Flavor Physics

Rusa Mandal
IIT Gandhinagar, India

Belle II Physics Week@KEK Tsukuba

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Outline

- Lecture I
  - Flavor of the Standard Model
  - Weak decays: Effective Theory: Operator Product Expansion
- Lecture II
  - Form factor, Penguin decays
  - Current tensions
- Lecture III
  - SMEFT, Minimal Flavor Violation
  - Flavor Model with BSM physics

- hep-ph/980647
review by A. J. Buras
- Gauge theories of weak decays
book by A. J. Buras

Aim of the lectures: to get familiar with the methods

and terms used in theory

The Standard Model

Gauge structure of the SM of Particle Physics

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

strong: color

weak: isospin

hypercharge

Fermions: three generations			$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
e_R	μ_R	τ_R	**1**	**1**	-1
$L_1 = (\nu_e, e_L)^\top$	$L_2 = (\nu_\mu, \mu_L)^\top$	$L_3 = (\nu_\tau, \tau_L)^\top$	**1**	**2**	$-\frac{1}{2}$
u_R	c_R	t_R	**3**	**1**	$\frac{2}{3}$
d_R	s_R	b_R	**3**	**1**	$-\frac{1}{3}$
$Q_1 = (u_L, d_L)^\top$	$Q_2 = (c_L, s_L)^\top$	$Q_3 = (t_L, b_L)^\top$	**3**	**2**	$\frac{1}{6}$
Gauge bosons: mediators					
	G_μ^a	$a = 1\text{–}8$	**8**	**1**	0
	W_μ^a	$a = 1, 2, 3$	**1**	**3**	0
	B_μ		**1**	**1**	0
Higgs					
	$\Phi = (\phi^+, \phi^0)^\top$		**1**	**2**	$\frac{1}{2}$

Lagrangian

$$\mathscr{L}_{\text{kin}} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D^{\mu}\Phi)^{\dagger}(D_{\mu}\Phi)$$

Fermion-gauge boson interaction

Higgs-gauge boson interaction

$$\psi = \{e_R, L, u_R, d_R, Q\}$$

$$F_{\mu\nu} = \{G^a_{\mu\nu}, W^a_{\mu\nu}, B_{\mu\nu}\}$$

$$\mathscr{L}_{\text{Yuk}} = -\bar{Q}\Phi Y_D d_R - \bar{Q}\Phi^c Y_U u_R - \bar{L}\Phi Y_E e_R$$

→ Higgs-fermion interaction

3 x 3 Yukawa matrices: flavour dynamics

Lagrangian

$$\mathscr{L}_{\rm kin} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D^\mu \Phi)^\dagger (D_\mu \Phi)$$

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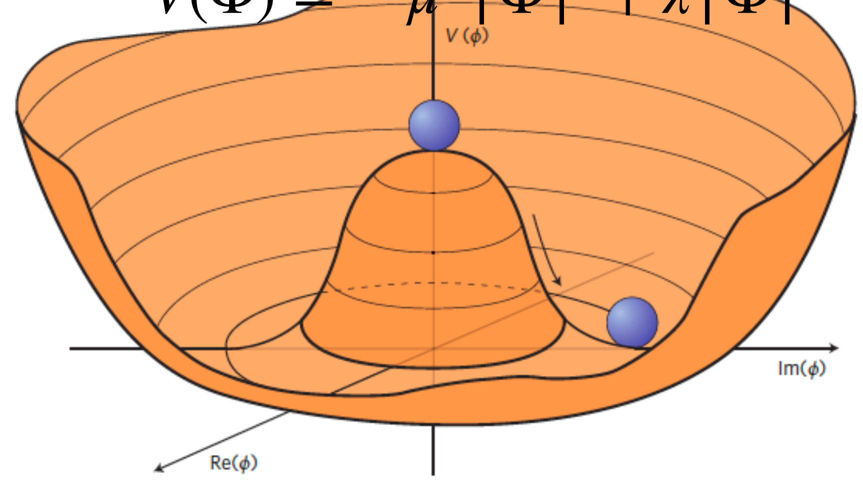
3 x 3 Yukawa matrices: flavour dynamics

No Yukawa for neutrinos → massless in SM

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_{\rm EM}$$

$$\langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$V(\Phi) = -\mu^2 |\Phi|^2 + \lambda |\Phi|^4$



[Courtesy: CERN document server]

