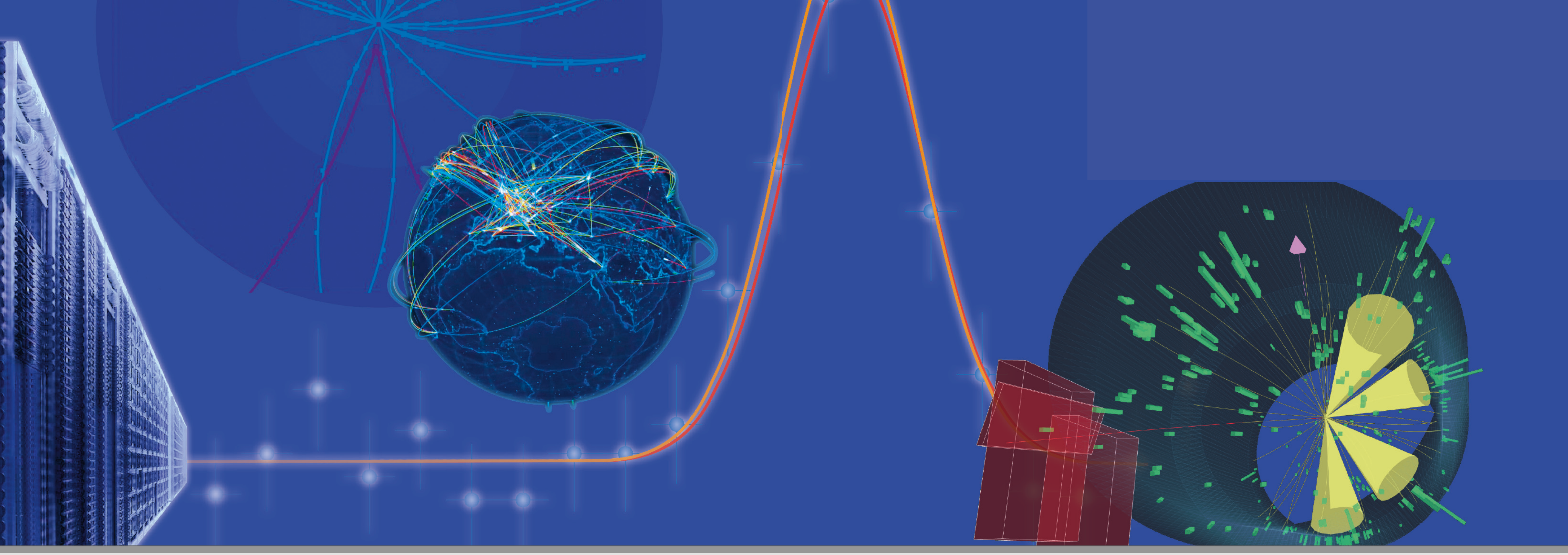

The Anatomy of semileptonic Decays

Belle II Physics Week 2023

Florian Bernlochner
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Disclaimer :

Topic is much more extensive of what I will cover in **O(1h + Questions)**

Reviews (RMPs) on the subject :

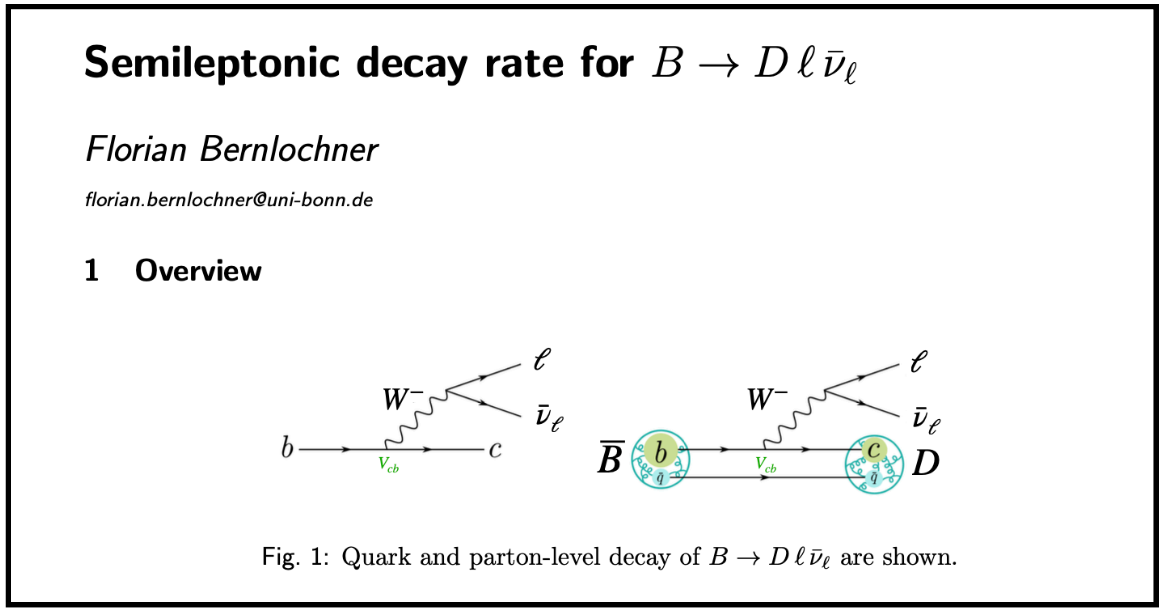
Mannel, Dingfelder

Richman, Burchat

Bernlochner, Robinson, Franco Sevilla, Wormser

Attached to the agenda :-)

In addition: some notes on $B \to D\ell\bar{\nu}_\ell$ are also attached



Semileptonic decay rate for $B \to D\,\ell\,\bar{\nu}_\ell$

Florian Bernlochner

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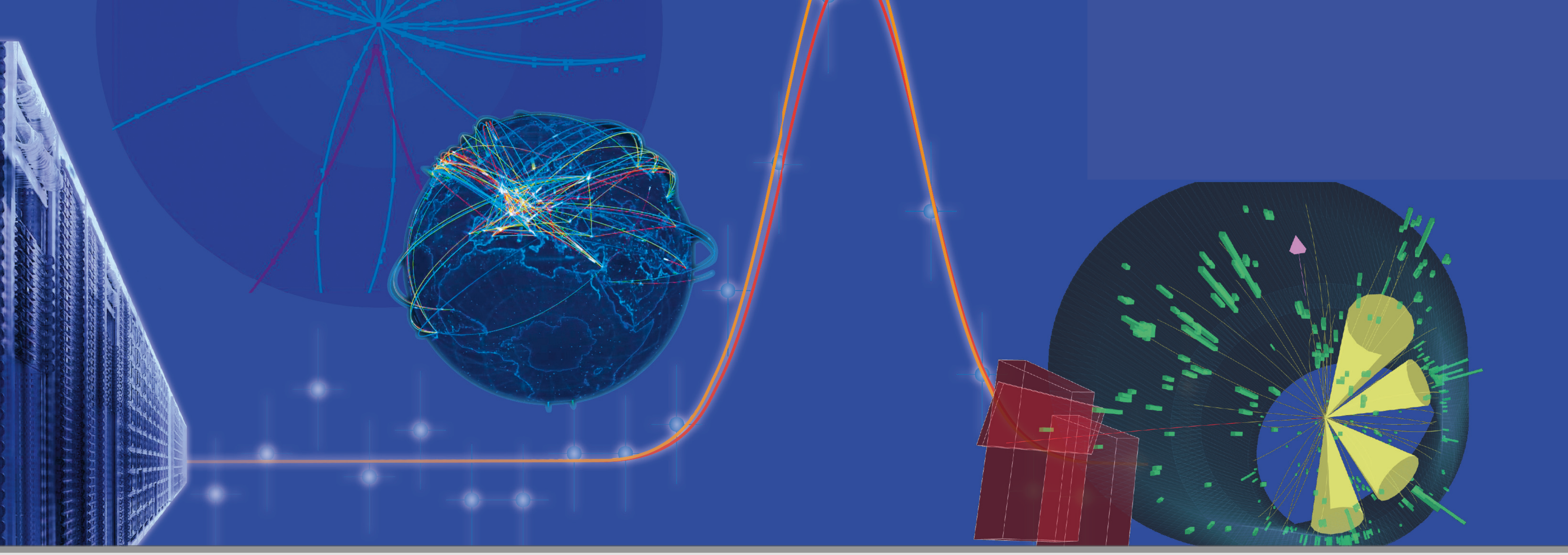
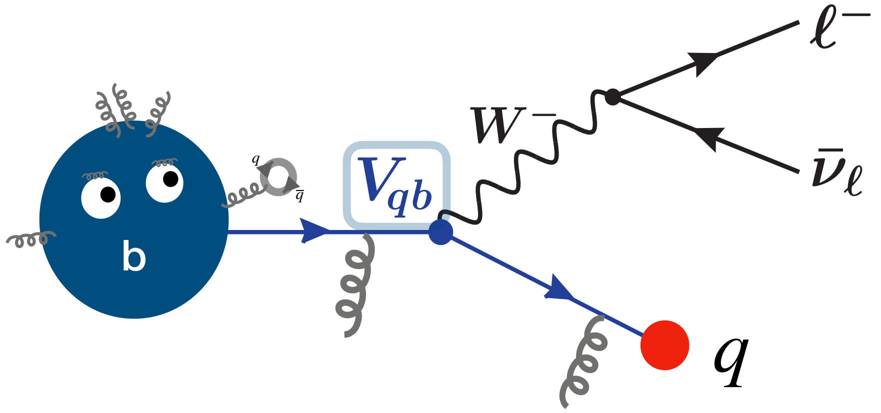
1 Overview

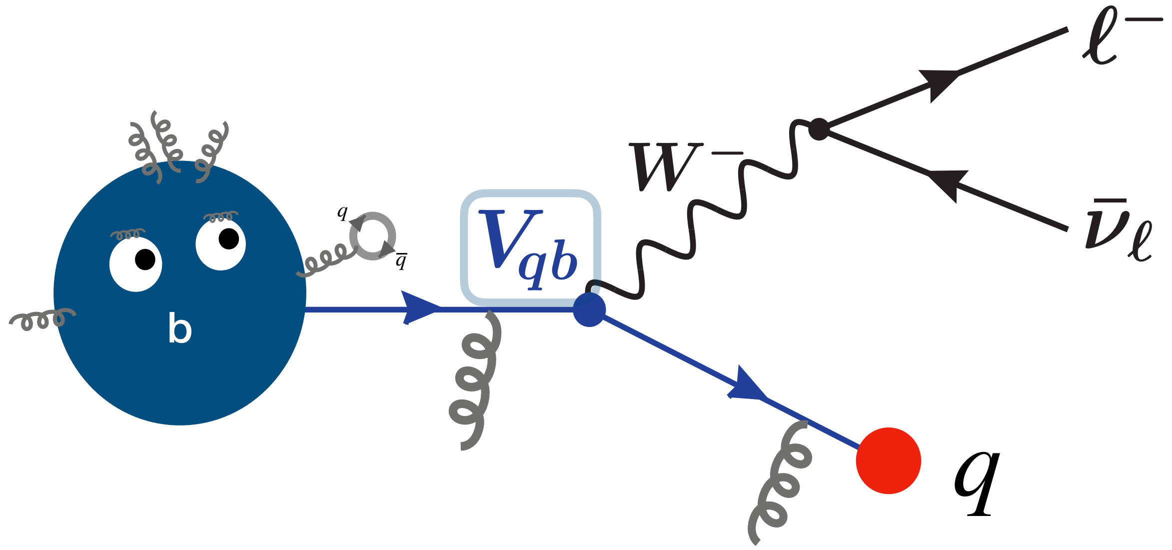
Fig. 1: Quark and parton-level decay of $B \to D\,\ell\,\bar{\nu}_\ell$ are shown.



1) Overview

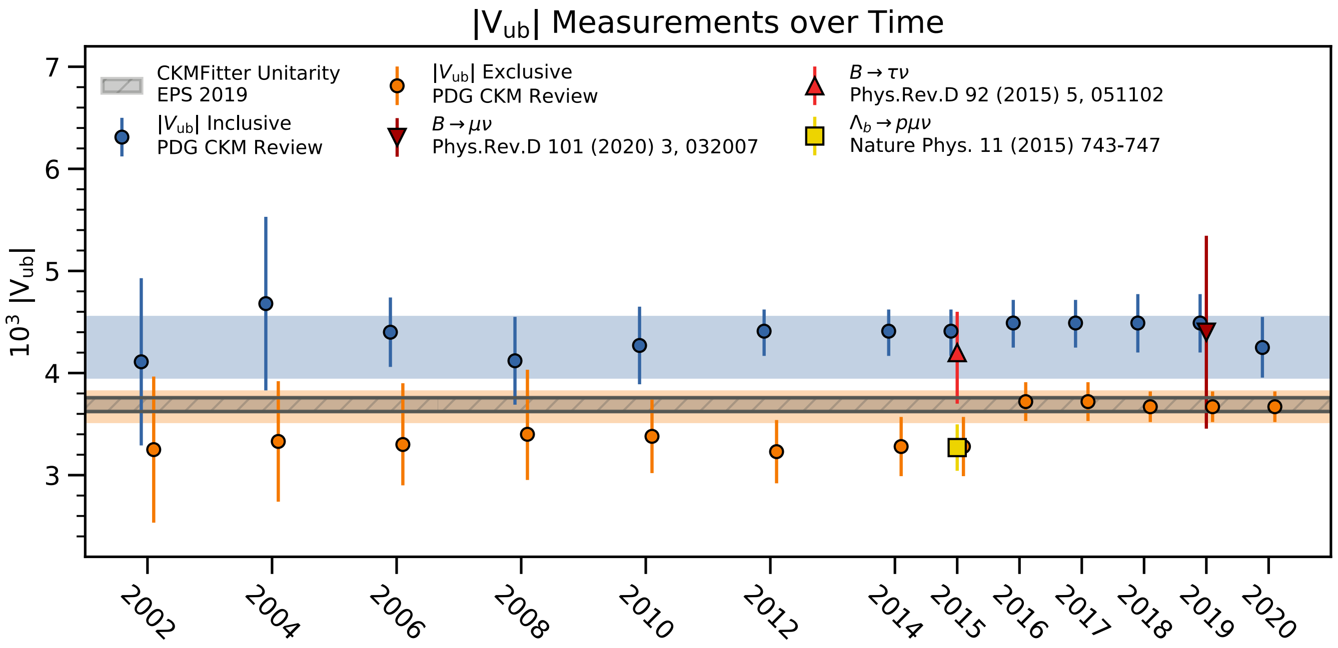


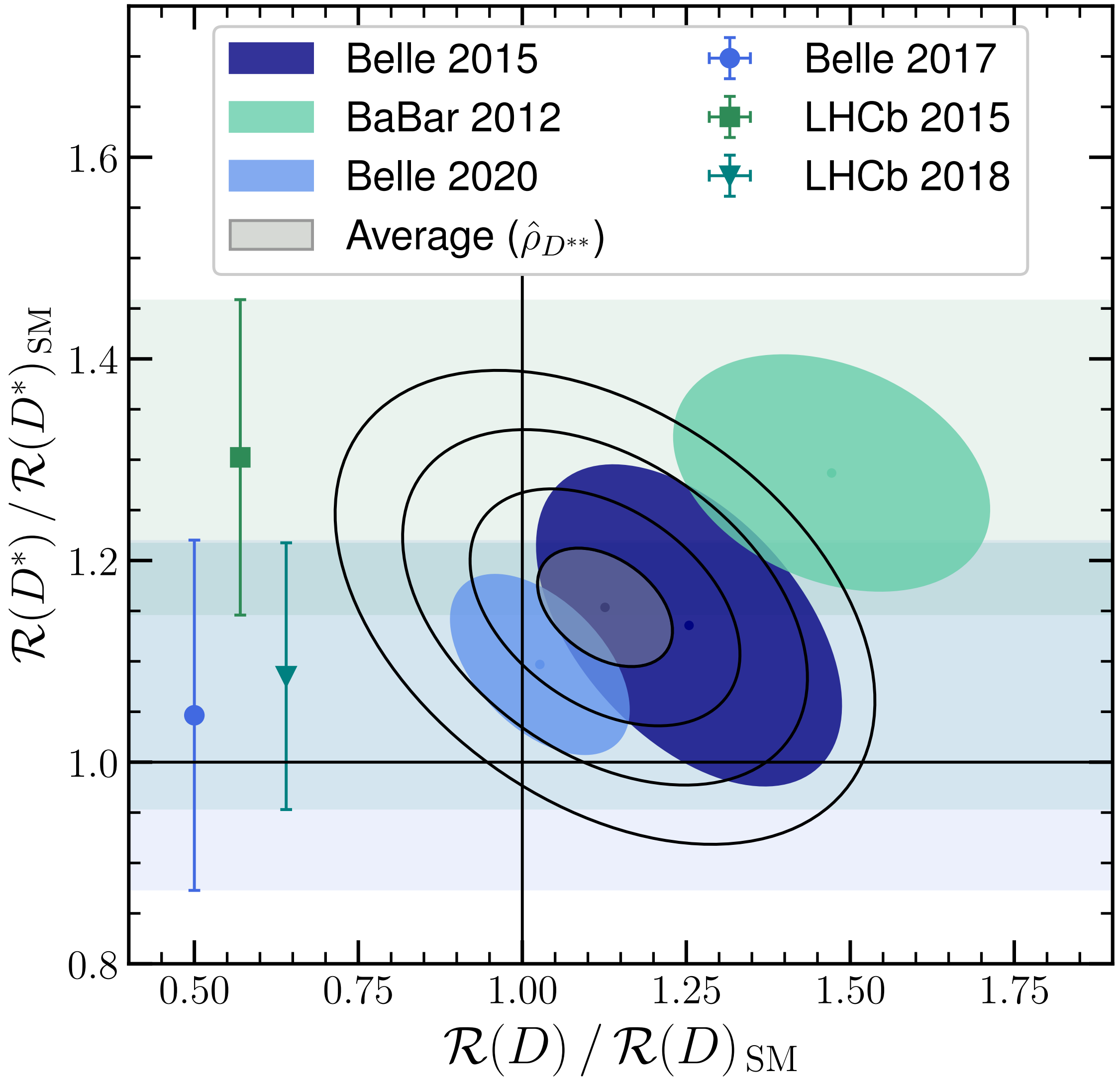
Let’s take a deep dive



They look **cute**, but
that could be **deceiving** …

… they are responsible for some of the long-standing **discrepancies** since about a decade





Why is it important to measure $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$?

$$\begin{array}{cc} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \end{array}$$

CKM Matrix

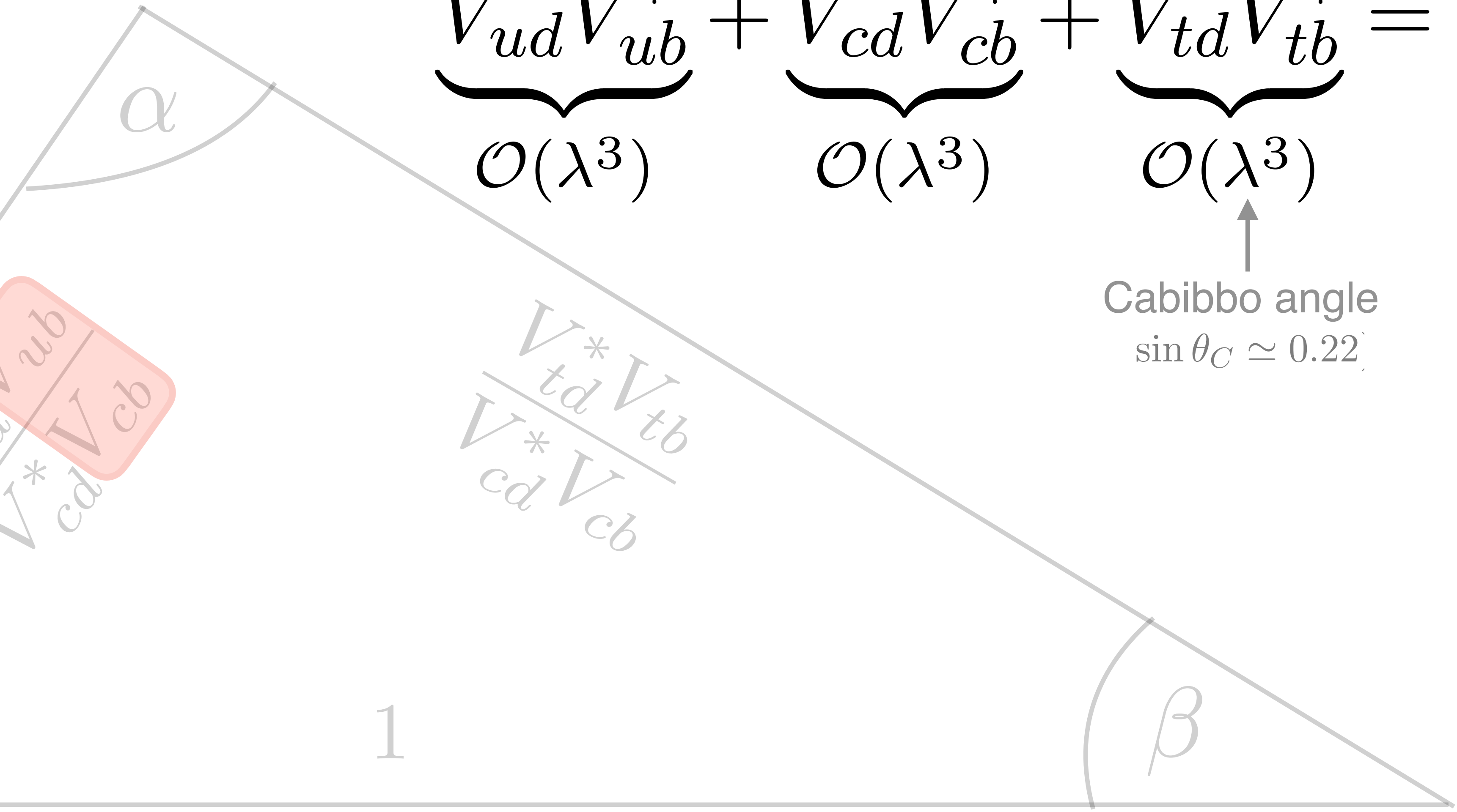
SM: Unitary 3x3 Matrix

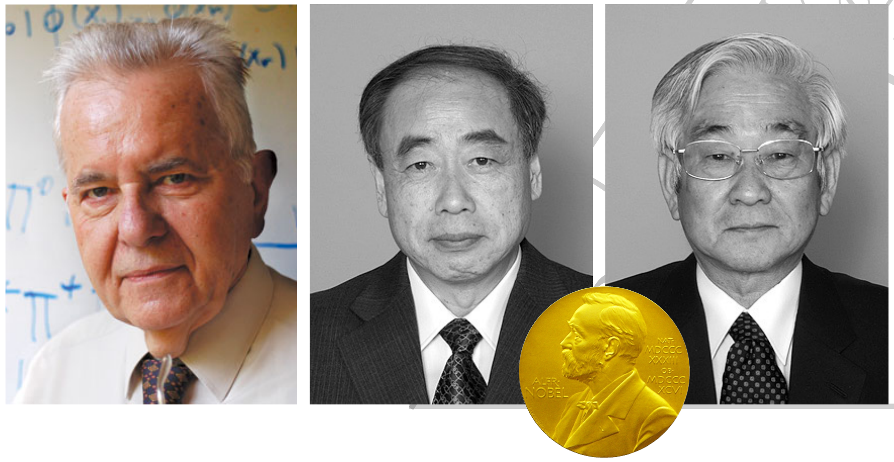
Over constrain Unitarity condition
→ Potent test of Standard Model

Unitarity
$CC^\dagger = 1$

$$\underbrace{V_{ud}V_{ub}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{cd}V_{cb}^*}_{\mathcal{O}(\lambda^3)} + \underbrace{V_{td}V_{tb}^*}_{\mathcal{O}(\lambda^3)} = 0$$

Cabibbo angle
$\sin\theta_C \simeq 0.22$

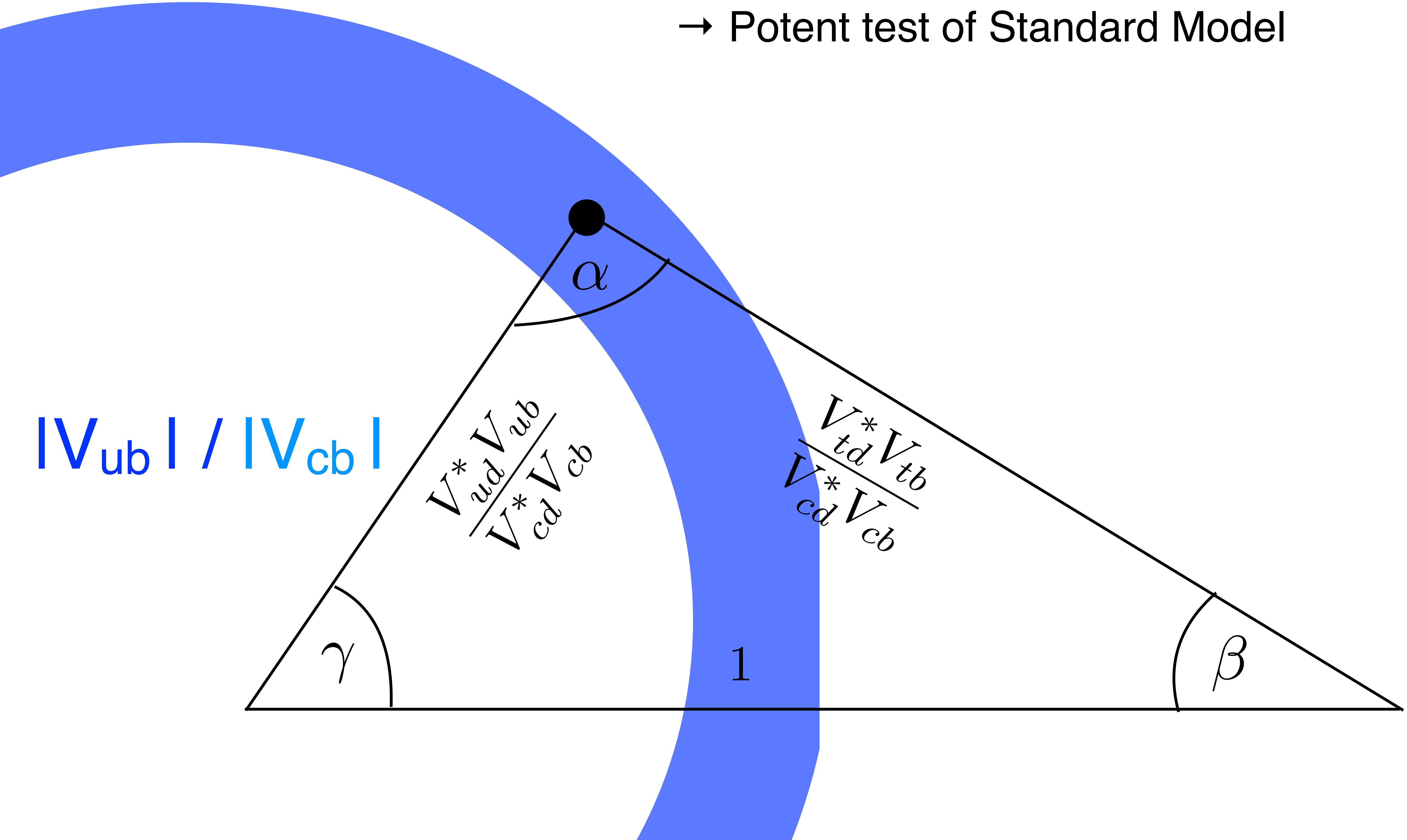




Nobel prize 2008

Why is it important to measure $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$?

