V_{cb} IN 3 YEARS AND BEYOND

Paolo Gambino

Università di Torino & INFN, Torino

2023 Belle II Physics Week, V_{cb} workshop
KEK 30.10-2.11, 2023

A LOT OF PROGRESS!

- The last 5-6 years have seen a ***burst of activity*** in semileptonic B decays
- Many new experimental analyses by Belle, Belle II, BaBar, LHCb incl and excl
- New pert calculations at $O(\alpha_s^3)$ by Fael et al. crucial progress for inclusive V_{cb}
- 3 new lattice calculations of $B \to D^*$ form factors beyond $w = 1$, inclusive on the lattice, new $B \to \pi$, …
- Many phenomenological studies with interesting ideas (RPI methods for incl, HQET studies of form factors, …)
- There is clear appreciation that ~1% uncertainties require a new approach
- Not glorious but ***work that needs to be done!*** (Bob)

Measurement of Angular Coefficients of $\bar{B} \to D^* \ell \bar{\nu}_\ell$: Implications for $|V_{cb}|$ and Tests of Lepton Flavor Universality

We measure the complete set of angular coefficients J_i for exclusive $\bar{B} \to D^* \ell \bar{\nu}_\ell$ decays ($\ell = e, \mu$). Our analysis uses the full $711\,\mathrm{fb}^{-1}$ Belle data set with hadronic tag-side reconstruction. The results allow us to extract the form factors describing the $B \to D^*$ transition and the Cabibbo-Kobayashi-Maskawa matrix element $|V_{\mathrm{cb}}|$. Using recent lattice QCD calculations for the hadronic form factors, we find $|V_{\mathrm{cb}}| = (41.0 \pm 0.7) \times 10^3$ using the BGL parameterization, compatible with determinations from inclusive semileptonic decays. We search for lepton flavor universality violation as a function of the hadronic recoil parameter w, and investigate the differences of the electron and muon angular distributions. We find no deviation from Standard Model expectations.

Belle [2310.20286]

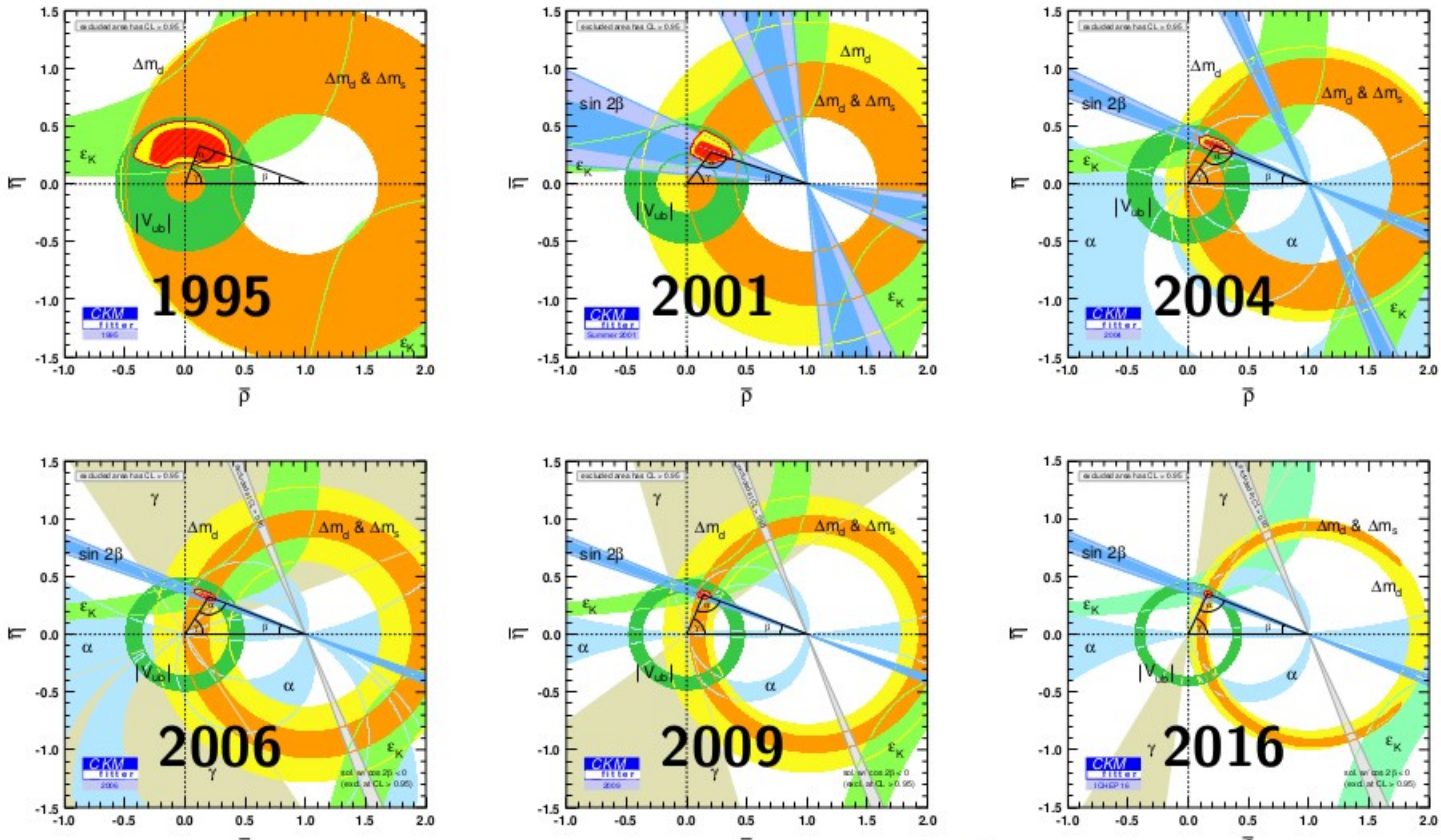
The New York Times

THE STANDARD MODEL COLLAPSES!

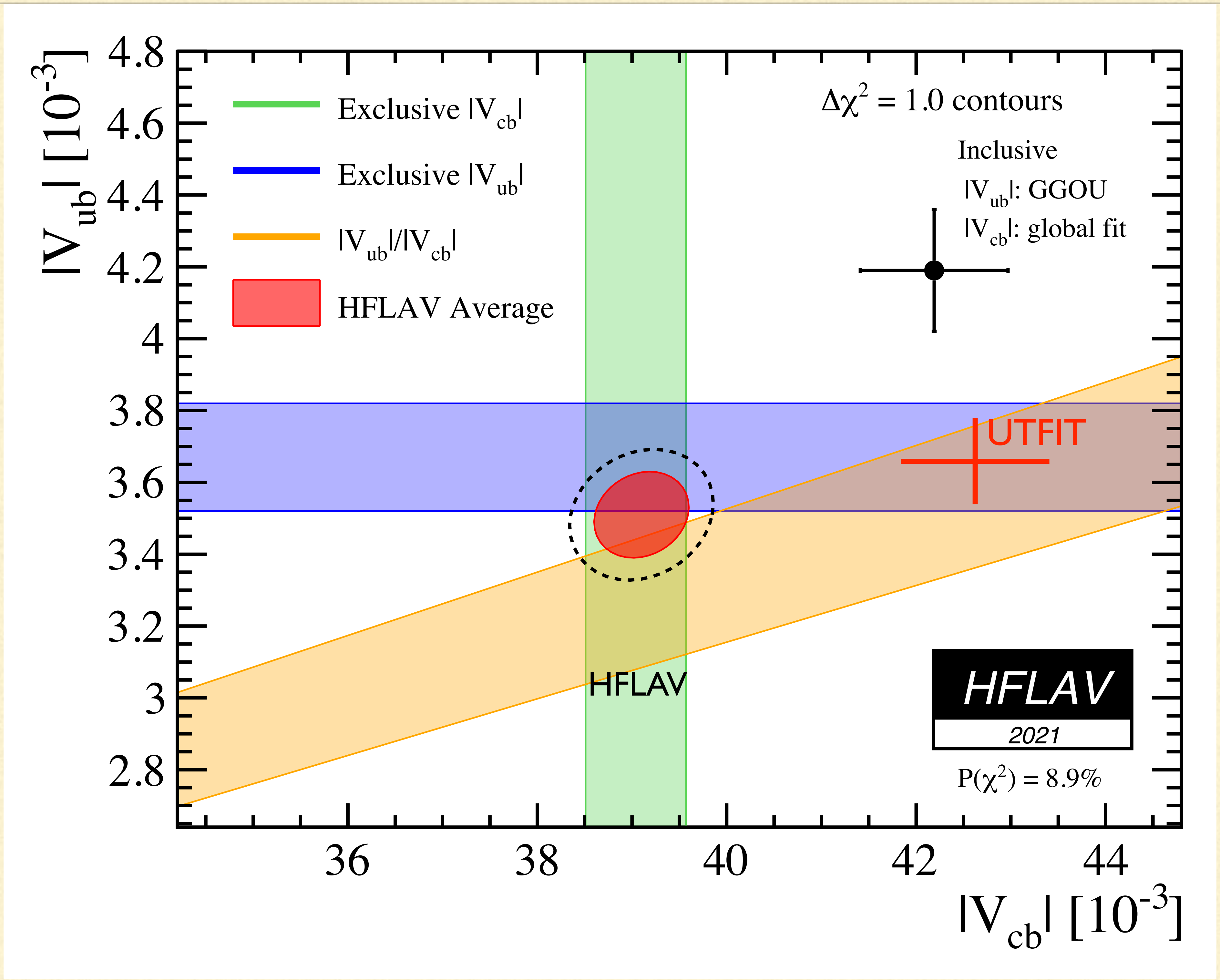
Progress over the years

Vale Silva

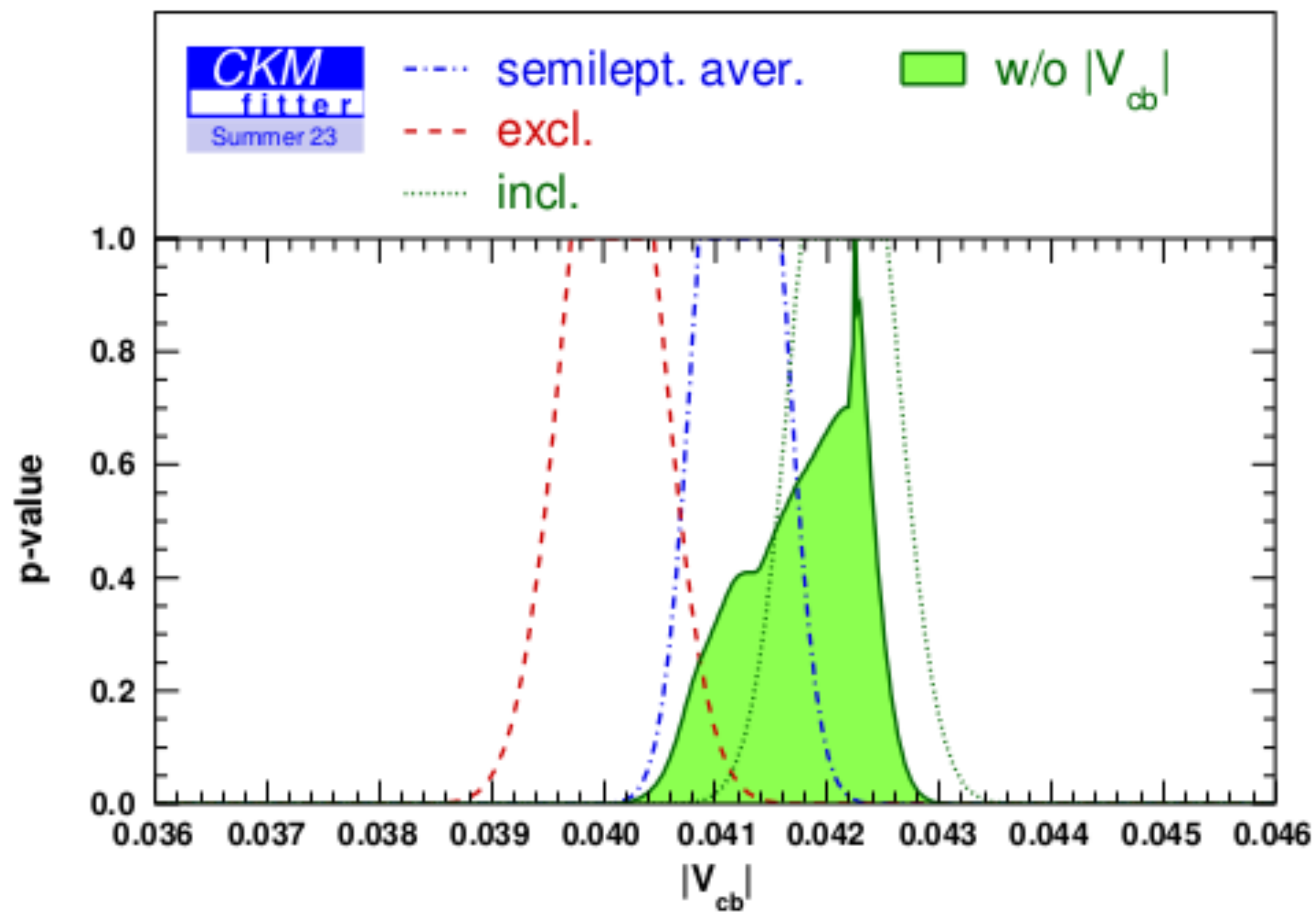
→ Long road for a better theoretical control (e.g., Lattice QCD), and more accurate data (LEP, KTeV, NA48, BaBar, Belle, CDF, DØ, LHCb, CMS, ...)



Since many years the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ diverge



Do we believe these errors?


CKM fitter
Summer 23
semilept. aver.
excl.
incl.
w/o $|V_{cb}|$
p-value
1.0
0.8
0.6
0.4
0.2
0.0
0.036
0.037
0.038
0.039
0.040
0.041
0.042
0.043
0.044
0.045
0.046
$|V_{cb}|$

Some reasons $|V_{cb}|$ matters

- $|V_{cb}|$ important to assess if there is an ε_K tension, predict $K \to \pi\nu\bar{\nu}$, $B \to (X)\ell\bar{\ell}$

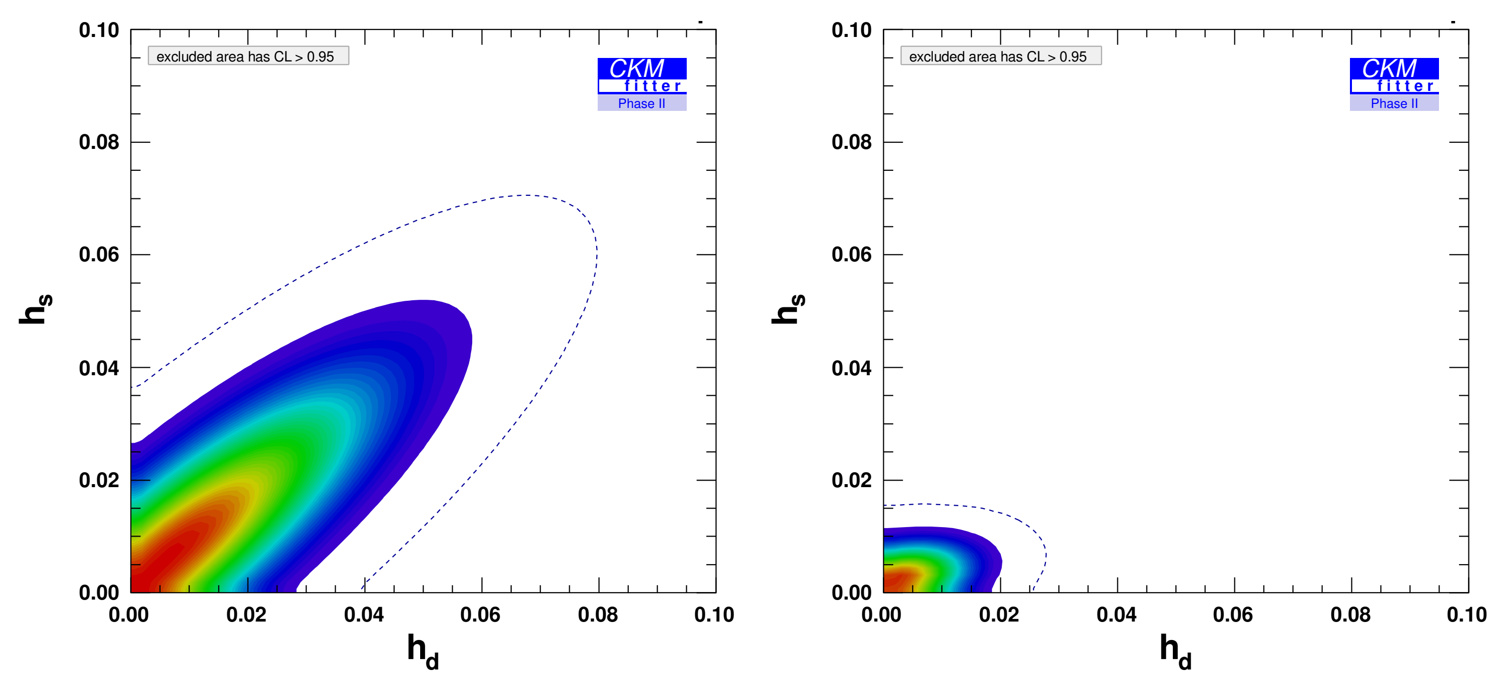
  SM predictions involve A^4, so 5% in $|V_{cb}|$ yields 20%

- The $b \to c\tau\bar{\nu}$ data should make $|V_{cb}|$ much better understood — are we there yet?