→ Massive gauge bosons: $W^\pm, Z$

Flavor sector

$\mathscr{L}_{\text{kin}}$ for fermions are invariant under $[U(3)]^5$

$$Q_L \to V_L^u\, Q, \quad u_R \to V_R^u\, u_R, \quad d_R \to V_R^d\, d_R,$$
$$L \to V_L^e\, L, \quad e_R \to V_R^e\, e_R,$$

$V_{L,R}^{u,d,e}$: 3x3 unitary matrices

$\mathscr{L}_{\text{Yuk}}$ breaks $[U(3)]^5$ → $[U(1)]^4$ Baryon no. *B*

& 3 lepton family nos. $L_{e,\mu,\tau}$

Flavor sector

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Can we use flavour symmetry to diagonalise all Yukawa matrices?

bi-unitary transformation $(V_L^d)^\dagger Y^D V_R^d = \hat{Y}^D, \quad (V_L^u)^\dagger Y^U V_R^u = \hat{Y}^U, \quad (V_L^e)^\dagger Y^E V_R^e = \hat{Y}^E.$

But only 3 matrices are available in quark sector: V_L^d missing

$$\mathscr{L}_{\rm Yuk} = -\bar{Q}\Phi (V_L^u)^\dagger V_L^d \hat{Y}^D d_R - \bar{Q}\Phi^c \hat{Y}^U u_R - \bar{L}\Phi \hat{Y}^E e_R$$

non-diagonal → Extra rotation for *d*-type quarks

Flavor sector

Mass basis → All Yukawa matrices are diagonal

$$d_L \to d'_L = (V^u_L)^\dagger V^d_L d_L \equiv V_{\rm CKM}\, d_L$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Flavor sector

Mass basis ➡ All Yukawa matrices are diagonal

$$d_L \to d'_L = (V_L^u)^\dagger V_L^d d_L \equiv V_{\text{CKM}}\, d_L$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Where do we see the effect of CKM rotation?

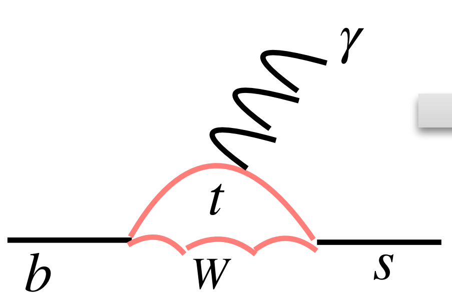
Kinetic term: $\bar{f} i \not{D} f$ ➡ $\mathscr{L}_I^{\text{SM}} \supset \dfrac{e}{\sin\vartheta_W \cos\vartheta_W}\left(T_3 - \sin^2\vartheta_W Q_f\right)\bar{f}\gamma^\mu Z_\mu f + eQ_f \bar{f}\gamma^\mu A_\mu f$

$$\sum_{j=1,2,3} \bar{d}_{Lj}\gamma_\mu d_{Lj} V^\mu \to \sum_{j,k=1,2,3} \bar{d}_{Lj} \underbrace{\left(V_{\text{CKM}}^\dagger V_{\text{CKM}}\right)_{jk}}_{=\delta_{jk}} \gamma_\mu d_{Lk} V^\mu$$

$V_\mu = \{G_\mu, Z_\mu, A_\mu\}$

neutral gauge bosons

No flavor changing neutral current@tree level



➡ possible at loop level

Flavor sector

Charged current: $\sum_{j-1,2,3} \bar{u}_j \gamma^\mu P_L d_j W_\mu^+ \longrightarrow \sum_{j,k-1,2,3} \bar{u}_j \gamma^\mu P_L V_{jk} d_k W_\mu^+ = \bar{u} \gamma^\mu V_{CKM} P_L d W_\mu^+$

→ flavor violation generated in gauge interaction via Yukawa interactions in mass basis

Flavor sector

Charged current: $\sum_{j-1,2,3} \bar{u}_j \gamma^\mu P_L d_j W_\mu^+ \longrightarrow \sum_{j,k-1,2,3} \bar{u}_j \gamma^\mu P_L V_{jk} d_k W_\mu^+ = \bar{u}\gamma^\mu V_{CKM} P_L d W_\mu^+$

flavor violation generated in gauge interaction via Yukawa interactions in mass basis

General parametrization of 3x3 unitary matrix → 3 angles + 6 phases

Not all phases physical—5 are rotated away $u_j^{L,R} \to e^{i\varphi_j^u} u_j^{L,R}, \quad d_j^{L,R} \to e^{i\varphi_j^d} d_j^{L,R}$