Over constrain Unitarity condition
→ Potent test of Standard Model



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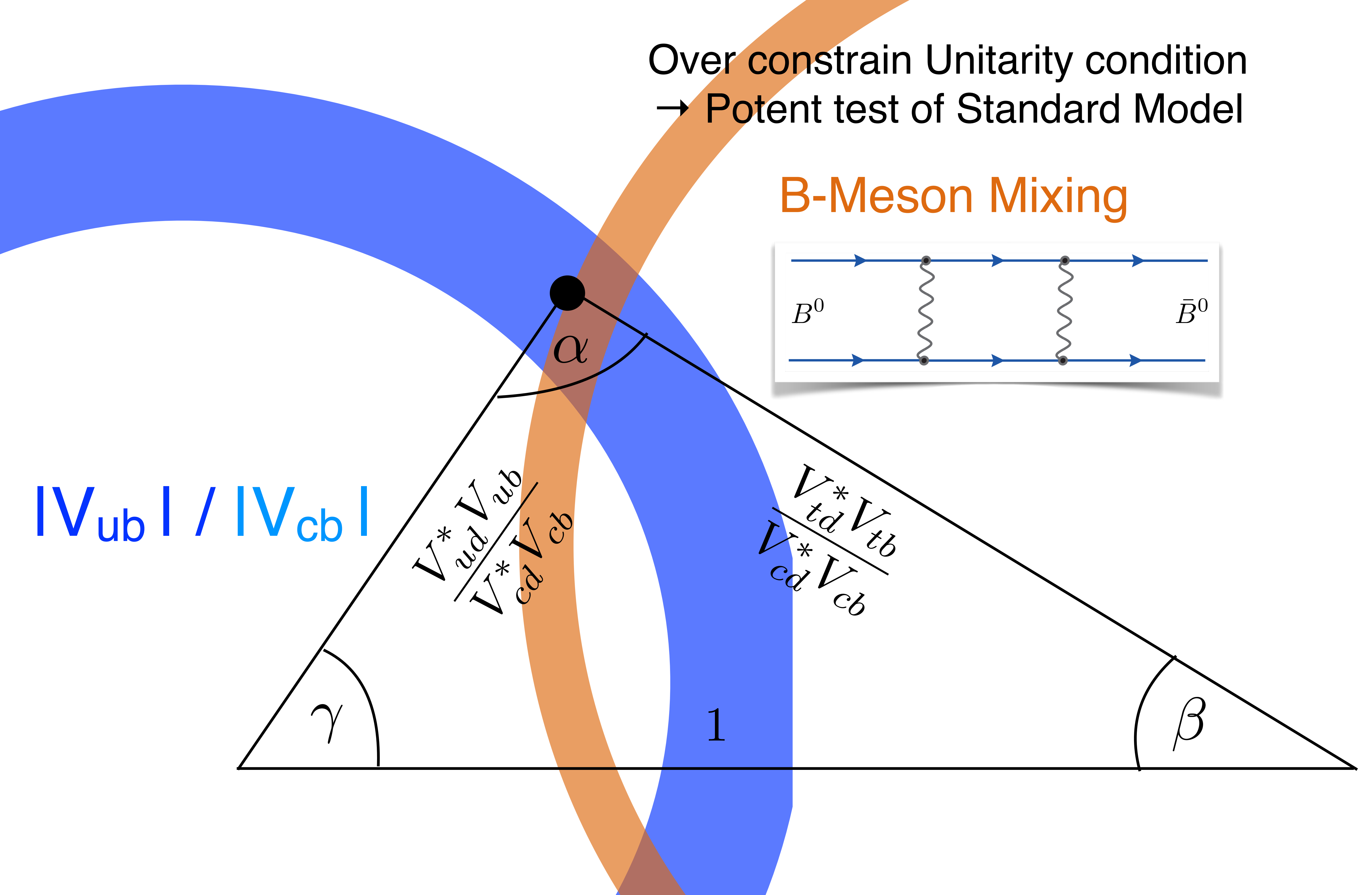
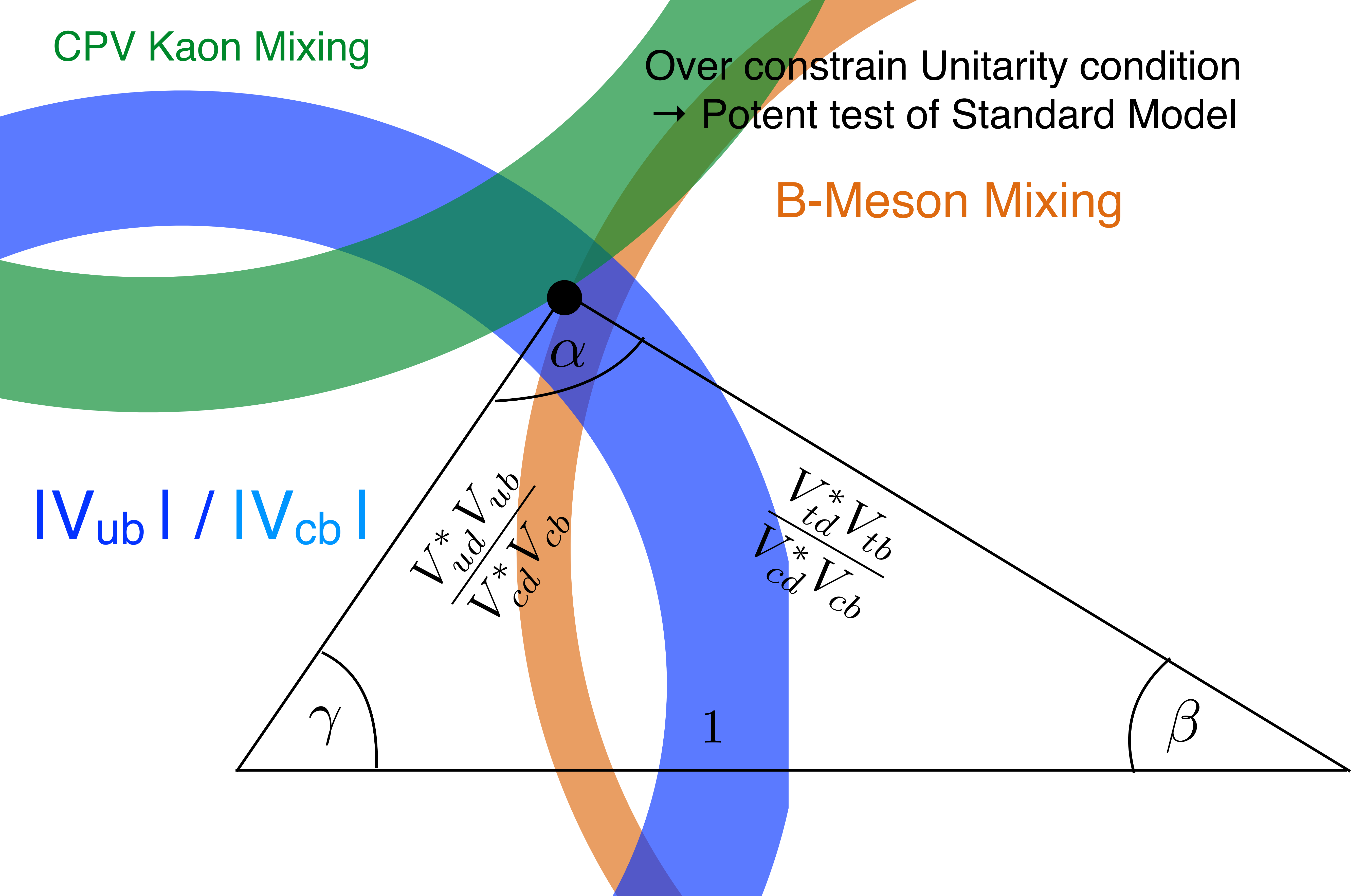


Why is it important to measure $\left|V_{ub}\right| \& \left|V_{cb}\right|$?

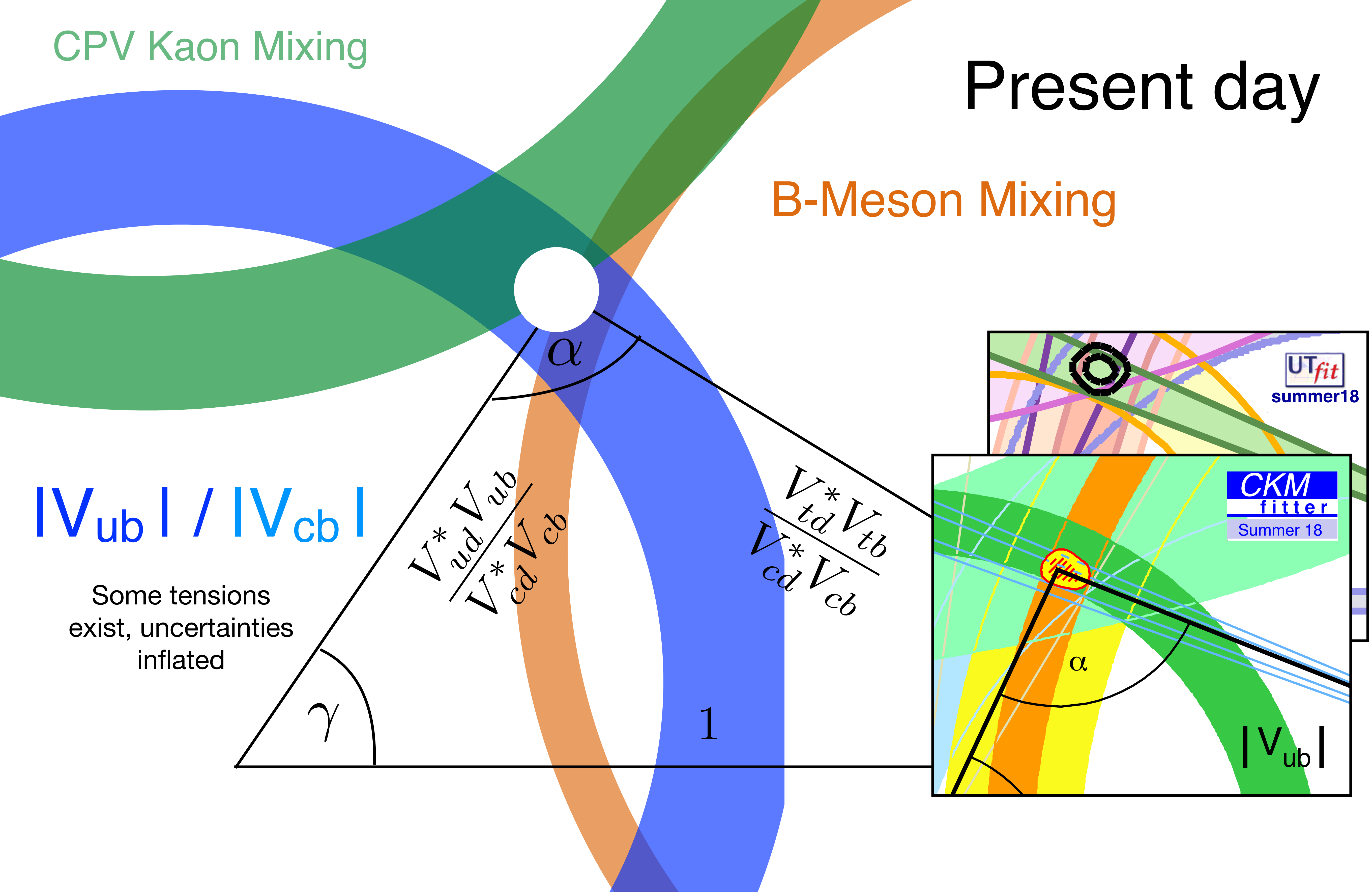
Over constrain Unitarity condition

→ Potent test of Standard Model



Why is it important to measure $|V_{ub}|$ & $|V_{cb}|$?

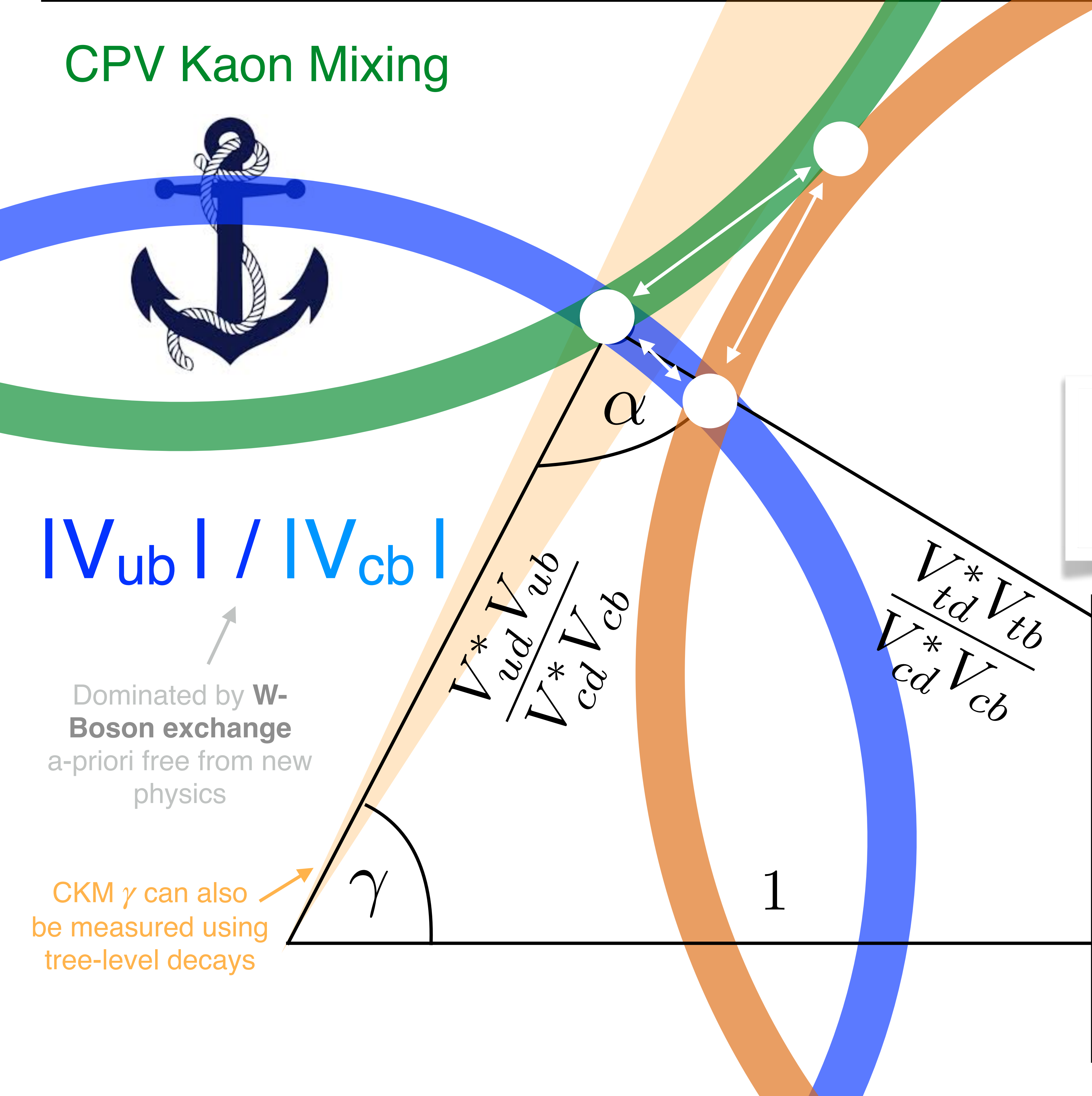
Present day



$|V_{ub}| / |V_{cb}|$

Some tensions exist, uncertainties inflated

Why is it important to measure $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$?

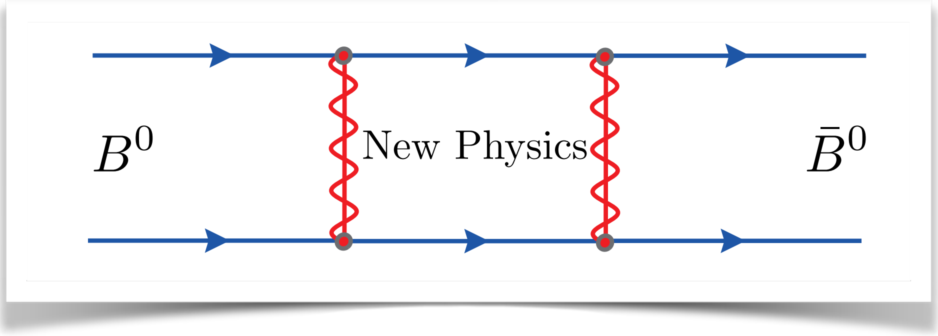


CPV Kaon Mixing

The future?

with Belle II & LHCb

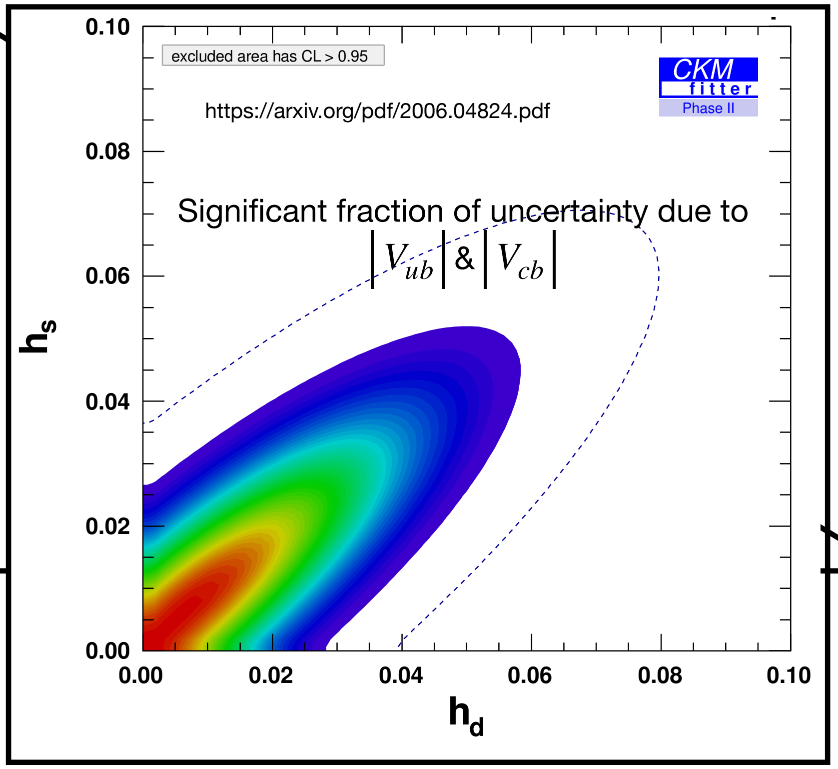
B-Meson Mixing



$|V_{ub}|$ / $|V_{cb}|$

Dominated by **W-Boson exchange** a-priori free from new physics

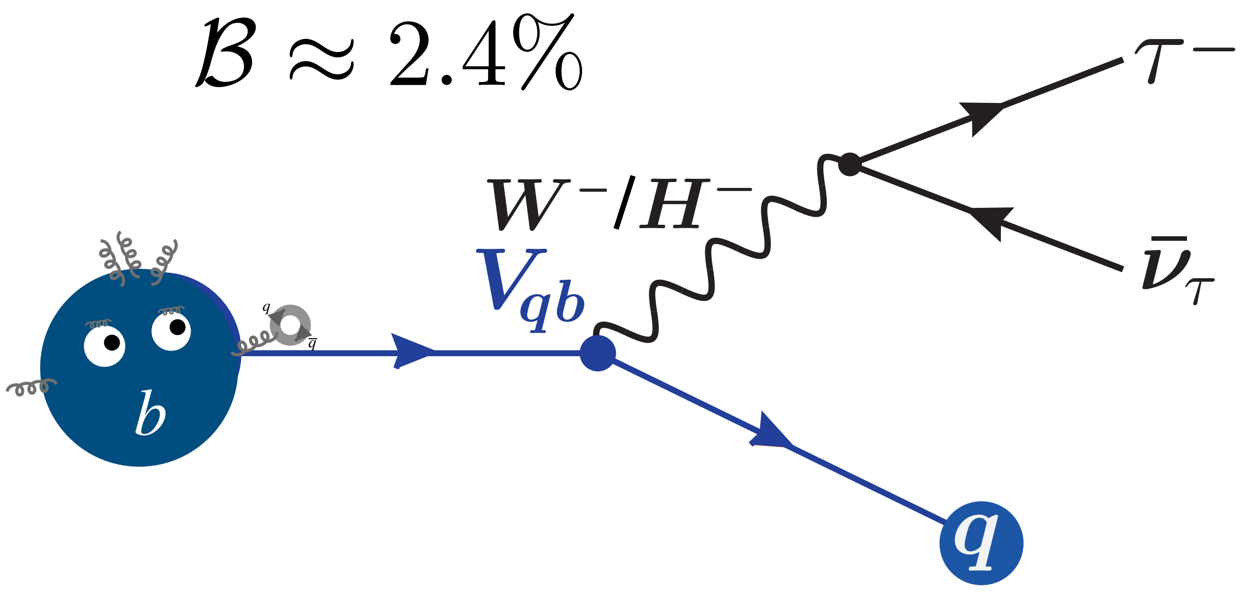
CKM γ can also be measured using tree-level decays

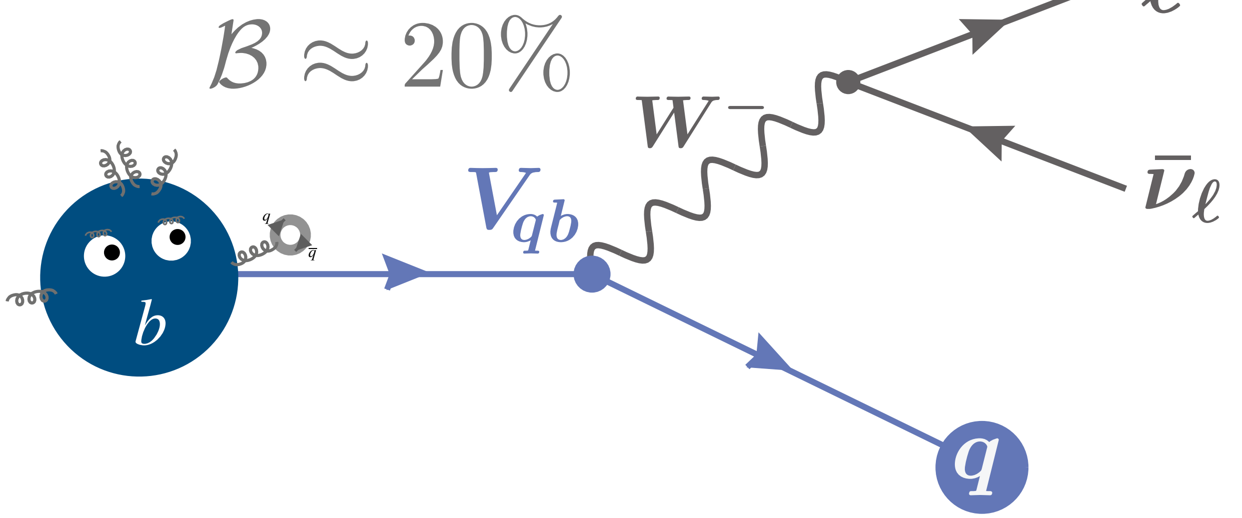


https://arxiv.org/pdf/2006.04824.pdf

Significant fraction of uncertainty due to $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$

Semileptonic decays with τ





Observable of choice:

$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

$\ell = e, \mu$

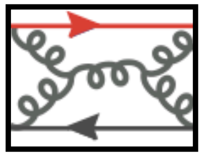
$$\downarrow$$

$$R(D^{(*)}, \pi, J/\psi)$$

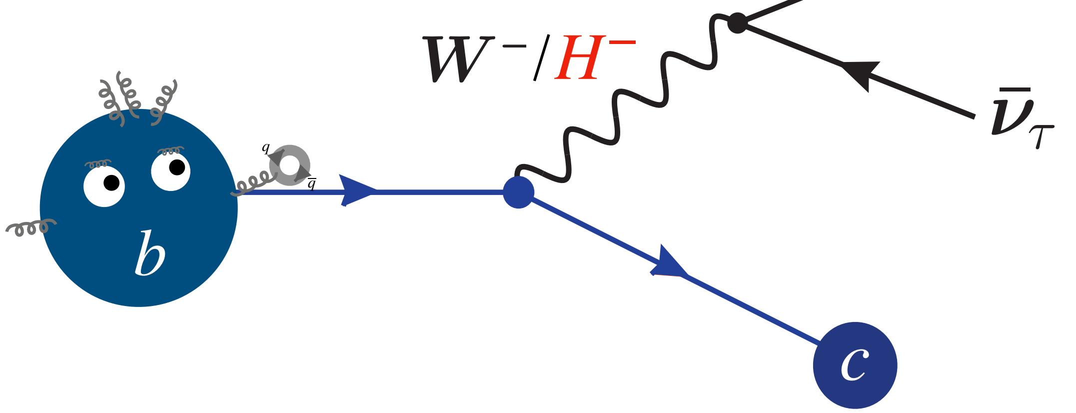
Benefits:

- Experimental systematics **cancel in ratio**
- Theory uncertainties **cancel in ratio**

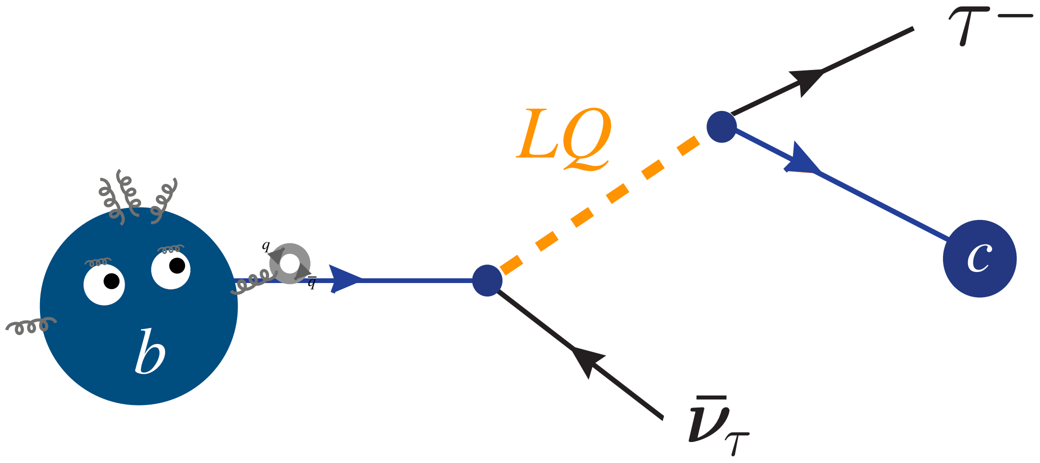
QCD:



charged Higgs bosons



Leptoquarks

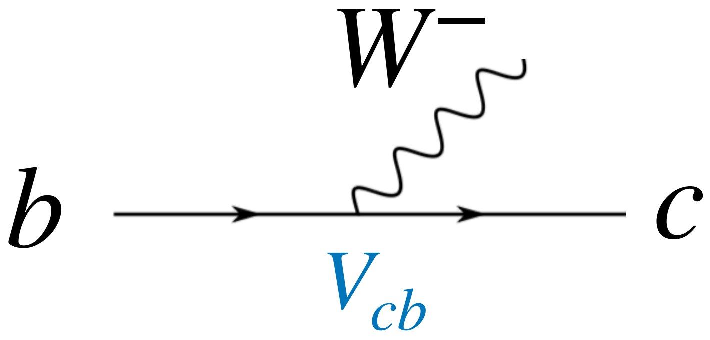


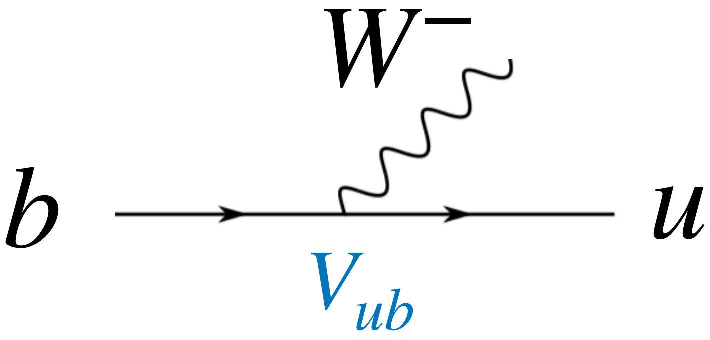
→ Not the focus of this talk; but I added you some introduction material nonetheless in case you are interested in these!

How do we determine $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$?