  To understand the τ mode thoroughly, must understand the e, μ modes better

- Recently: $|V_{cb}|$ uncertainty limits future improvements in the sensitivity to NP in B and B_s mixing

  "Phase II" (LHCb upgrade 2 and Belle II upgrade) with / without $|V_{cb}|$ uncertainty, maybe early 40s

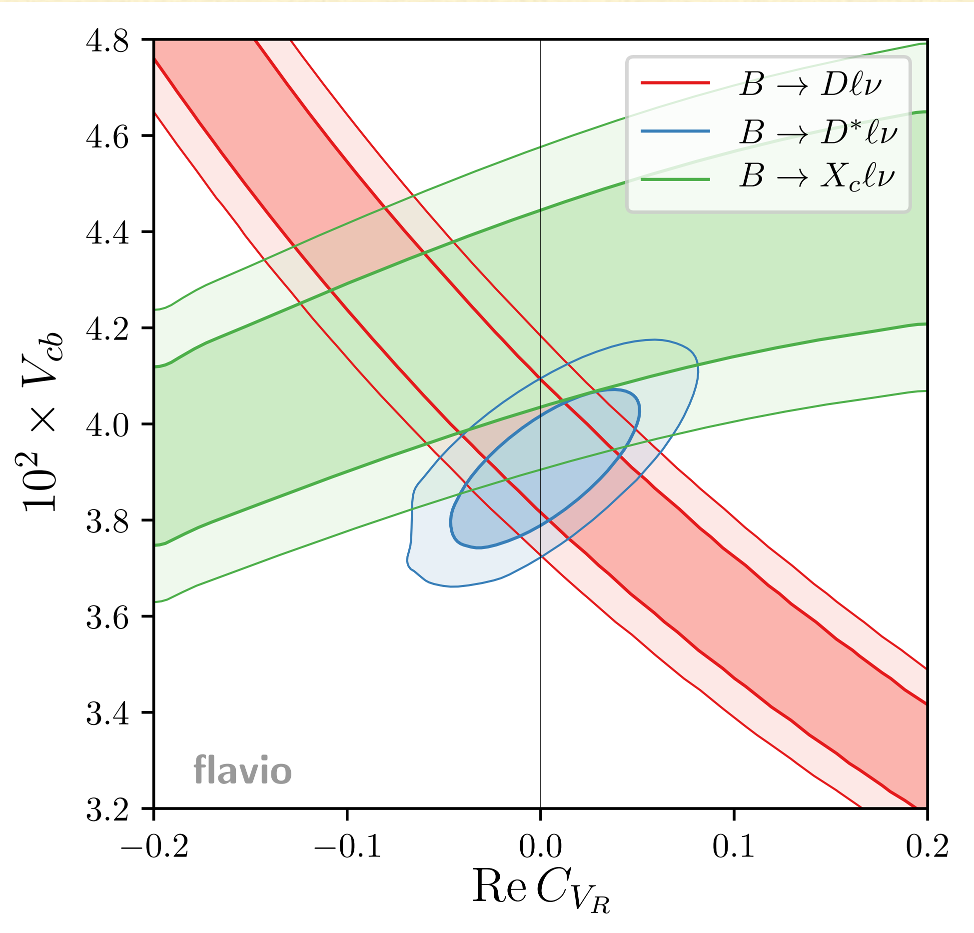


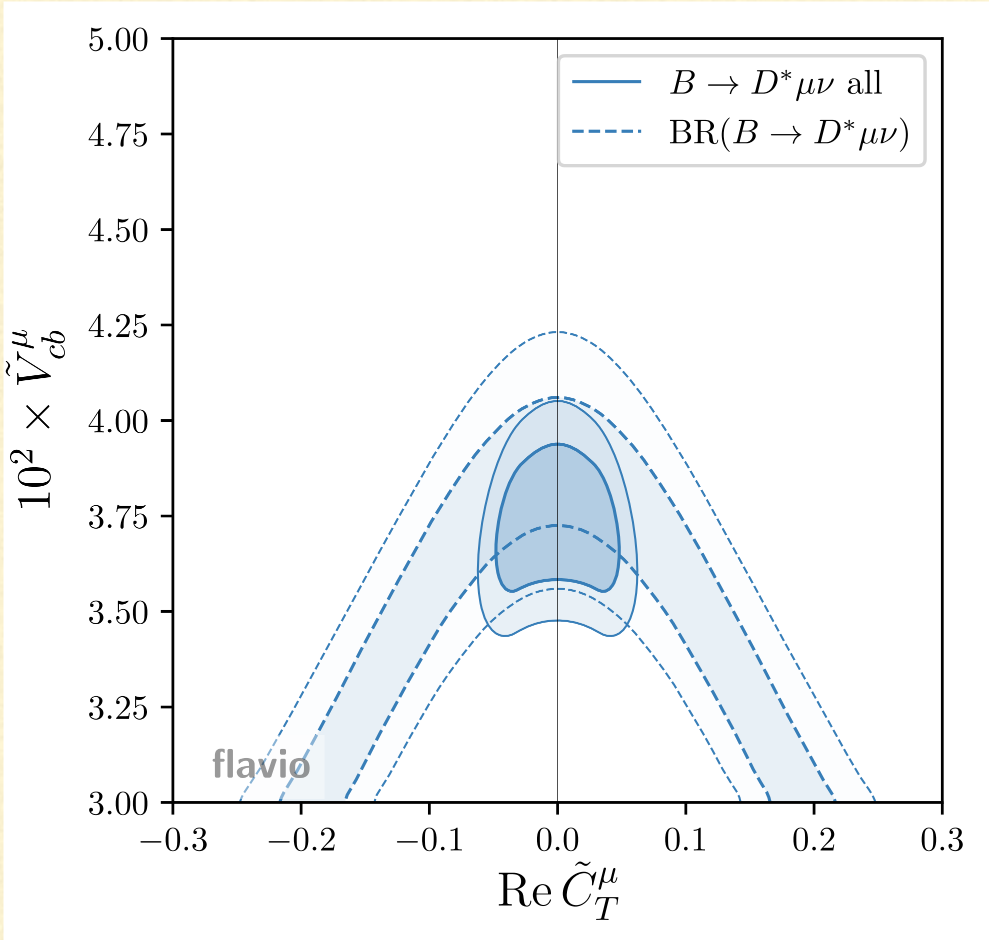
[Charles, Descotes-Genon, ZL, Monteil, Papucci, Trabelsi, Vale Silva, 2006.04824]

V_{cb} puzzle is a limiting factor in indirect NP searches

NEW PHYSICS?

Jung & Straub, 1801.01112

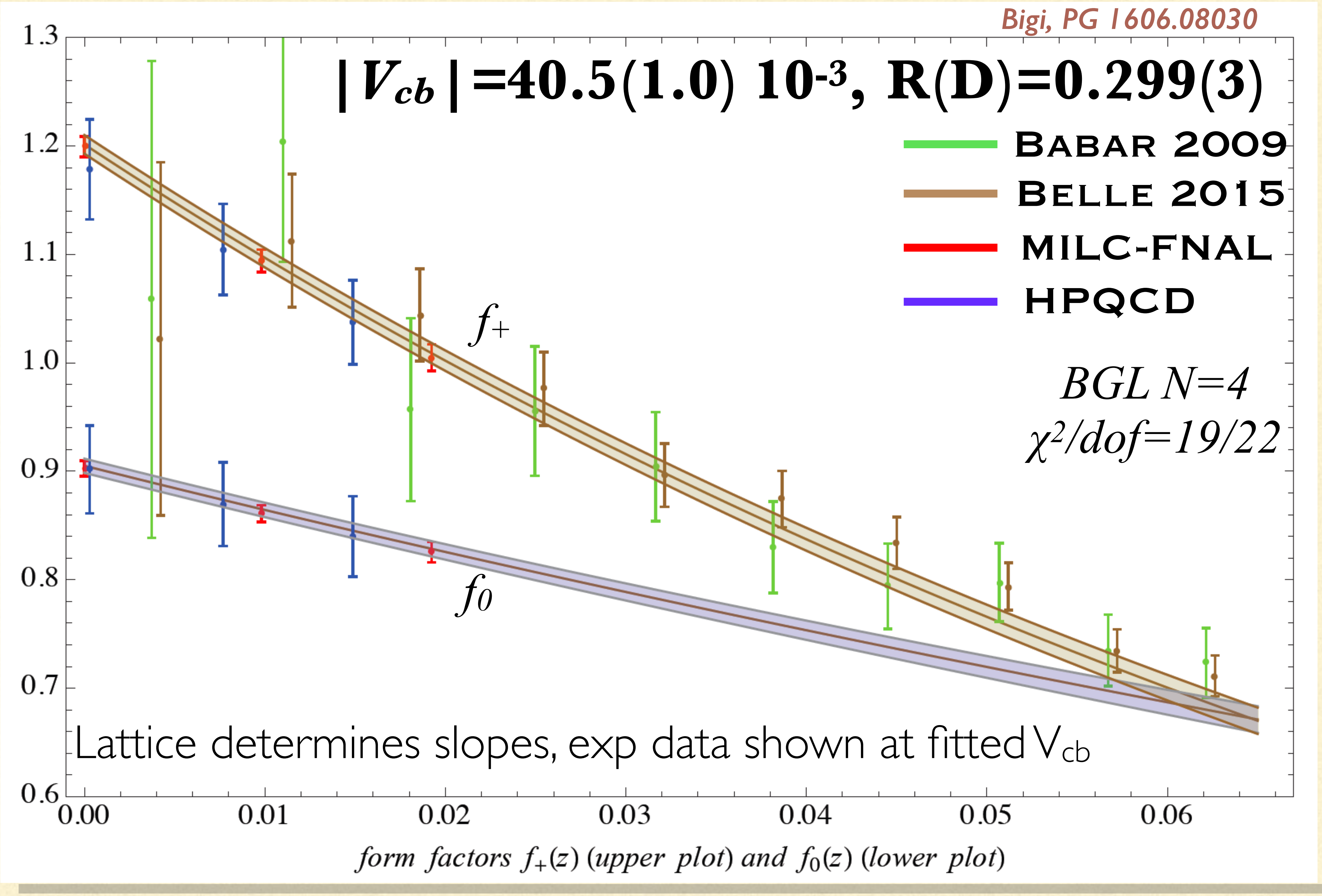




Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub…

EXCLUSIVE DECAYS

LATTICE + EXP IN $B \to D\ell\nu$



R(D)=0.299(3)

1.3σ from exp

FLAG has very similar results

CLN cannot fit both ff

D'AGOSTINI BIAS

Watch out!

Standard χ^2 fits sometimes lead to paradoxical results

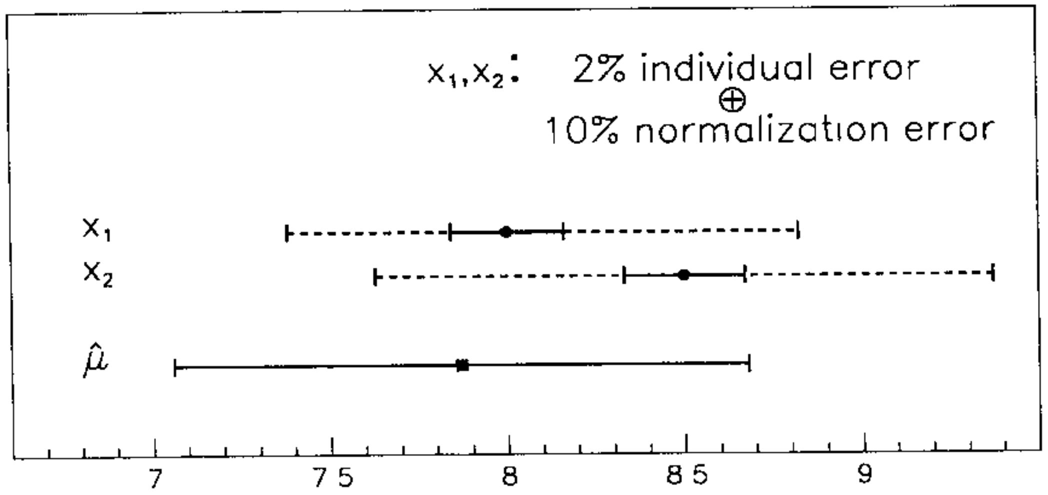


Fig. 1. Best estimate of the true value from two correlated data points, using in the χ^2 the empirical covariance matrix of the meaurements. The error bars show individual and total errors.

$$\hat{k} = \frac{x_1\sigma_2^2 + x_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (x_1 - x_2)^2\sigma_f^2},$$

Many exp systematics are highly correlated. Bias is stronger with more bins

On the use of the covariance matrix to fit correlated data

G. D'Agostini

Dipartimento di Fisica, Università "La Sapienza" and INFN, Roma, Italy

(Received 10 December 1993; revised form received 18 February 1994)

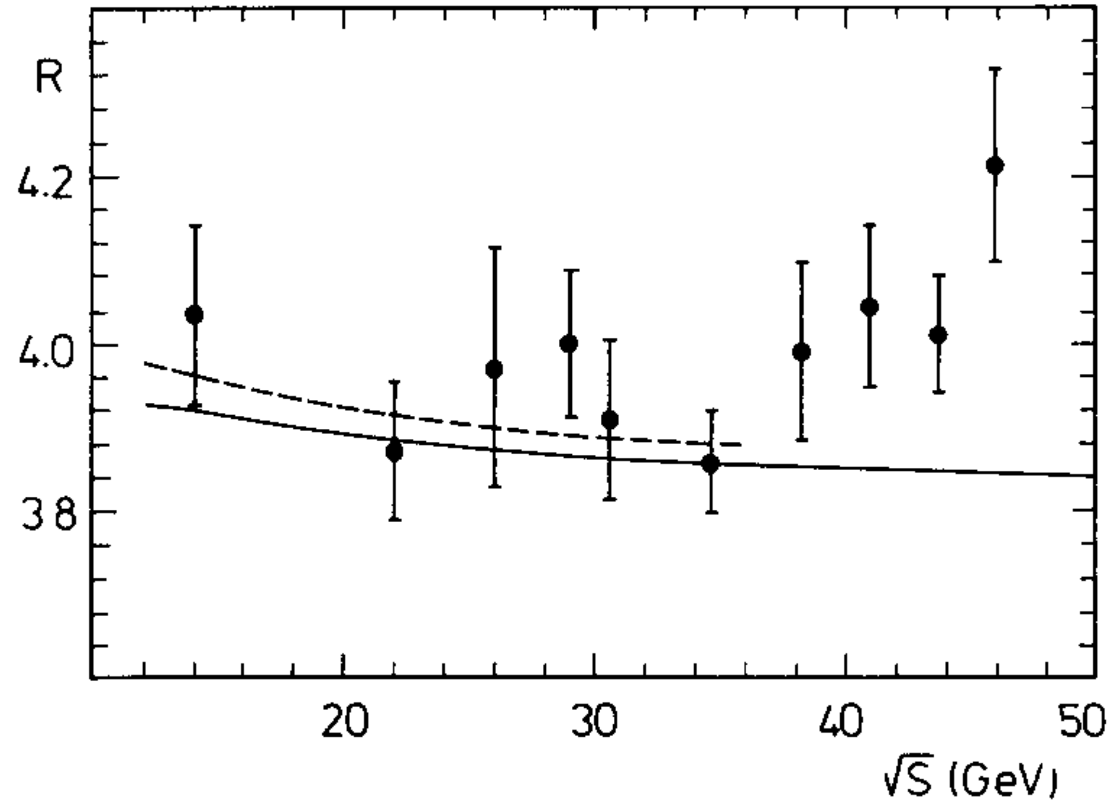
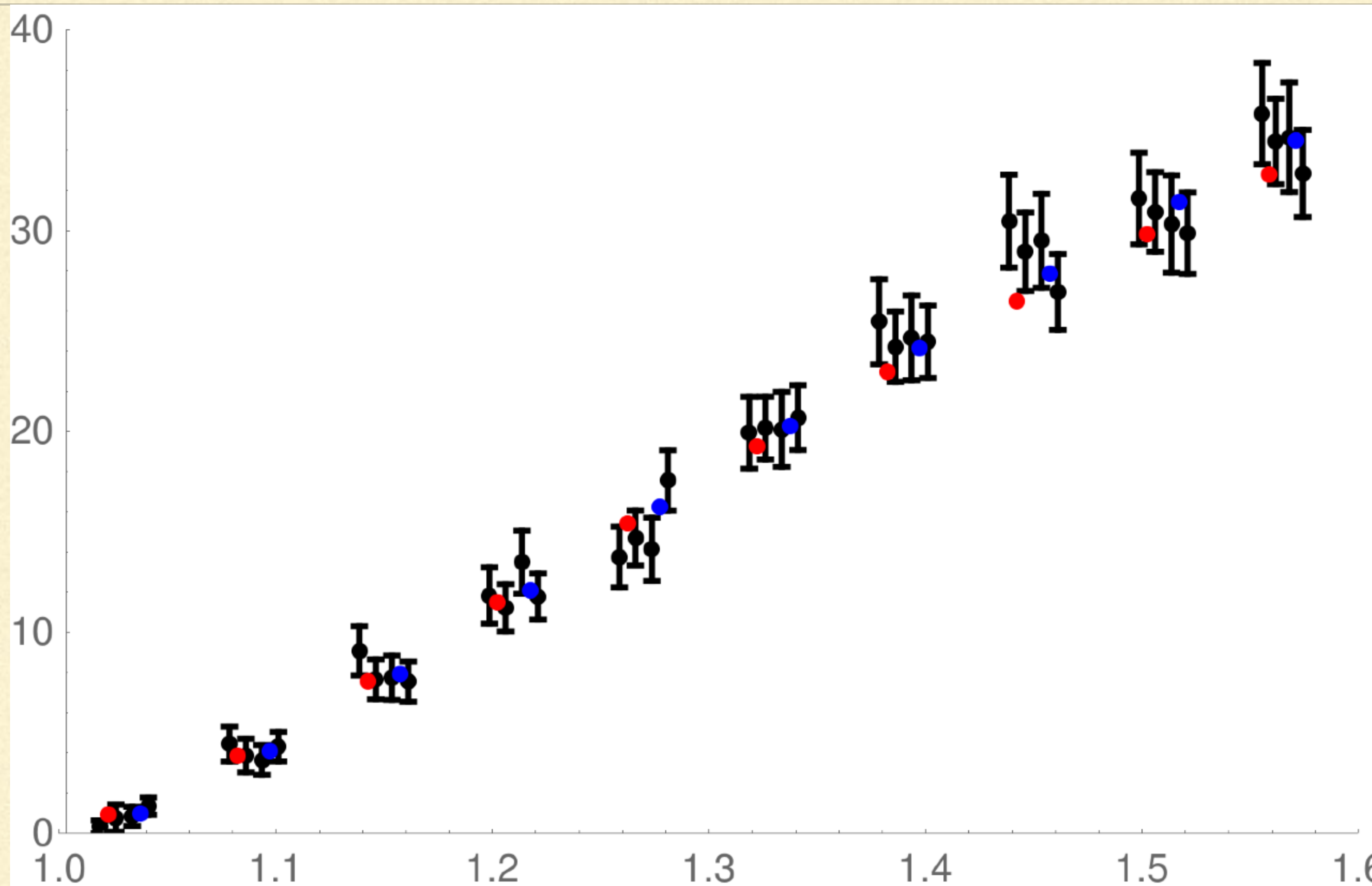


