in Python, fast
- Used to fit SM exclusive $b \to c\ell\nu$, $b \to u\ell\nu$ differential distributions
- Used to construct SM $b \to u\ell\nu$ hybrid MC weights
- Transparent / easy to modify code: good for quick on the fly comparisons
- Standard Model only, works at level of (differential) rates, no taus
- "actively maintained, but developed on demand"

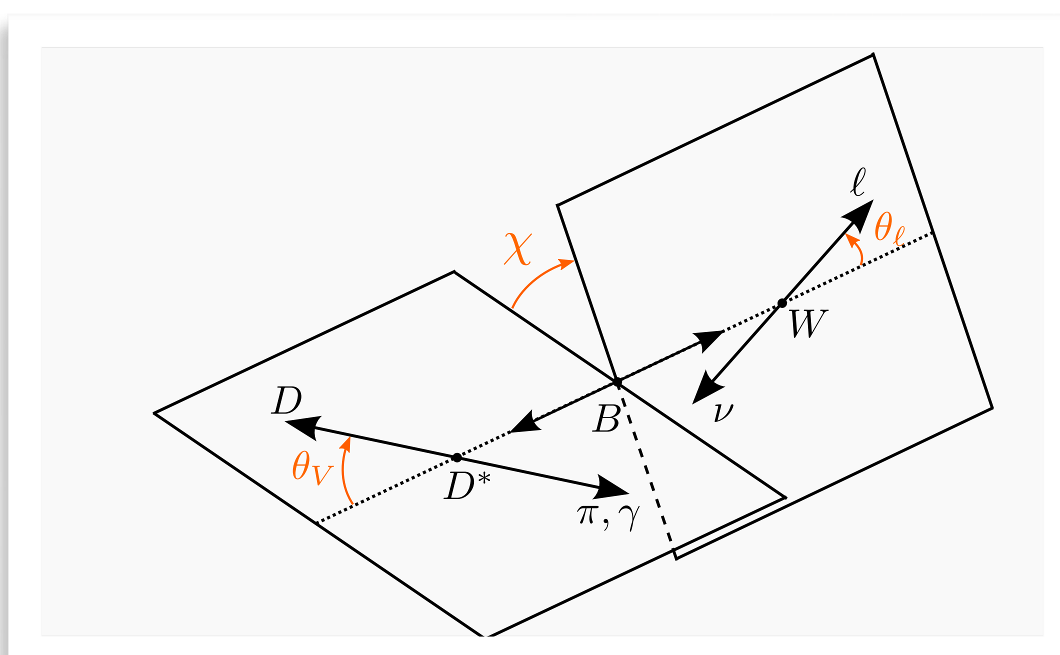
EFFORT V2

- Processes / FF available:
  - $B \to D^{(*)}\ell\nu$ w/ CLN, BGL, BLPR(XP)
  - $B \to \pi\ell\nu$ w/ BCL
  - $B \to \eta^{(')}\ell\nu$ w/ ISGW2, LCSR
  - $B \to (\rho,\omega)\ell\nu$ w/ BSZ
- $B \to V\ell\nu$:

$$\frac{d^4\Gamma}{dw\, d\cos\theta_\ell\, d\cos\theta_V\, d\chi}$$

- $B \to P\ell\nu$

$$\frac{d^2\Gamma}{dw\, d\cos\theta_\ell}$$



HAMMER

(Bernlochner, Duell, Ligeti, MP, Robinson, 2002.00020)

Helicity Amplitude Module for Matrix Element Reweighting

- Reweigh events to "any" form factor parameterization and to SM + New Physics
- Can compute total rates (for BR reweighing)
- Keeps spin correlations down to D's,π's,γ's
- Designed to save computational time → fast, can be further optimized
- Internal code is C++. APIs: C++, Python and ROOT "interface"
- Currently three customers: Belle II, LHCb, CMS

https://hammer.physics.lbl.gov/

HAMMER'S LIBRARY (V. 1.3)

Helicity Amplitude Module for Matrix Element Reweighting

➤ Available Amplitudes & Form Factor parameterizations:

Process	FF parametrizations
$B \to D^{(*)}\ell\nu$	ISGW2* [16, 17], BGL*‡ [13–15], CLN*‡ [18], BLPR‡ [19], BLPRXP‡ [20]
$B \to (D^* \to D\pi)\ell\nu$	ISGW2*, BGL*‡, CLN*‡, BLPR‡, BLPRXP‡
$B \to (D^* \to D\gamma)\ell\nu$	ISGW2*, BGL*‡, CLN*‡, BLPR‡, BLPRXP‡
$B \to D_0^*\ell\nu$	ISGW2*, LLSW* [21, 22], BLR‡ [23, 24]
$B \to D_1^*\ell\nu$	ISGW2*, LLSW*, BLR‡
$B \to D_1\ell\nu$	ISGW2*, LLSW*, BLR‡
$B \to D_2^*\ell\nu$	ISGW2*, LLSW*, BLR‡
$B \to (\rho \to \pi\pi)\ell\nu$	ISGW2*, BSZ‡ [25]
$B \to (\omega \to \pi\pi\pi)\ell\nu$	ISGW2*, BSZ‡
$\Lambda_b \to \Lambda_c\ell\nu$	PCR* [26], BLRS‡ [27, 28]
$\Lambda_b \to \Lambda_c^*\ell\nu$	PCR*, LSPR‡ [29, 30]
$B_c \to (J/\psi \to \ell\ell)\ell\nu$	Kiselev* [31], EFG* [32], BGL*‡ [33], ...
$B \to \pi\ell\nu$	ISGW2*, BCL*‡ [34], GKvD [35]
$\tau \to \pi\nu$	—
$\tau \to \ell\nu\nu$	—
$\tau \to 3\pi\nu$	RCT* [36–38]
$D_1 \to (D^* \to D\pi/\gamma)\pi$	PW
$D_2^* \to (D^* \to D\pi/\gamma)\pi$	PW
$D_2^* \to D\pi$	PW

$\ell = e, \mu, \tau$,

$B = B^0, B^+, B_s$,

Etc.

Other processes / FFs on the way + added upon request

HAMMER'S IDEA

HAMMER
Helicity Amplitude Module
for Matrix Element Reweighting

- Compute helicity amplitudes instead of squared matrix elements for general SM+NP (speed: O(n) vs O(n^2) terms)
- Keep full spin correlation & interference* effects in decays
- Tensorialize amplitudes:

$$\mathscr{M} = \mathscr{M}_{\alpha,i} FF_{\alpha} C_i$$

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Event kinematics dependent, FF parameterization & NP independent

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q^2 & FF param. dependent, NP independent can be further expanded in (linearized) uncertainties/parameters:
$FF_\alpha(q^2, \vec{a}) = FF_{\alpha,\lambda}(q^2, \vec{a}_0)\delta a_\lambda$, *with* δa_λ *event independent*

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C_i are NP Wilson Coefficients, $C_i = (1, V_{LL}, V_{RL}, V_{RL}, V_{RR}, S_{LL}, S_{RL}, S_{LR}, S_{RR}, T_{LL}, T_{RR})$
$FF_{\alpha,\lambda}$ are central value and gradient w.r.t. to parameters, $\delta a_\lambda = (1, \overrightarrow{\delta a})$ *(e.g. coefficients of Taylor expansions of IW functions, BGL parameters, …)*

HAMMER'S IDEA

Helicity Amplitude Module for Matrix Element Reweighting

- Squared matrix element is

$$M^{2} = \left(C_i C_{i'}^{\dagger}\right)\left(\delta a_{\lambda}\delta a_{\lambda'}\right)\left(FF_{\alpha,\lambda}FF^{\dagger}_{\alpha',\lambda'}\right)\left(\mathscr{M}_{\alpha,i}\mathscr{M}^{\dagger}_{\alpha',i'}\right)$$

- Scalar event weight is $W = M_{new}^{2} / M_{old}^{2}$
- Define NP-independent, FF-independent tensor event weight

$$\mathscr{W}_{\alpha\alpha',ii'} = \left(\mathscr{M}_{\alpha,i}\mathscr{M}^{\dagger}_{\alpha',i'}\right) / M_{old}^{2}$$

- Pre-compute once & store $\mathscr{W}_{\alpha\alpha',ii'}$, $FF_{\alpha,\lambda}$ (and/or $\mathscr{W}_{\lambda\lambda',ii'}$) (per event, using truth level 4-momenta; data format will always be backward compatible)
- Scalar weight W for a given choice of C_i, δa_{λ} can then be retrieved from simple (and fast) dot products
- Same philosophy for tensor decay rates: $\Gamma_{\lambda\lambda',ii'}/\Gamma_{old}$ (necessary for e.g. reweighing branching ratios)

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NP & FF-independent

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FF uncert' — *NP & FF-independent*

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FF parameterizations

$$|M|^2 = \left(C_i C_{i'}^\dagger\right)\, \left(\delta a_\lambda \delta a_{\lambda'}\right) \left(FF_{\alpha,\lambda} FF^\dagger_{\alpha',\lambda'}\right) \left(\mathscr{M}_{\alpha,i} \mathscr{M}^\dagger_{\alpha',i'}\right)$$

NP   FF uncert'   NP & FF-independent

- Scalar event weight is $W = |M_{new}|^2 / |M_{old}|^2$
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HAMMER'S IDEA