At first glance fairly straightforward:

Step 1: Identify a process, in which you have a $b \to cW^-$ or $b \to uW^-$ vertex





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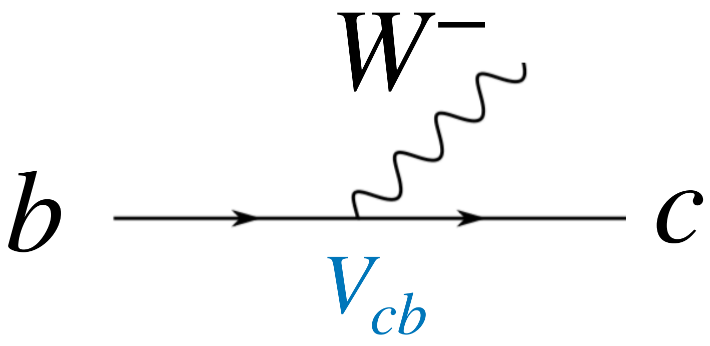
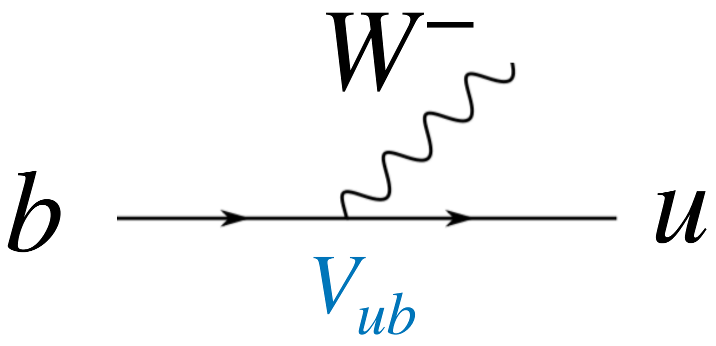




Step 2: Measure how often such a process occurs

Branching fraction

$q = c \text{ or } u$

$$\mathcal{B}(b \to qW)$$

and compare this with the expectation from theory w/o CKM factors (or $V_{qb} = 1$)

Mathematically: $\boxed{\mathscr{B}(b \to qW) \propto |V_{qb}|^2}$

Predicted partial rate sans CKM factors (or with $V_{qb} = 1$) $\boxed{\Gamma(b \to qW)}$

Both quantities are connected as

$$\boxed{|V_{qb}|^2 \frac{\Gamma(b \to qW)}{\Gamma(b \to \text{Everything})} = \mathscr{B}(b \to qW)}$$

so we can solve this using $\tau_b = \hbar / \Gamma(b \to \text{Everything})$

$$|V_{qb}| = \sqrt{\frac{\mathscr{B}(b \to qW)}{\tau_b \Gamma(b \to qW)}}$$

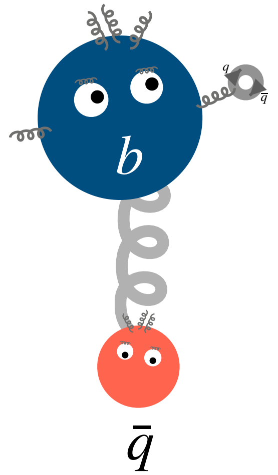
Measured by experiment

Predicted from theory

Great, now we only have to identify suitable processes for this:

1. Complication: Quarks are not free particles

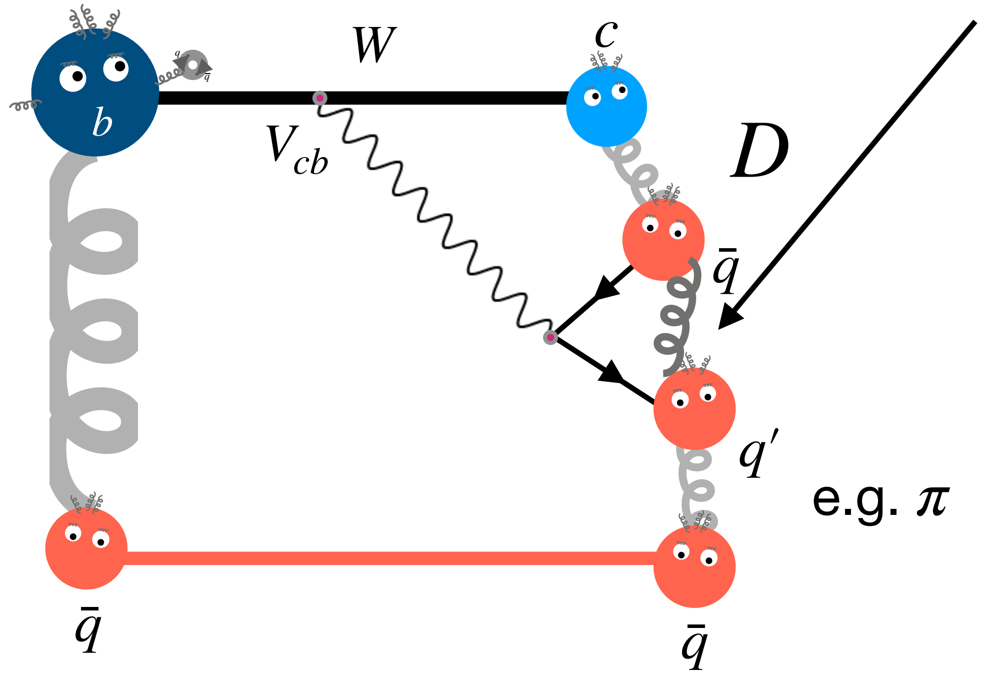
i.e. initial and final state quarks will be bound in hadrons (mesons or baryons)



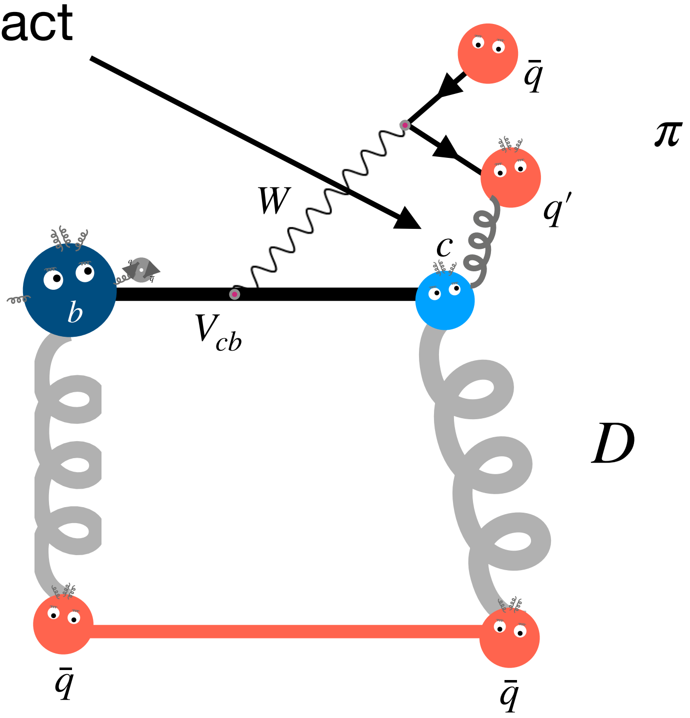
2. Complication: We need a process, we can describe well from a theory point of view

final states involving $W^- \to q\bar{q}'$ introduce additional CKM factors (a priori fine), but also have **color charged constituents**

i.e. final states can strongly interact



+ multiple diagrams



So what are the choices?

1) Hadronic decays

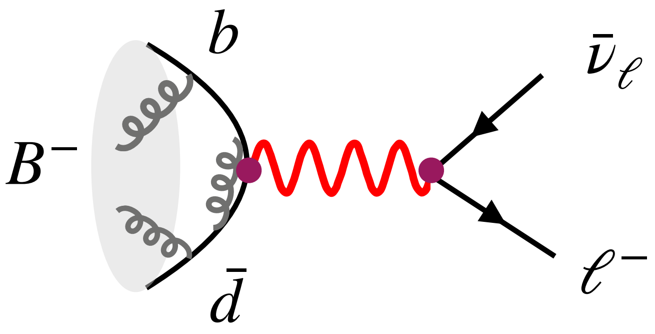
→ theory very hard, experimentally "easy"

2) Leptonic decays

→ theory "easy"
experimentally very hard

$$\mathscr{B}(B \to \mu \bar{\nu}_{\mu}) \sim 10^{-7}$$

$$\mathscr{B}(B \to \tau \bar{\nu}_{\tau}) \sim 10^{-4}$$



3) Semileptonic decays

→ theory doable, experimentally doable

So what are the choices?

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$$\mathscr{B}(B \to \mu \bar{\nu}_\mu) \sim 10^{-7}$$

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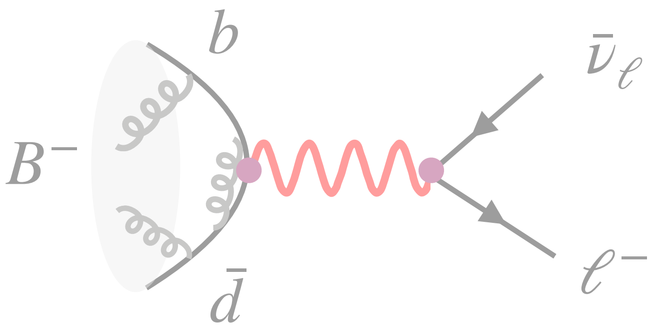
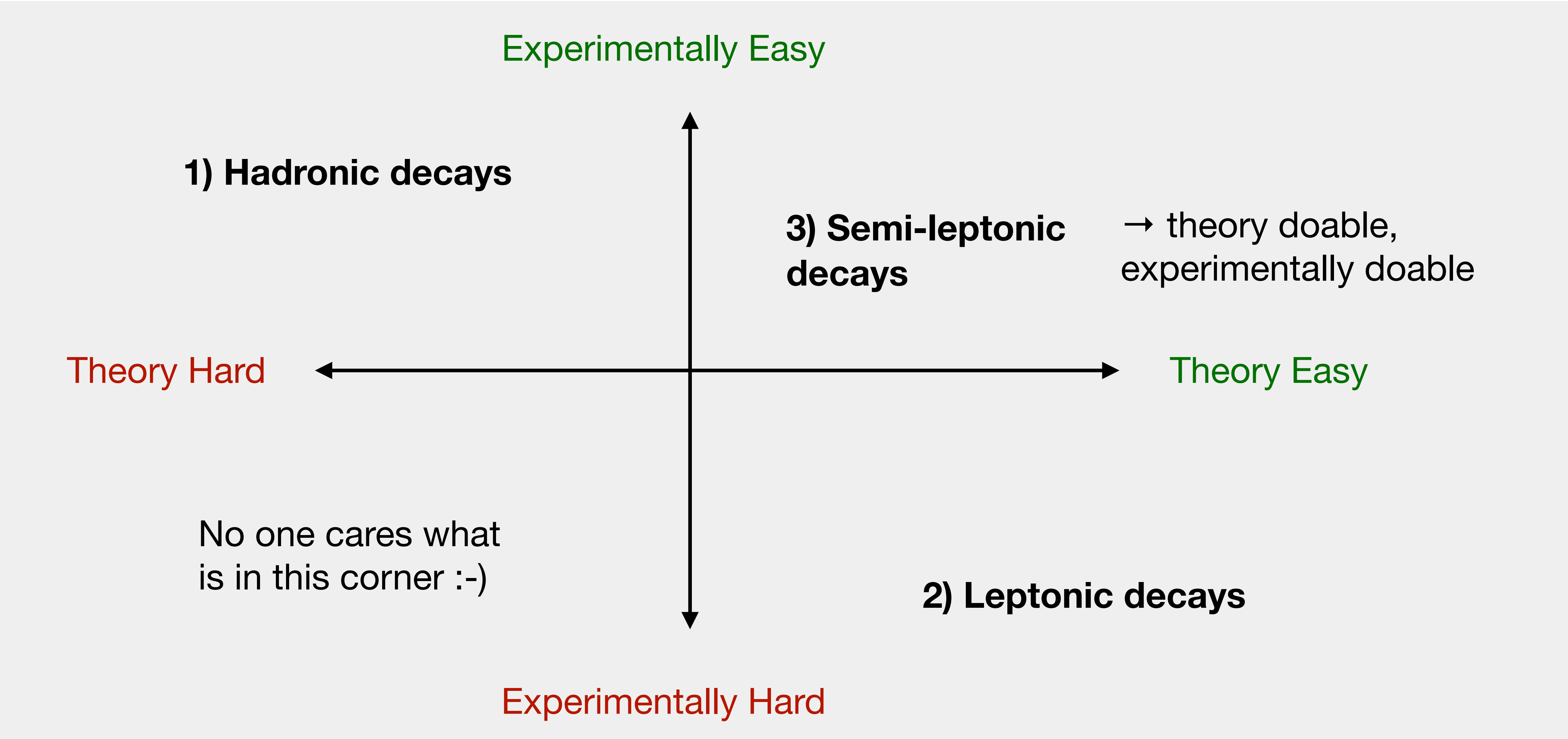


3) Semileptonic decays → theory doable, experimentally doable



Experimentally Easy

1) Hadronic decays

3) Semi-leptonic decays → theory doable, experimentally doable

Theory Hard

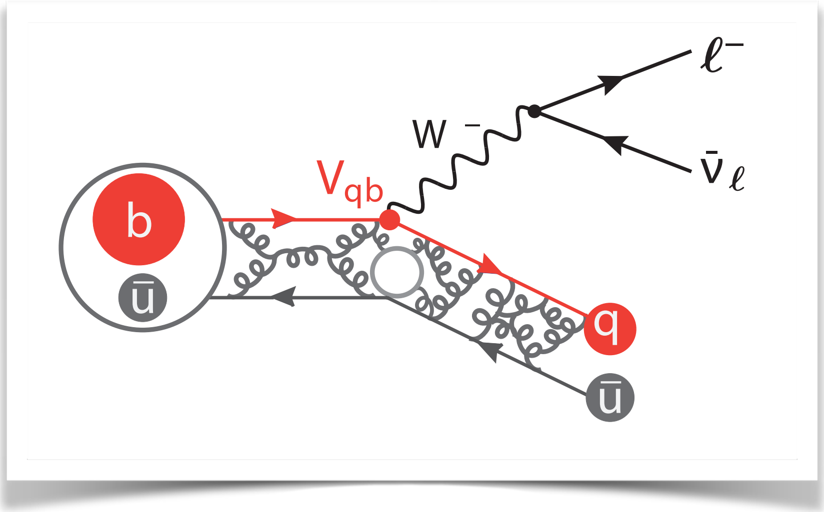
Theory Easy

No one cares what is in this corner :-)

2) Leptonic decays

Experimentally Hard

A quick boot-camp: how do we determine $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$?



Inclusive $|V_{ub}|$

$$\bar B \to X_u\,\ell\,\bar\nu_\ell$$

\+ Fermi Motion / Shape Function

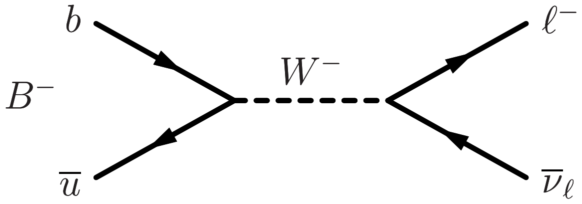
Inclusive $|V_{cb}|$

$$\bar B \to X_c\,\ell\,\bar\nu_\ell$$

Operator Product Expansion

$$\boxed{\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\,\ell\,\bar\nu_\ell) + 1/m_{c,b} + \alpha_s + \ldots\right]}$$

'Leptonic' $|V_{ub}|$



$$\boxed{\mathcal{B} \propto |V_{ub}|^2\, f_B^2\, m_\ell^2}$$

B-Meson decay constant

Exclusive $|V_{ub}|$

$$\bar B \to \pi\,\ell\,\bar\nu_\ell, \Lambda_b \to p\,\mu\,\bar\nu_\mu$$

Exclusive $|V_{cb}|$

$$\bar B \to D\,\ell\,\bar\nu_\ell, \bar B \to D^*\,\ell\,\bar\nu_\ell$$

$$\boxed{\mathcal{B} \propto |V_{cb}|^2\, f^2}$$

Form Factors

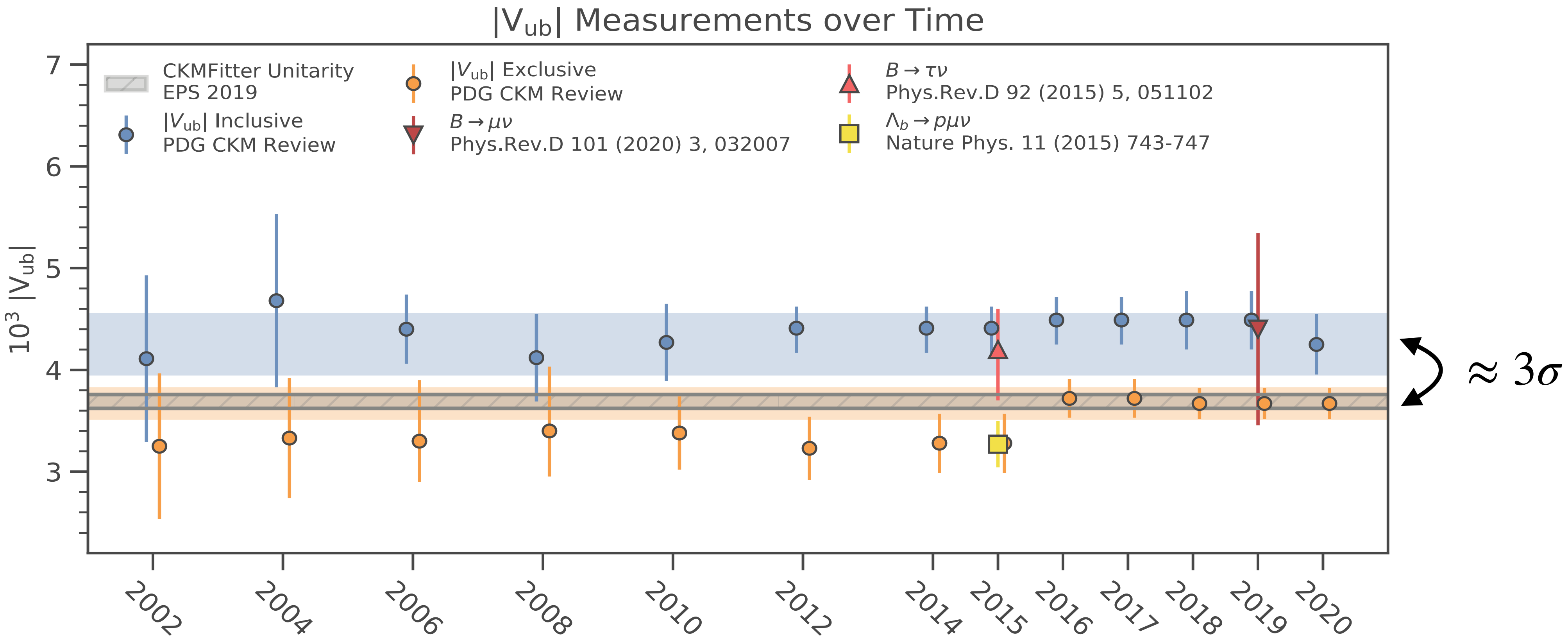
$$\langle B|H_\mu|P\rangle = (p+p')_\mu\, f_+$$

How are we doing?

$|V_{ub}|$

Inclusive

Exclusive



$|V_{cb}|$

Inclusive

Exclusive

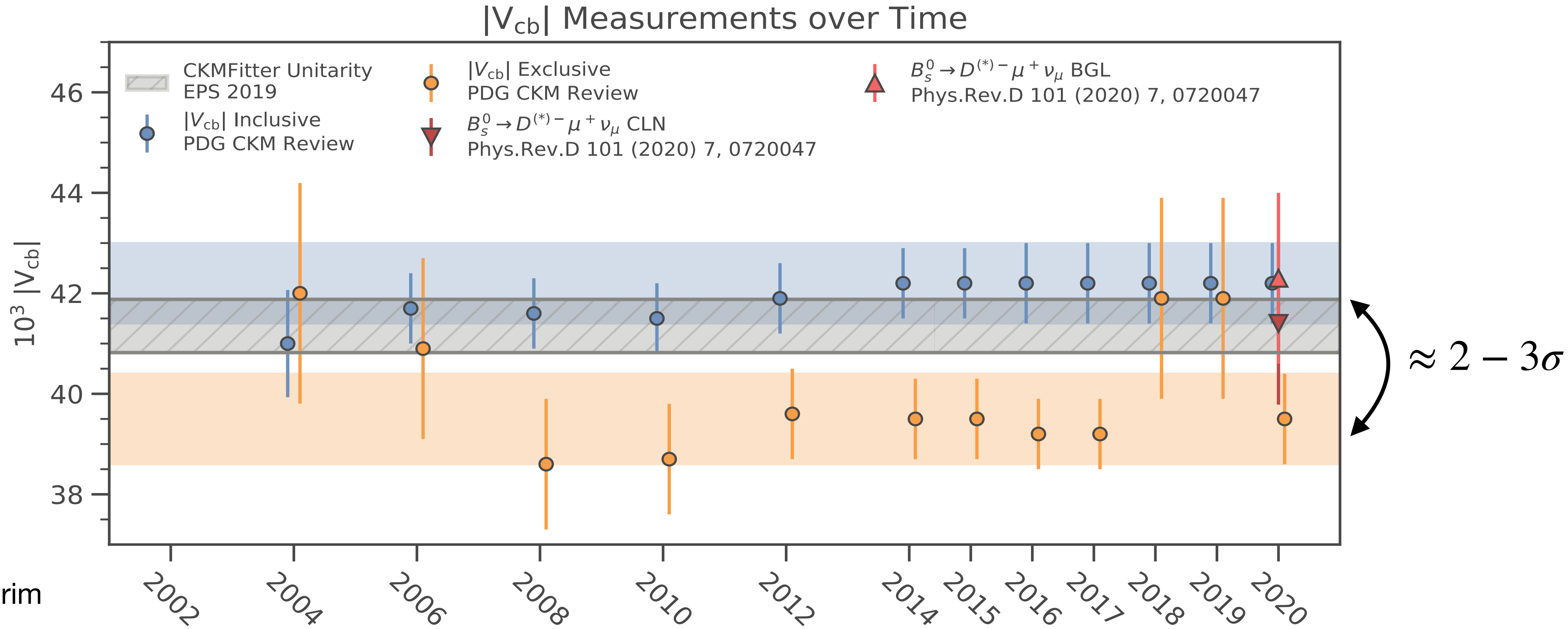
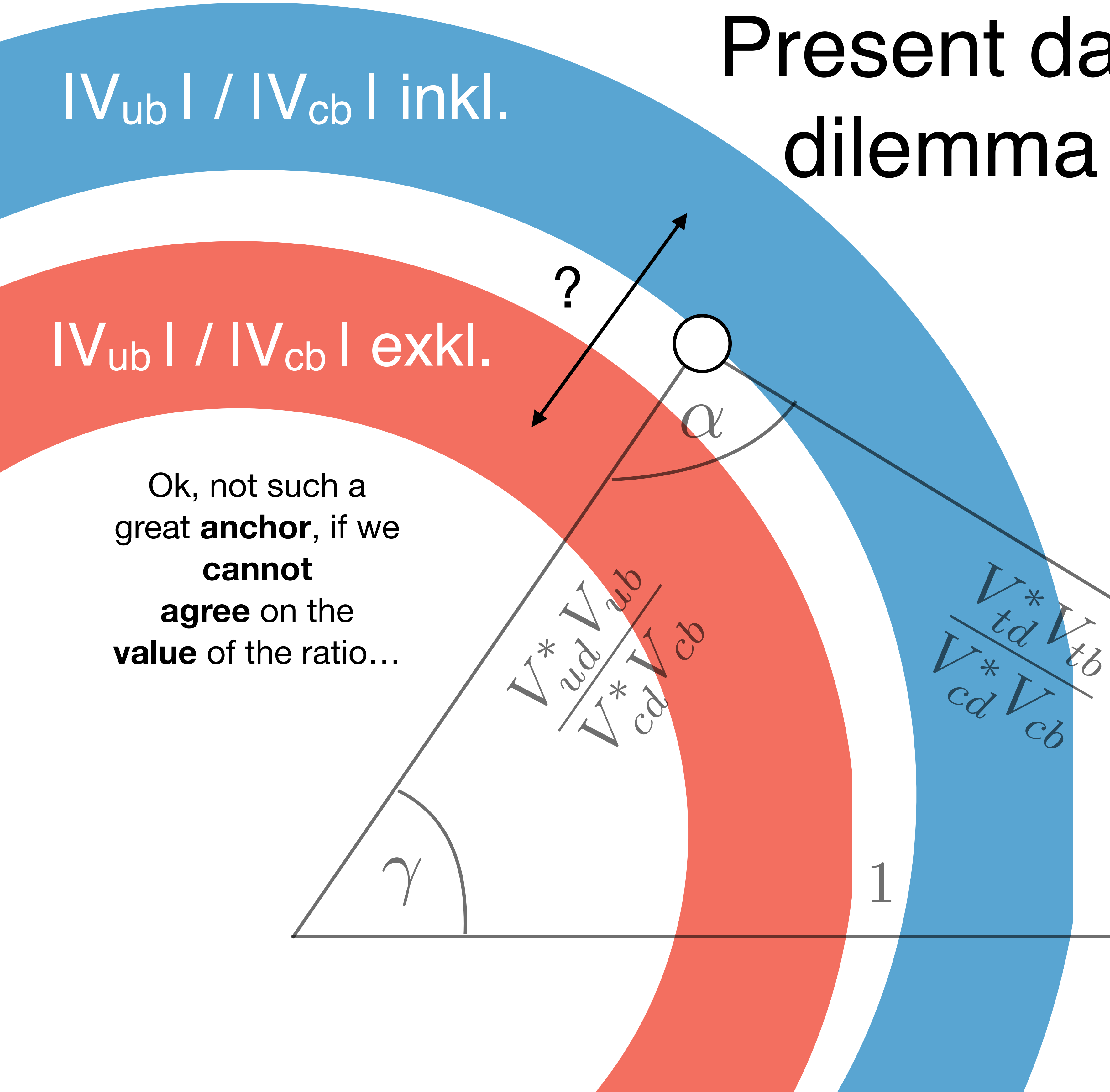


Image credit: Markus Prim

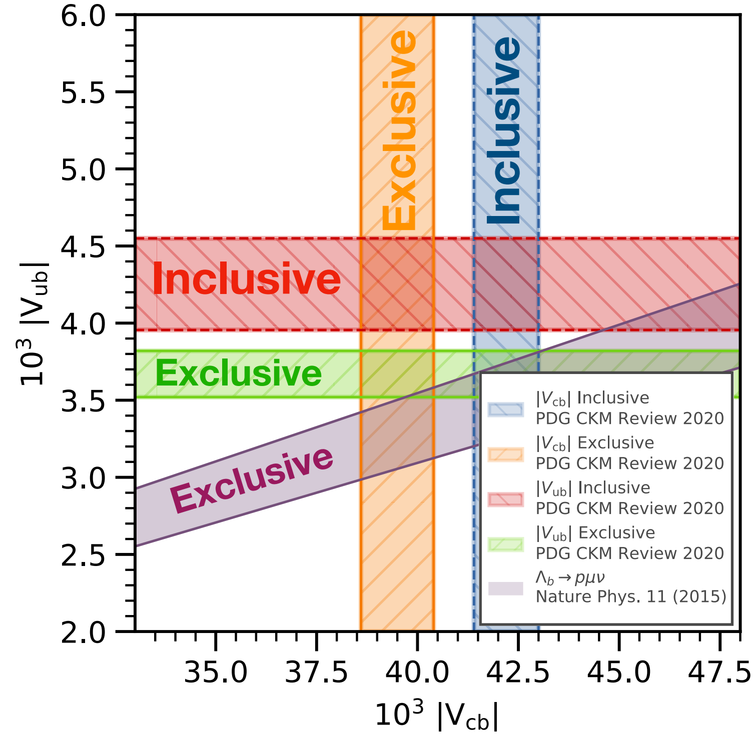
Why is it important to measure $|V_{ub}|$ and $|V_{cb}|$?