Fig. 2. R measurements from PETRA and PEP experiments with the best fits of QED + QCD to all the data (full line) and only below 36 GeV (dashed line). All data points are correlated (see text).

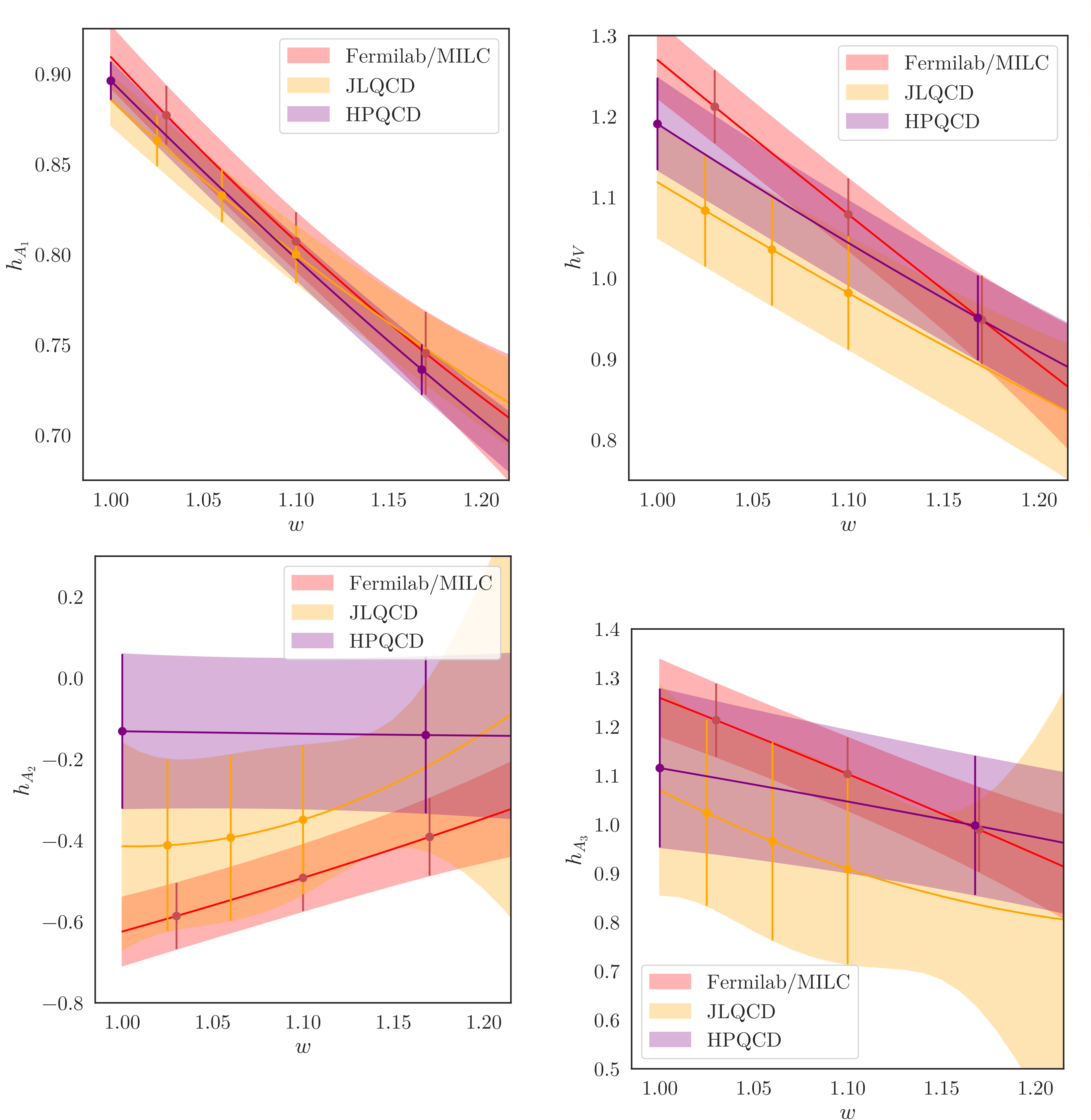
w DISTRIBUTION for $B \to D\ell\nu$



Belle 2015 consider 4 channels $(B^{0,+}, e, \mu)$ for each bin.
Average (red points) usually lower than all central values. **_D'Agostini bias?_**
Blue points are average of normalised bins.

Standard fit to Belle15+FNAL+HPQCD: $|V_{cb}| = 40.9(1.2)\,10^{-3}$ Jung, PG
Fit to normalised bins+width Belle15+FNAL+HPQCD: $|V_{cb}| = 41.9(1.2)\,10^{-3}$

LATTICE FORM FACTORS FOR $B \to D^*$



No major discrepancy

but differences may get amplified in certain combinations of ffs

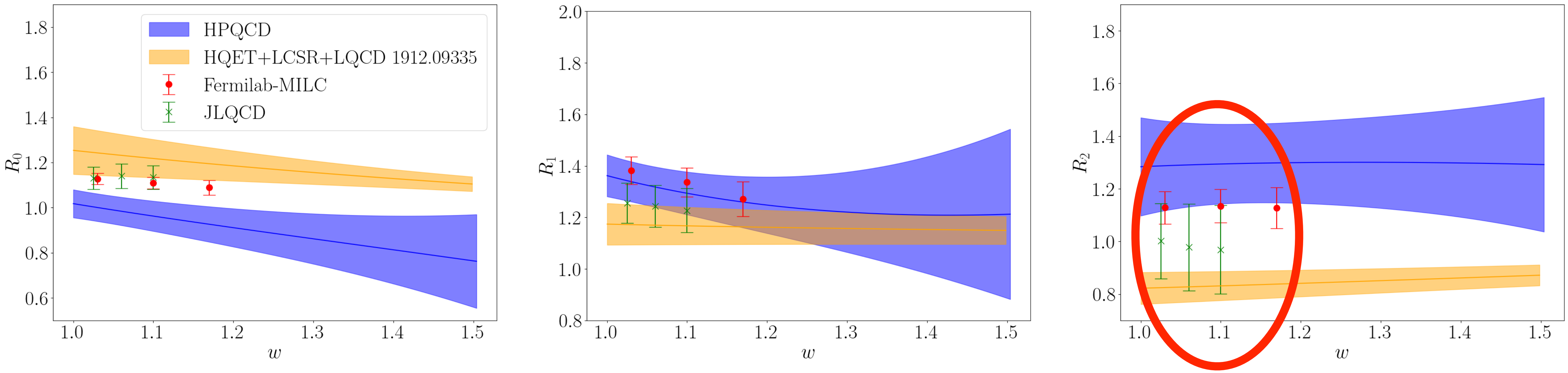
However, ratios do not seem to agree so well $\rightarrow$ correlations?

Judd

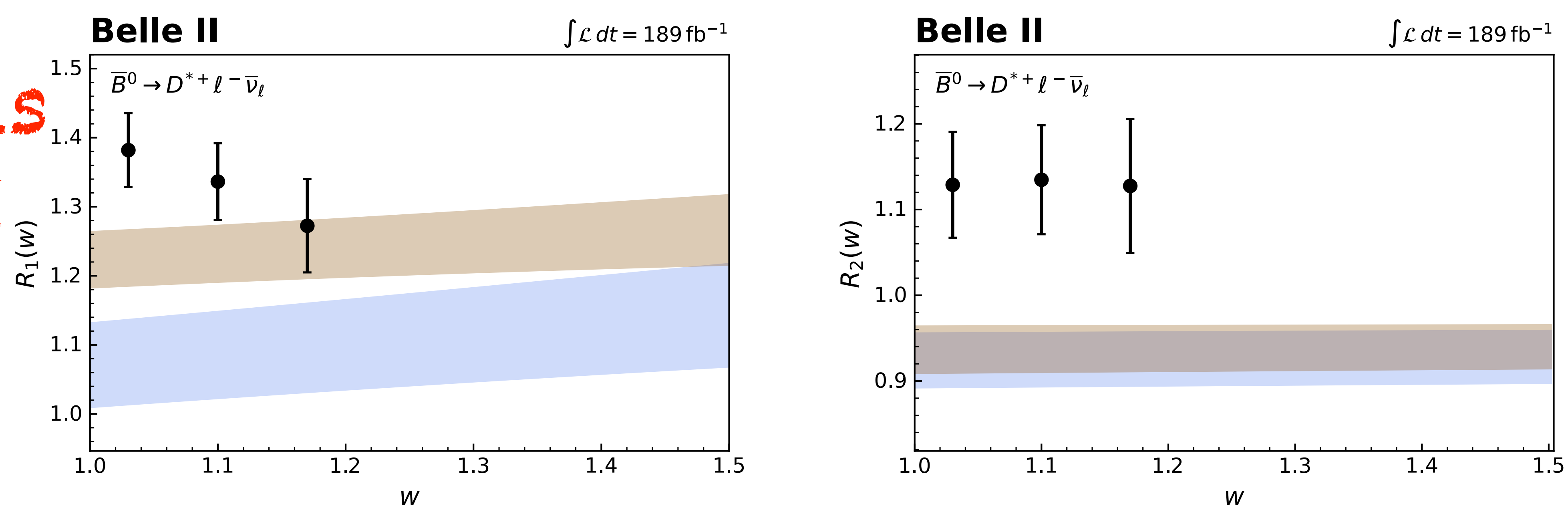
$$R_0 = \frac{1}{1+r}\left(w + 1 + w\frac{rh_{A_2} - h_{A_3}}{h_{A_1}} - \frac{h_{A_2} - rh_{A_3}}{h_{A_1}}\right), \quad R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{rh_{A_2} + h_{A_3}}{h_{A_1}}$$

Here yellow band is HQE analysis by Bordone et al.

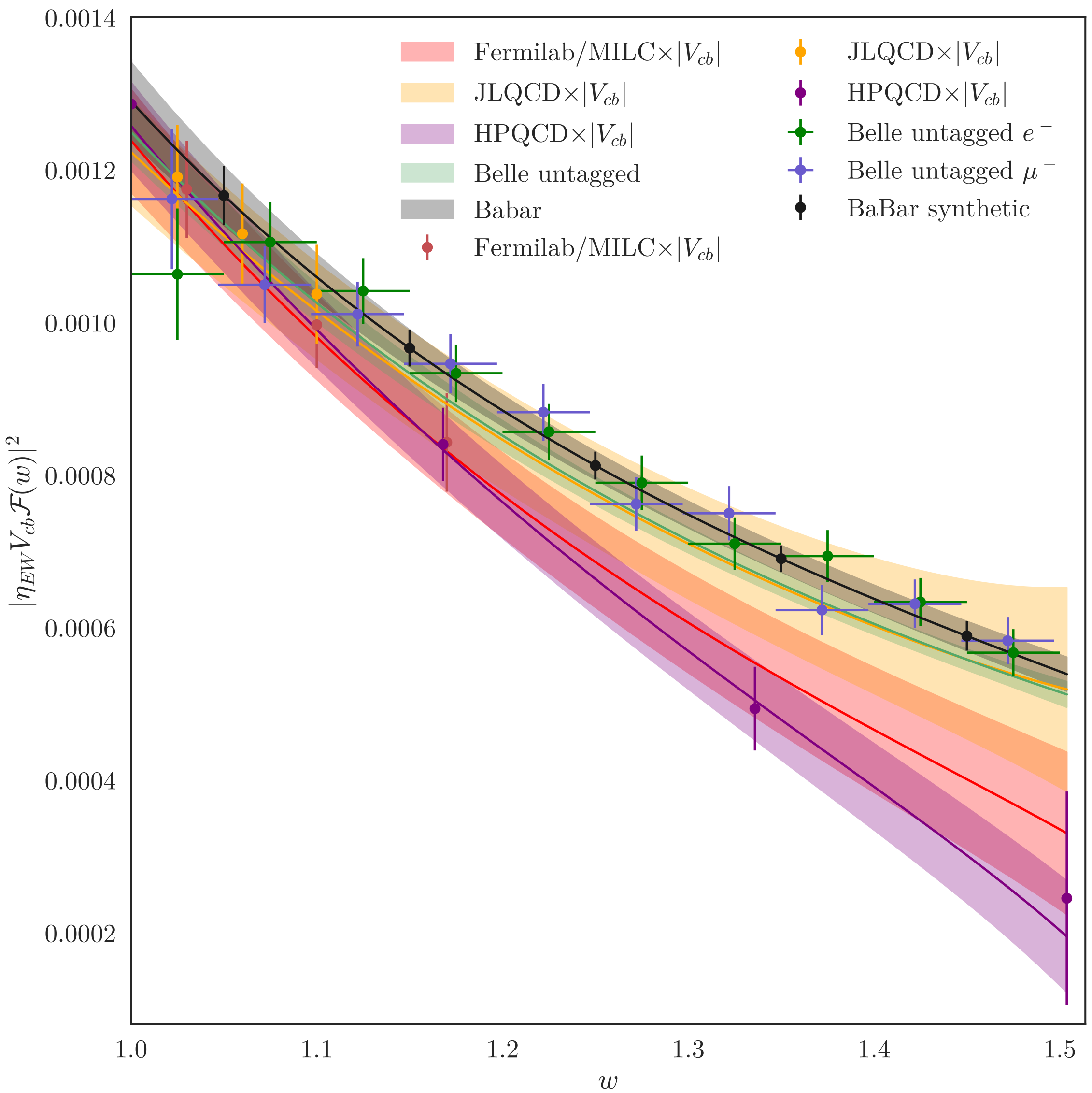
We need to check consistency with heavy quark expansion