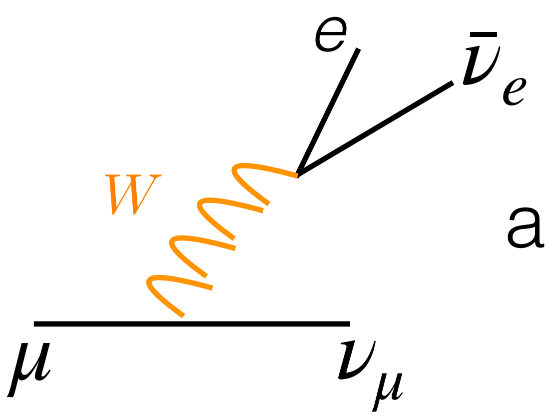
$V_{ij}^{\mathrm{CKM}} \to e^{i(\varphi_j^d - \varphi_i^u)} V_{ij}^{\mathrm{CKM}}$ → 3 angles + 1 phases

only source of CP violation

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$
$c_{ij} = \cos\theta_{ij}$

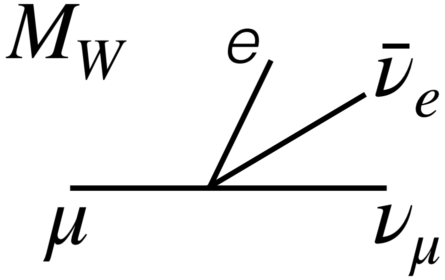
Weak decays of muons



amplitude= $-\frac{1}{8}\frac{g_2^2}{k^2 - M_W^2}[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu][\bar{e}\gamma^\mu (1-\gamma_5)\nu_e],$

$k^2 \ll M_W^2$ is good approximation as $m_\mu \ll M_W$

$\Rightarrow \frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu][\bar{e}\gamma^\mu (1-\gamma_5)\nu_e],$

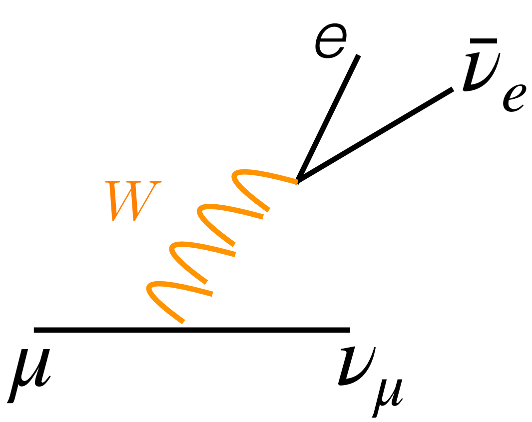


matching with Fermi theory with 4-point effective interaction $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}$

$\mathrm{BR}(\mu \to e\nu_\mu \bar{\nu}_e) \sim 100\,\%$ $\Rightarrow$ decay width of muon

used to evaluate G_F

Weak decays of muons



$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)$$

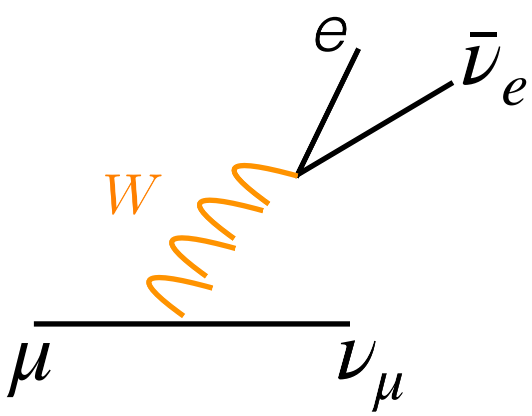
@LO with phase space factor

$$\tau_\mu^{\text{theo}} = 2.18776 \times 10^{-6}\,\text{s}$$

$$\tau_\mu^{\text{expt}} = 2.1969811(22) \times 10^{-6}\,\text{s}$$

→ very closely in agreement

Weak decays of muons



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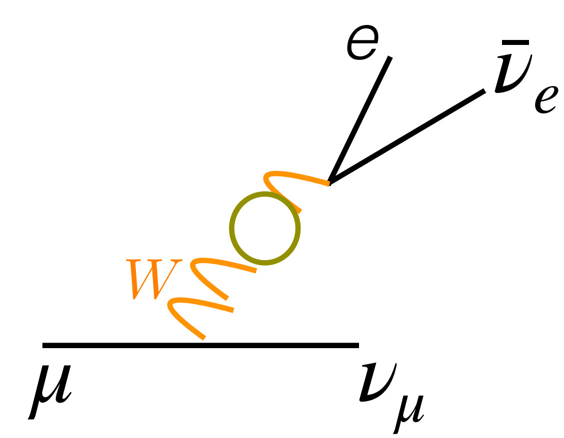
Including electro-weak corrections