Present day dilemma



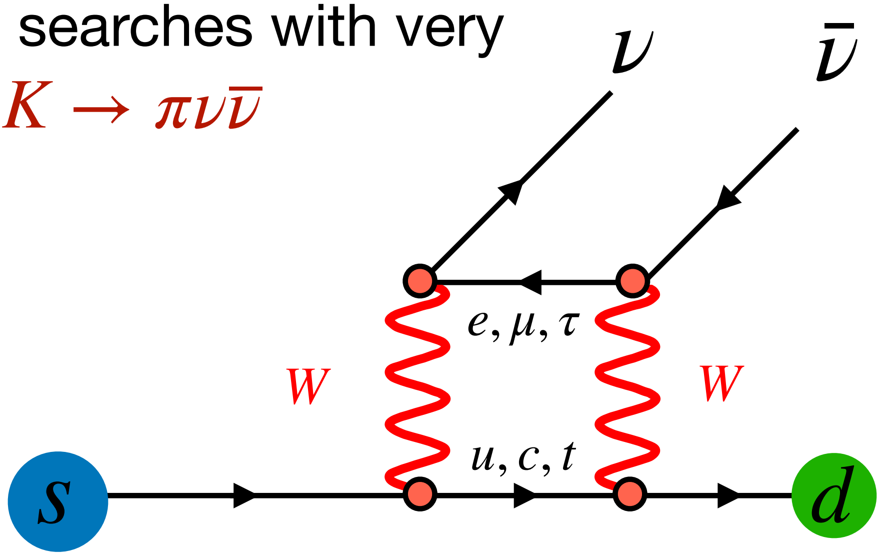
Ok, not such a great **anchor**, if we **cannot agree** on the **value** of the ratio…

Image credit: Markus Prim



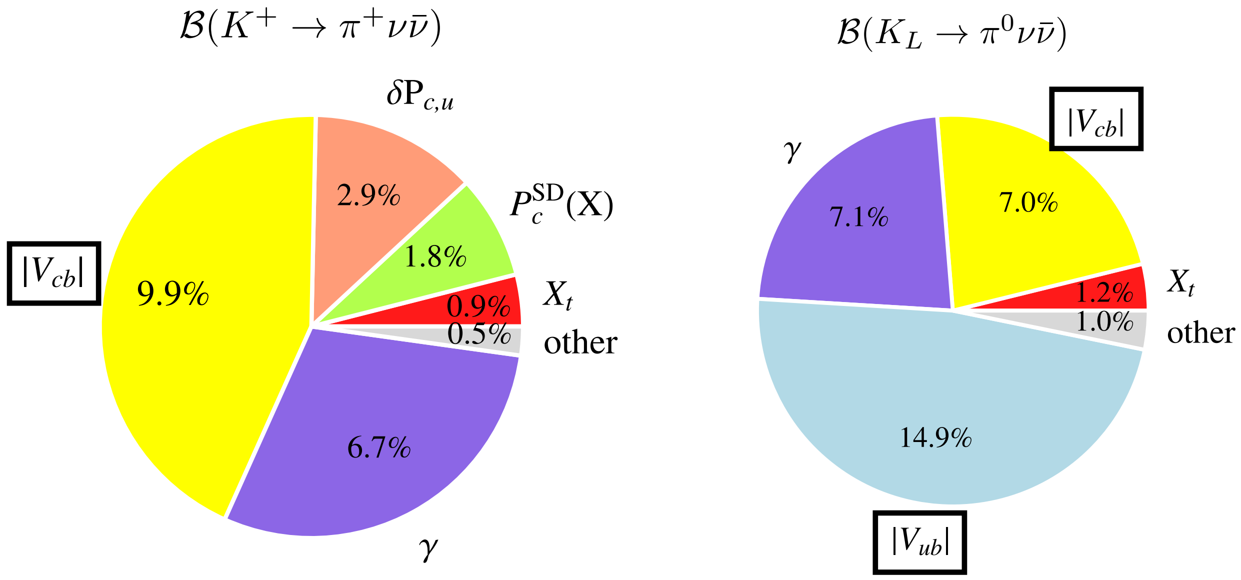
Yet another angle on $\left|V_{ub}\right|$ & $\left|V_{cb}\right|$:

New Physics searches with very rare decays: $K \rightarrow \pi\nu\bar{\nu}$



SM: $$\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (8.4 \pm 1.0) \times 10^{-11},$$
$$\mathcal{B}(K_L \rightarrow \pi^0\nu\bar{\nu}) = (3.4 \pm 0.6) \times 10^{-11}.$$

Uncertainty Sources



Experimental status:

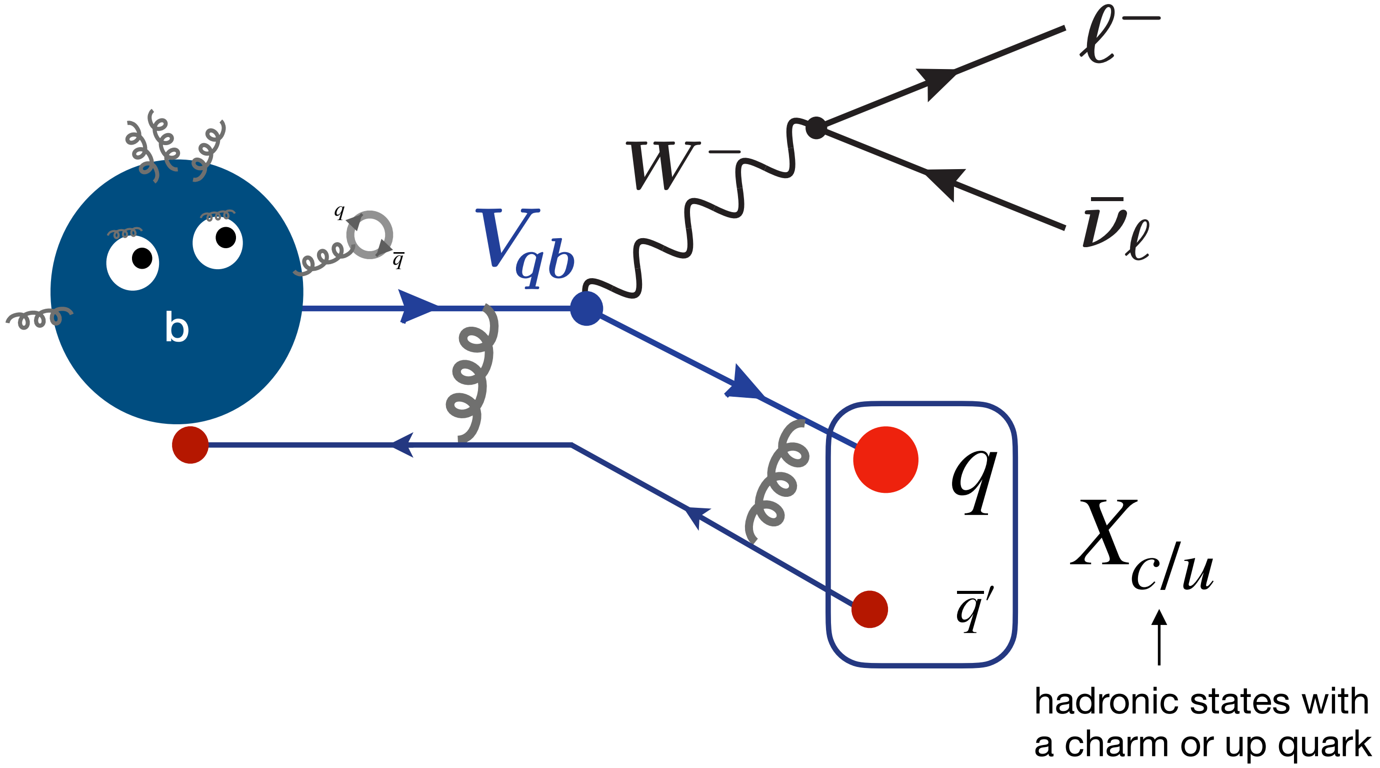
$$\mathscr{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = \left(10.6^{+4.0}_{-3.5} \pm 0.3\right) \times 10^{-11}$$

or in the future KLEVER

K_LEVER

K_L Experiment for VEry Rare events

Let’s first have a look at some of the kinematics



hadronic states with a charm or up quark

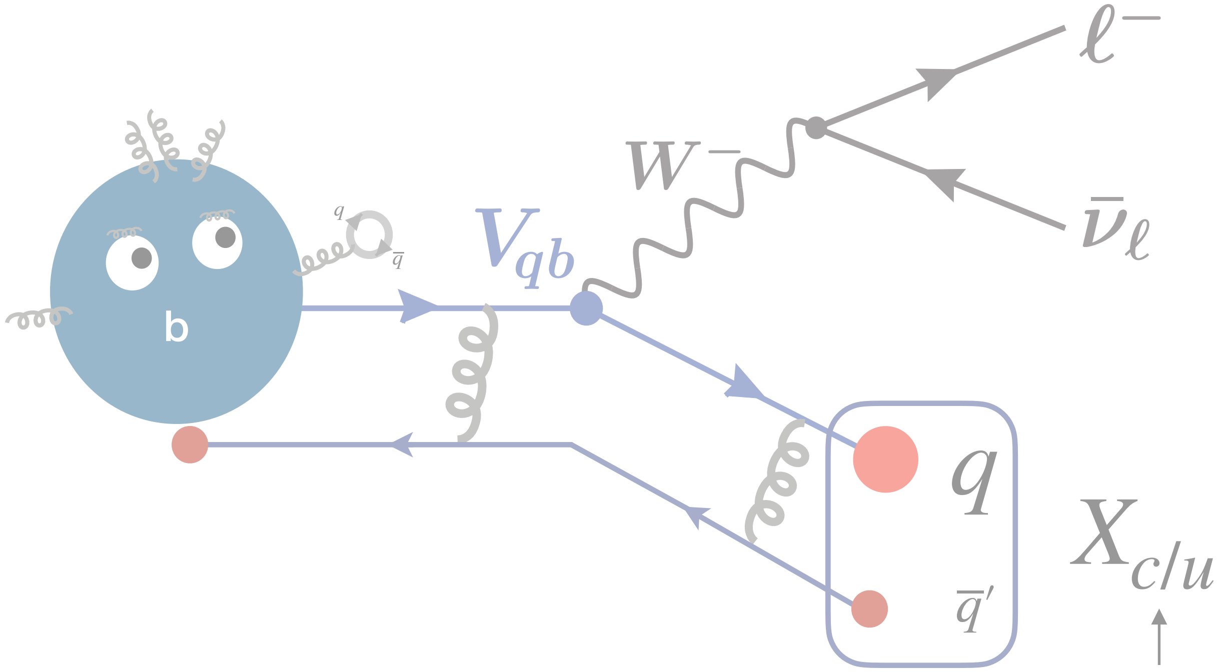
$$p_B = p_X + p_\ell + p_\nu$$

or

$$\begin{pmatrix} E_B \\ \mathbf{p}_B \end{pmatrix} = \begin{pmatrix} E_X \\ \mathbf{p}_X \end{pmatrix} + \begin{pmatrix} E_\ell \\ \mathbf{p}_\ell \end{pmatrix} + \begin{pmatrix} E_\nu \\ \mathbf{p}_\nu \end{pmatrix}$$

$$p_B^2 = m_B^2\,, \quad p_X^2 = m_X^2\,, \quad p_\ell^2 = m_\ell^2\,, \quad p_\nu^2 = 0$$

Let's first have a look at some of the kinematics



hadronic states with
a charm or up quark

$$p_B = p_X + p_\ell + p_\nu$$

or

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$$p_B^2 = m_B^2\,, \quad p_X^2 = m_X^2\,, \quad p_\ell^2 = m_\ell^2\,, \quad p_\nu^2 = 0$$

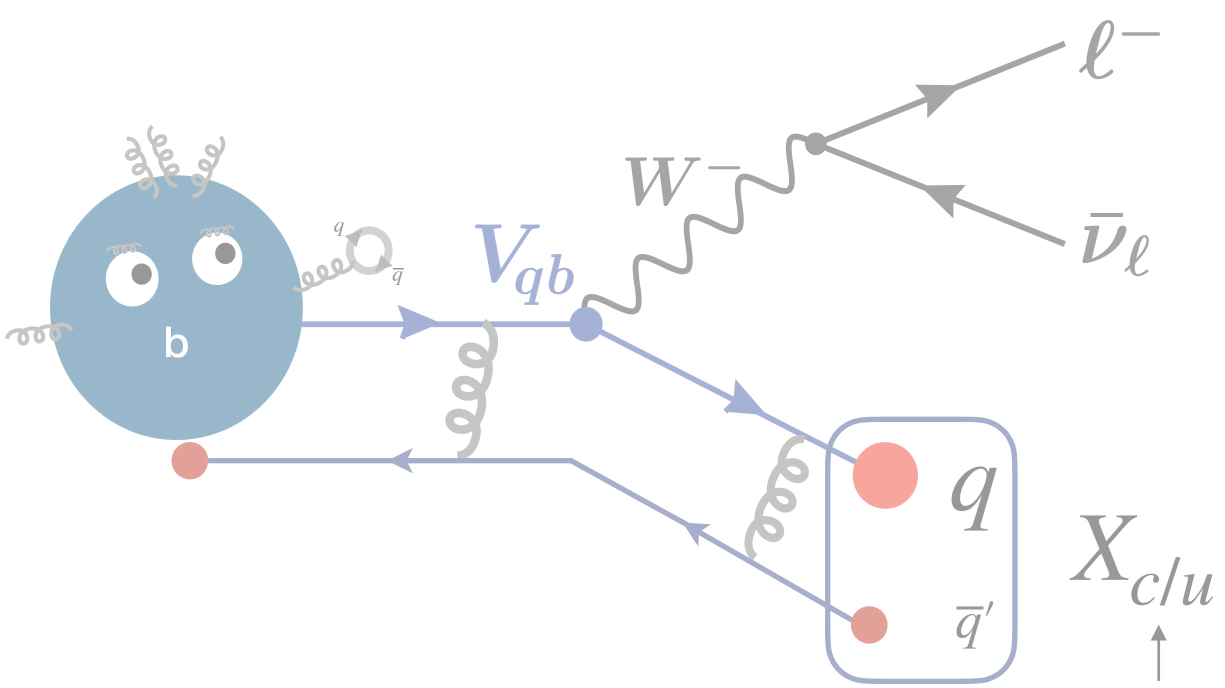
Let's assume we are in the rest frame of the B: $\begin{pmatrix} E_B \\ \mathbf{p}_B \end{pmatrix} = \begin{pmatrix} m_B \\ 0 \end{pmatrix}$

Which variables describe the final state?
Let's for now assume we look at a final state that is a resonance

$$X_c \in \{D, D^*, D^{**}, \dots\}$$

$$X_u \in \{\pi, \rho, f_0, \dots\}$$

Let’s first have a look at some of the kinematics



hadronic states with a charm or up quark

$$p_B = p_X + p_\ell + p_\nu$$

or

$$\begin{pmatrix} E_B \\ \mathbf{p}_B \end{pmatrix} = \begin{pmatrix} E_X \\ \mathbf{p}_X \end{pmatrix} + \begin{pmatrix} E_\ell \\ \mathbf{p}_\ell \end{pmatrix} + \begin{pmatrix} E_\nu \\ \mathbf{p}_\nu \end{pmatrix}$$

$$p_B^2 = m_B^2 \,, \quad p_X^2 = m_X^2 \,, \quad p_\ell^2 = m_\ell^2 \,, \quad p_\nu^2 = 0$$

If we look at final states with a **fixed mass** m_X , we can describe them with **two** kinematic quantities :

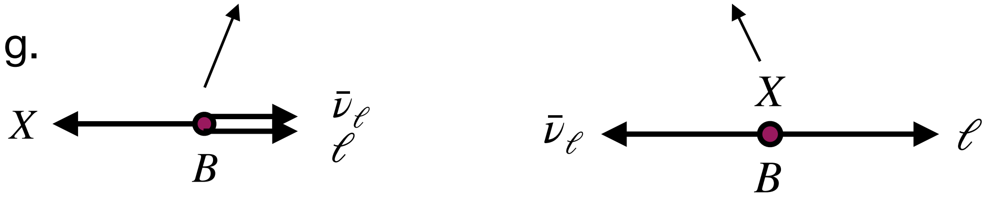
$$q^2 = (p_\ell + p_\nu)^2 = (p_B - p_X)^2$$

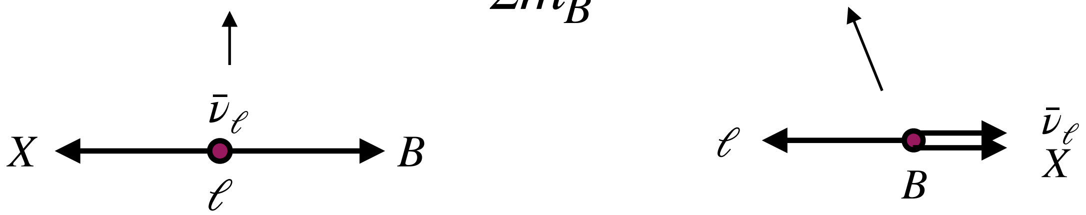
$$E_\ell = \frac{p_B p_\ell}{m_B}$$

$$m_\ell^2 \leq q^2 \leq (m_B - m_X)^2$$

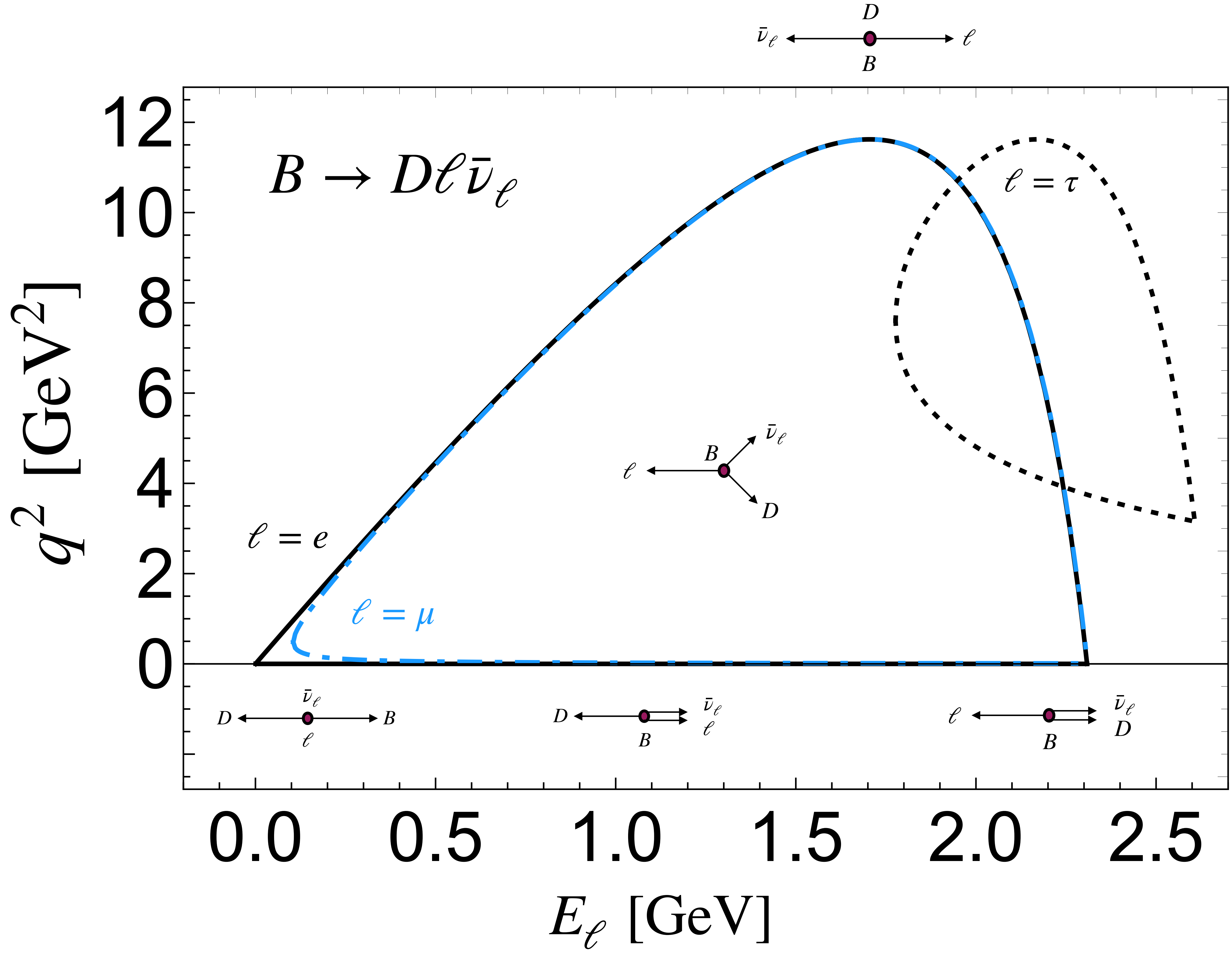
$$m_\ell \leq E_\ell \leq \frac{1}{2 m_B} \left(m_B^2 - m_X^2 + m_\ell^2 \right)$$

e.g.

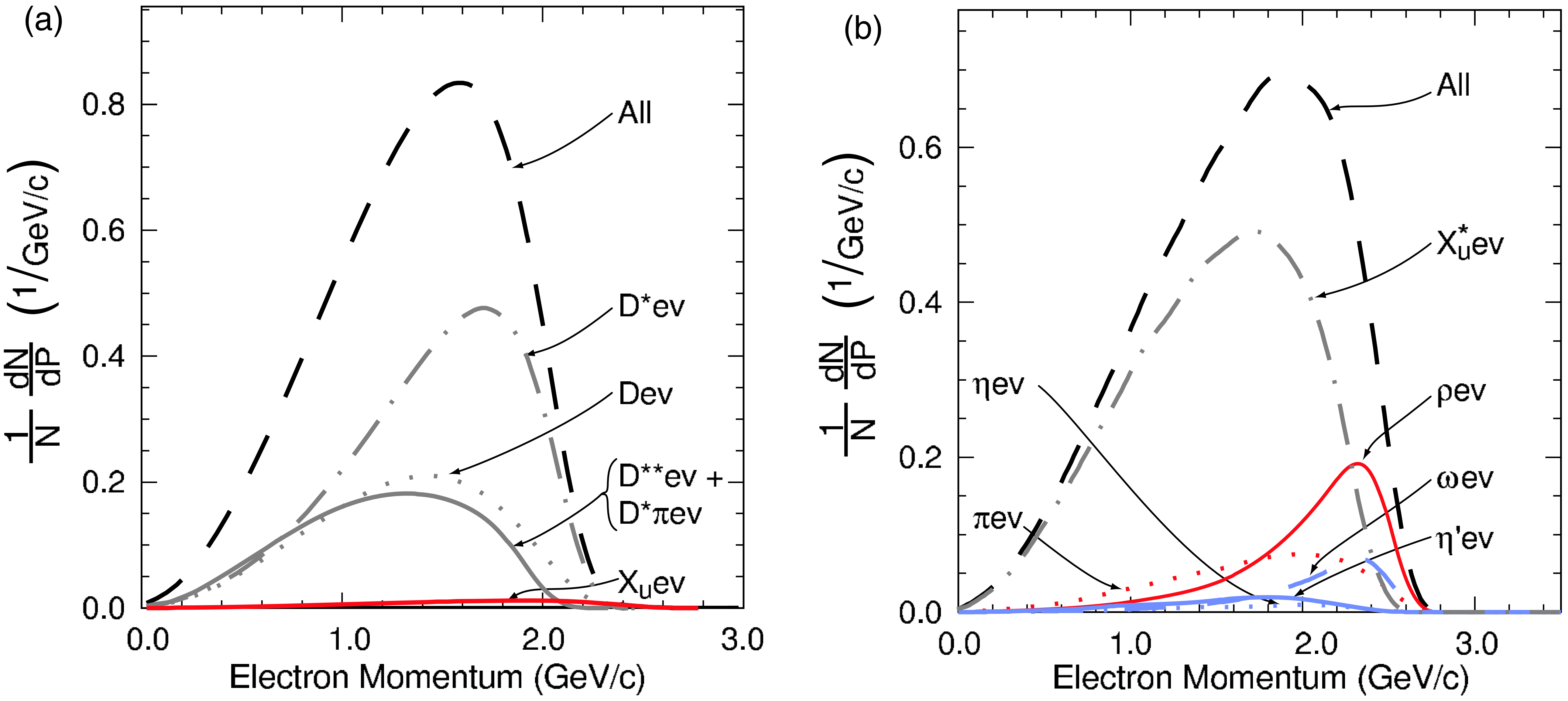




But q^2 : E_ℓ **not** independent :



The various semileptonic modes have spectra with **different endpoints**, e.g. for $B \to X_c \ell \bar{\nu}_\ell$ and $B \to X_u \ell \bar{\nu}_\ell$:



These already can give you some **experimental intuition**: e.g. if you want to measure $B \to X_u \ell \bar{\nu}_\ell$ its much easier beyond the endpoint of $B \to X_c \ell \bar{\nu}_\ell$

In the context of the **heavy-quark expansion**, it is convenient to introduce **velocities** instead of momenta.

E.g. for the case of heavy mesons like B and D^* one defines

$$v_B = \frac{p_B}{m_B}\,, \qquad v_{D^{(*)}} = \frac{p_{D^{(*)}}}{m_{D^{(*)}}}\,,$$

$$\boxed{w = v_B\, v_{D^{(*)}}}$$

Here w is the scalar product of the two velocities and used instead of q^2

They are related via $q^2 = m_B^2 + m_{D^{(*)}}^2 - 2 m_B m_{D^{(*)}}\, w$

Note that :

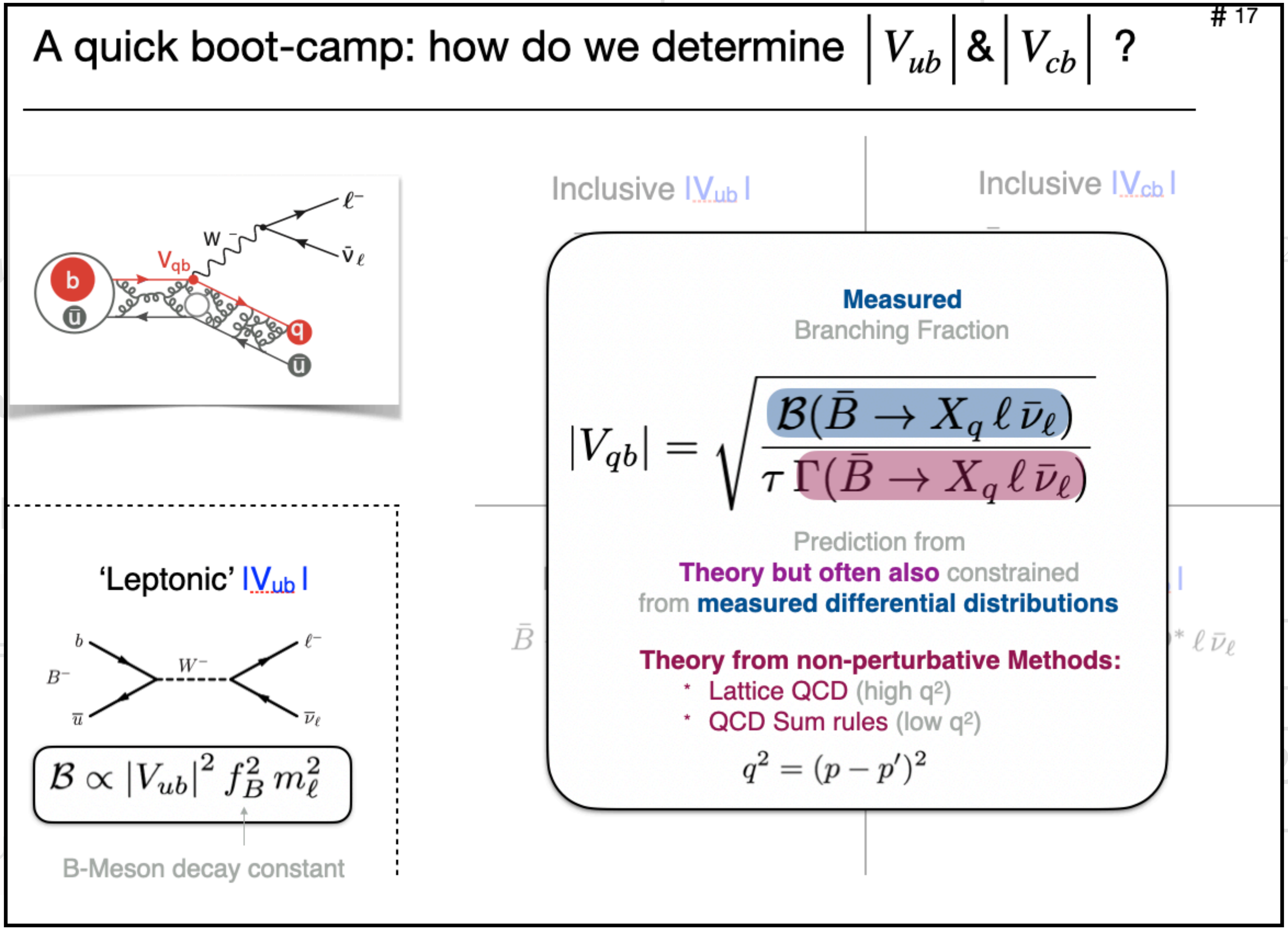
$$w = 1 \quad \longleftrightarrow \quad q^2_{\text{max}} = \left(m_B - m_{D^{(*)}}\right)^2$$

While $q^2 = m_\ell^2 \approx 0$ for light leptons results in the maximal value of w

$$\boxed{\rightarrow 1 \leq w \leq \frac{m_B^2 + m_{D^{(*)}}^2 - m_\ell^2}{2 m_B m_{D^{(*)}}}}$$

In the context of the **heavy-quark expansion**, it is convenient to introduce

All these quantities are useful, since they **encode** the **non-perturbative decay dynamics**, i.e. you can combine **differential shapes** (or **moments of differential spectra**) with predictions from theory to determine or constrain non-perturbative QCD



A quick boot-camp: how do we determine $|V_{ub}|$ & $|V_{cb}|$?