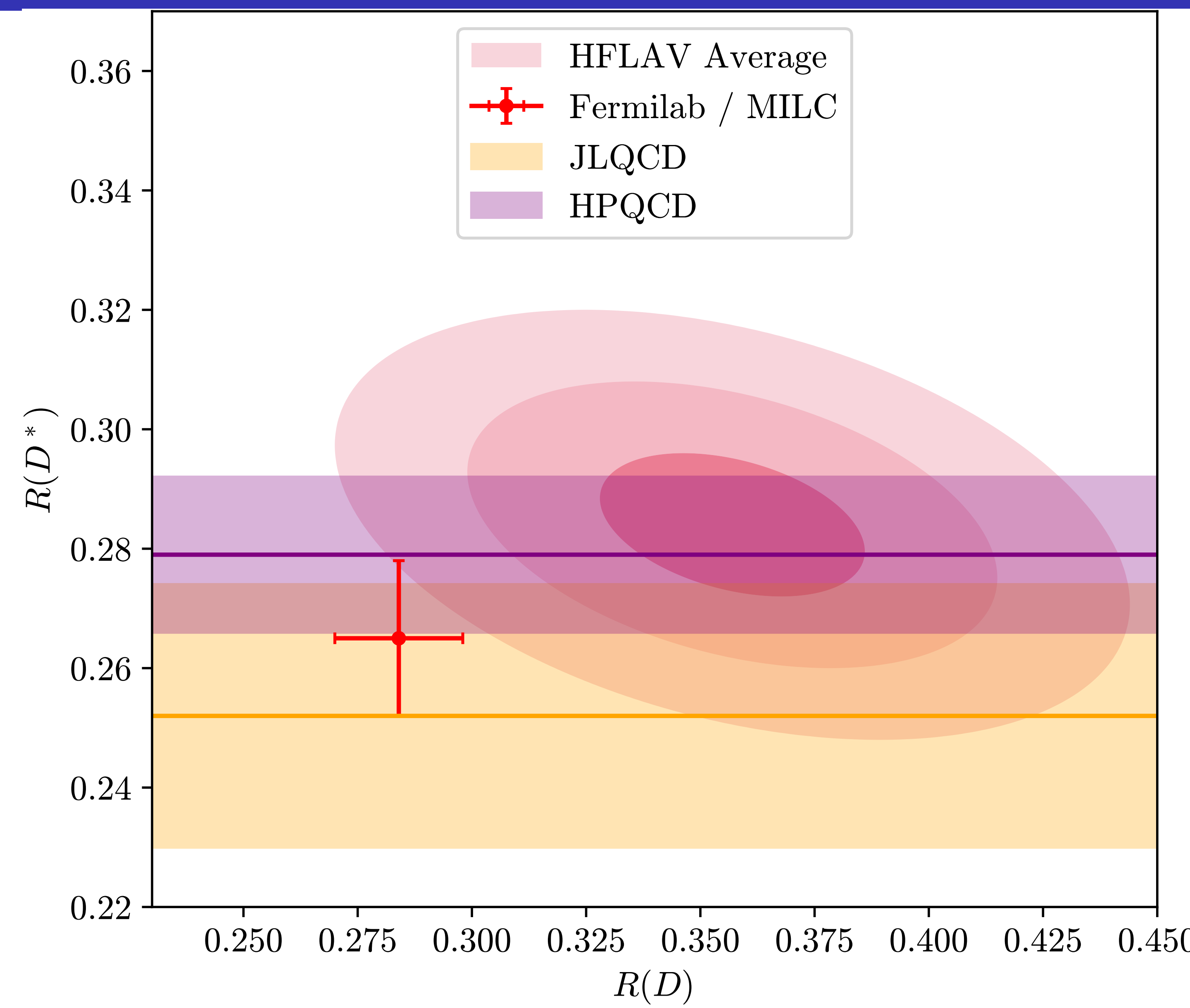


Exp agrees with HQE



Semileptonic B decays on the lattice: HPQCD





$$|V_{cb}|^{\mathrm{HPQCD}} = 39.31(74) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{JLQCD}} = 39.19(90) \times 10^{-3}$$

$$|V_{cb}|^{\mathrm{FerMILC}} = 38.17(85) \times 10^{-3}$$

$$R(D^*)^{\mathrm{HPQCD}} = 0.279(13)$$

$$R(D^*)^{\mathrm{JLQCD}} = 0.252(22)$$

$$R(D^*)^{\mathrm{FerMILC}} = 0.265(13)$$

- Fit to Belle dataset WITH the Coulomb factor

$B \to D^*\ell\bar{\nu}$: Things to look into

Issue seems to be with h_{A2}, h_{A3} - need to look for possible biases in chiral-continuum fit.

- Are we including enough kinematic terms/estimating truncation errors correctly/using broad enough priors?
- Are we including discretisation effects consistently? e.g. do different parameterisations allow for $(ap)^2$ effects?

Other things we can improve:

- HPQCD and JLQCD have been conservative with lattice heavy quark mass - can get up to the B in future updates.
- HPQCD can use time reversed 3pt functions together with lattice rotations of current setup, to improve statistics.
- Fermilab heavy quark + HISQ calculation in not too distant future.
- Work to incoorporate HQE information into HPQCD chiral continuum extrapolation.

Bottom line: 2 out of 3 lattice analyses in tension with both exp data and HQE, but they can explain $R(D^*)$: New Physics in the light lepton sector?

Can we reproduce everything introducing NP in light leptons?

The DM FF offer the unique possibility to employ NP in light leptons to address anomalies (forbidden in other scenarios due to CKM limits)

Could this fix the issue?

Only evidence found for g_{V_L}; however F_L^ℓ and A_{FB}^ℓ are ratios, hence insensitive to it!

The absence of an hint for scalar/tensor WCs is due to more precise measurements in light lepton channel, together with m_ℓ suppression in interference terms with SM

$$g_{V_L} = -0.054 \pm 0.015$$

$$g_{V_R} \in [-0.04, 0.01]$$

$$g_{S_L} \in [-0.07, 0.02]$$

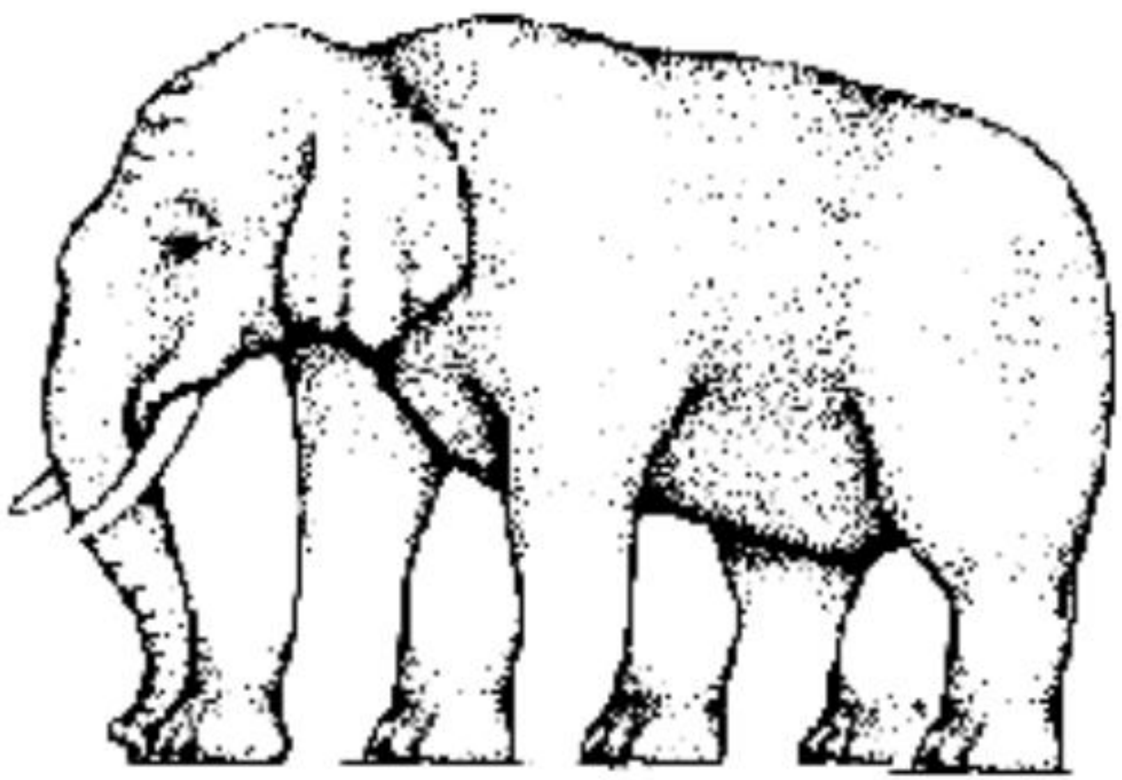
$$g_{S_R} \in [-0.05, 0.03]$$

$$g_T \in [-0.01, 0.02]$$

$\Rightarrow$ If the FF prediction for F_L^ℓ and A_{FB}^ℓ does not reproduce data, this cannot be fixed by introducing NP effects in light leptons as could be done for $R(D^*)$!

2305.15457
MF, Blanke, Crivellin, Iguro, Nierste, Simula, Vittorio

The how-to-fit saga



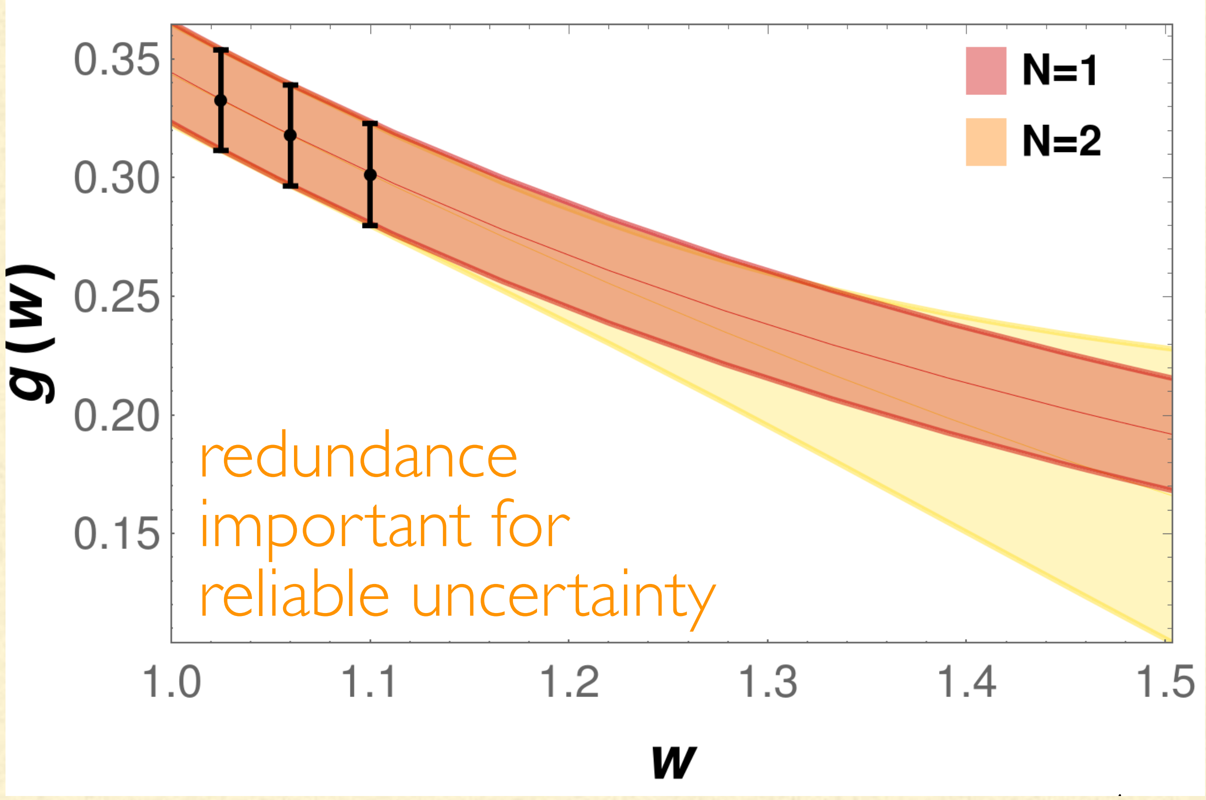
"With four parameters I can fit an elephant, and with five
I can make him wiggle his trunk." (John von Neumann)

Overfitting? Truncation orders? Additional inputs/constraints from HQET, LQCD, unitarity?

Zoltan

Model independence vs overfitting

$$\phi(z) = \sum_i^{\infty} a_i z^i, \qquad \sum_i^{\infty} a_i^2 < 1$$



Where do we truncate the series? How can we include unitarity constraints? These questions are related.

until you have lots of precise data...

Different options with various pro/cons:

1. Frequentist fits with strong χ^2 ***penalty*** outside unitarity; increase BGL order till χ^2_{min} is stable. Can compute CL intervals Bigi, PG, 1606.08030, Jung,Schacht,PG 1905.08209 ***New: Feldman-Cousins***.
2. Frequentist fit with ***Nested Hypothesis Test or AIC*** to determine optimal truncation order: go to order $N+1$ if $\Delta\chi^2 = \chi^2_{min,N} - \chi^2_{min,N+1} \geq 1{,}2$ Check unitarity a posteriori Bernlochner et al, 1902.09553
3. ***Bayesian inference*** using unitarity constraints as prior with BGL Flynn, Jüttner, Tsang 2303.11285 or in the ***Dispersive Matrix approach (which avoids truncation!)***, Martinelli, Simula, Vittorio et al. 2105.02497

While there is probably no best method, viable methods should address overfitting *and* take into account unitarity. Detailed comparison would be very useful.

Issue likely irrelevant with more data and lattice results… but let's do it.

Benchmark procedure steps:

Step 1 : Produce a **large number** of **possible BGL shapes** as true underlying distributions **that respect unitarity** and a **given true $|V_{cb}|$** value

Step 2 : Use these shapes and **produce toys** / replica measurements **with** our **current** (or a future) experimental precision / covariance

Step 3 : Apply the different procedures (NHT, AIC, Feldman/Cousins, stability by eye, …) to determine FFs, $|V_{cb}|_{toy}$ and σ_{toy}

Step 4 : Study pulls of toys : $(|V_{cb}|_{toy} - |V_{cb}|_{true}) / \sigma_{toy}$

OUR BGL FITS

Jung, Schacht, PG in progress

With Belle 2018 only

FNAL/MILC

$|V_{cb}|=39.4(9)\ 10^{-3}$ $(\chi^2_{min} = 50)$ *using only total rate* $|V_{cb}|=42.2^{+2.8}_{-1.7}\ 10^{-3}$

JLQCD

$|V_{cb}|=40.7(9)\ 10^{-3}$ $(\chi^2_{min} = 33)$ *using only total rate* $|V_{cb}|=40.8^{+1.8}_{-2.3}\ 10^{-3}$

HPQCD

$|V_{cb}|=40.4(8)\ 10^{-3}$ $(\chi^2_{min} = 50)$ *using only total rate* $|V_{cb}|=44.4 \pm 1.6\ 10^{-3}$

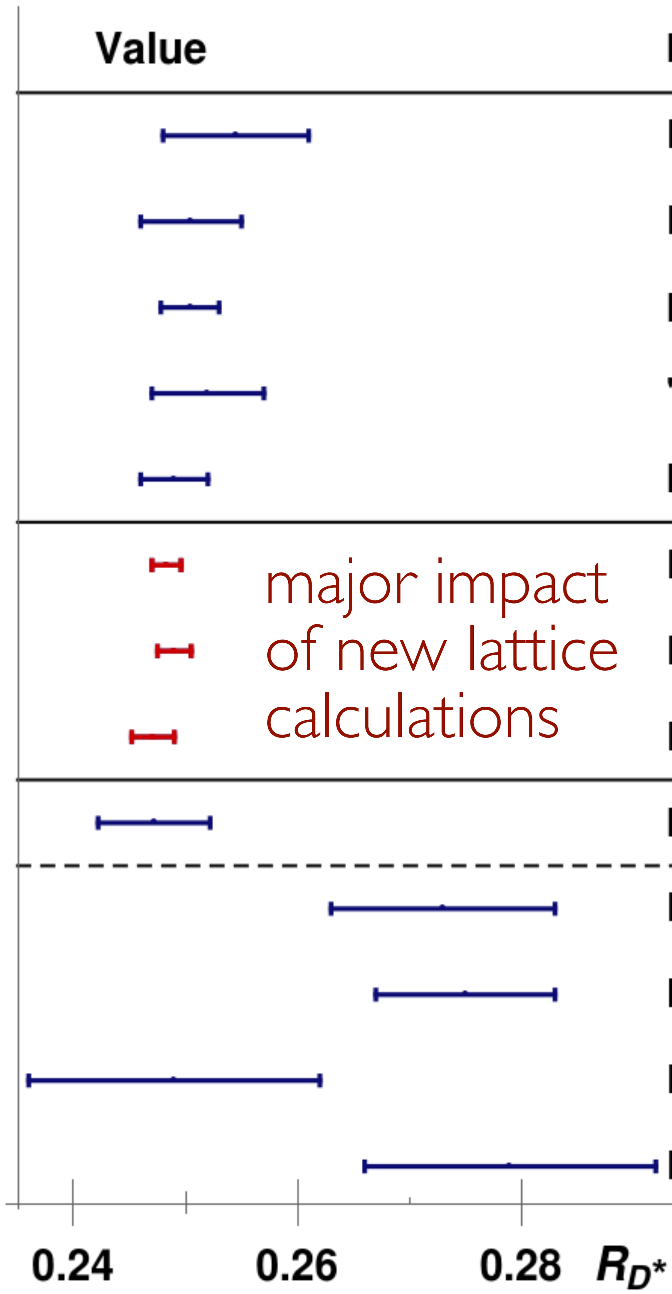
HPQCD and FNAL are not well compatible: adding 16 points increases χ^2 *by 35*

Global BGL fit to Belle18+FNAL+JLQCD+HPQCD data:

$|V_{cb}|=40.3(7)\ 10^{-3}$ $(\chi^2_{min} = 91)$ *using only total rate* $|V_{cb}|=42.4(1.0)\ 10^{-3}$