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)$$



$$\tau_\mu^{\rm theo} = 2.19699 \times 10^{-6}\,{\rm s}$$

$$\tau_\mu^{\rm expt} = 2.1969811(22) \times 10^{-6}\,{\rm s}$$

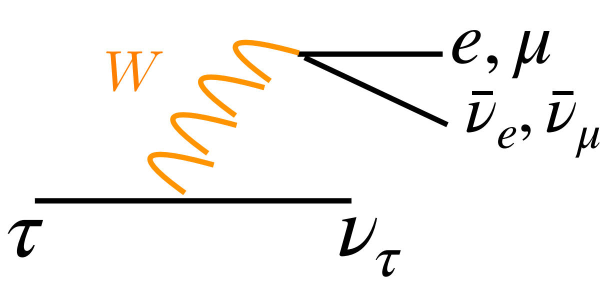
→ in perfect agreement

Weak decays of tau

Total decay width of fermion $\Gamma_f^{\rm tot} = \frac{G_F^2 m_f^5}{192\pi^3}\left[f(m_{f'}/m_f) + \cdots\right]$

phase space + higher order in α_{EM}

BR~35%



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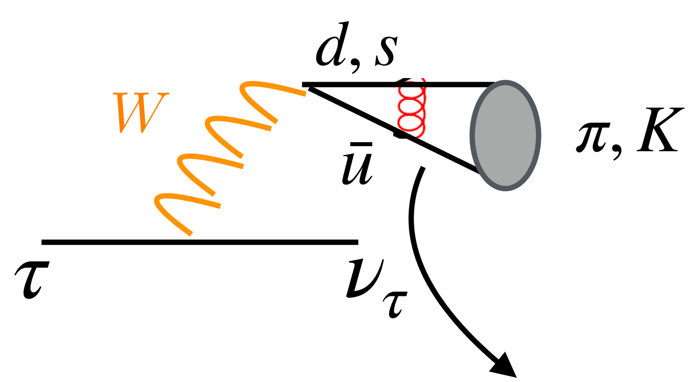


$\tau_\tau^{\rm theo} = 3.26707 \times 10^{-13}\,{\rm s}$

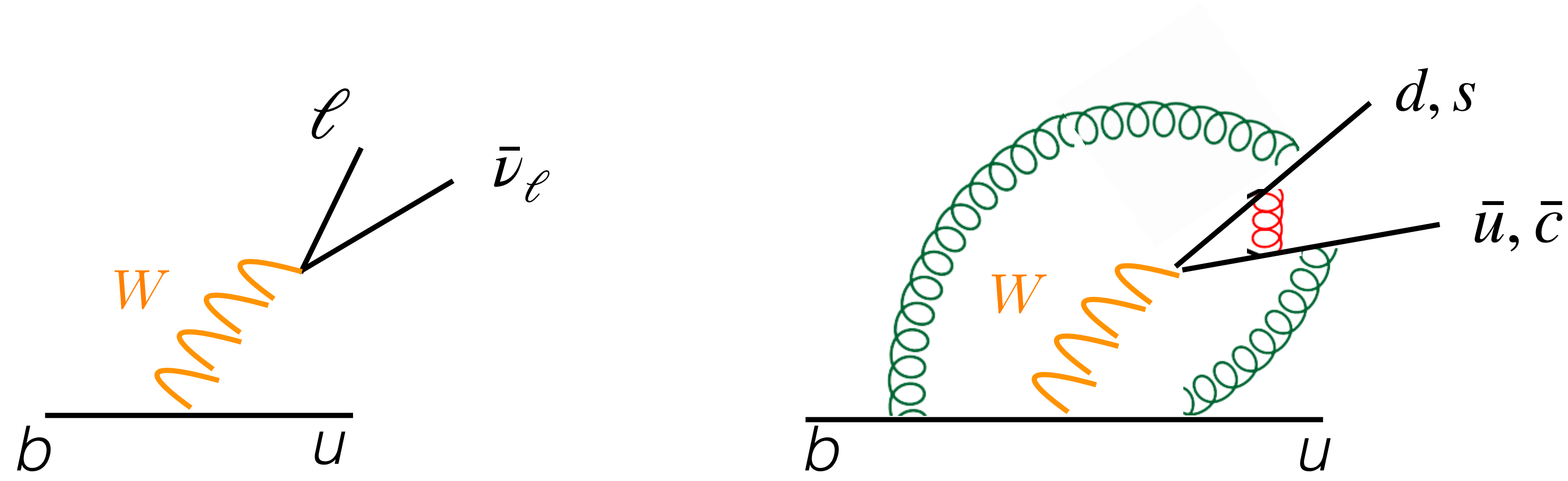
$\tau_\tau^{\rm expt} = 2.906(1) \times 10^{-13}\,{\rm s}$