Inclusive |Vub|

Inclusive |Vcb|

Measured
Branching Fraction

$$|V_{qb}| = \sqrt{\frac{\mathcal{B}(\bar{B} \to X_q\,\ell\,\bar{\nu}_\ell)}{\tau\,\Gamma(\bar{B} \to X_q\,\ell\,\bar{\nu}_\ell)}}$$

Prediction from
Theory but often also constrained
from **measured differential distributions**

Theory from non-perturbative Methods:

- Lattice QCD (high q²)
- QCD Sum rules (low q²)

$$q^2 = (p - p')^2$$

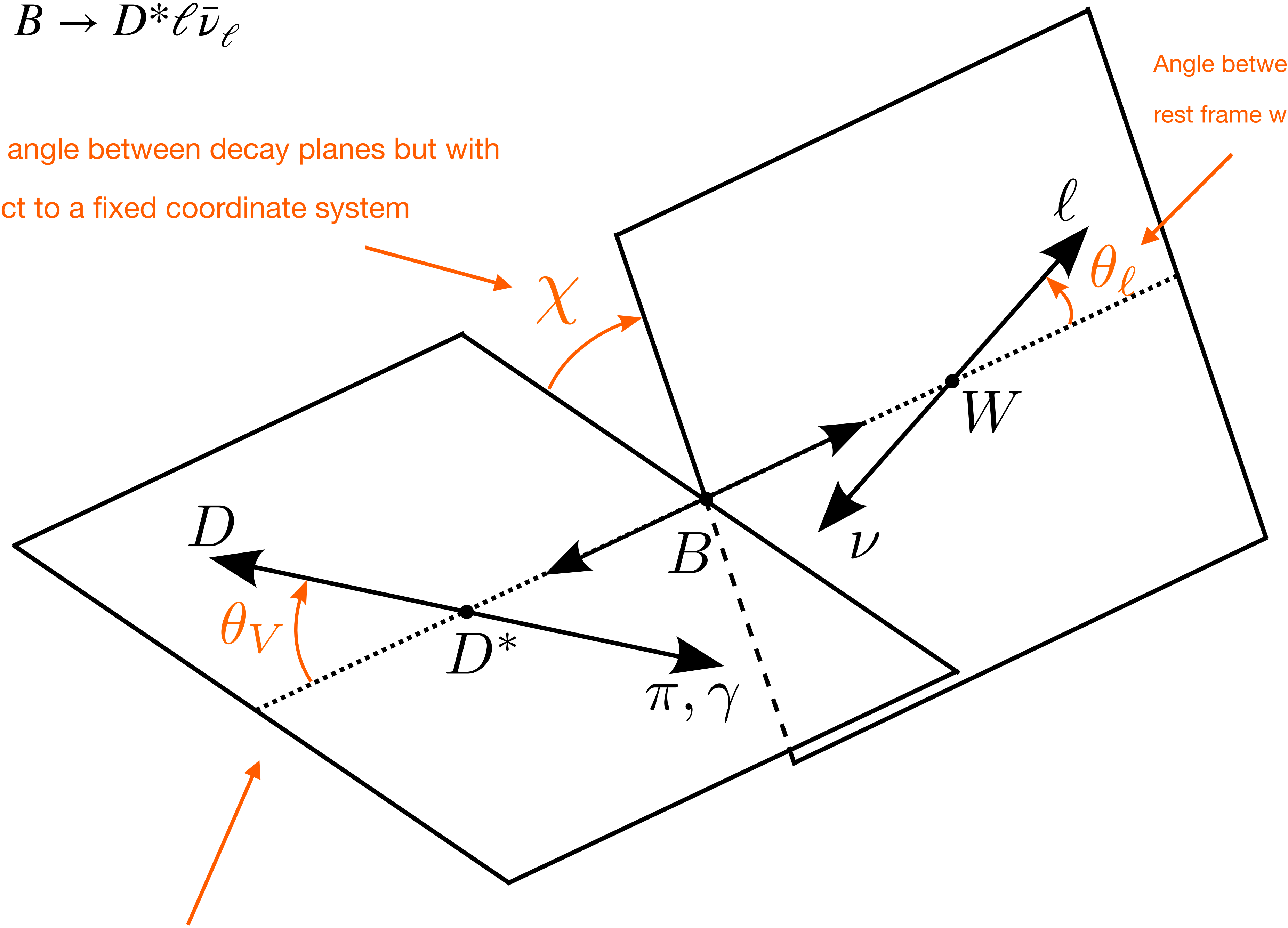
'Leptonic' |Vub|

$$\boxed{\mathcal{B} \propto |V_{ub}|^2\, f_B^2\, m_\ell^2}$$

B-Meson decay constant

If the final state meson carries spin, **information** is also encoded into the **decay angles**

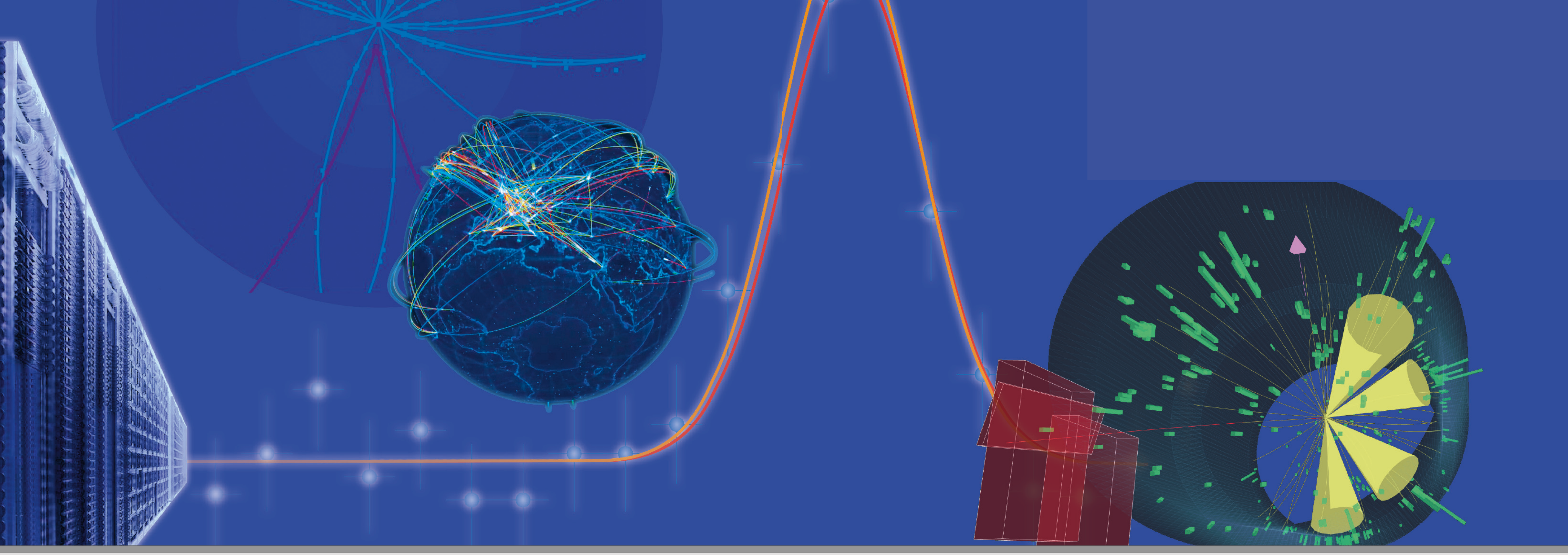
E.g. $B \to D^* \ell \bar{\nu}_\ell$



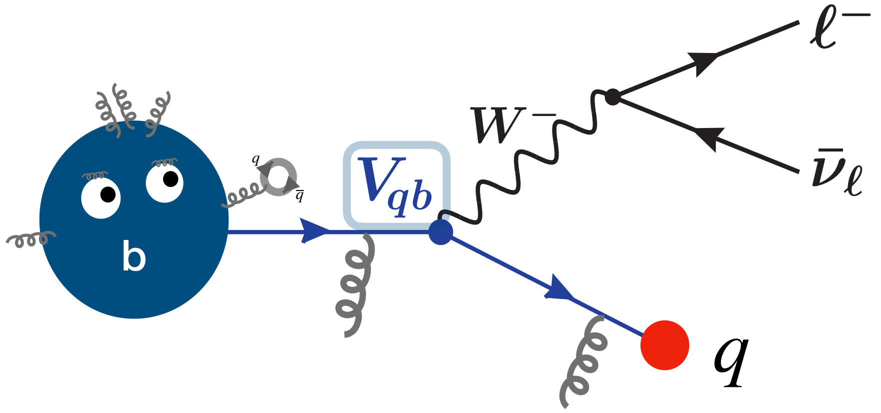
tilting angle between decay planes but with respect to a fixed coordinate system

Angle between lepton flight direction in W* rest frame with respect to W* direction in B frame

Angle between D flight direction in D* rest frame with respect to D* direction in B rest frame



2) Touch and go



Inclusive |Vub|

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive |Vcb|

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive |Vub|

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

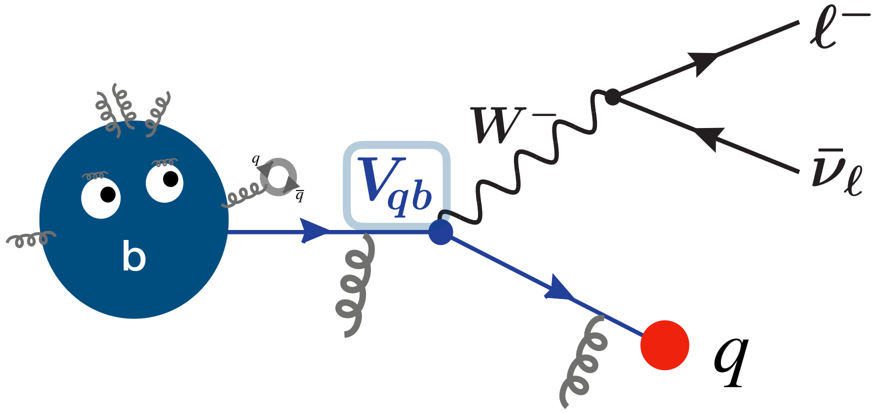
Exclusive |Vcb|

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

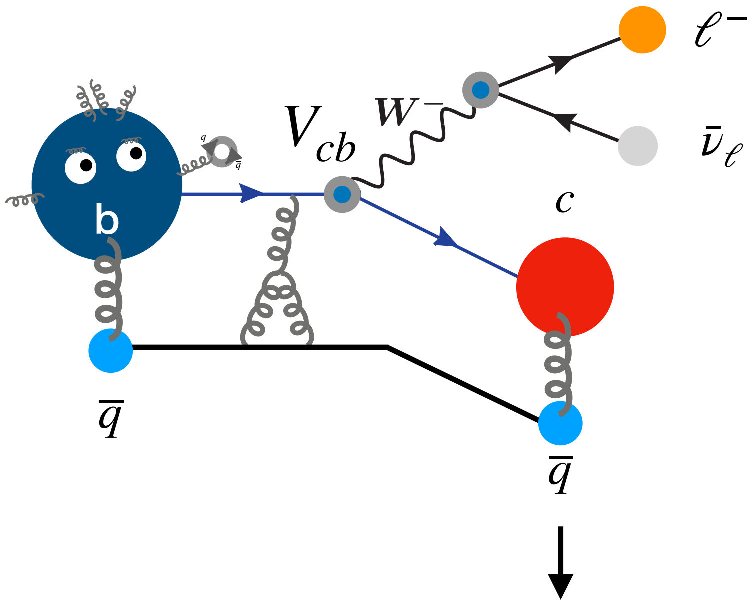
$$\mathcal{B} \propto |V_{qb}|^2 \, f^2$$

Form Factors

$$\langle B|H_\mu|P\rangle = (p + p')_\mu \, f_+$$



Overview

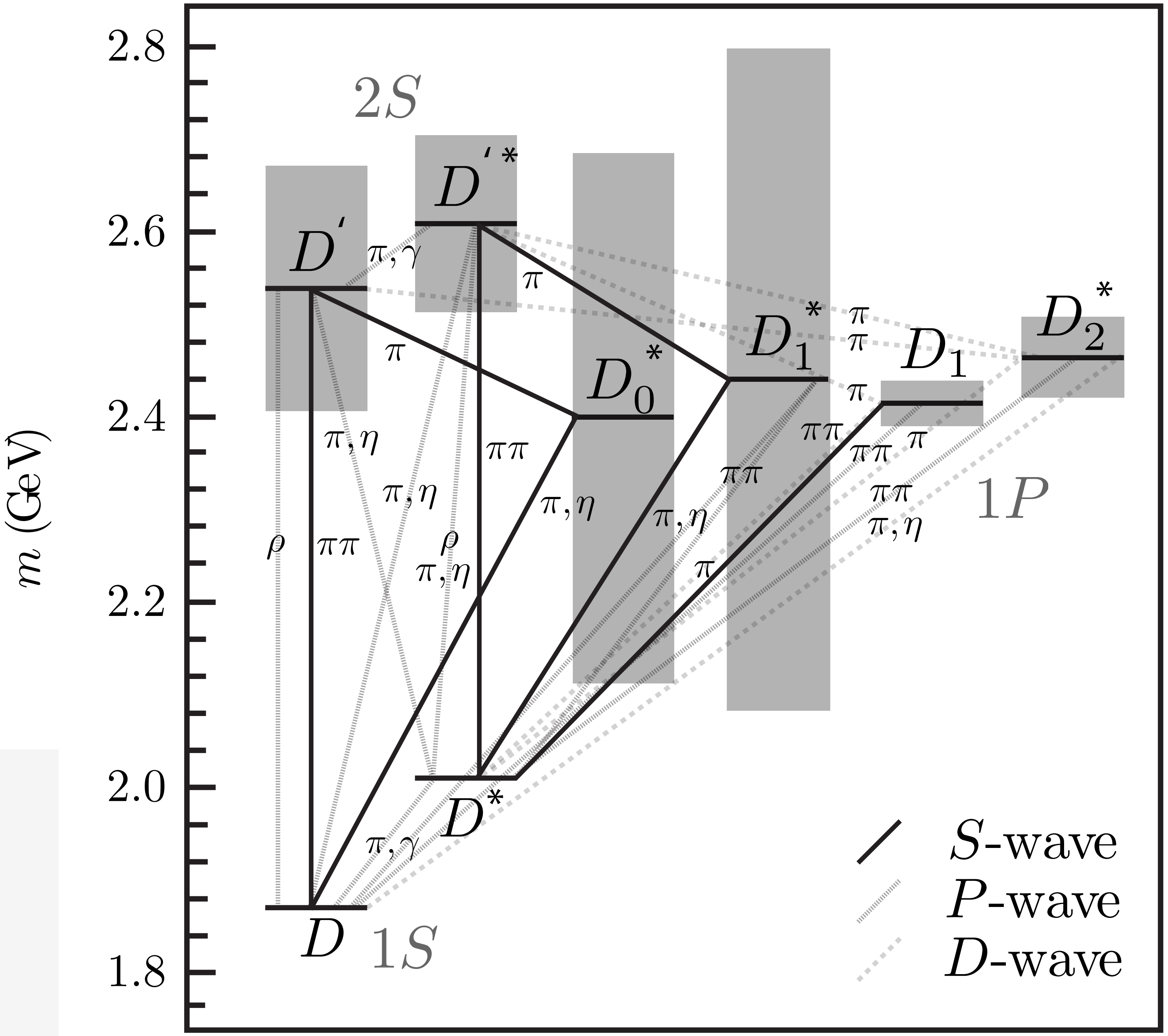


Mesons containing a heavy quark Q are made up of a heavy quark and a light antiquark $\bar{q}$ (and gluons and $q\bar{q}$ pairs)

2 L = 0 ground states: D/D^*

4 L = 1 ground states: D_0, D_1', D_1, D_2

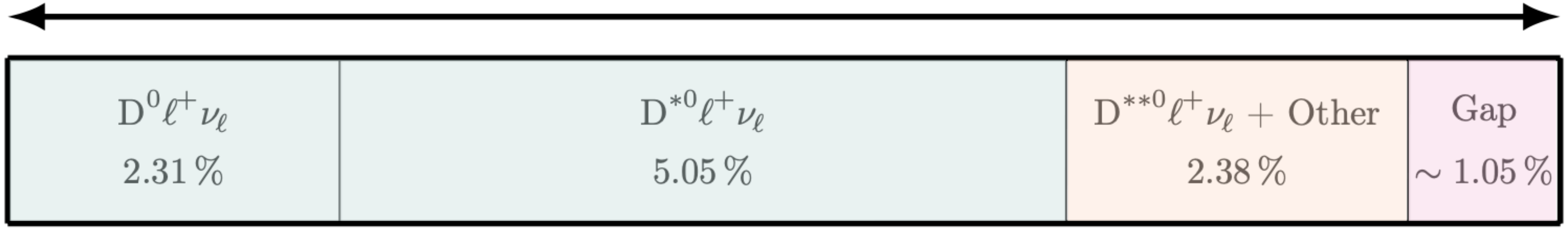
(or sometimes D_0^*, D_1^*, D_1, D_2^* or just D^{**})



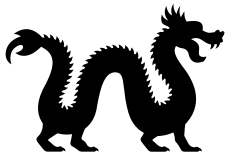
D/D^* saturate **~75%** of the inclusive $B \to X_c \ell \bar{\nu}_\ell$ rate and are the **principal route** to V_{cb}

D^{**} saturate **~15%** of the inclusive $B \to X_c \ell \bar{\nu}_\ell$ rate, mostly are perceived as background

$$\mathcal{B}(\mathrm{B}^+ \to X_c^0\,\ell^+\nu_\ell) \approx 10.79\,\%$$



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \to D\,\ell^+\,\nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \to D^*\,\ell^+\,\nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \to D_1\,\ell^+\,\nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \to D_2^*\,\ell^+\,\nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \to D_0^*\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \to D_1'\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \to D\pi\pi\,\ell^+\,\nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \to D^*\pi\pi\,\ell^+\,\nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \to X_c\ell\nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$


?

Fairly well known.
Some iso-spin tension.

Broad states based on
3 measurements.
(BaBar, Belle, DELPHI)

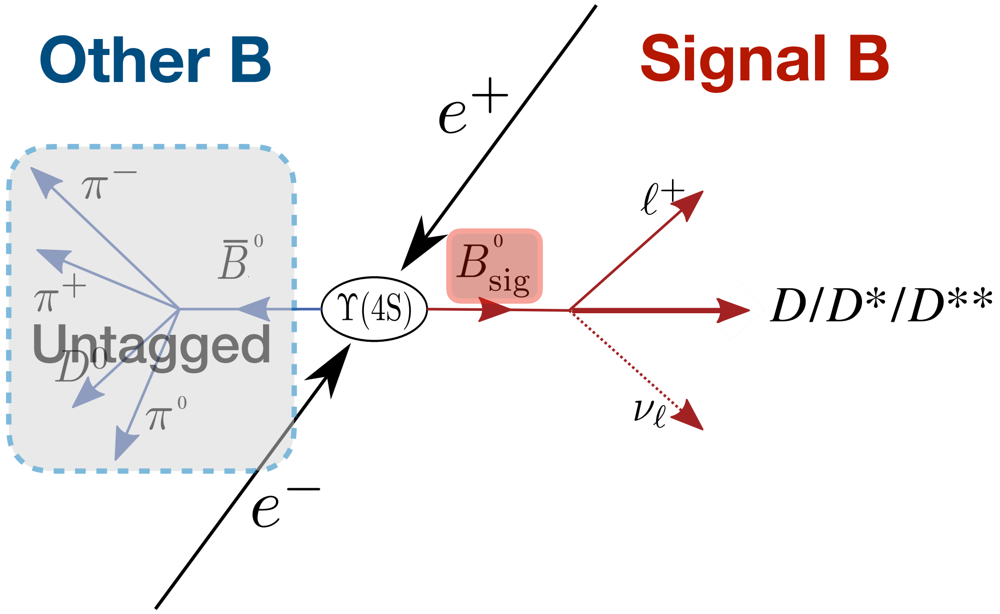
Some hints from
the BaBar result.

+ Belle

Check my slides

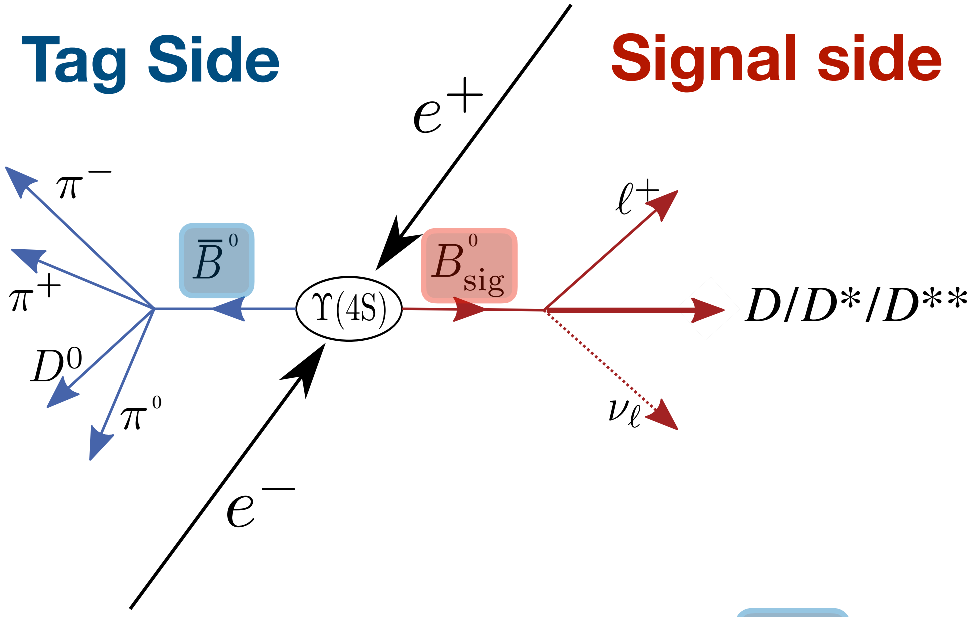
Image credit: F. Metzner

Let’s strategize



\+ Very high efficiency
\+ Measurement of absolute branching fractions straightforward
(depends on total # of $N_{B\bar{B}}$, understanding efficiencies)

\- Less experimental control, e.g. more background from $e^+e^+ \to q\bar{q}$
\- Cannot directly access signal B rest frame, need tricks



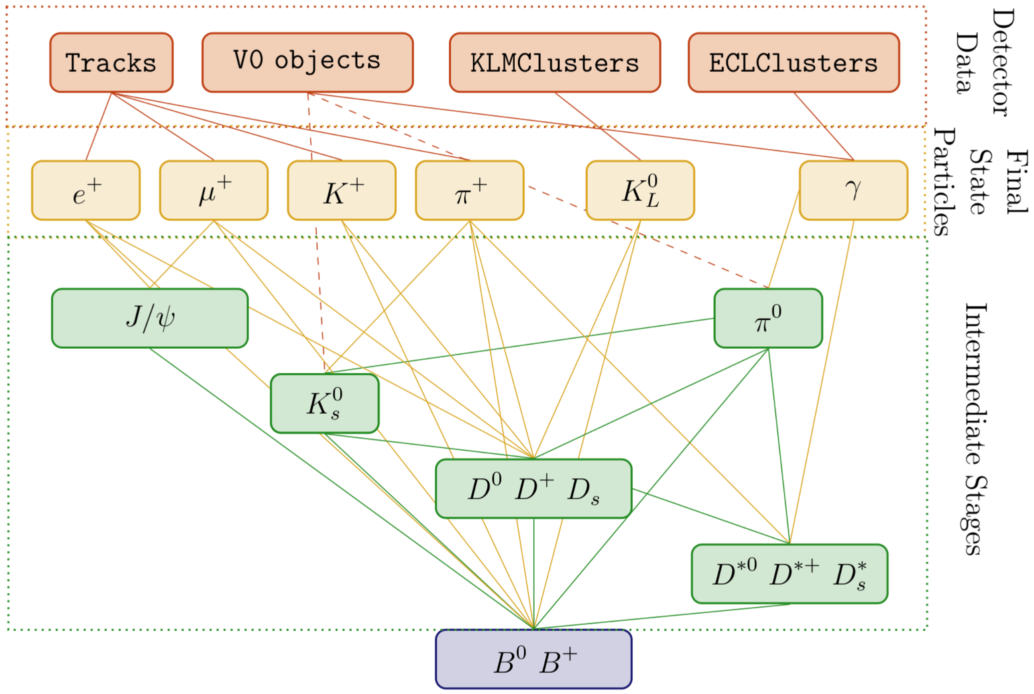
$$p_{\text{sig}} = p_{e^+e^-} - p_{\text{tag}}$$

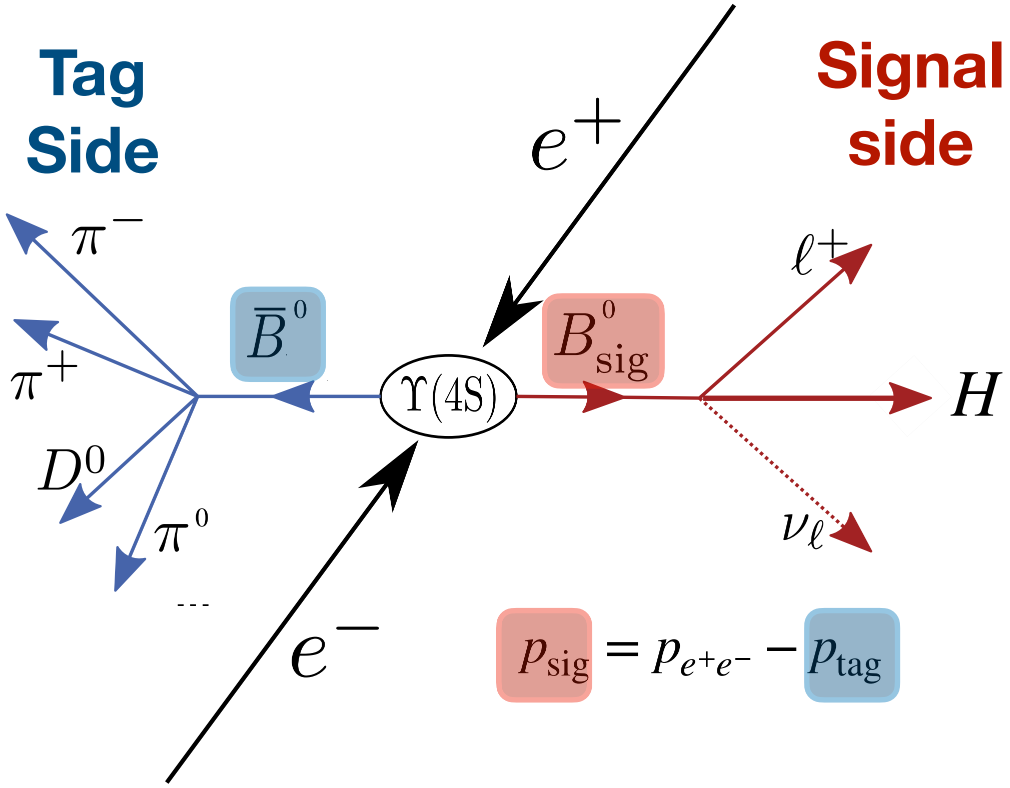
\+ High degree of experimental control, e.g. can identify all final state particles with either the signal or the tag side
\+ If hadronic modes for tagging are used, can reconstruct B rest frame

\- Understanding efficiencies is difficult
\- Low efficiency reduces the effective statistical power

Tagging in a nutshell

https://arxiv.org/abs/1807.08680





Candidates reconstructed with **hierarchical** approach via e.g. **neural networks (FR) or boosted decision trees (FEI)**

Over 10'000 decay cascades with an **efficiency** of **0.28% / 0.18%** for $B^{\pm}$ and $B^0/\bar{B}^0$

E.g. train a classifier to identify correctly reconstructed electron candidates:

Input variables: all four momenta & particle identification scores

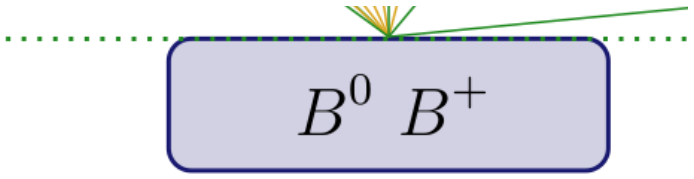
Output: Score $\mathcal{O}_e$

Apply mild selection on $\mathcal{O}_e$ to reduce # of candidate particles

Then train a classifier to identify correctly reconstructed J/ψ candidates

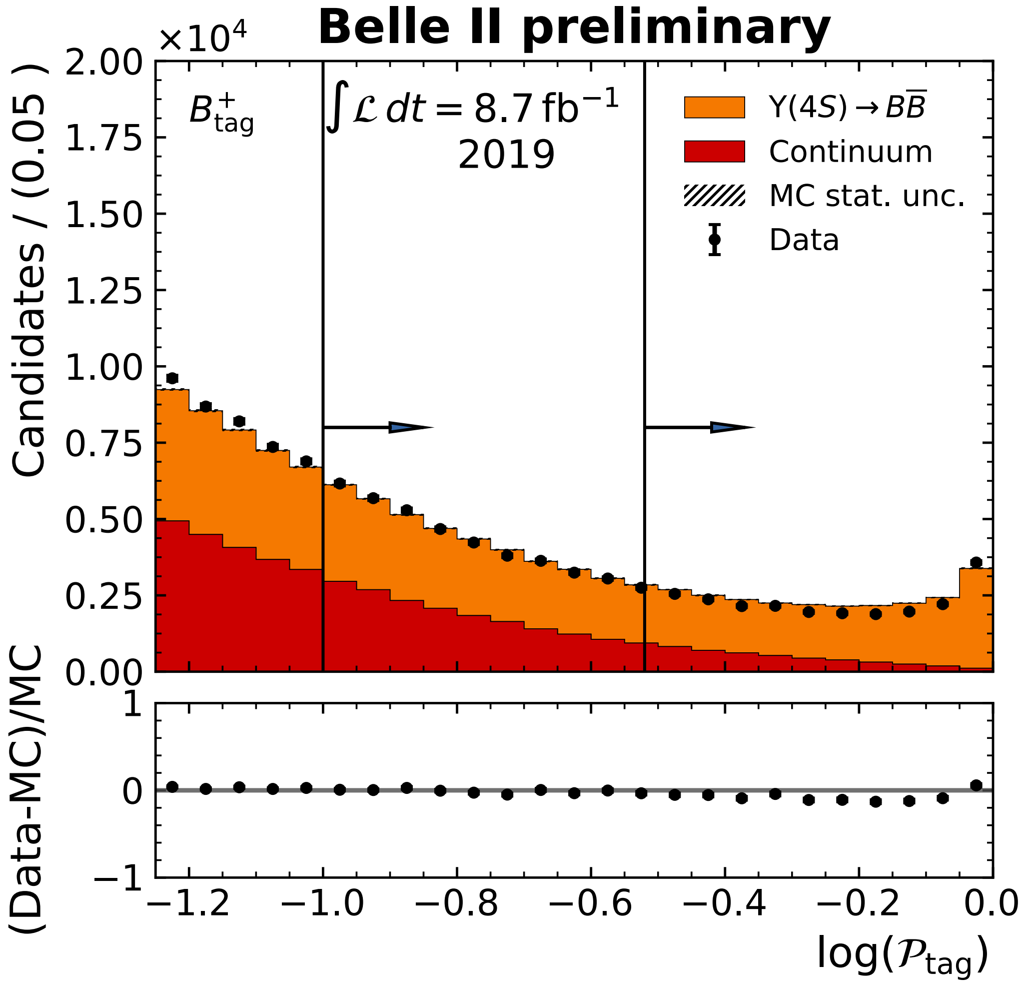
Input variables: all four momenta and output scores of previous layer

Output variable: $\mathcal{O}_{J/\psi}$ […]



$\mathscr{P}_{\text{tag}}$ =

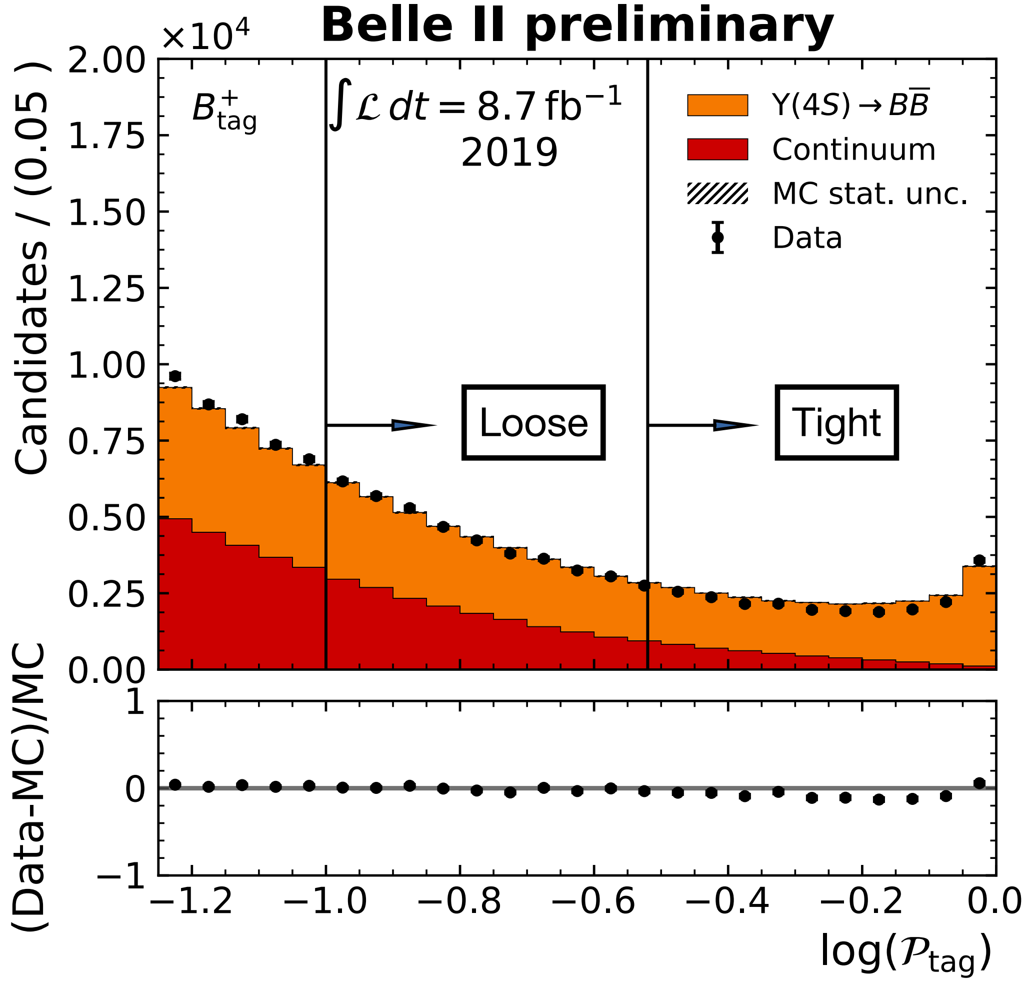
Output classifier = Measure of how well we reconstructed the B-Meson decay



beam constrained mass

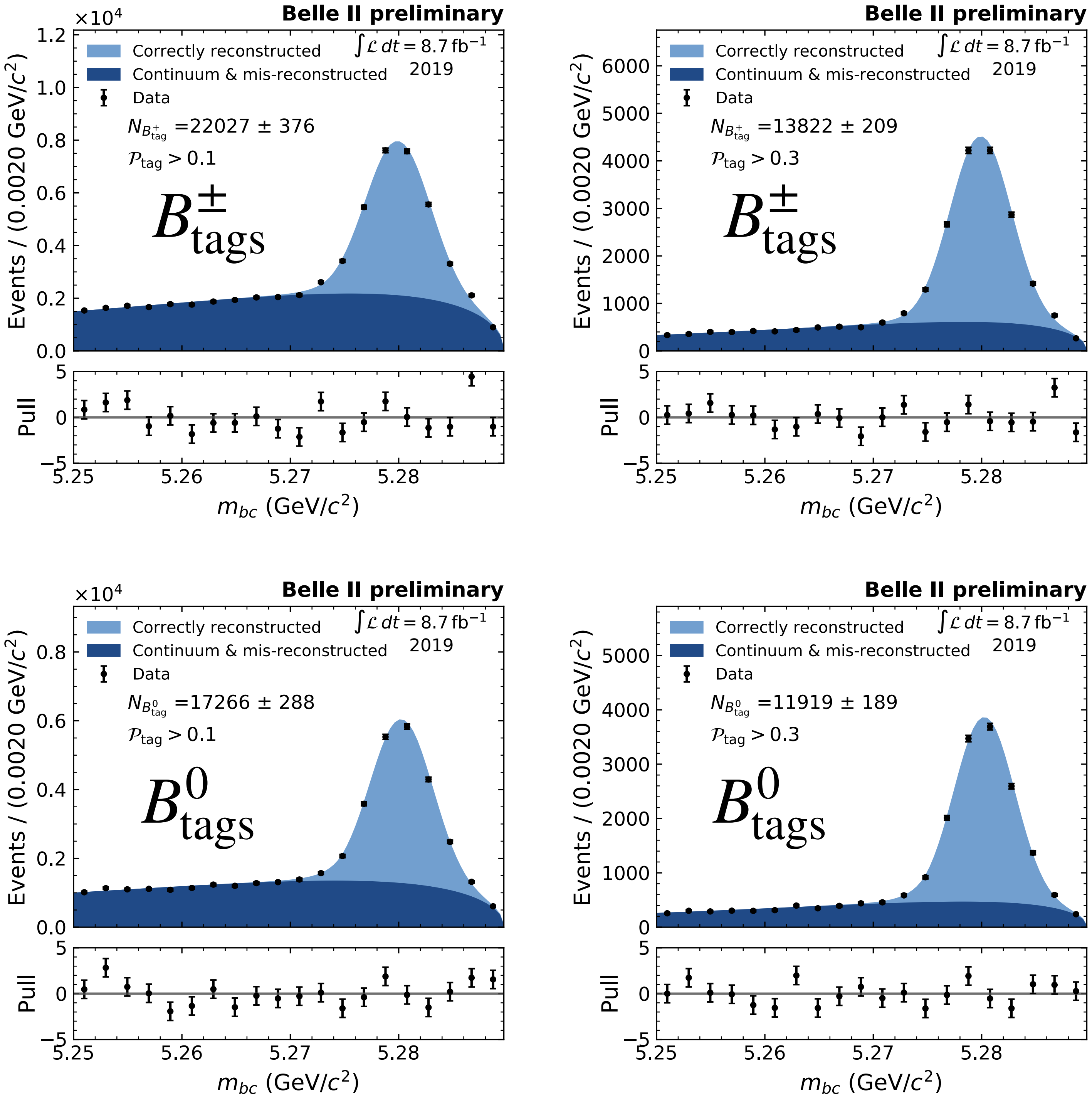
ca. 5.279 GeV

$$m_{bc} = \sqrt{E_{\text{beam}}^2/4 - |\overrightarrow{p}_{B_{\text{tag}}}|^2} \simeq m_B$$

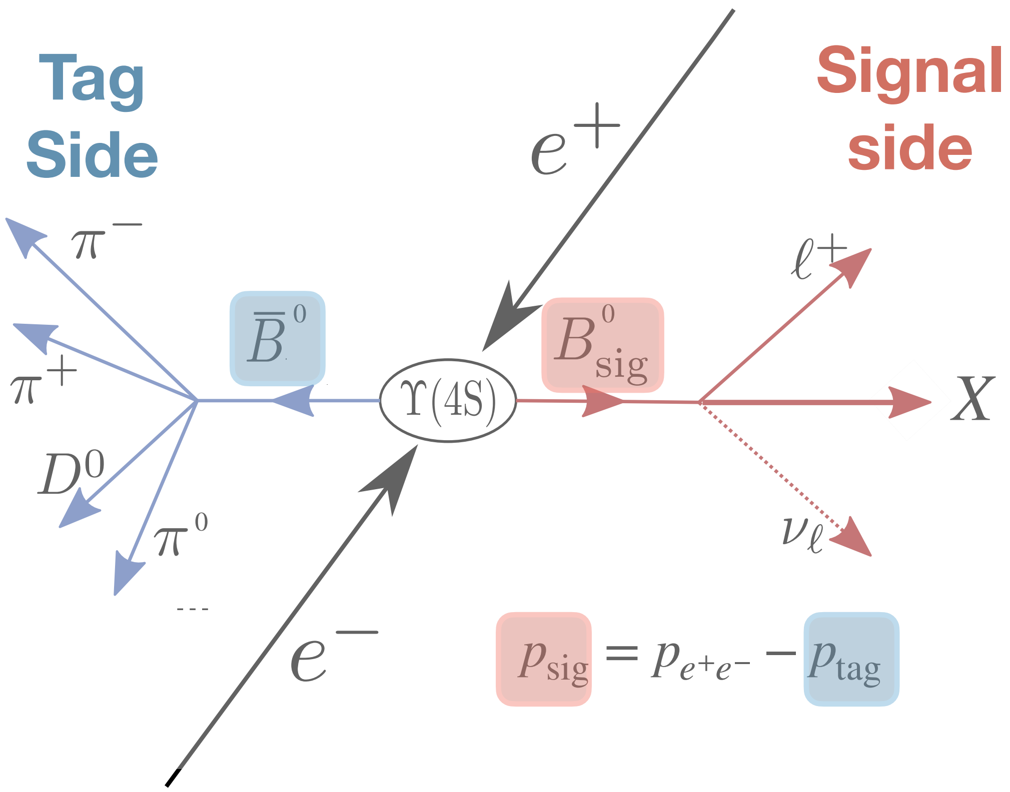


Loose Selection

Tight Selection



Efficiency can be calibrated, but this has caveats



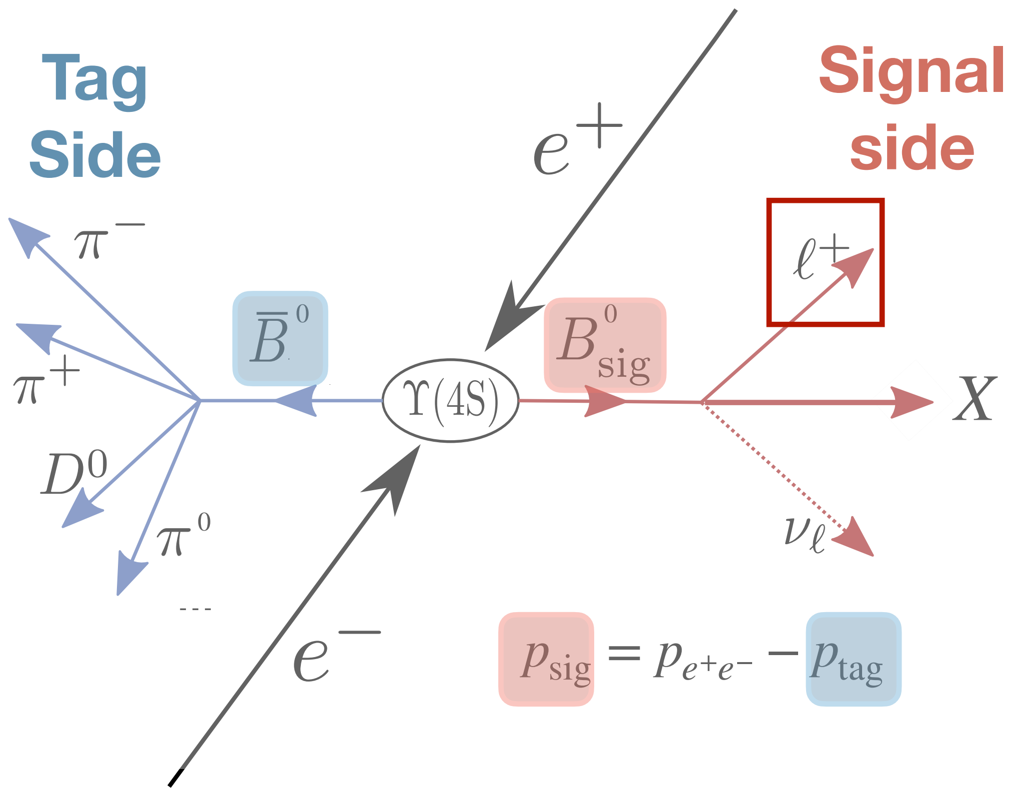
Why is the efficiency different? Use 10'000 different decays, use uncalibrated detector information, line-shapes differ in simulation
→ all aggregated in $\mathscr{P}_{\text{tag}}$

<u>Strategy</u>: use a well measured process, add it to your MC with its measured BF and compare

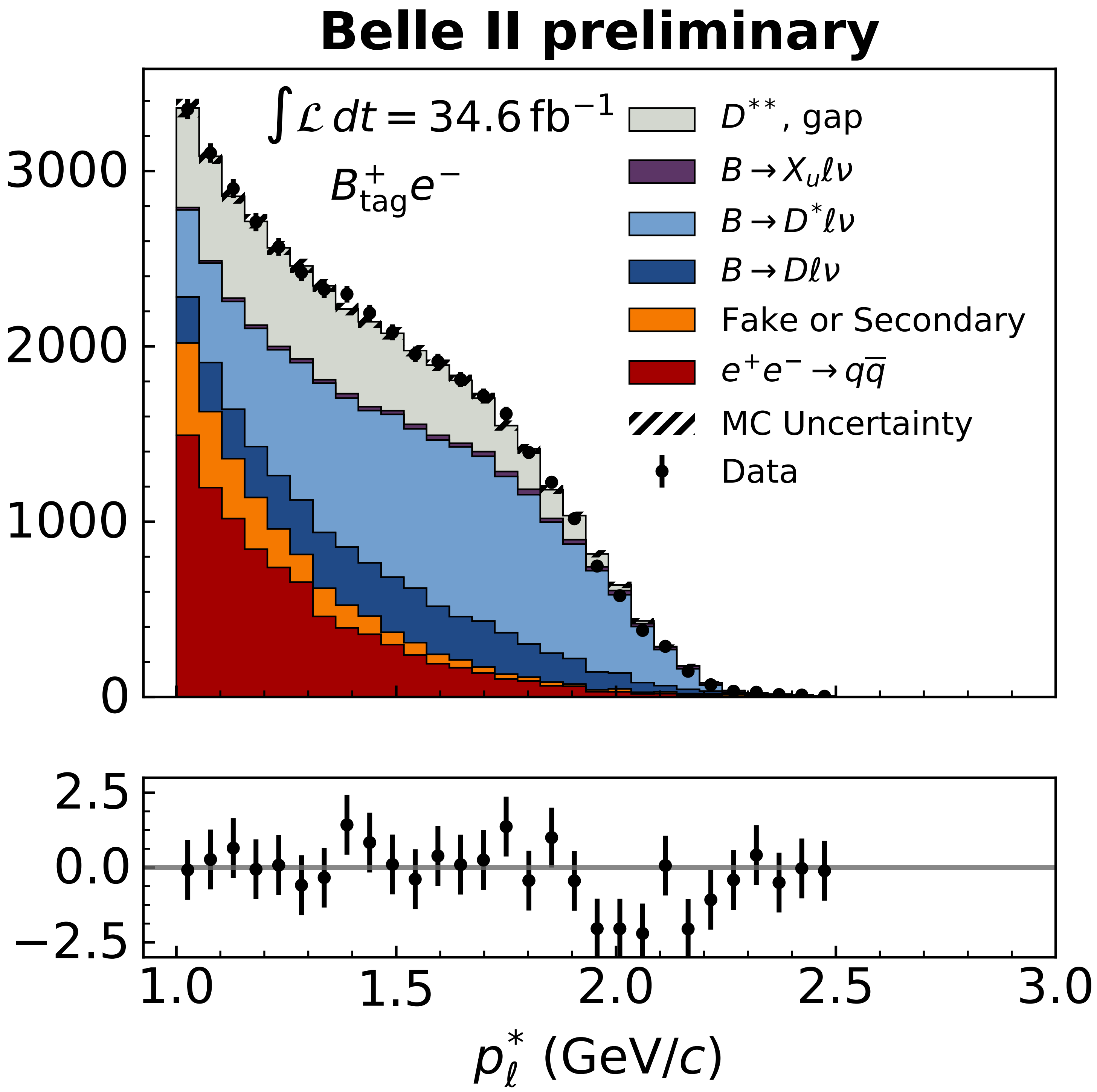
$$\frac{N^{\text{Data}}_{X\ell\bar{\nu}_\ell}}{N^{\text{MC}}_{X\ell\bar{\nu}_\ell}}$$

Belle II Collaboration, BELLE2-CONF-PH-2020-005, [arXiv:2008.06096]

Efficiency can be calibrated, but this has caveats



e.g.



Why is the efficiency different? Use 10’000 different decays, use uncalibrated detector information, line-shapes differ in simulation

→ all aggregated in $\mathscr{P}_{\rm tag}$

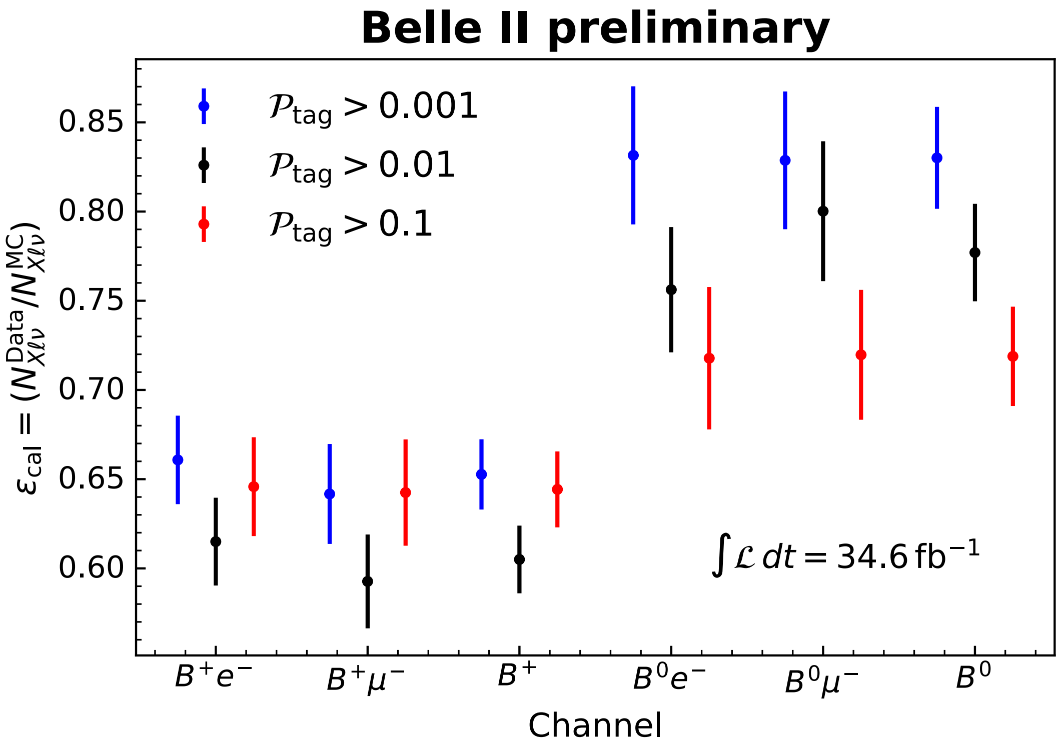
<u>Strategy</u>: use a well measured process, add it to your MC with its measured BF and compare

$$\frac{N^{\rm Data}_{X\ell\bar{\nu}_\ell}}{N^{\rm MC}_{X\ell\bar{\nu}_\ell}}$$

Belle II Collaboration, BELLE2-CONF-PH-2020-005, [arXiv:2008.06096]

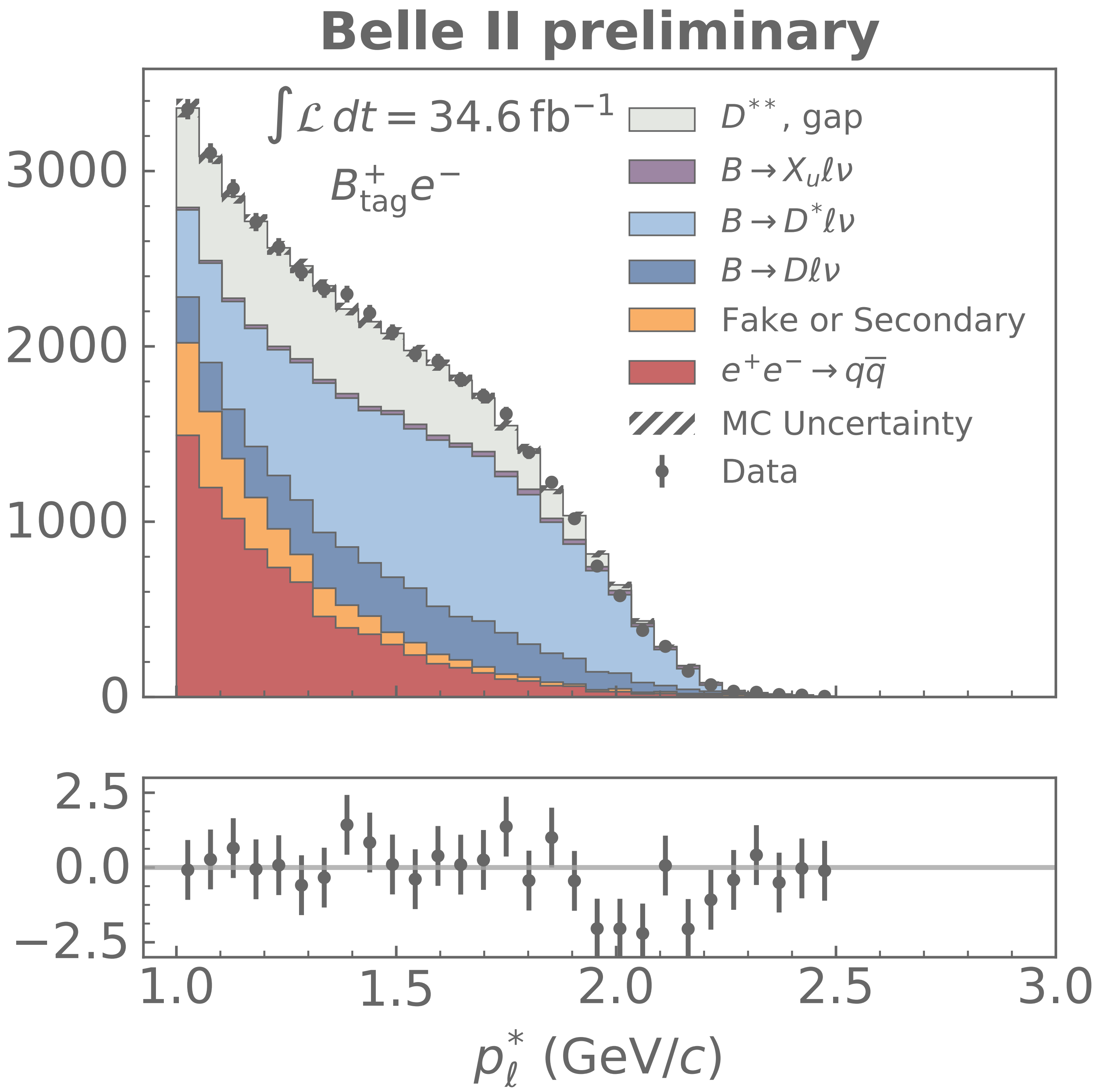
Efficiency Calibration

$$\epsilon_{\mathrm{cal}} = \frac{N^{\mathrm{Data}}_{X\ell\bar{\nu}_\ell}}{N^{\mathrm{MC}}_{X\ell\bar{\nu}_\ell}}$$



B^+		
$\mathcal{P}_{\mathrm{tag}} >$	ϵ	uncertainty [%]
0.001	0.65 ± 0.02	3.0
0.01	0.61 ± 0.02	3.1
0.1	0.64 ± 0.02	3.3

B^0		
$\mathcal{P}_{\mathrm{tag}} >$	ϵ	uncertainty [%]
0.001	0.83 ± 0.03	3.4
0.01	0.78 ± 0.03	3.5
0.1	0.72 ± 0.03	3.9