I would not sell this as our best value

Overview over predictions for $R(D^*)$



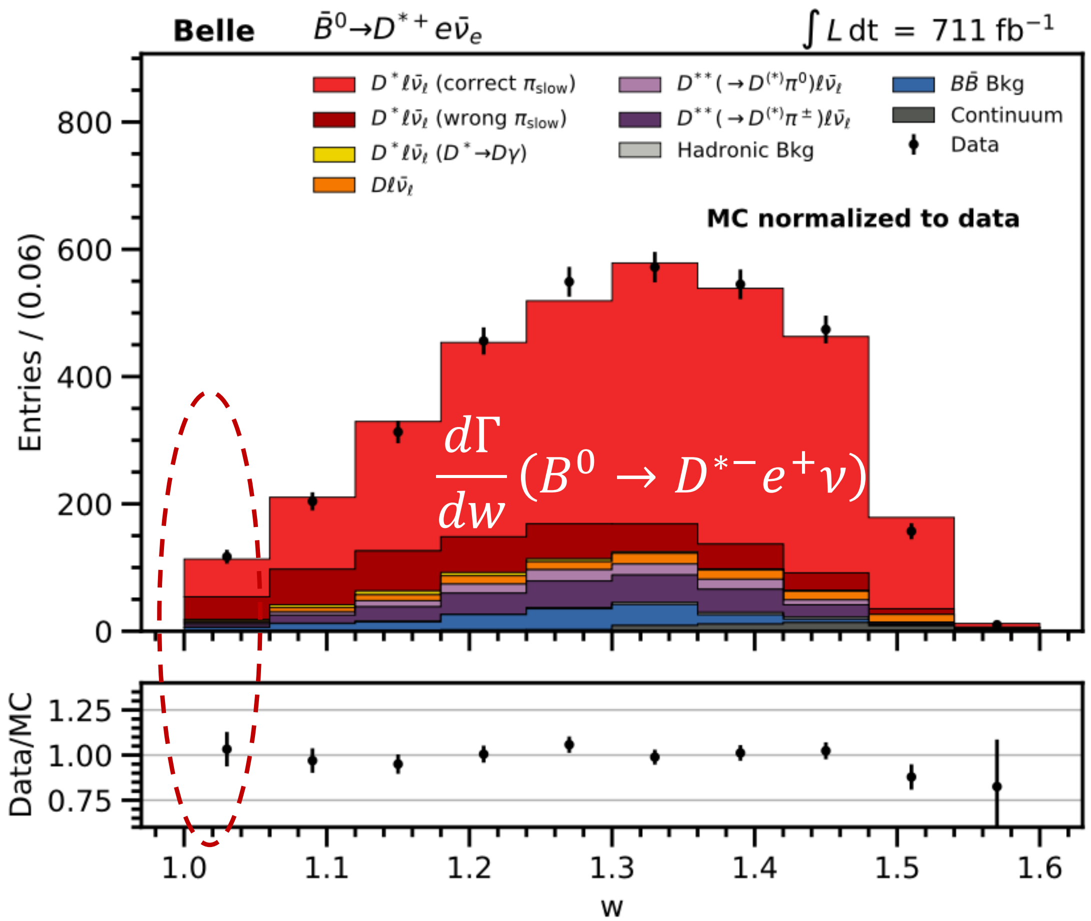
Method	Input Theo	Input Exp	Reference
BGL	Lattice, HQET	Belle'17'18	Gambino et al.'19
BGL	Lattice, HQET	Belle'18	Jaiswal et al.'20
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	Belle'17'18	Bordone et al.'20
"Average"			HFLAV'21
$\mathrm{HQET_{RC}}$@$1/m^2,\alpha_s^{(2)}$	Belle'17'18	Lattice	Bernlochner et al.'22
BGL	Lattice	Belle'18, Babar'19	Vaquero et al.'21v2
BGL	Lattice	Belle'18	JLQCD prel. (MJ)
BGL	Lattice	Belle'18	Davies, Harrison'23
HQET@$1/m_c^2,\alpha_s$	Lattice, LCSR, QCDSR	---	Bordone et al.'20
BGL	Lattice	---	Vaquero et al.'21v2
DM	Lattice	---	Martinelli et al. FNAL/MILC
BGL	Lattice	---	JLQCD prel. (MJ)
BGL	Lattice	---	Davies, Harrison'23

M.Jung

Predictions based only on Fermilab & HPQCD lead to larger R(D*), in better agreement with exp, mostly because of the suppression at high w of the denominator. ***No reason not to use experimental data for a SM test***, especially in presence of tensions in lattice data.

Exclusive $|V_{cb}|$ analyses - overview

- Total and partial rates for $B \to D^{(*)}\ell\nu$ depend on $|V_{cb}|^2$
- Soft QCD enters through form factors $FF(q^2)$ – we can only predict the FF size with lattice QCD near the zero recoil (max q^2) point ($D^{(*)}$ at rest in the B frame)
- The rate there is phase-space suppressed and can't be measured directly (extrapolation)
- **Challenge:** need to measure **both FF shape and overall normalization (BF/lifetime)** to determine $|V_{cb}|$
- In practice, combined experiment+lattice(+BF) fits are used. Different analyses are best adapted to measuring shape versus normalization



PRD 108, 012002 (2023)

Shapes from tagged analyses, BF from untagged analyses

What will the limiting factors be in a few years?

- FF shape information will be improved from both experiment and LQCD; it will remain an important but not dominant uncertainty on $|V_{cb}|$
- Normalization uncertainties (efficiencies, BFs) will be prominent:
  - N_{BB}
  - f_{00} (avoid by combining B^+, B^0 results)
  - D meson BFs
  - Lepton, kaon ID efficiencies

- MC statistics
- Modeling uncertainties

- Residual backgrounds will be important in some analyses:
  - unmeasured $B \to X_c \ell \nu$ modes
  - $B \to \bar{D}^{(*)} D_{(s)}^{(*)}(X)$
  - $e^+e^- \to q\bar{q}$ continuum

Belle II result [2310.01170]: $\mathscr{B}(\bar{B}^0 \to D^{*+}\ell\nu) = (4.922 \pm 0.023 \pm 0.220)\,\%$

w.a. $4.97 \pm 0.12\,\%$

still a long way to improve significantly…

The 'Gap'

Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \to D\,\ell^+\,\nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \to D^*\,\ell^+\,\nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \to D_1\,\ell^+\,\nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \to D_2^*\,\ell^+\,\nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \to D_0^*\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \to D_1'\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \to D\pi\pi\,\ell^+\,\nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \to D^*\pi\pi\,\ell^+\,\nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \to D\eta\,\ell^+\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to D^*\eta\,\ell^+\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Consistently counted
$D_1 \to D\pi\pi$ contribution

$$\mathcal{B}(B^+ \to X_c\,\ell\bar{\nu}_\ell) - \mathcal{B}(B^+ \to D^{(*)}\,\ell\bar{\nu}_\ell) - \mathcal{B}(B^+ \to D^{**}(1P)\,\ell\bar{\nu}_\ell) - \mathcal{B}(B^+ \to D^{(*)}\pi\pi\,\ell\bar{\nu}_\ell) = \mathcal{B}(B^+ \to X_c^{\text{Gap}}\,\ell\bar{\nu}_\ell)$$

$$\rightarrow \boxed{\mathcal{B}(B^+ \to X_c^{\text{Gap}}\,\ell\bar{\nu}_\ell) = \left(0.8 \pm 0.5\right) \times 10^{-2}}$$

Since we have no clue what populates this 'gap' a 100% error seems prudent, a possible candidate is $B \to D^{(*)}\eta\ell\bar{\nu}_\ell$

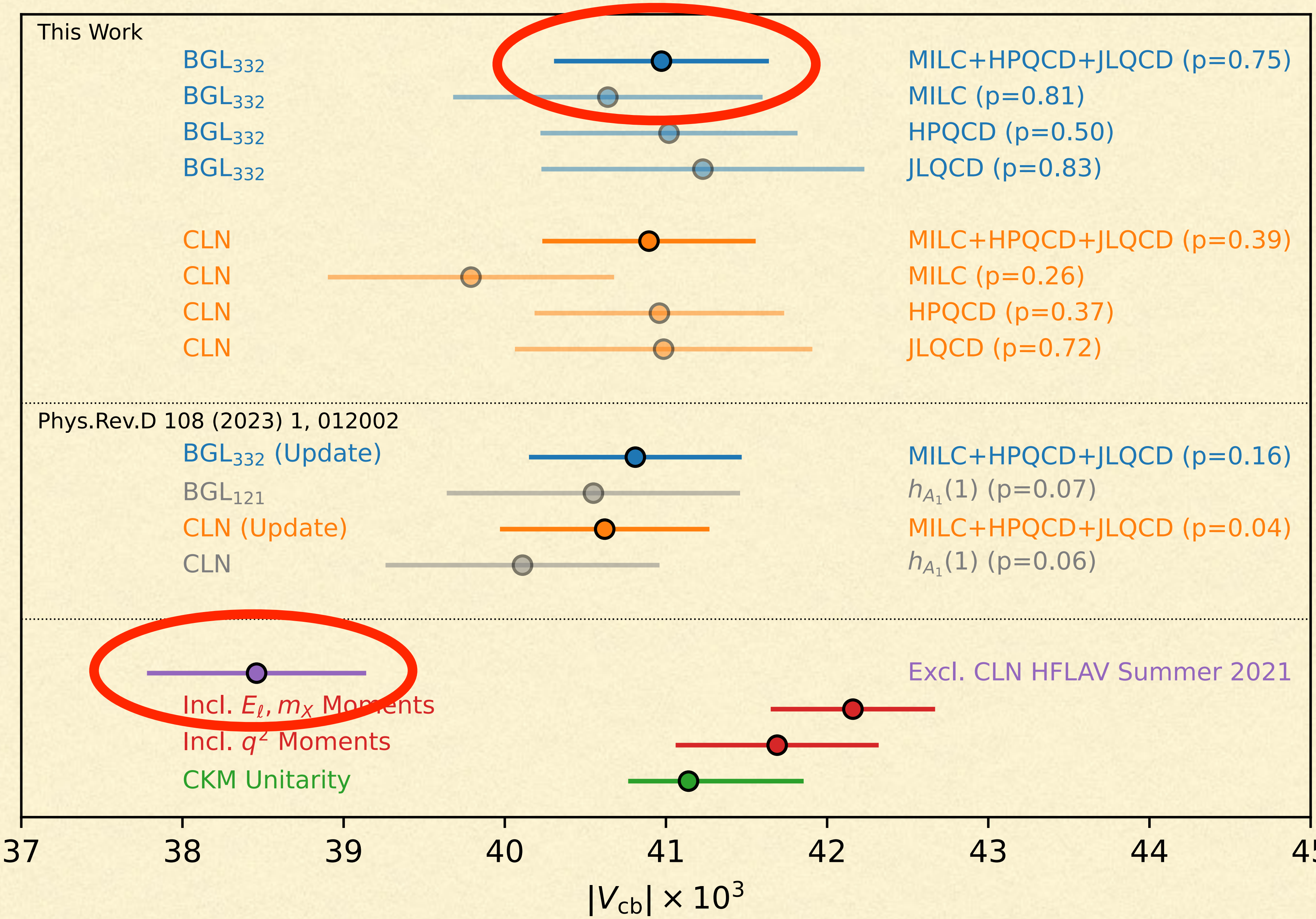
Detailed proposal of new measurements

Florian

NEW!

[2310.20286]

$$|V_{cb}| = 41.0(7) \times 10^{-3}$$



Do we understand why they differ so much?
Should we average them?

INCLUSIVE V_{cb}

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub},...)

Reliability of the method depends on our control of higher order effects.
Quark-hadron duality violation would manifest itself as inconsistency in the fit.

State-of-the art

- **Total Rate**
  - NNLO Czarnecki, Pak, *Phys.Rev.D* 78 (2008) 114015, *Phys.Rev.Lett.* 100 (2008) 241807
  - N3LO ($b \to c$) **MF**, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003.
  - N3LO ($b \to u$) **NEW** **MF**, Usovitsch, hep-ph/2310.03685
- **M_X and E_l moments**
  - NLO differential rate Aquila, Gambino, Ridolfi, Uraltsev, Nucl.Phys.B 719 (2005) 77
  - NNLO for moments with $E_{\text{cut}} < E_l$, numerical results for specific E_{cut} and $\rho = m_c/m_b$ Biswas, Melnikov, JHEP 02 (2010) 089; Gambino, JHEP 09 (2011) 055. Gambino, JHEP 09 (2011) 055.
  - NLO for μ_π^2 and μ_G^2 Alberti, Gambino, Nandi, *Nucl.Phys.B* 870 (2013) 16, *JHEP* 01 (2014) 147
  - N3LO for moments without cuts **MF**, Schönwald, Steinhauser, JHEP 08 (2022) 039.
- **q^2 moments with a lower cut on q^2**
  - NLO up to ρ_D^3 Moreno, Mannel, Pivovarov, *Phys.Rev.D 105 (2022) 5, 054033*
  - NNLO for moments with $q^2_{\text{cut}} \leq q^2$ **NEW** **MF**, Herren, in preparation
- **QED effects**

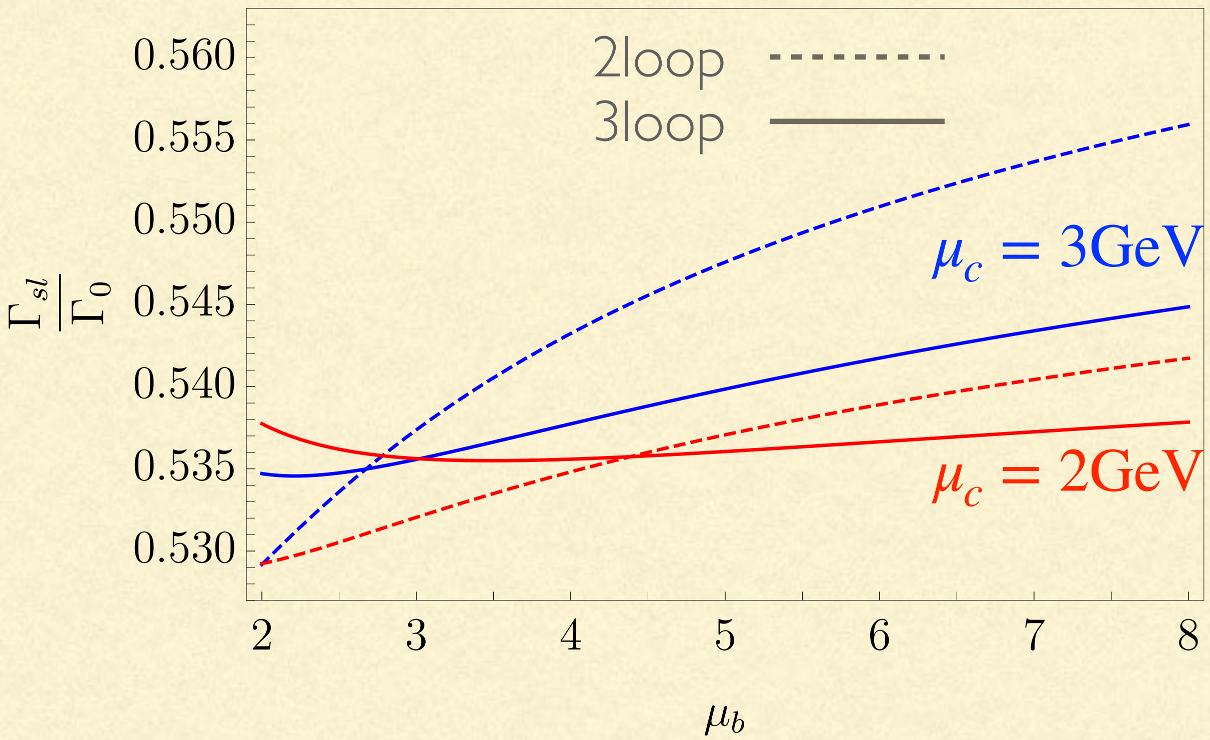
Bordone, Gambino, Haisch, Piccione, hep-ph-2309.02849

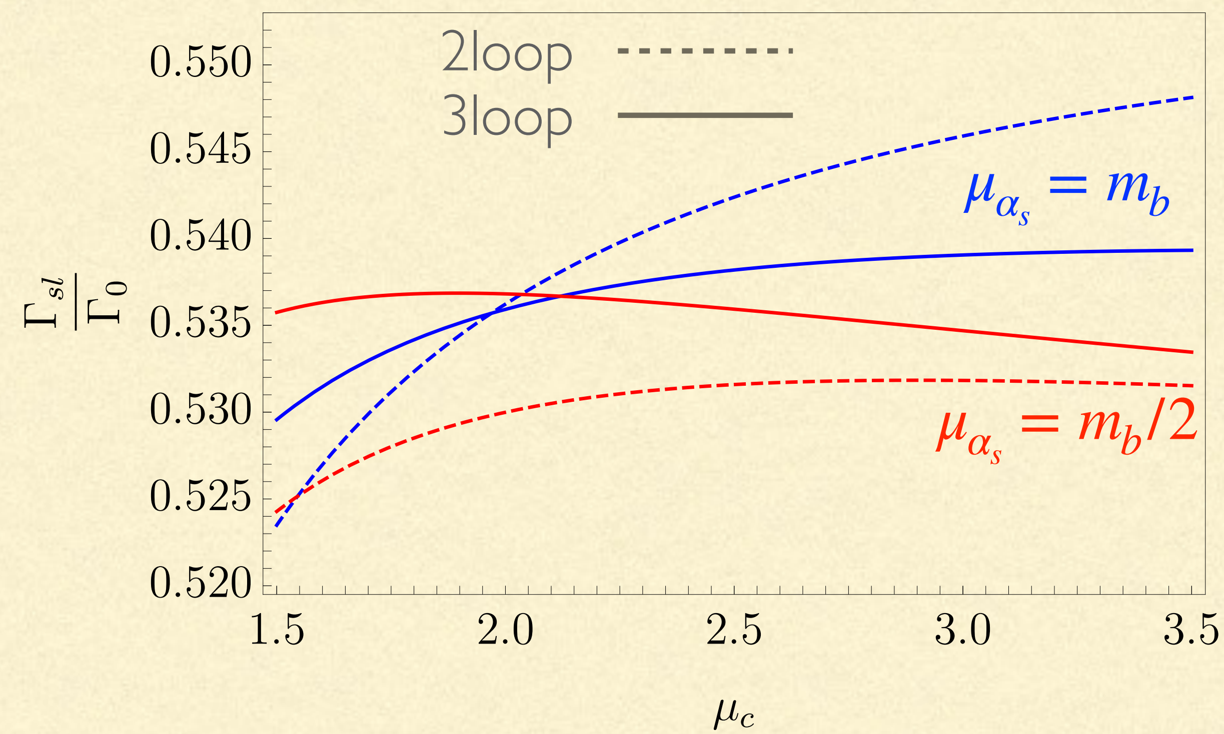
Also important: 3loop relation kinetic-pole mass

Calculations based on new sophisticated methods

RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604





Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty $\pm 0.7\,\%$ (max spread), central values at $\mu_c = 2\mathrm{GeV}, \mu_{\alpha_s} = m_b/2$.