Gluon exchange within quarks

QED effects under control but not QCD

Tau decay is used to evaluate α_s— strong coupling constant

Weak decays of quarks



Total decay width $\Gamma_f^{\rm tot} = \frac{G_F^2 m_f^5}{192\pi^3}\left[f(m_{f'}/m_f) + \cdots\right]$

heavy quark masses enter
— depend on scheme

$$\begin{aligned} m_c^{\rm Pole} &= 1.471 \ \text{GeV}, & m_b^{\rm Pole} &= 4.650 \ \text{GeV} \\ \bar{m}_c(\bar{m}_c) &= 1.277 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) &= 4.248 \ \text{GeV} \\ \bar{m}_c(\bar{m}_b) &= 0.997 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) &= 4.248 \ \text{GeV} \end{aligned}$$

→ Wide range of theory predictions for $\tau_b^{\rm theo}$

$\tau_b^{\rm theo} = 2.60 \times 10^{-15}\,\text{ps}$ $@\bar{m}_{c,b}(\bar{m}_b)$ — differs from $\tau_b^{\rm expt}$

Loops

Actual physics lies in loops! → Accuracy check of tree level & validity of perturbation theory

Several decays start@1-loop

Divergent pieces in loop integrals → Renormalization

Rusa Mandal, IIT Gandhinagar

Loops

Actual physics lies in loops! → Accuracy check of tree level & validity of perturbation theory

Several decays start@1-loop

Divergent pieces in loop integrals → Renormalization → energy scale induced

↓

Large Logarithms

$\alpha_s(m_b)\ln(m_b^2/\mu^2)$

0.2×1.3

Resummation with RG equations ←

	Leading Log	Next-to leading Log	NNLL
Tree	1		
1-loop	$\alpha_s \ln$	α_s	
2-loop	$\alpha_s^2 \ln^2$	$\alpha_s^2 \ln$	α_s^2

OPE

Weak decays of quarks involve different scales

$\mu = \mathcal{O}(M_W)$ → fundamental scale of weak interaction— small α_s

$\mathcal{O}(1\,\mathrm{GeV}) \leq \mu \leq M_W$ → α_s variation is significant
resummation of large Logs necessary

$\mu \leq \mathcal{O}(1\,\mathrm{GeV})$ → confinement effects has to be included

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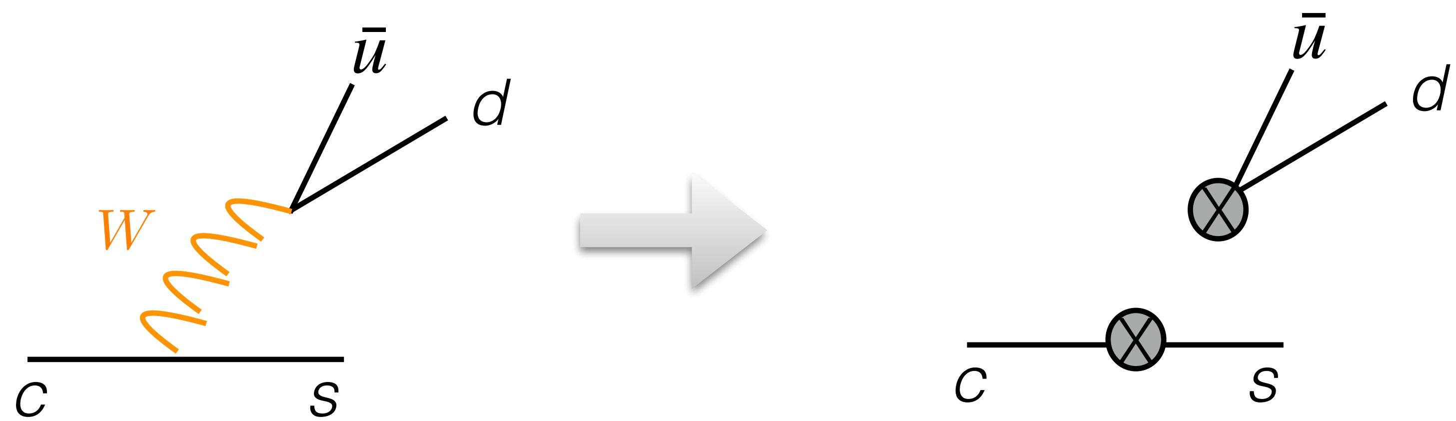
An example:

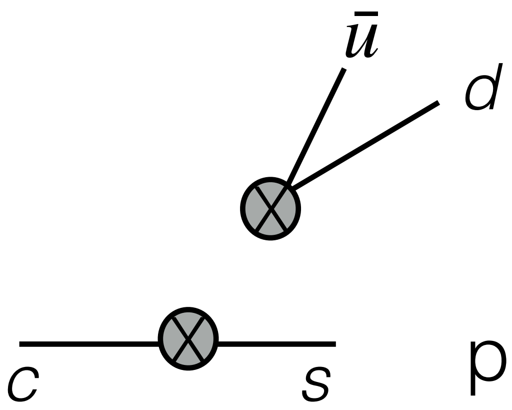


$$A = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{M_W^2}{k^2 - M_W^2} (\bar{s}c)_{V-A}(\bar{u}d)_{V-A}$$

$$(\bar{f}f)_{V-A} \equiv \bar{f}\gamma_\mu(1-\gamma_5)f$$

$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A}(\bar{u}d)_{V-A} + O\left(\frac{k^2}{M_W^2}\right)$$

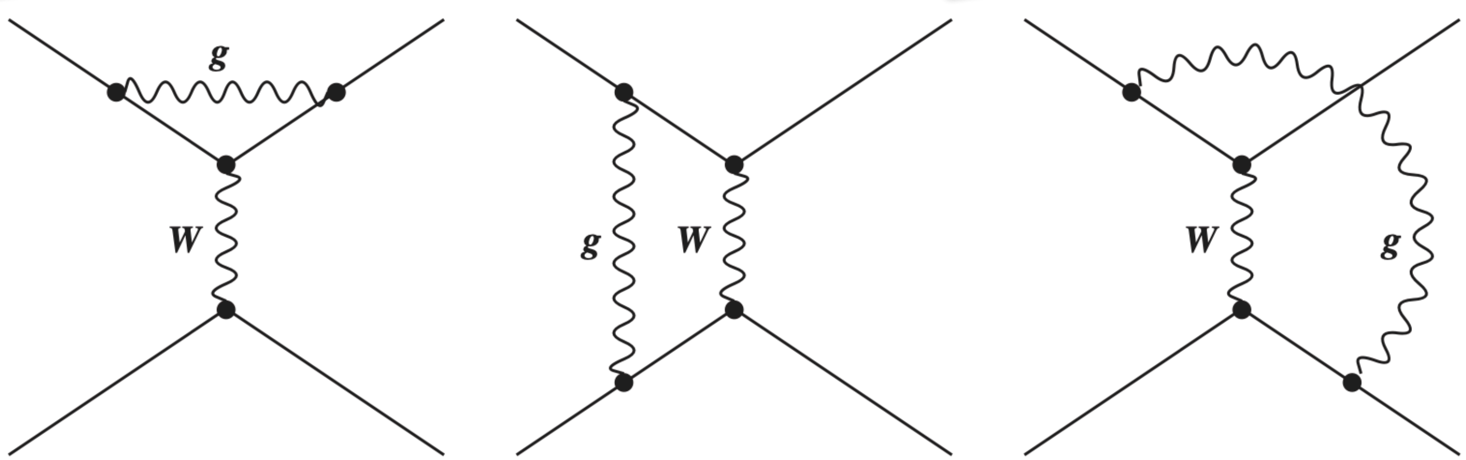
OPE



$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud}\, C \mathscr{Q} + \text{higher D}\,; \quad \mathscr{Q} \equiv (\bar{s}c)_{V-A}(\bar{u}d)_{V-A}$$

product of two currents expanded in series of local operators weighted by effective coupling constants—Wilson coefficients *C*

C=1 altered by QCD corrections + new operators induced



different colour structure

$$T^a_{\alpha\beta} T^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$$

OPE

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1(\mu) Q_1 + C_2(\mu) Q_2),$$