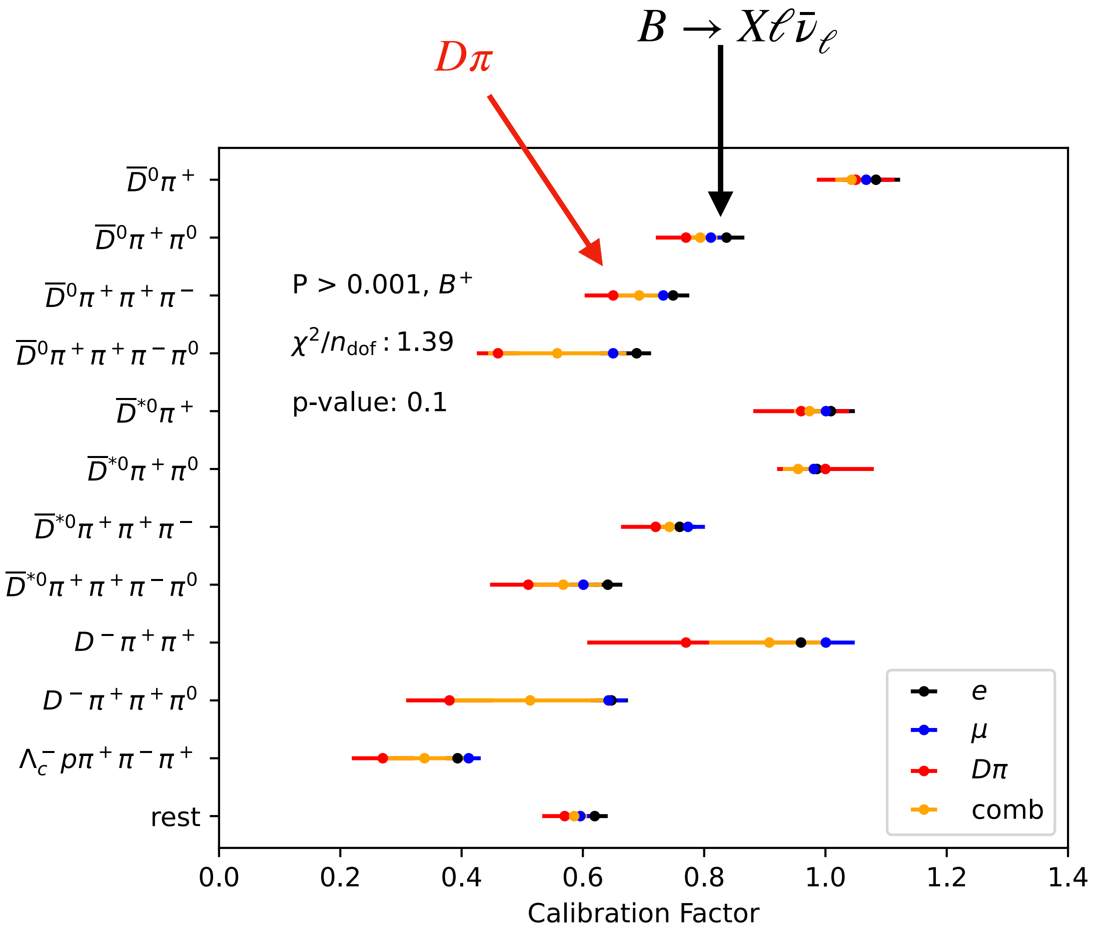


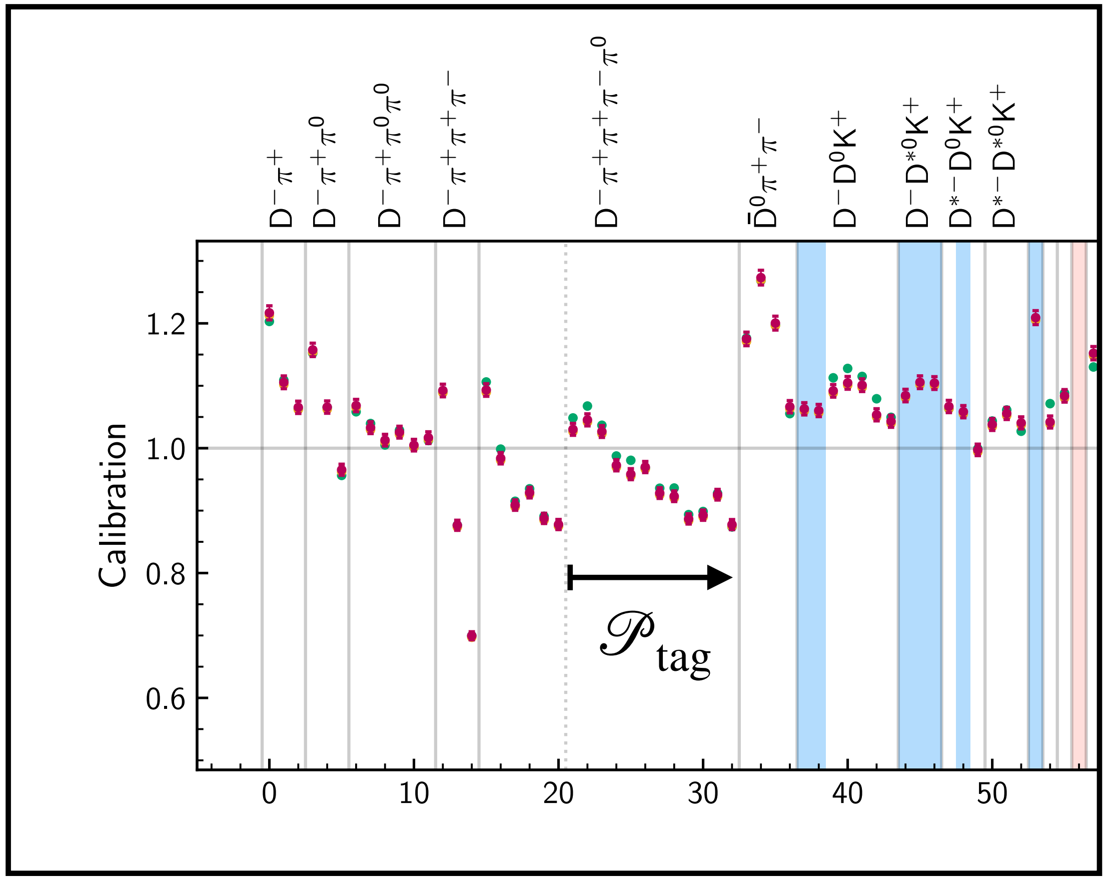
→ Also look at BELLE2-NOTE-PH 2023-004)
and BELLE2-NOTE-PH-2023-008

Unbiased calibration very challenging :

Calibration shows **signal side dependence**

Calibration also dependent on **composition** of **tag-side candidates** and fraction of **good** versus **bad tags**





One needs to carefully check these issues; best to carry out **self calibration** whenever possible

See e.g. PhD thesis of Kilian Lieret: https://edoc.ub.uni-muenchen.de/30193/1/Lieret_Kilian.pdf

Tagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

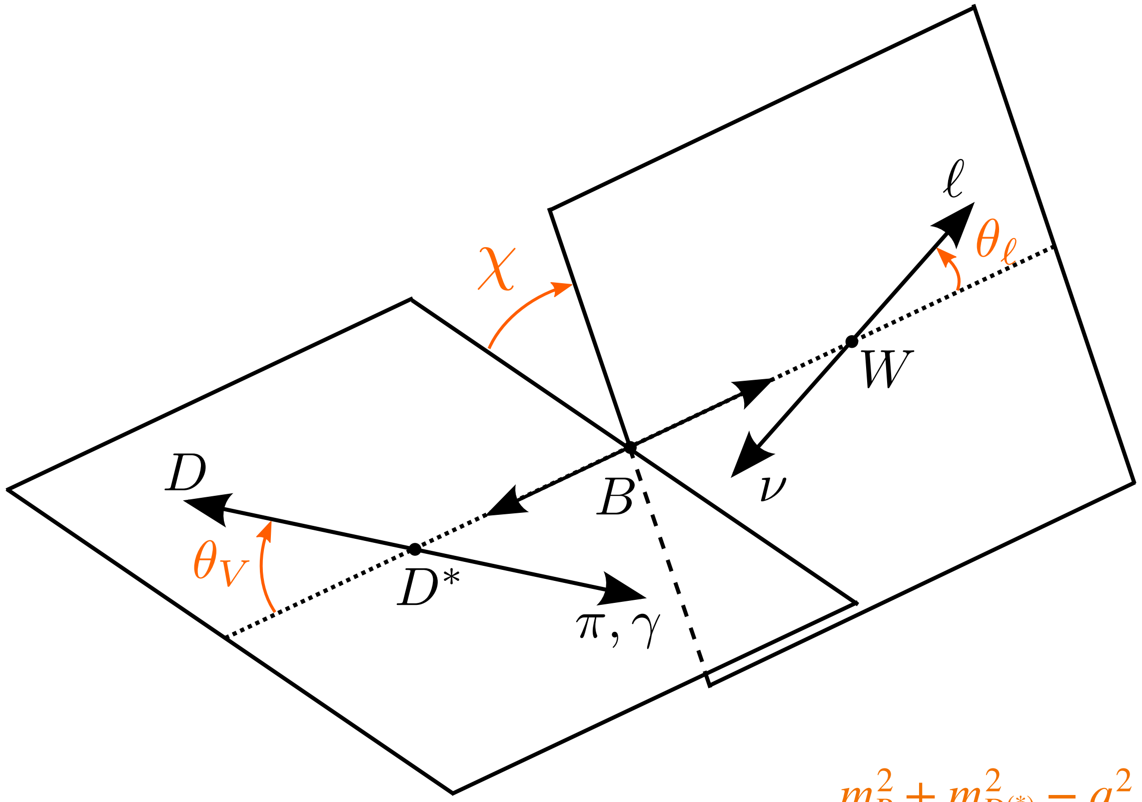
Target B^0 and B^+ and reconstruct D in many modes :

$$D^+ \to K^-\pi^+\pi^+,\ D^+ \to K^-\pi^+\pi^+\pi^0,$$
$$D^+ \to K^-\pi^+\pi^+\pi^+\pi^-,\ D^+ \to K^0_S\pi^+,\ D^+ \to K^0_S\pi^+\pi^0,$$
$$D^+ \to K^0_S\pi^+\pi^+\pi^-,\ D^+ \to K^0_S K^+,\ D^+ \to K^+K^-\pi^+,$$
$$D^0 \to K^-\pi^+,\ D^0 \to K^-\pi^+\pi^0,\ D^0 \to K^-\pi^+\pi^+\pi^-,$$
$$D^0 \to K^-\pi^+\pi^+\pi^-\pi^0,\ D^0 \to K^0_S\pi^0,\ D^0 \to K^0_S\pi^+\pi^-,$$
$$D^0 \to K^0_S\pi^+\pi^-\pi^0, \text{ and } D^0 \to K^-K^+.$$

Reconstruct $D^{*+} \to D^0\pi^+, D^{*+} \to D^+\pi^0, D^{*0} \to D^0\pi^0$

In principle also can do $D^{*0} \to D^0\gamma$ but has different Lorentz structure & angular distributions

Tagged measurement can directly reconstruct **B rest frame** & access $\{w, \cos\theta_\ell, \cos\theta_V, \chi\}$



$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

Tagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

Target B^0 and B^+ and reconstruct D in many modes : $D^+ \to K^-\pi^+\pi^+$, $D^+ \to K^-\pi^+\pi^+\pi^0$, $D^+ \to K^-\pi^+\pi^+\pi^+\pi^-$, $D^+ \to K^0_S\pi^+$, $D^+ \to K^0_S\pi^+\pi^0$, $D^+ \to K^0_S\pi^+\pi^+\pi^-$, $D^+ \to K^0_S K^+$, $D^+ \to K^+K^-\pi^+$, $D^0 \to K^-\pi^+$, $D^0 \to K^-\pi^+\pi^0$, $D^0 \to K^-\pi^+\pi^+\pi^-$, $D^0 \to K^-\pi^+\pi^+\pi^-\pi^0$, $D^0 \to K^0_S\pi^0$, $D^0 \to K^0_S\pi^+\pi^-$, $D^0 \to K^0_S\pi^+\pi^-\pi^0$, and $D^0 \to K^-K^+$.

Reconstruct $D^{*+} \to D^0\pi^+, D^{*+} \to D^+\pi^0, D^{*0} \to D^0\pi^0$

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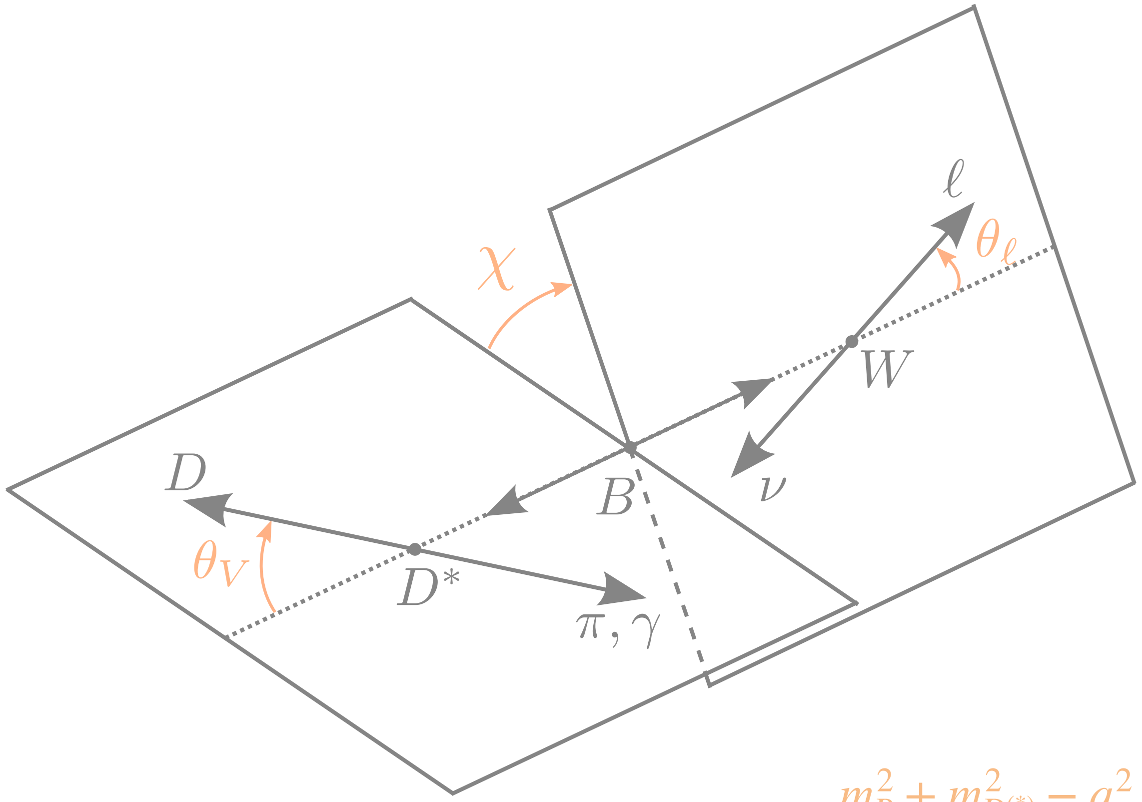
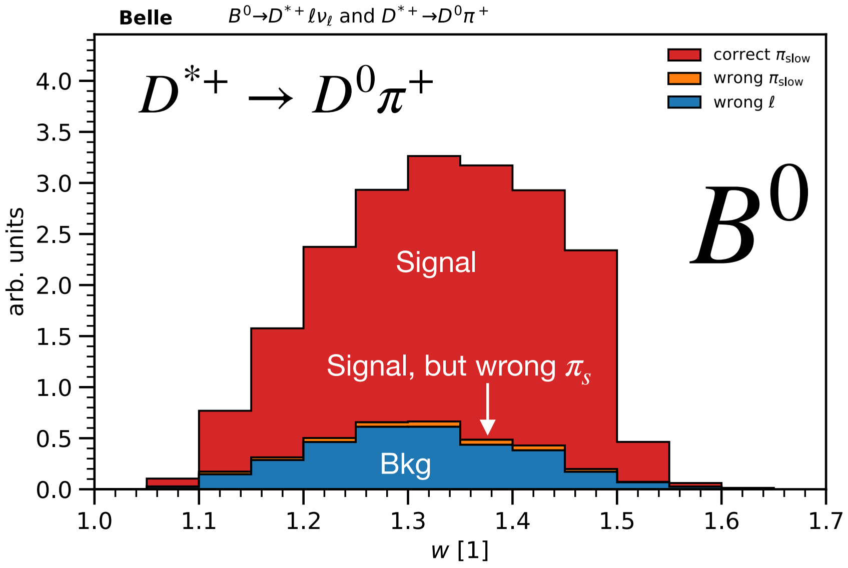
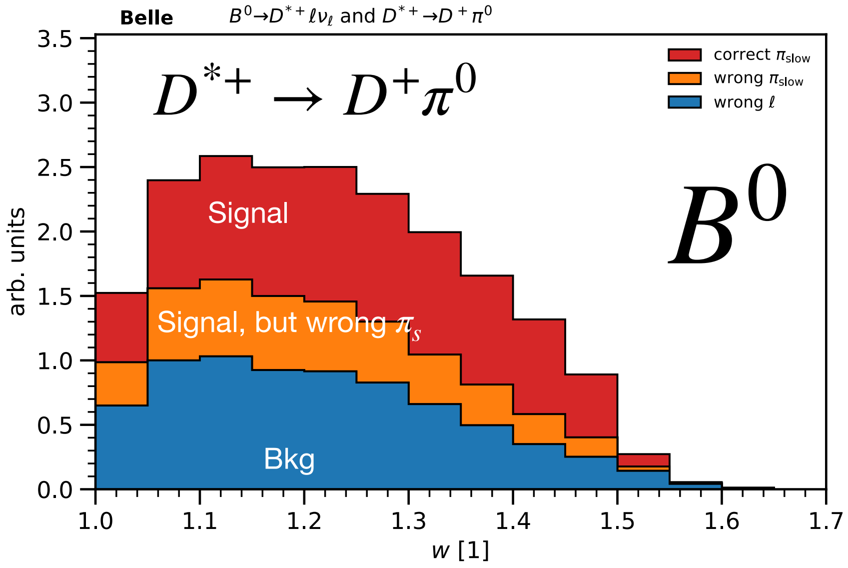
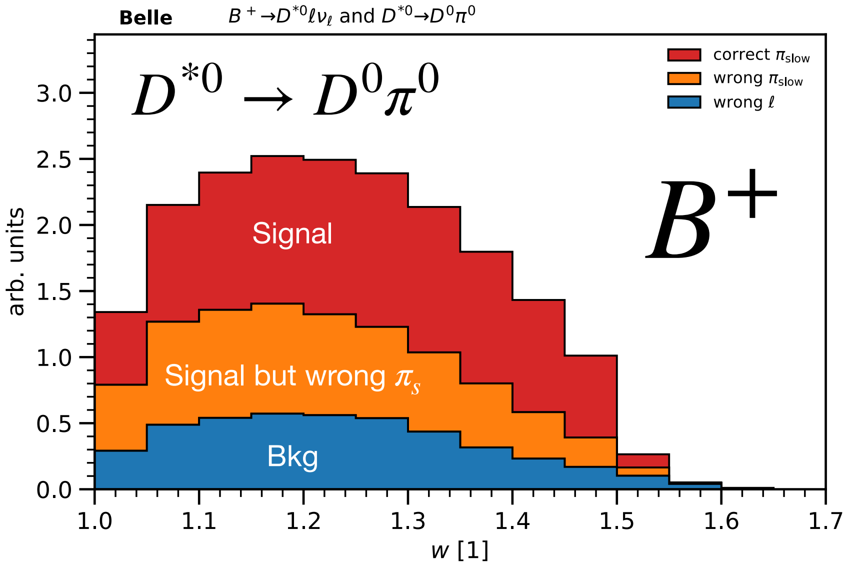


$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$







Background subtraction:

Need to subtract residual **background** contributions:

- From other SL decays ($B \to D^{**}\ell\bar{\nu}_\ell$ or $B \to D\ell\bar{\nu}_\ell$)
- From other B decays (with fake or real leptons)
- From Continuum ($e^+e^- \to q\bar{q}$)

Key idea :

$$p_{B_{\rm sig}} = p_{e^+e^-} - p_{B_{\rm tag}}$$

Background subtraction:

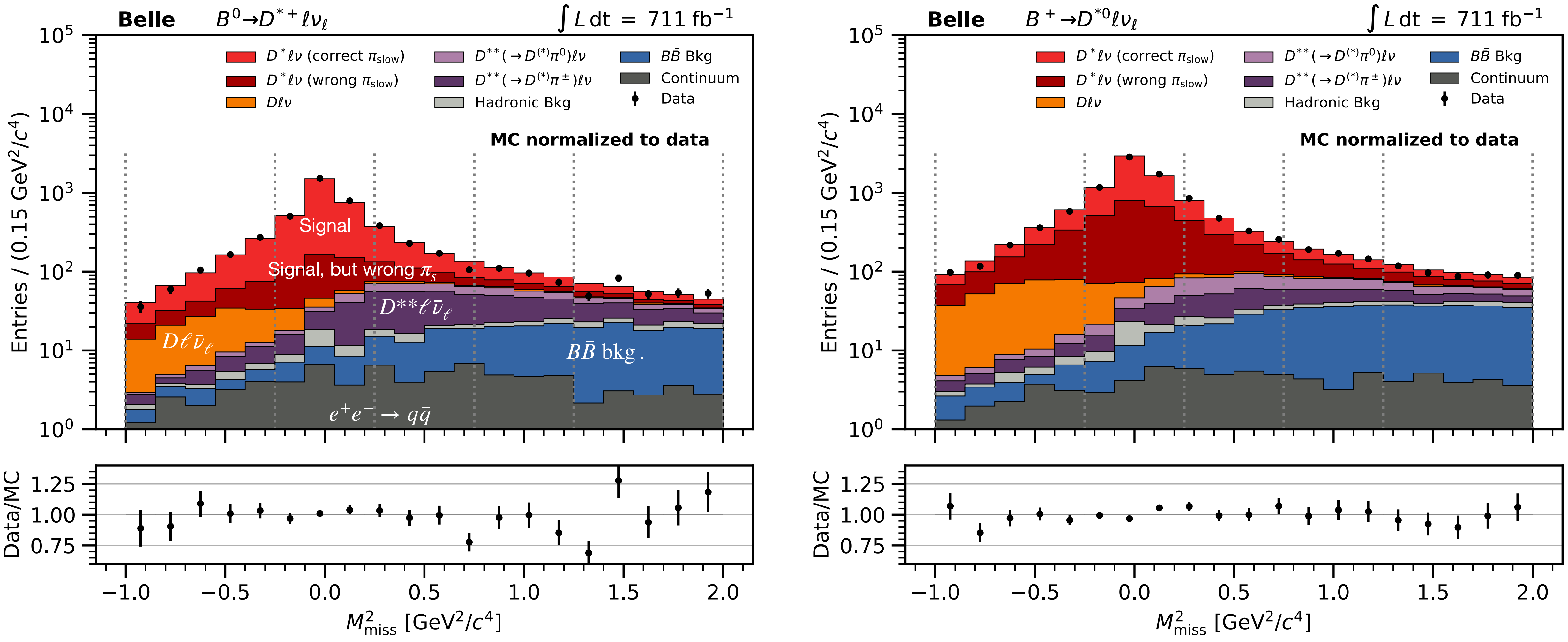
Need to subtract residual **background** contributions:

- From other SL decays ($B \to D^{**}\ell\bar{\nu}_\ell$ or $B \to D\ell\bar{\nu}_\ell$)
- From other B decays (with fake or real leptons)
- From Continuum ($e^+e^- \to q\bar{q}$)

Key idea :

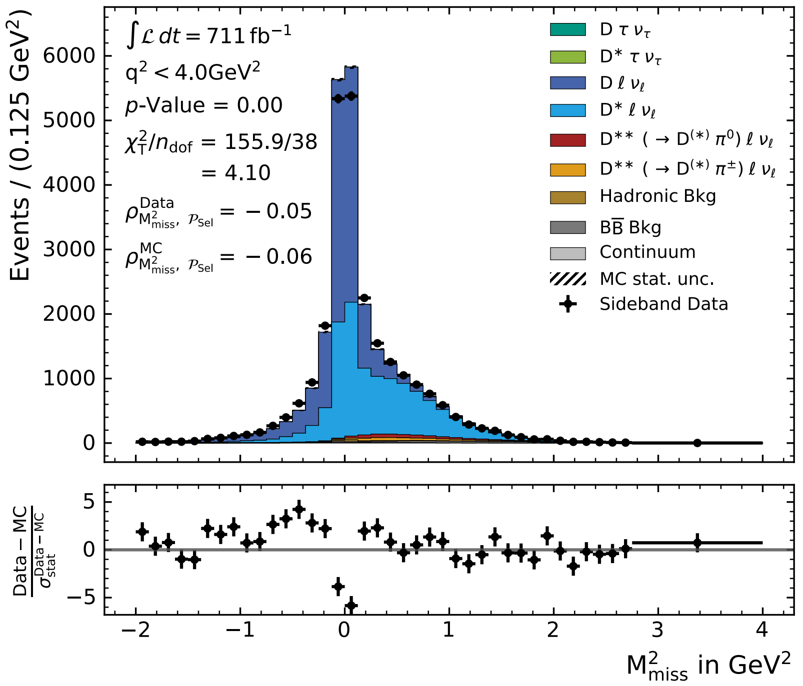
$$p_{B_{\mathrm{sig}}} = p_{e^+e^-} - p_{B_{\mathrm{tag}}}$$

Use: $0 = m_\nu^2 \simeq M_{\mathrm{miss}}^2 = (E_{\mathrm{miss}}, \mathbf{p}_{\mathrm{miss}})^2 = (p_B - p_{D^*} - p_\ell)^2$ or $U = E_{\mathrm{miss}} - |\,\mathbf{p}_{\mathrm{miss}}\,|$

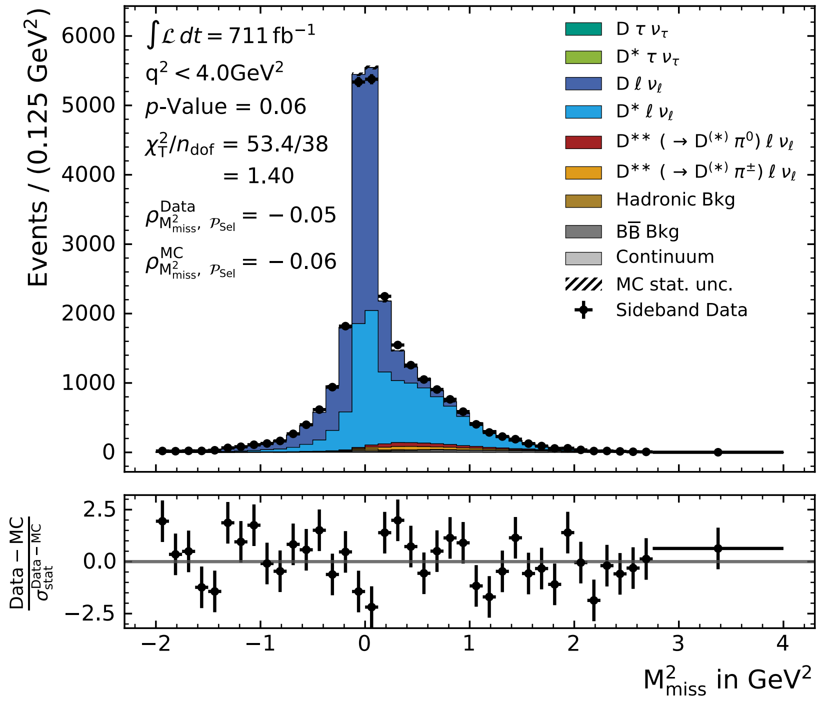


MC modelling of $M^2_{\rm miss}$ challenging

Need to apply additional corrections to match actual resolution



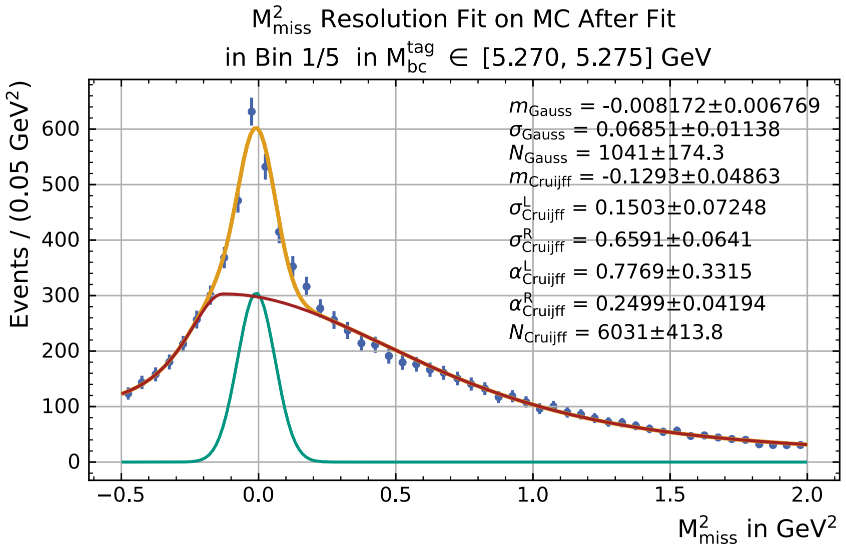
$\Downarrow$



E.g. use an appropriate smearing function

(e.g. asymmetric Laplace distribution and as a function of $m_{\rm bc}$)

$$f_{\rm AL}(x; m, \lambda, \kappa) = \frac{\lambda}{\kappa + 1/\kappa} \begin{cases} \exp\left((\lambda/\kappa)(x-m)\right) & \text{if } x < m, \\ \exp(-\lambda\kappa(x-m)) & \text{if } x \geq m, \end{cases}$$



Also other issues which cannot be necessarily solved by smearing alone, e.g. in inclusive analyses the modeling of e.g. D mesons is extremely important