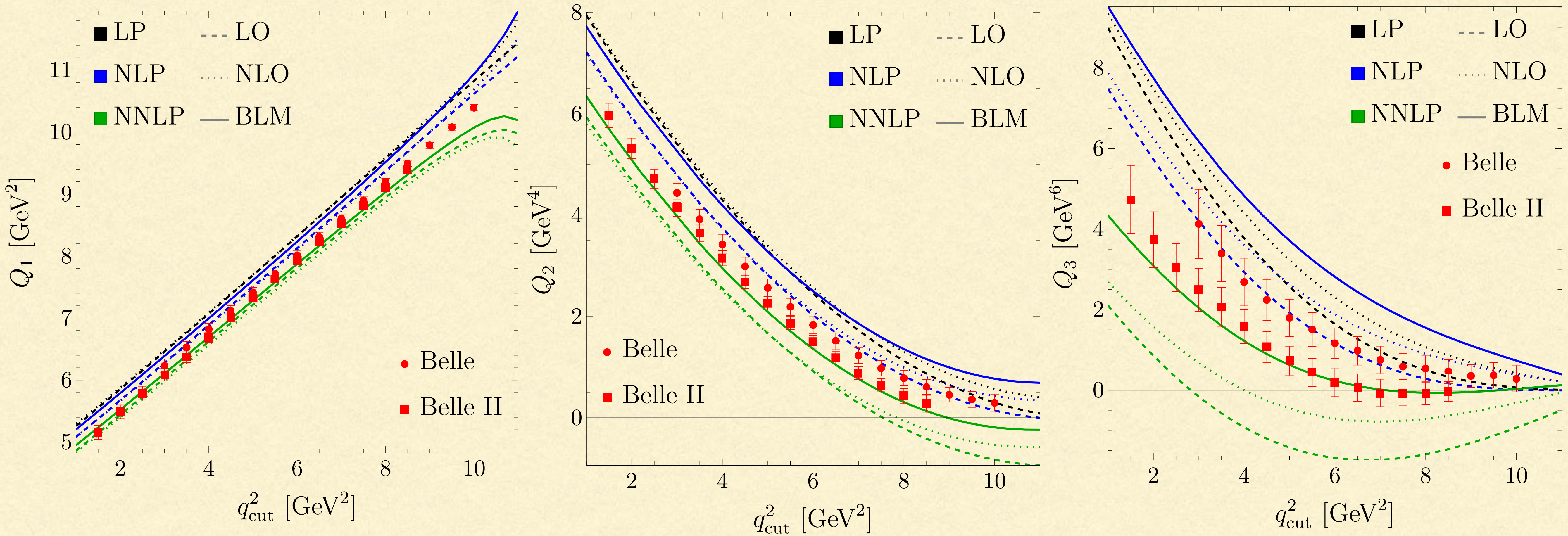
$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3 m_c)$, duality violation.

Conservatively: 1.2% overall theory uncertainty in Γ_{sl}

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s\Lambda^2/m_b^2)$

NEW: $O(\alpha_s^2\beta_0)$ CORRECTIONS TO q^2 MOMENTS

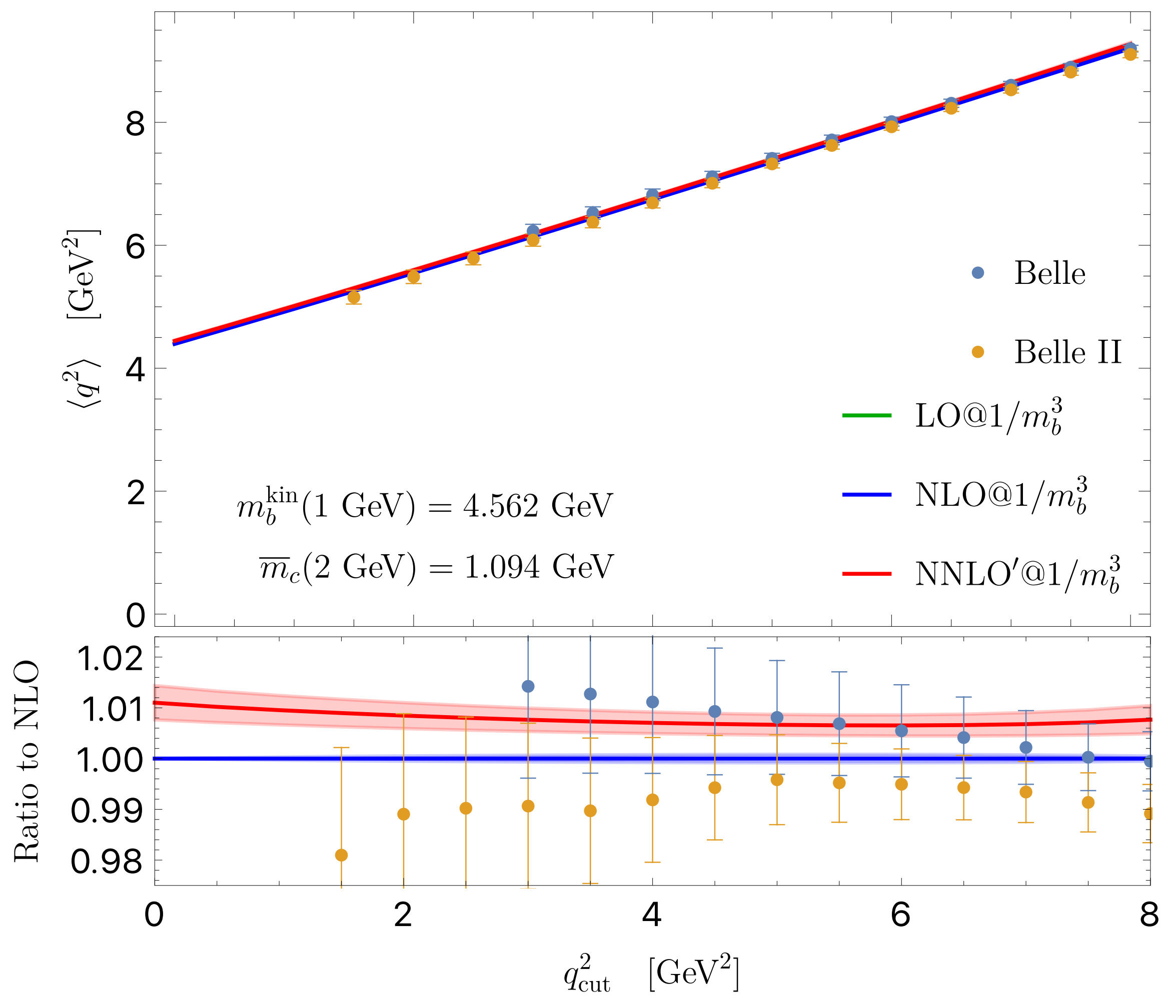
Finauri, PG 2310.20324

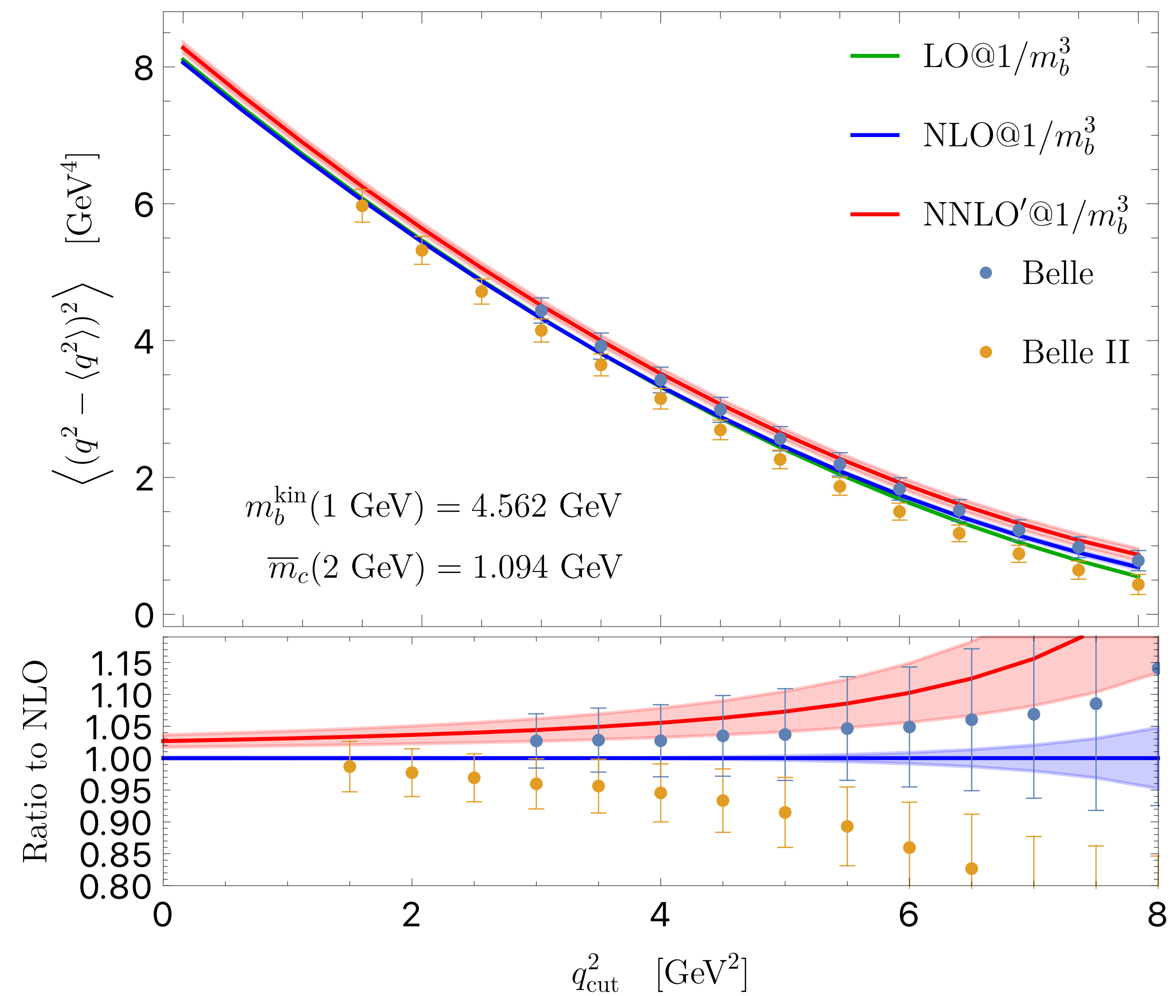


sizeable for 2nd and 3rd moments

Belle and Belle II moments differ by $\sim 2\sigma$

NEW: complete $O(\alpha_s^2)$ corrections to q^2 moments by Fael & Herren



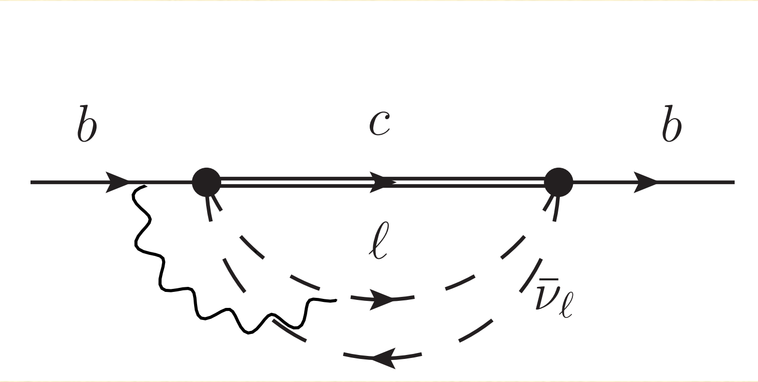


HQE parameters from: Bernlochner et al, JHEP 10 (2022) 068

QED CORRECTIONS

Bigi, Bordone, Haisch, Piccione PG
2309.02849

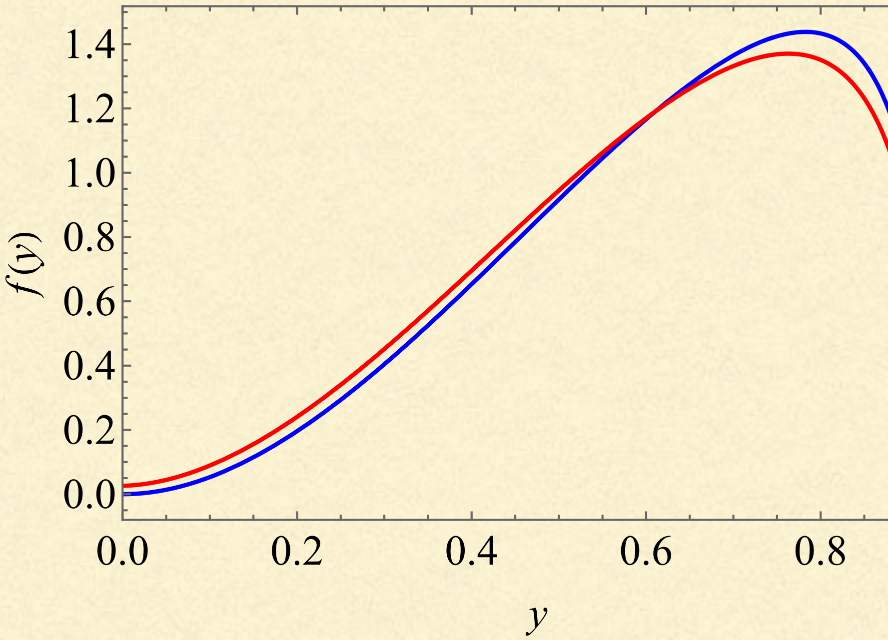
In the presence of photons, ***OPE valid only for total width*** and moments that do not resolve lepton properties (E_ℓ, q^2). Expect mass singularities and $O(\alpha\Lambda/m_b)$ corrections.



Leading logs $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)}\left(\frac{y}{x}\right) \left(\frac{d\Gamma}{dx}\right)^{(0)}$$

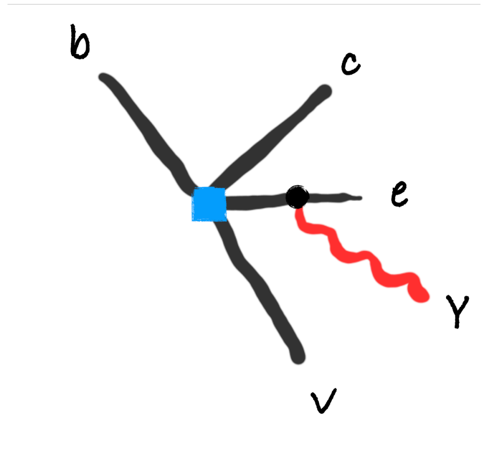
$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z}\right]_+$$



Electron energy spectrum

QED Leading contributions

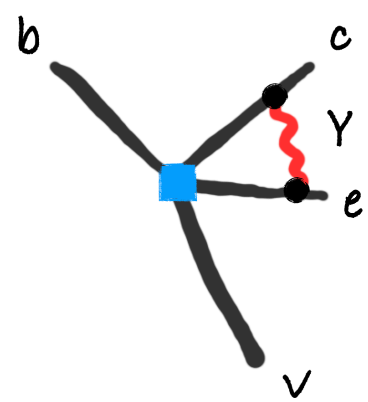
1. **Collinear logs**: captured by splitting functions



also at subleading power!