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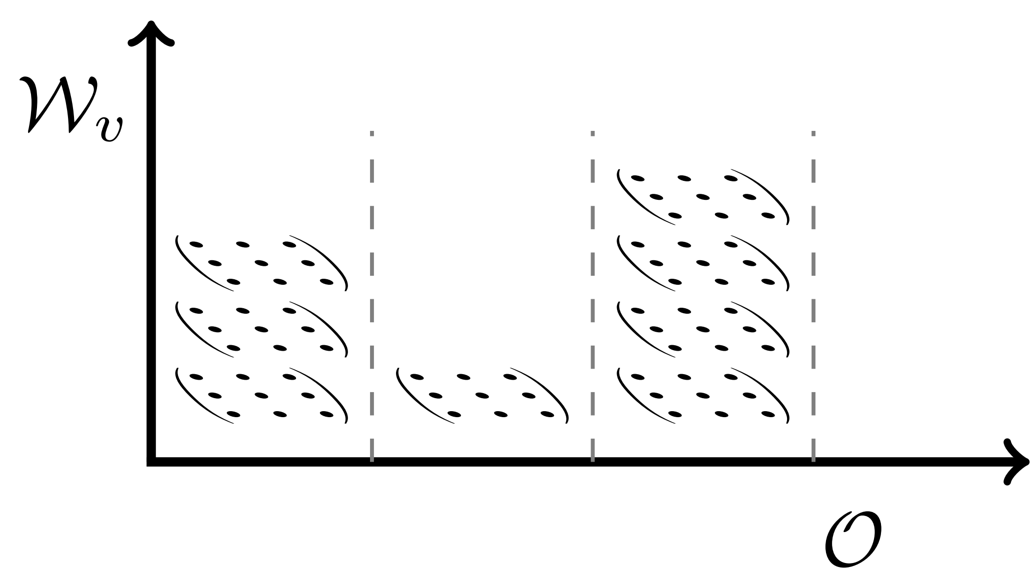
- Event reweighing alone brings you only so far:

$$N_{\text{weights}} \sim N_{\text{events}} * (N_{\text{variations}})^D$$

Still very large (→ N_{events}) *Weight tensorialization helps here* (→ $N_{\text{variations}}$)

- Large number of computations for large statistics samples
- For binned analyses further help is possible: Tensor histograms!
  - Bin tensors directly (weights and squared weights) and collapse to conventional histogram at the end when contracting with external vectors



- Trade $N_{\text{events}} \rightarrow N_{\text{bins}}$, space for speed

WHAT'S NEXT?

AKA: what do you really need from us?

NEAR FUTURE DIRECTIONS

- Improving the modeling of the "X" in $B \to D^{(*)}\ell\nu X$:
  - Theoretical work needed is in progress
    - On-shell vs off-shell resonance heavy meson form factors (two independent approaches ongoing see Florian's talk)
  - Goal is to get to modeling $B \to D^{(*)}\ell\nu\pi(+\pi)$ in terms of
    - Form factors of known resonances defined "on the pole" controlling their contributions (on-shell and off-shell)
    - "UV" non-resonant contribution parameterized by another form factor suppressed by $v^{(\prime)} \cdot p_\pi/\Lambda$, with $\Lambda \sim 1\text{GeV}$
    - A bunch of hadronic coupling constants controlling interactions of pions (and η, ρ) with D,D*,D** (need to be measured)
  - "Cocktail" model based on EFT (chiral and heavy quark symmetries) → in principle systematically improvable (although # parameters may get out of hand quickly…)

NEAR FUTURE DIRECTIONS

- $B \to D^{(*)}\ell\nu\pi(+\pi)$ modeling will be added into HAMMER → can use independently of how MC samples were generated (as long as they cover phase space)
- HAMMER implementation → v2.0:
  - Will require some code restructuring (but no backward compatibility breaks)
  - Tackling interference between different D** requires some "plumbing" work to maintain performance
    - Bonus: this code restructuring will also improve performance for scans on a reduced set of parameters (Wilson coeffs, Form Factor params). Already a feature request

NEAR FUTURE DIRECTIONS

- Radiative semileptonic $B \to D^{(*)} \ell \nu + \gamma$
  - Current HAMMER procedure: use the 4-momenta before γ radiation by "undoing" PHOTOS emissions
  - Plan to transition to different model:
    - Don't undo PHOTOS but reweigh 0γ, 1γ, … with QED corrected amplitudes
    - Go beyond soft photon approx and include structure dependent contributions: 4 unknown functions, constrained by HQET (MP Wise Trickle 2110.13154)
    - Only Belle II can measure γ spectra (with enough statistics) Test of HQET in different regime (+ help for LHCb)

WHAT ELSE?

- Is there something that you would like to do with HAMMER but cannot be presently done?
- Processes? Form Factors? Decay Modes?
- Is the current API sufficient for integration in Belle II software framework?
- Is there a need for more programmatic access to tensor histograms?
- Specific performance issues needing improvement?

(We can always be reached at hammer-support@lbl.gov)