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$

$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

Amplitude of full theory should match with the amplitude produced from effective theory Hamiltonian → matching condition

$$A_{\text{full}} = A_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1 \langle Q_1 \rangle + C_2 \langle Q_2 \rangle).$$

$$A_{\text{full}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Big[\left(1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2} \right) \right) \right) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln\left(\frac{M_W^2}{-p^2} \right) S_2 - 3 \frac{\alpha_s}{4\pi} \ln\left(\frac{M_W^2}{-p^2} \right) S_1 \Big].$$

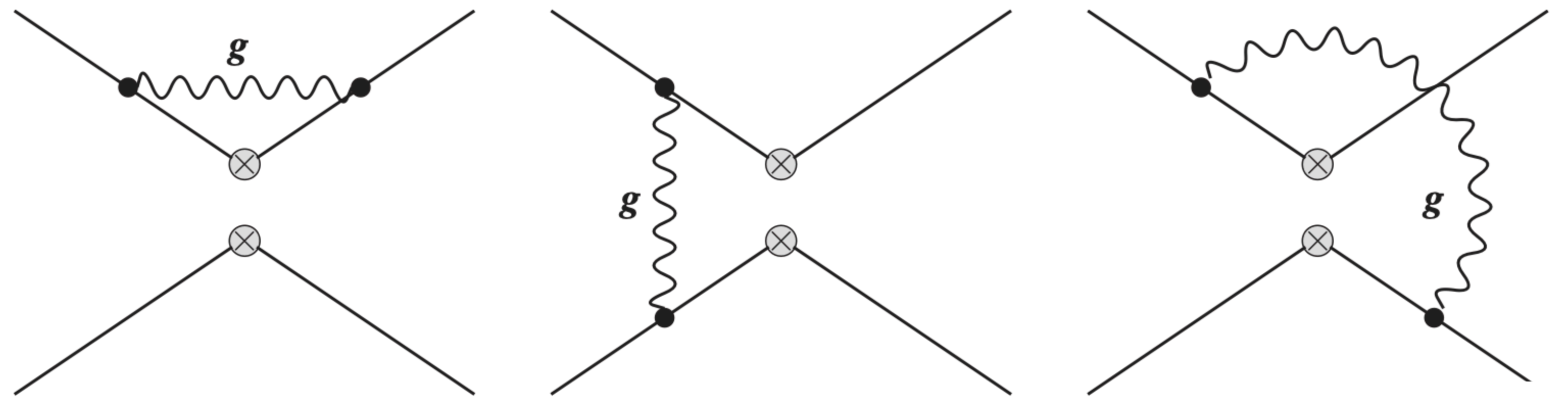
Divergent pole can be absorbed in field redefinition

$$S_1 \equiv \langle Q_1 \rangle_{tree} = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A},$$

$$S_2 \equiv \langle Q_2 \rangle_{tree} = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A},$$

tree level matrix element

Matrix element



$$\langle Q_1\rangle^{(0)} = \left(1 + 2C_F \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)\right) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_1 \; - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_2$$

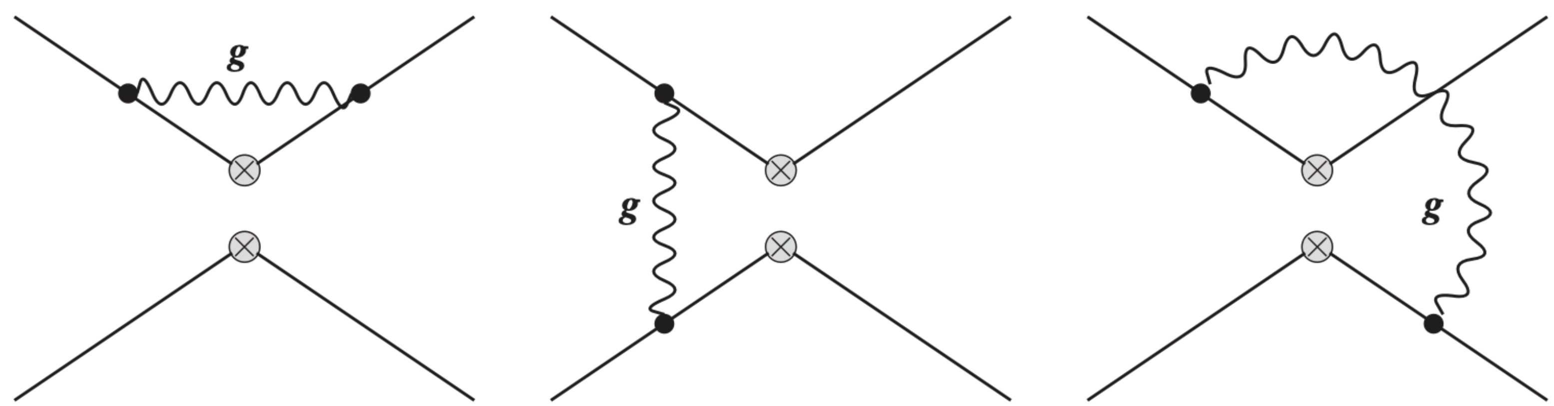
$$\langle Q_2\rangle^{(0)} = \left(1 + 2C_F \frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)\right) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_2 \; - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_1$$

divergences in 1st two terms absorbed in field renormalization.