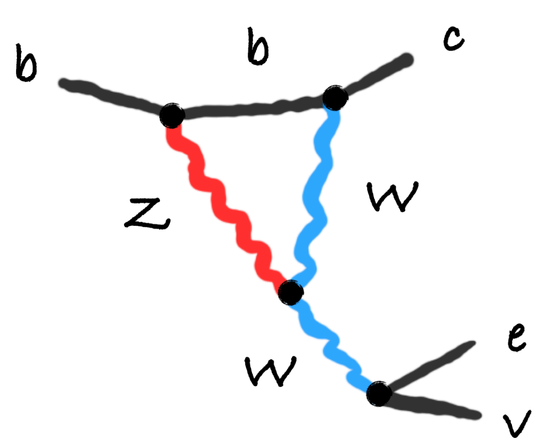
$$\sim \frac{\alpha_e}{\pi} \log^2\left(\frac{m_b^2}{m_e^2}\right)$$

2. **Threshold effects** or Coulomb terms



$$\sim \frac{4\pi\alpha_e}{9}$$

3. **Wilson Coefficient**



$$\sim \frac{\alpha_e}{\pi}\left[\log\left(\frac{M_Z^2}{\mu^2} - \frac{11}{6}\right)\right]$$

Marzia

A GLOBAL FIT

Finauri, PG 2310.20324

$m_b^{\rm kin}$	$\overline{m}_c(2\,{\rm GeV})$	μ_π^2	$\mu_G^2(m_b)$	$\rho_D^3(m_b)$	ρ_{LS}^3	${\rm BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.090	0.454	0.288	0.176	−0.113	10.63	41.97
0.012	0.010	0.043	0.049	0.019	0.090	0.15	0.48

Includes all leptonic, hadronic, and q^2 moments

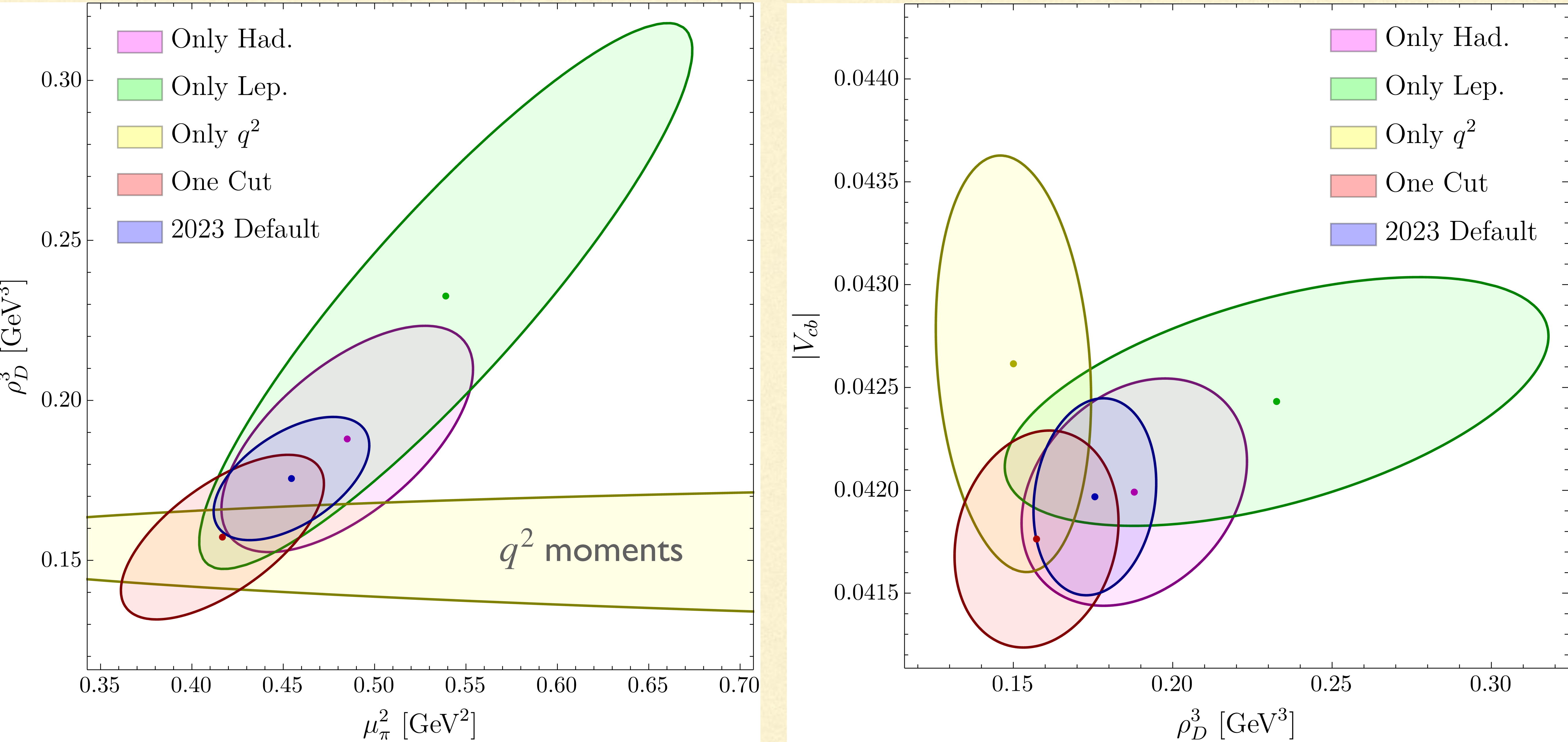
Up to $O(\alpha_s^2)$, $O(\alpha_s/m_b^2)$, $O(1/m_b^3)$ for M_X, E_ℓ moments
Up to $O(\alpha_s^2\beta_0)$, $O(\alpha_s/m_b^3)$ for q^2 moments
Subtracts QED effects beyond those computed by PHOTOS (only BaBar BR and lept moments) $\delta|V_{cb}| \sim -0.2\,\%$

Employs $\overline{m}_b(\overline{m}_b) = 4.203(11)\mathrm{GeV}$ and $\overline{m}_c(3\mathrm{GeV}) = 0.989(10)\mathrm{GeV}$(FLAG)
$\chi^2_{min}/dof = 0.55$

$$|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_\Gamma) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}$$

comparison of different datasets

Finauri, PG 2310.20324



Theory correlations are no longer an issue

$$\Gamma \propto |V_{cb}|^2 m_b^5 \left[\Gamma_0 + \Gamma_0^{(1)} \frac{\alpha_s}{\pi} + \Gamma_0^{(2)} \left(\frac{\alpha_s}{\pi}\right)^2 + \Gamma_0^{(3)} \left(\frac{\alpha_s}{\pi}\right)^3 + \frac{\mu_\pi^2}{m_b^2} \left(\Gamma^{(\pi,0)} + \frac{\alpha_s}{\pi}\Gamma^{(\pi,1)}\right) \right.$$

$$\left. + \frac{\mu_G^2}{m_b^2} \left(\Gamma^{(G,0)} + \frac{\alpha_s}{\pi}\Gamma^{(G,1)}\right) + \frac{\rho_D^3}{m_b^3} (\Gamma^{(D,0)} + \Gamma_0^{(1)} \left(\frac{\alpha_s}{\pi}\right)) + \mathcal{O}\left(\frac{1}{m_b^4}\right) + \cdots \right)$$

What is next?

- Include higher-order α_s corrections [Talk by Matteo Fael]
- Add higher-order non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

Higher power corrections Wilson coefficients are known at LO. One can use the Lowest Lying State Saturation Approximation (Mannel,Turczyk,Uraltsev 1009.4622) as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers but no q^2 moments (yet)

$$|V_{cb}| = 42.00(53) \times 10^{-3}$$

Bordone, Capdevila, PG 2107.00604
Healey, Turczyk, PG 1606.06174

The advantage of q^2 moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177

- Standard lepton energy and hadronic mass moments are not RPI quantities
- New q^2 moments are RPI!

Reparametrization invariant quantities:

- Setting up the HQE: momentum of b quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of v not unique: Reparametrization invariance (RPI)

$$v_\mu \to v_\mu + \delta v_\mu$$

$$\delta_{RP}\, v_\mu = \delta v_\mu \;\text{ and }\; \delta_{RP}\, iD_\mu = -m_b \delta v_\mu$$

  - links different orders in $1/m_b \to$ reduction of parameters
  - up to $1/m_b^4$: 8 parameters (previous 13)

- q^2 moments enable (?) a full extraction up to $1/m_b^4$

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.63) \times 10^{-3}$$

Important consistency check

What's next for moments?

- Measure all kin. moments simultaneously as a function of q^2 (E_l^B) thresholds in

  $B \to X\ell\nu$: $q^2, E_l^B, M_X, \cos\theta_\ell$, combined variables $n_X^2\,(M_X^2, E_X)$, $P_X^\pm\,(M_X, E_X)$

- Full experimental correlations will be derived => important for global analysis
- Only shape observation (drop tagging eff. calibration, separate from $\mathscr{B}$ measurement)

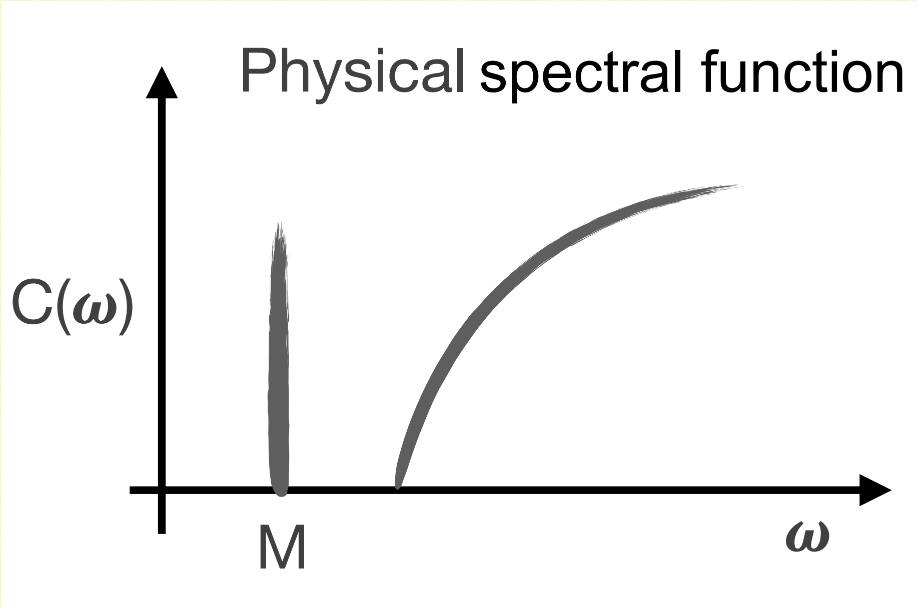
yes please

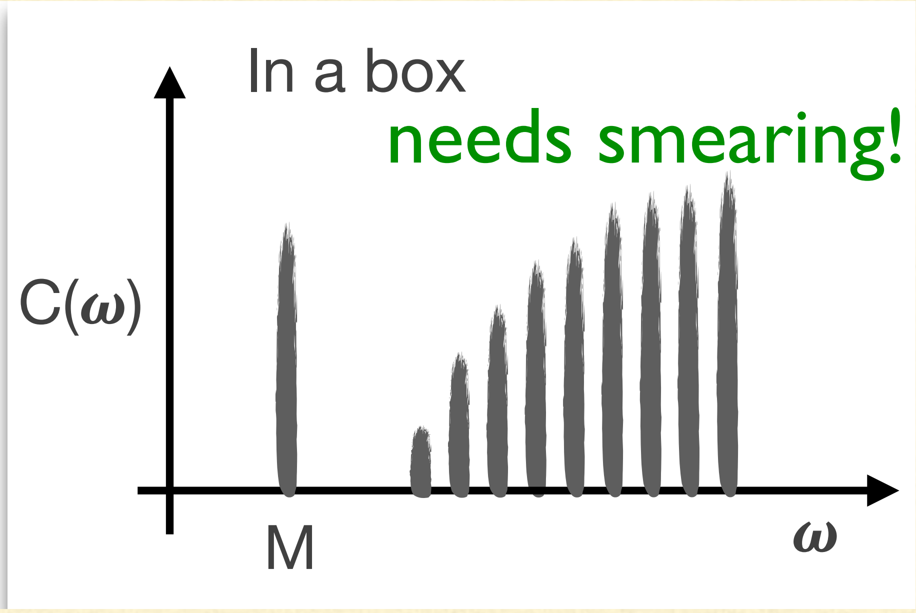
INCLUSIVE DECAYS ON THE LATTICE

- Inclusive processes *impractical* to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \to$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.

see Judd's talk

- While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa





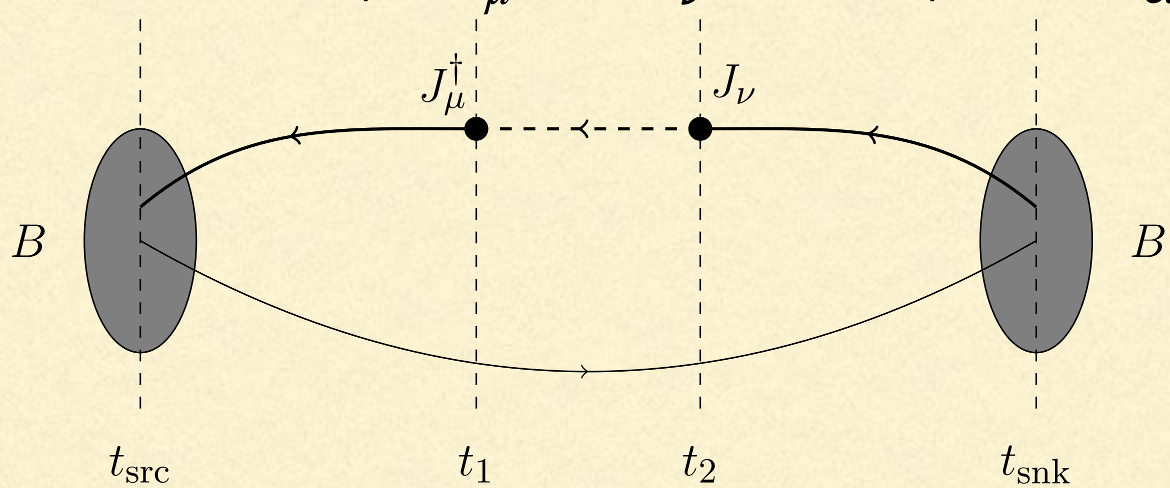
W. Jay @Snowmass workshop

A NEW APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

$$\sim \langle B | J_\mu^\dagger(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle$$

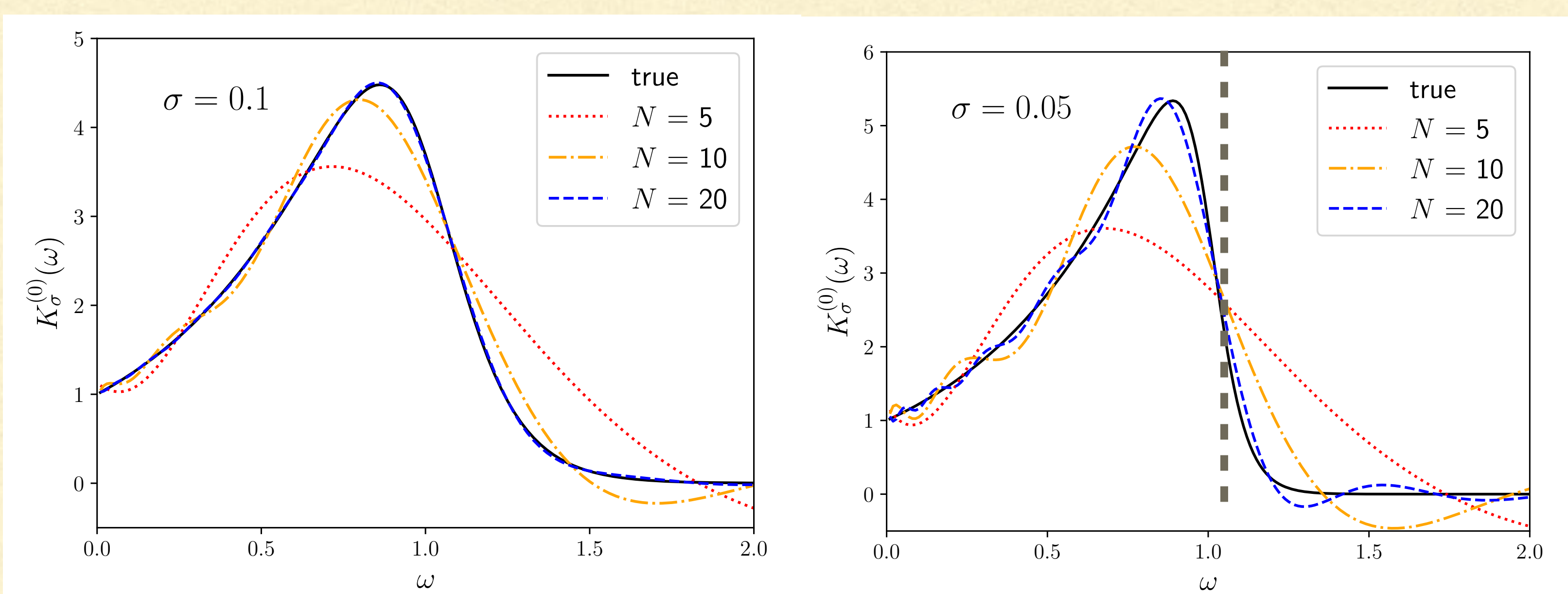


$$d\Gamma \sim L^{\mu\nu} W_{\mu\nu}, \quad W_{\mu\nu} \sim \sum_X \langle B | J_\mu^\dagger | X \rangle \langle X | J_\nu | B \rangle$$

$$\int d^3x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_B} \langle B | J_\mu^\dagger(\mathbf{x},t) J_\nu(\mathbf{0},0) | B \rangle \sim \int_0^\infty d\omega W_{\mu\nu} e^{-t\omega}$$

smearing kernel $f(\omega) = \sum_n a_n e^{-na\omega}$

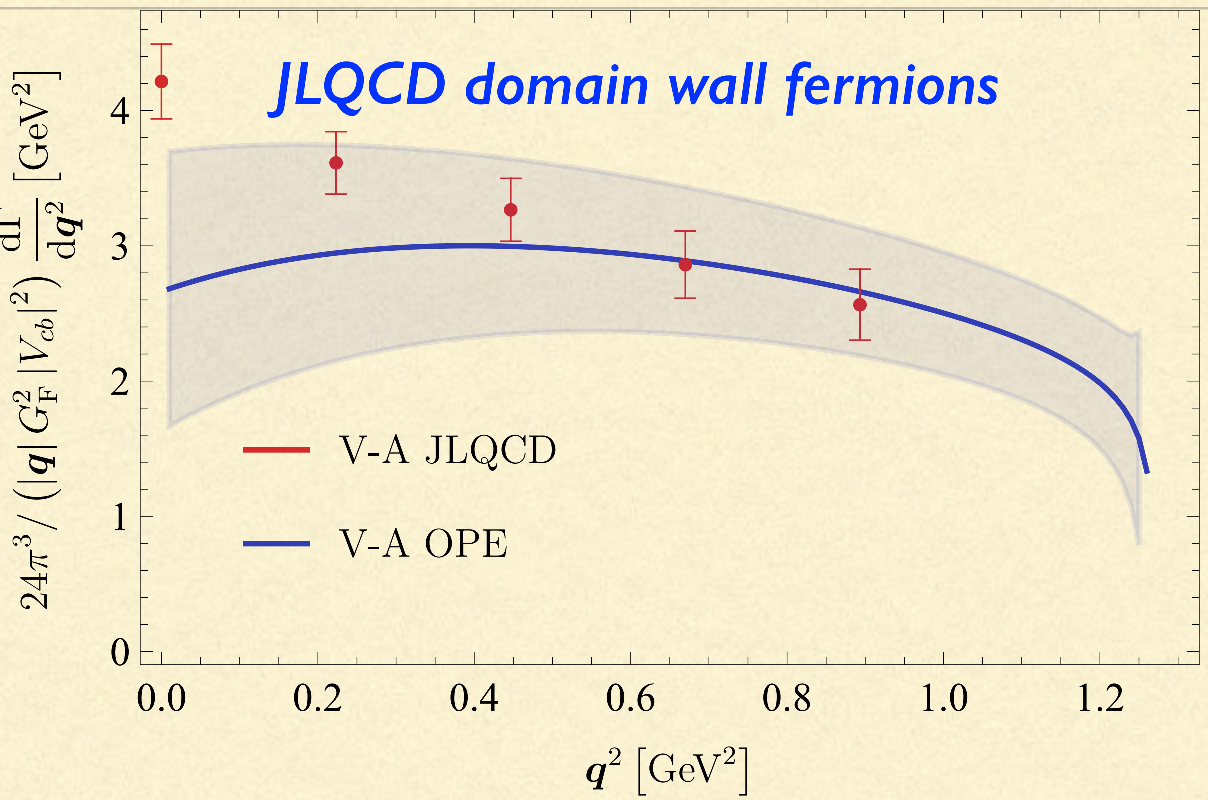
The necessary smearing is provided by phase space integration over the hadronic energy, which is cut by a θ with a sharp hedge: sigmoid $1/(1+e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$ for $\sigma \to 0$. Larger number of polynomials needed for small σ

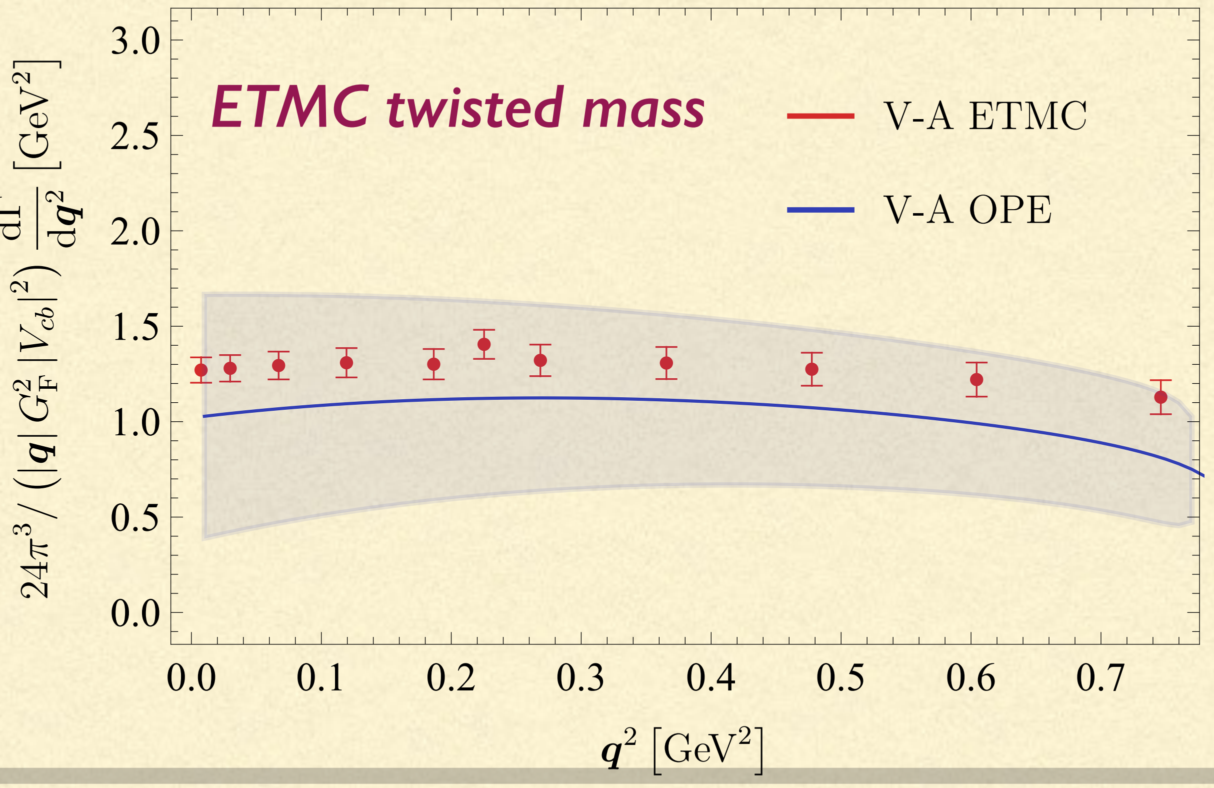


Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

$$\lim_{\sigma \to 0} \lim_{V \to \infty} X_\sigma$$

LATTICE VS OPE





m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2\text{ GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2\text{ GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2\text{ GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2+\mathbf{q}^2} \sim 1-1.5\,\text{GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

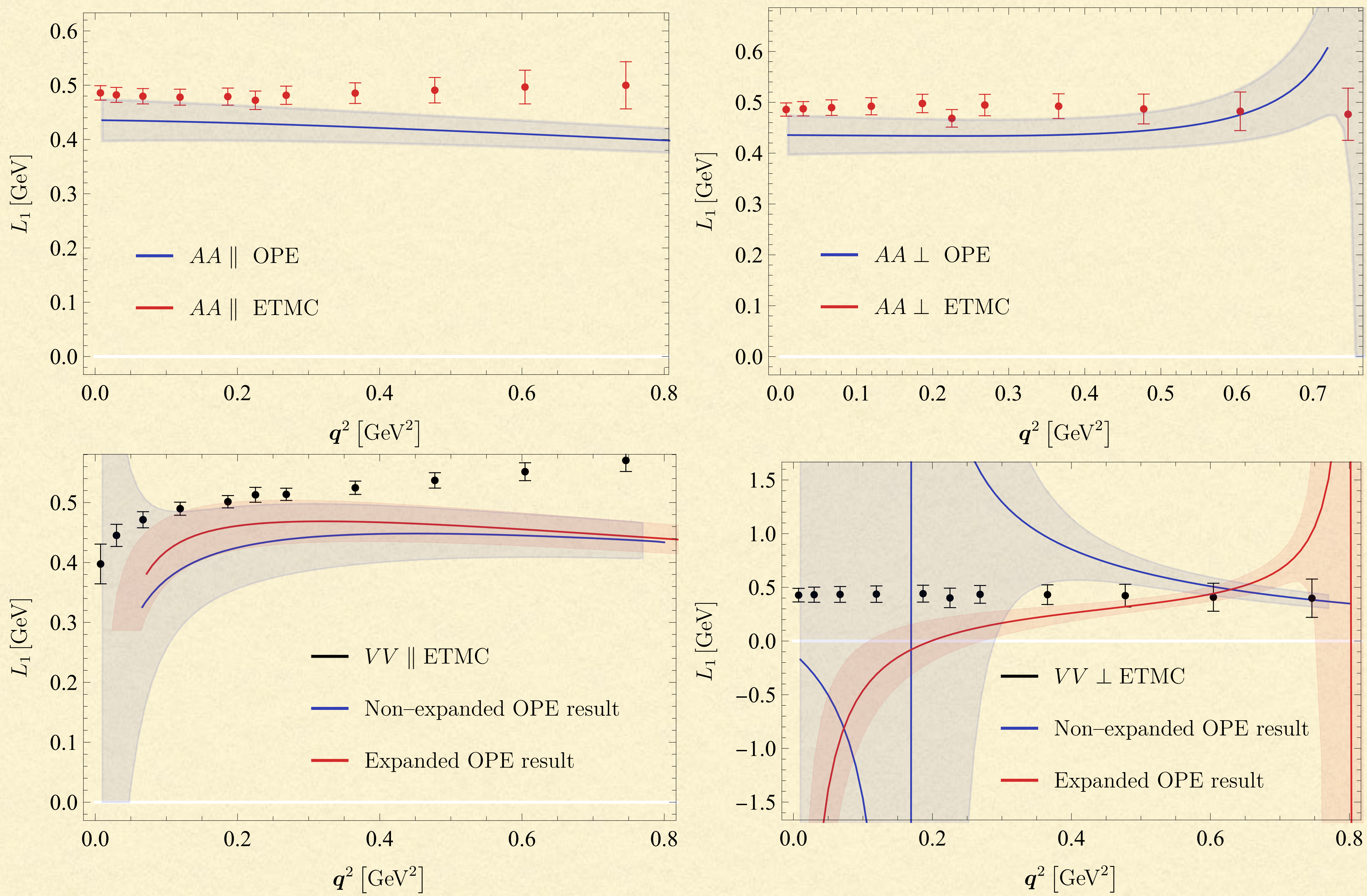
Twisted boundary conditions allow for any value of $\vec{q}^{\,2}$

Smaller statistical uncertainties

MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

$$L_1 = \langle E_\ell(\mathbf{q}^2) \rangle$$



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

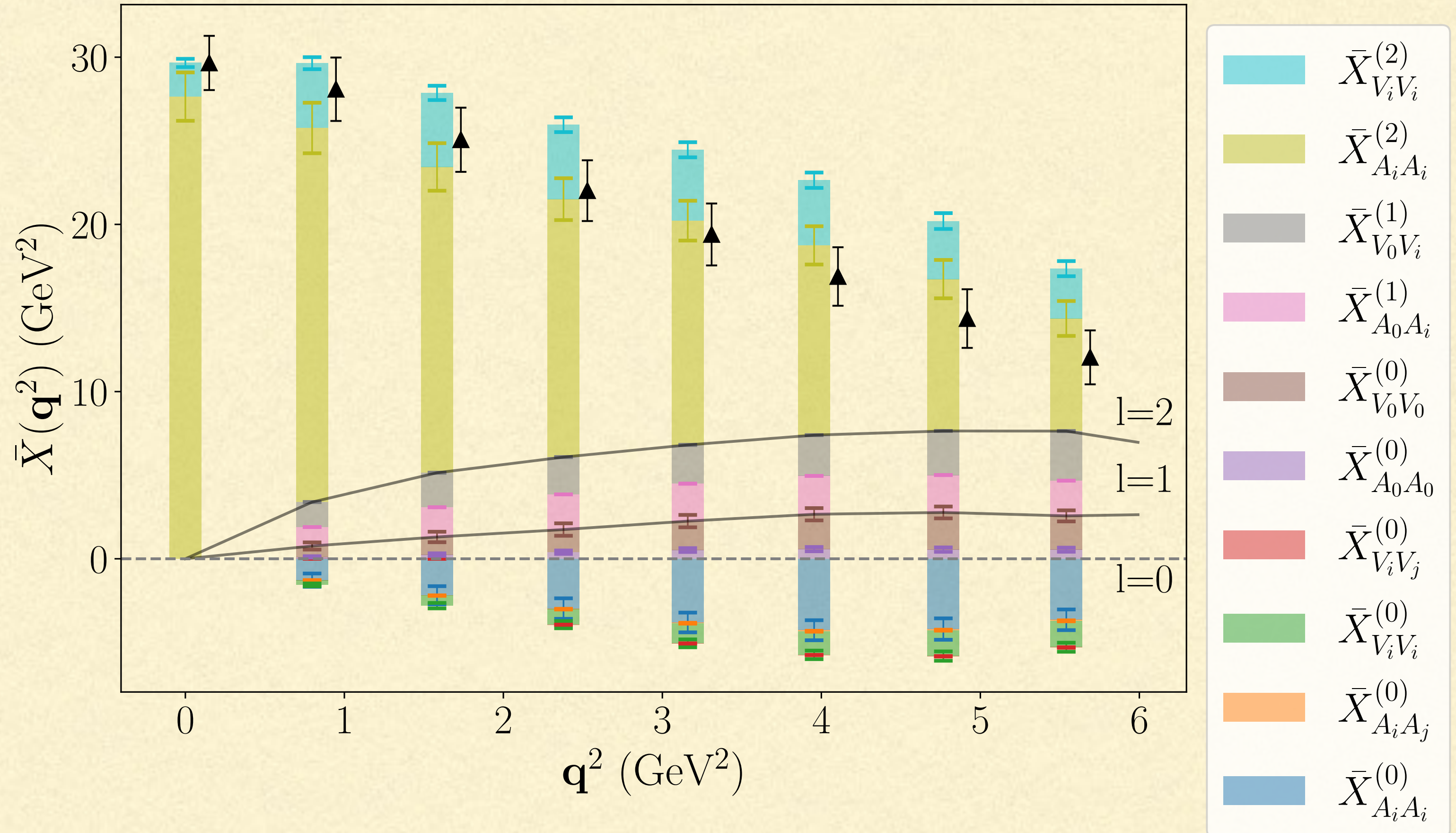
First results at the physical *b* mass

Relativistic heavy quark effective action for b

B_s decays,
domain wall fermions,
improved implementation of Chebychev polynomials and Backus-Gilbert

qualitative study
~5% statistical uncertainty on total width

possibly better to compare with partial width at low $\vec{q}^2$



Barone, Hashimoto, Juttner, Kaneko, Kellermann, 2305.14092

Ongoing work on semileptonic D_s decays by two collaborations

SUMMARY AND OUTLOOK

- *Exclusive: we learned that parametrisations matter and the related uncertainties require careful consideration. Uncertainties were often underestimated. We agree BGL is an appropriate framework for fits. Ongoing discussions on how exactly use it.*
- *Lattice form factors: situation still unclear, 2 calculations in conflict with exp and HQE. It's normal: don't underestimate their difficulty.*
- *I am impressed by the many new ideas on how to improve the exp analyses and reduce/control errors*
- *Inclusive $b \to c$ is robust: q^2 moments consistent with leptonic and adronic ones; $O(\alpha_s^3)$ effects show perturbation theory OK; higher powers appear small. But don't dream of going below 1%…*
- *Calculations of inclusive semileptonic meson decays on the lattice have started. Precision to be seen, but you can count they will, at some point, contribute.*
- *Are there unknown unknowns?*
- *We have a plan. It's a good one, let's execute it with care and passion and we'll see in 3 years time. I bet some cloud will have disappeared.*