More divergent than full theory—effective theory is nonrenormalizable

need additional constants —operator renormalization

Matrix element



$$\langle Q_i \rangle^{(0)} = Z_q^{-2} \hat{Z}_{ij} \langle Q_j \rangle$$

Quark field renormalization

Operator renormalization

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

Renormalized operators:

$$\langle Q_1 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right) S_1 - 3\frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right) S_2$$

$$\langle Q_2 \rangle = \left(1 + 2C_F \frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right)\right) S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right) S_2 - 3\frac{\alpha_s}{4\pi} \ln\left(\frac{\mu^2}{-p^2}\right) S_1$$

Wilson coefficients

Matching between full and EFT amplitudes gives

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right), \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right)$$

remember when NO QCD: $C_1(M_W) = 0, \;\; C_2(M_W) = 1$

Operator renormalization similar to coupling constant renormalization if Wilson coefficients are thought as bare coupling constants in $\mathscr{H}_{eff}$

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Operator renormalization similar to coupling constant renormalization if Wilson coefficients are thought as bare coupling constants in $\mathcal{H}_{eff}$

Factorisation of energy scales@ $\mathcal{O}(\alpha_s)$

$$\left(1 + \alpha_s r \ln\left(\frac{M_W^2}{-p^2}\right)\right) \doteq \left(1 + \alpha_s r \ln\left(\frac{M_W^2}{\mu^2}\right)\right) \cdot \left(1 + \alpha_s r \ln\left(\frac{\mu^2}{-p^2}\right)\right)$$

full theory= WC (short distance) matrix element (long distance)

$$\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}$$

Wilson coefficients

- WCs are independent of external states

  p^2 dropped from the expressions

  — need to be careful while regularising infrared divergences

- Operators mix under renormalization: $\hat{Z}$ is non-diagonal

→ Counter term for $\mathcal{Q}_2$ depends on the constants for both $\mathcal{Q}_2$ & $\mathcal{Q}_1$

diagonal basis: $Q_\pm = \dfrac{Q_2 \pm Q_1}{2}, \qquad C_\pm = C_2 \pm C_1$

$$C_\pm(\mu) = 1 + \left(\frac{3}{N} \mp 3\right) \frac{\alpha_s}{4\pi} \ln\left(\frac{M_W^2}{\mu^2}\right)$$

4%

Large log $\mu = 1\,\mathrm{GeV}$

Total 1st order correction amounts 60-130%

→ Naive breakdown of perturbative series

Rusa Mandal, IIT Gandhinagar

Wilson coefficients

Resum large logs via RG eqn: $$\frac{dC_\pm(\mu)}{d\ln\mu} = \gamma_\pm(g)C_\pm(\mu)$$

anomalous dimension

depends on renormalization constants

Similar to running of α_s ➡ $$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{1-\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right)} \approx \alpha_s(M_Z)\left[1-\sum_{n=1}^{\infty}\left(\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right)\right)^n\right]$$

In RG improved perturbation theory: $$C_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{\gamma_\pm^{(0)}}{2\beta_0}} C_\pm(M_W)$$

$C_\pm(M_W) \to 1$

@b-mass scale $C_+(\mu_b) = 0.847$ and $C_-(\mu_b) = 1.395$

➡ departure from the value 1 due to QCD

Prescription

- Step-1: Matching in perturbation theory

amplitude in full theory matched to operator matrix element in effective theory → extraction of WCs $C_i(\Lambda)$

→ mass of heavy particles integrated out

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using anomalous dimension of operators compute WCs at any lower scale via RG evolution $C_i(\mu) = U(\mu, \Lambda) C_i(\Lambda)$

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- Step-3: Non-perturbative calculation

hadronic matrix elements at the lower scale via methods: Lattice gauge theory, QCD sum rules

factorization between short & long distance physics

$C_i(\mu) : \mu >$ ← short → $< \mu : \langle \mathcal{Q}(\mu) \rangle$