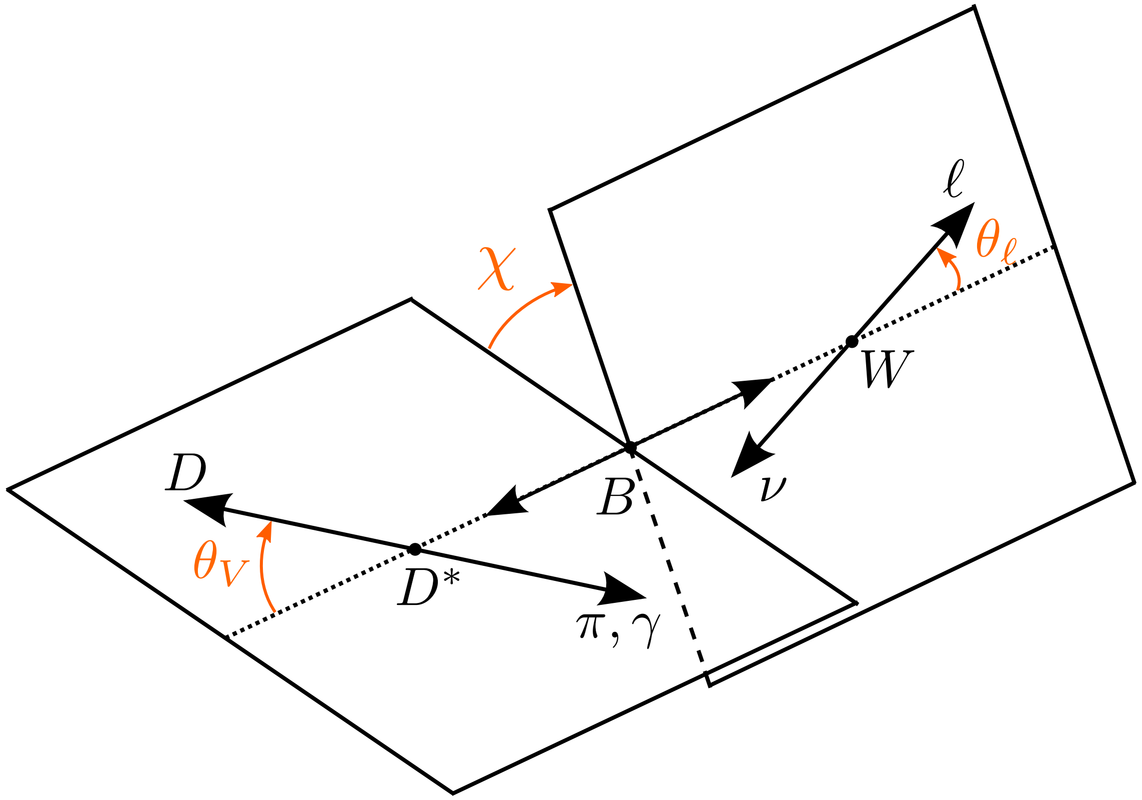
see e.g. Belle II R(X) measurement in preparation

Fit in Bins of $\{w, \cos\theta_\ell, \cos\theta_V, \chi\}$

E.g. Can use **binned likelihood** fit to **1D distributions**
(good to use coarse binning to reduce modelling dependence (Bkg shape, resolution))

4D fit also possible; but binned approach suffers from course of dimensionality

→ **better unbinned (but then need to worry about efficiency & migrations)**



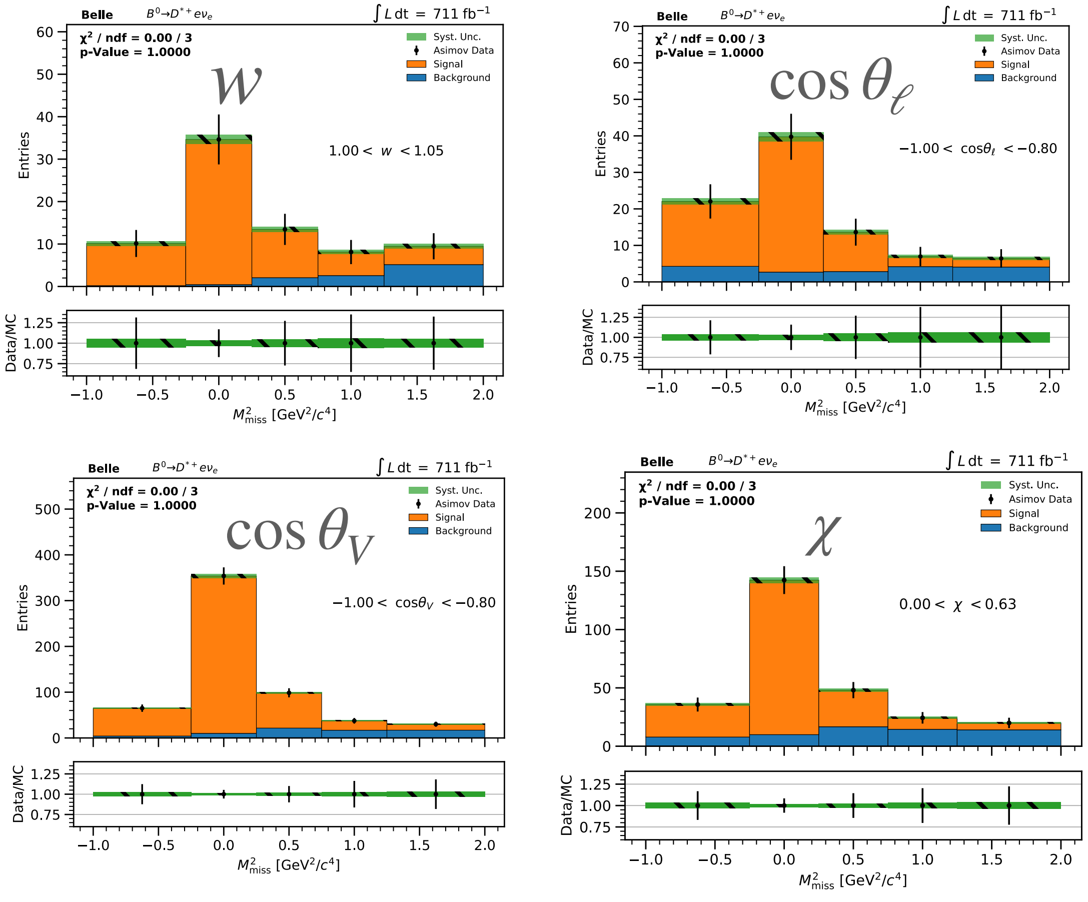
Fit in Bins of $\{w, \cos\theta_\ell, \cos\theta_V, \chi\}$

E.g. Can use **binned likelihood** fit to **1D distributions**
(good to use coarse binning to reduce modelling dependence (Bkg shape, resolution))

4D fit also possible; but binned approach suffers from course of dimensionality

→ **better unbinned (but then need to worry about efficiency & migrations)**

Example **1D fits** to MC (Asimov fits)



Best approach: use folding to extract relevant information

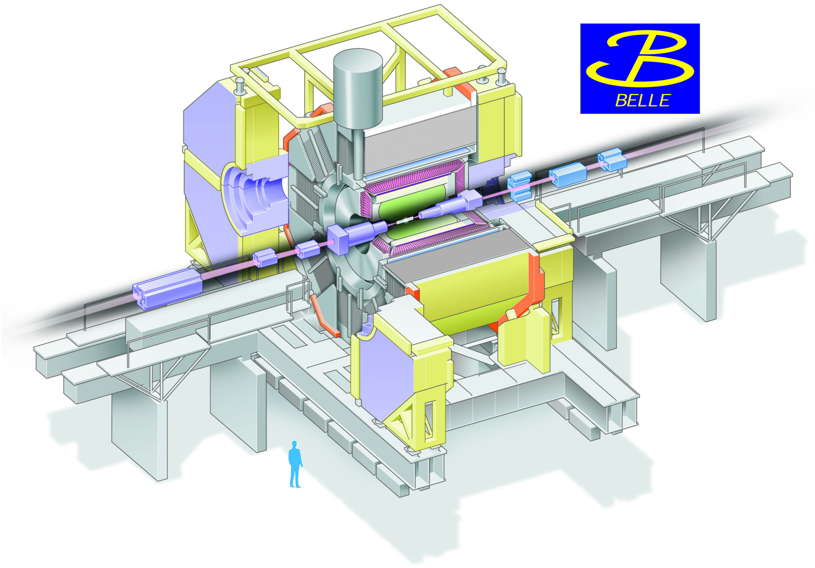
$$\frac{d^4\Gamma}{dq^2\, d\cos\theta^*\, d\cos\theta_\ell\, d\chi} = \frac{9}{32\pi}\Big[\big(I_1^s \sin^2\theta^* + I_1^c\cos^2\theta^*\big) + \big(I_2^s\sin^2\theta^* + I_2^c\cos^2\theta^*\big)\cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta^*\sin^2\theta_\ell\cos 2\chi + I_4 \sin 2\theta^* \sin 2\theta_\ell \cos\chi + I_5 \sin 2\theta^*\sin\theta_\ell\cos\chi$$
$$+ \big(I_6^c\cos^2\theta^* + I_6^s\sin^2\theta^*\big)\cos\theta_\ell + I_7\sin 2\theta^*\sin\theta_\ell\sin\chi$$
$$+ I_8 \sin 2\theta^*\sin 2\theta_\ell\sin\chi + I_9\sin^2\theta^*\sin^2\theta_\ell\sin 2\chi\Big]\,,$$

I.e. by building smart asymmetries, can project out the relevant 12 terms (integrated over a certain q^2 range)

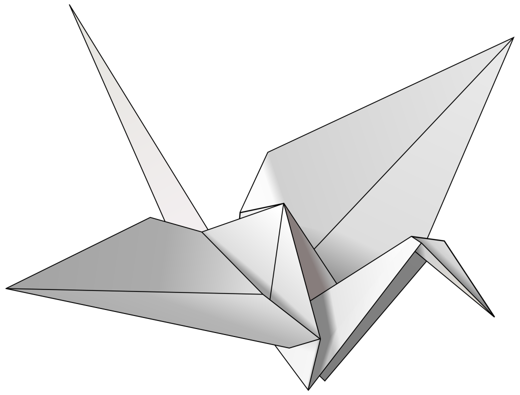
→ See e.g. Markus Prim's Belle Analysis (in preparation)

Detector migrations

X: True distribution



M: Detector response



Y: Measured distribution

An event reconstructed in a given *bin i,* might not have had a “true” value corresponding to a *bin j*

Can be parametrized as a **migration matrix:**

$$\mathscr{M}_{ij} = \mathscr{P}(\text{reco. in bin } i \,|\, \text{true value in bin } j)$$

parametrize detector migrations
as **conditional probability**

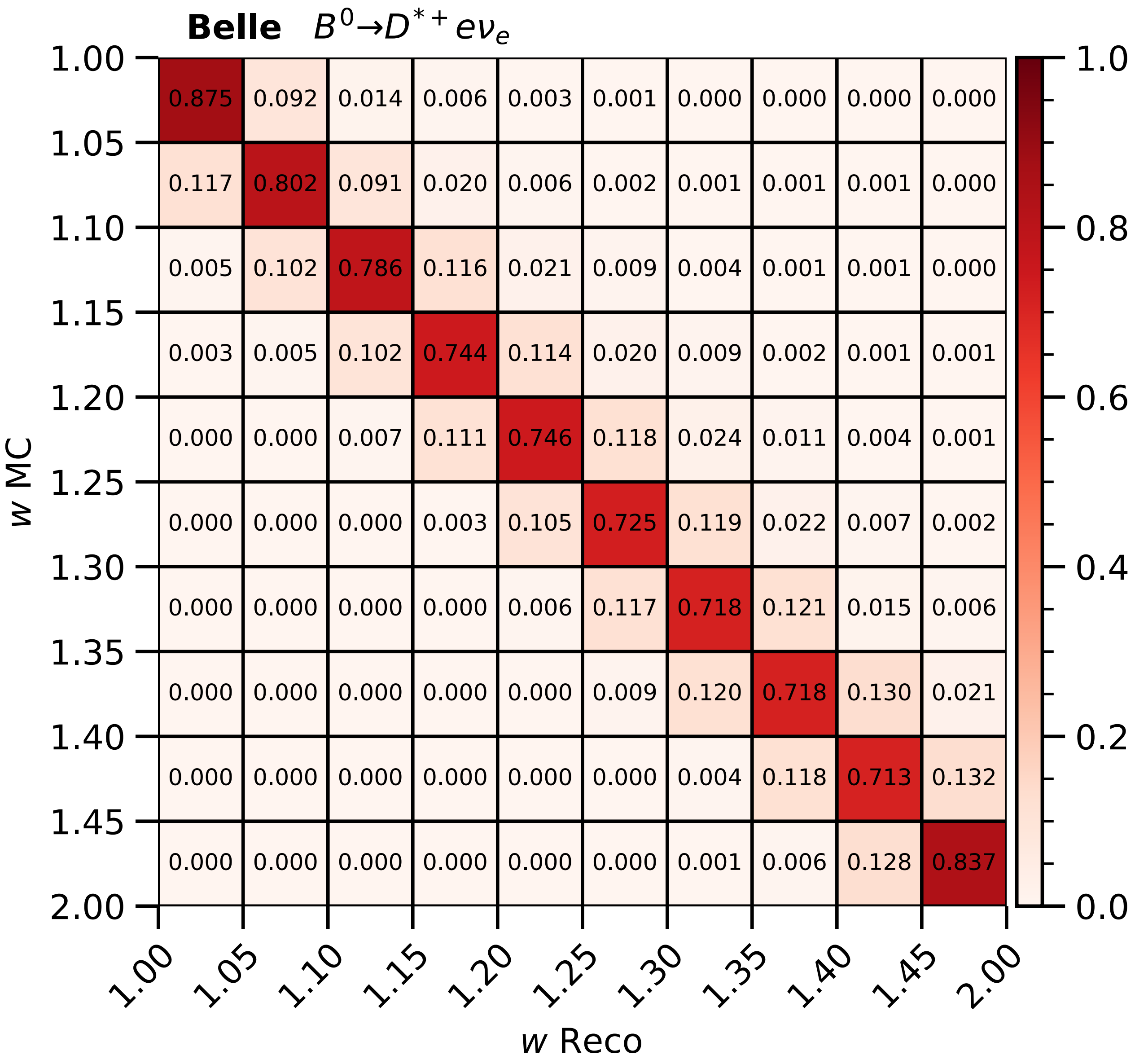
Check Markus Prim’s slides

Detector migrations

An event reconstructed in a given *bin i,* might not have had a “true” value corresponding to a *bin j*

Can be parametrized as a **migration matrix:**

$$\mathscr{M}_{ij} = \mathscr{P}(\text{reco. in bin } i \,|\, \text{true value in bin } j)$$



0.875	0.092	0.014	0.006	0.003	0.001	0.000	0.000	0.000	0.000
0.117	0.802	0.091	0.020	0.006	0.002	0.001	0.001	0.001	0.000
0.005	0.102	0.786	0.116	0.021	0.009	0.004	0.001	0.001	0.000
0.003	0.005	0.102	0.744	0.114	0.020	0.009	0.002	0.001	0.001
0.000	0.000	0.007	0.111	0.746	0.118	0.024	0.011	0.004	0.001
0.000	0.000	0.000	0.003	0.105	0.725	0.119	0.022	0.007	0.002
0.000	0.000	0.000	0.000	0.006	0.117	0.718	0.121	0.015	0.006
0.000	0.000	0.000	0.000	0.000	0.009	0.120	0.718	0.130	0.021
0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.118	0.713	0.132
0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.128	0.837

Can recover estimates for true values via “unfolding” determined yields, mapping $\text{reco} \to \text{true}$

Simplest version: migration matrix inversion

$$\mathbf{x}_{\text{true}} = \mathscr{M}_{ij}^{-1}\, \mathbf{x}_{\text{reco}}$$

Many approaches to dampen impact of increase in variance

(mostly a problem with large migrations → true bin is then the sum of many reco bins with high weights)

or to reduce impact of MC prior

(here less an issue; but Bayesian unfolding can propagate the observed shape to MC to minimize model dependencies)

Acceptance × Efficiency

After migration effects are corrected, need to correct also for selection effects (Acceptance x Efficiency)

$$\Delta\mathscr{B}/\Delta\mathbf{x} = \boxed{\left(\epsilon_{\mathrm{reco}} \times \epsilon_{\mathrm{tag}}\right)^{-1}} \times \mathscr{M}^{-1}\,\mathbf{x}_{\mathrm{reco}} \times \frac{1}{4\,N_{B\overline{B}}}$$

2 if $e+\mu$

# of charged or neutral B meson pairs (other factor of 2)

Actually a matrix

$$\left(\epsilon_{\mathrm{reco}} \times \epsilon_{\mathrm{tag}}\right) = \mathrm{diag}\left(\mathscr{A}(\text{true bin } i)\right)$$

Although it’s acceptance × efficiency,
we just call this acceptance
in the figure on the next slide

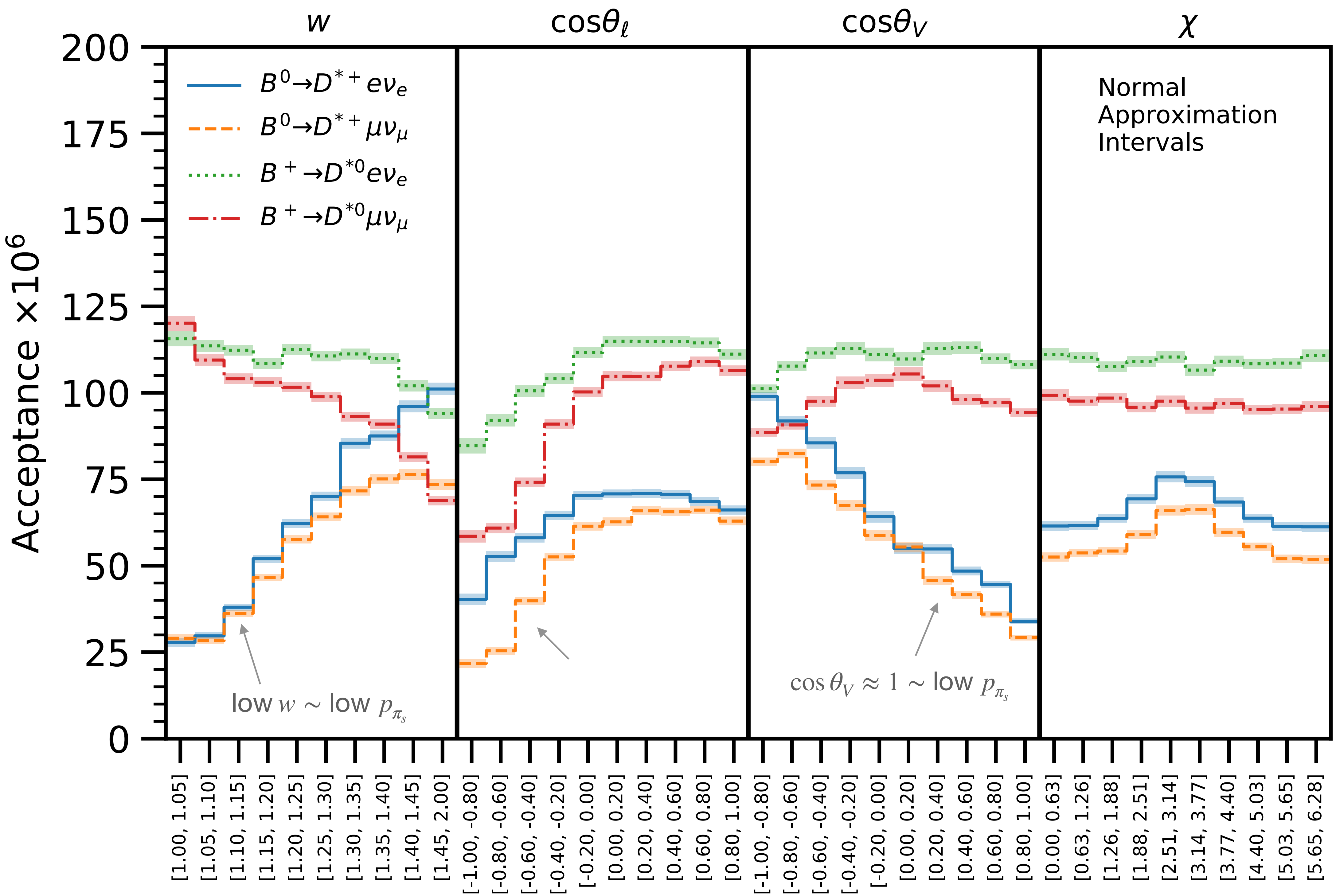
$$2\,N_{B\overline{B}} = \left(1+f_{+0}\right) N_{B^0} = \left(1+f_{+0}^{-1}\right) N_{B^+}$$

$$f_{+0} = \frac{\mathscr{B}(\Upsilon(4S)\to B^+B^-)}{\mathscr{B}(\Upsilon(4S)\to B^0\overline{B}^0)}$$

Check Jim’s slides

Acceptance × Efficiency

After migration effects are corrected, need to correct also for selection effects (Acceptance x Efficiency)



A word on Efficiencies

Efficiencies can be are a large source of uncertainties

Two examples very relevant for semileptonic decays:

- Lepton Identification Uncertainty

Often based on a global likelihood (or a multivariate classifier) using individual likelihoods (or input features) to calculate a score how likely the identified particle is an electron or a muon

Symbolically:

$$\mathscr{L} = \mathscr{L}_{\text{CDC}} \times \mathscr{L}_{\text{ECL}} \times \mathscr{L}_{\text{TOP}} \times \mathscr{L}_{\text{KLM}}$$

Ionization energy loss

$E / |\vec{p}\,|$

Information from Cherenkov light angles

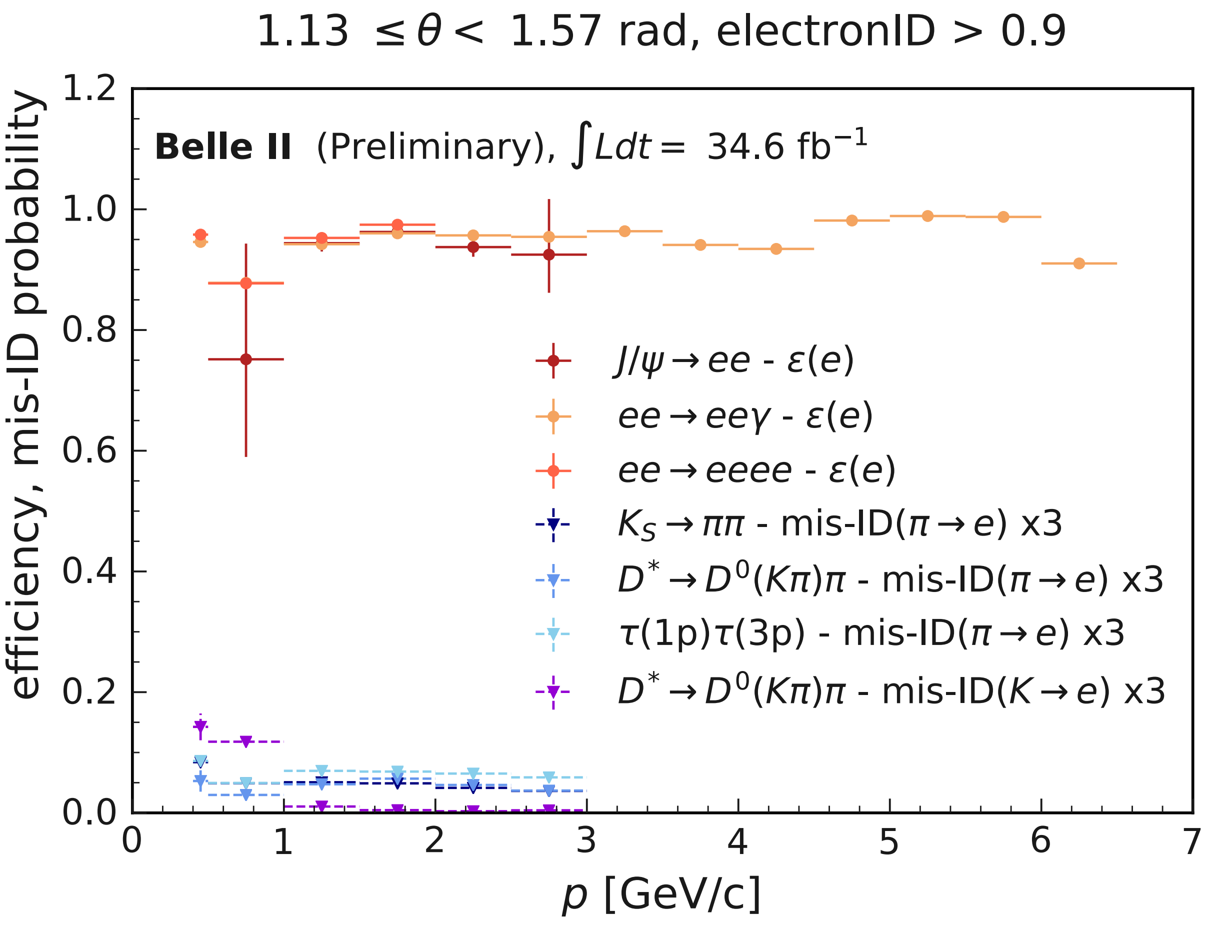
Matched KLM cluster hit?

Use clean physics sample to correct MC efficiencies and fake rates

E.g. $e^+e^- \to \mu\mu\gamma, e^+e^- \to e^+e^-\gamma, J/\psi \to \ell\ell, \ldots$

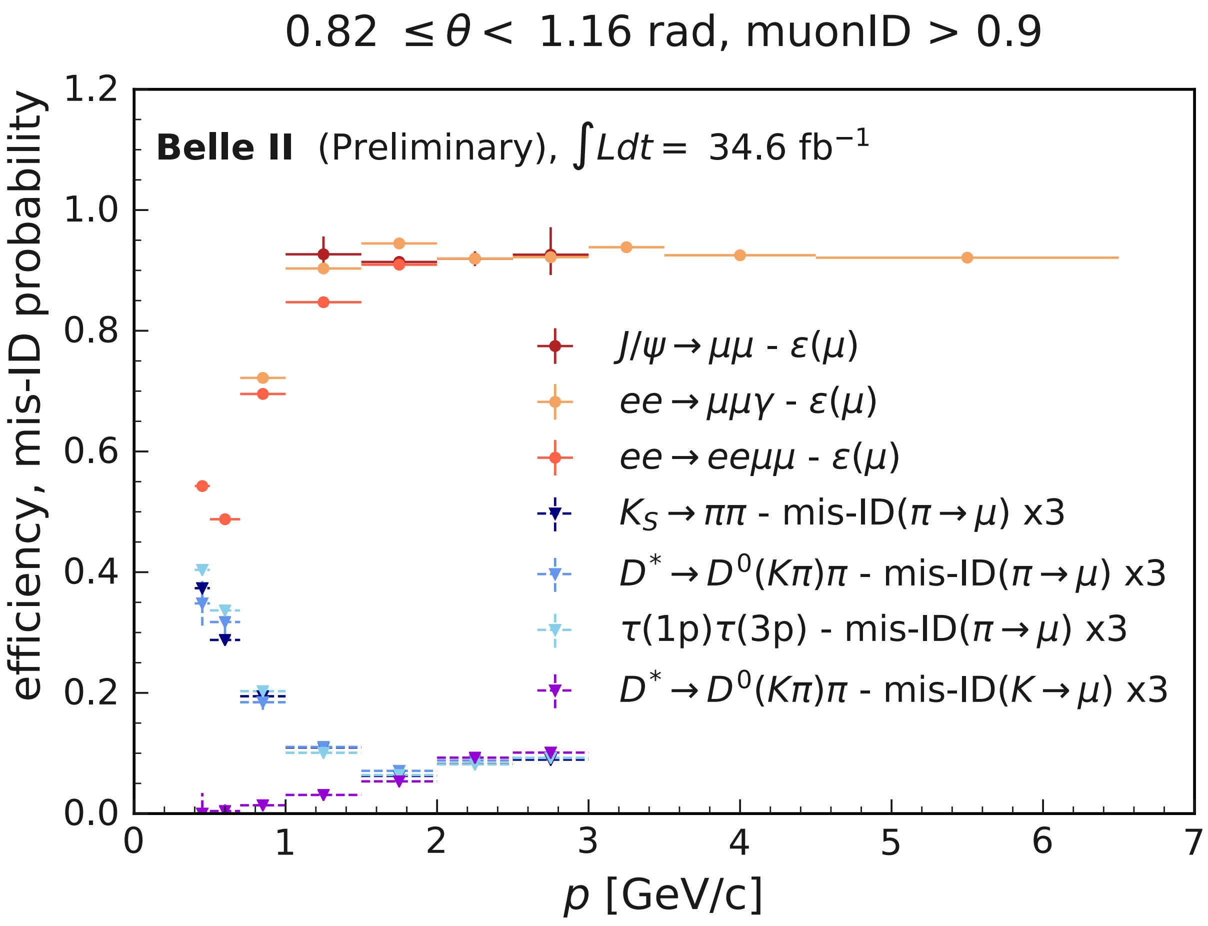
Construct likelihood ratio for Lepton ID: $\ell\mathrm{ID} = \mathscr{L}_e/[\mathscr{L}_e + \mathscr{L}_\mu + \mathscr{L}_\pi + \mathscr{L}_K + \mathscr{L}_p]$

Electrons



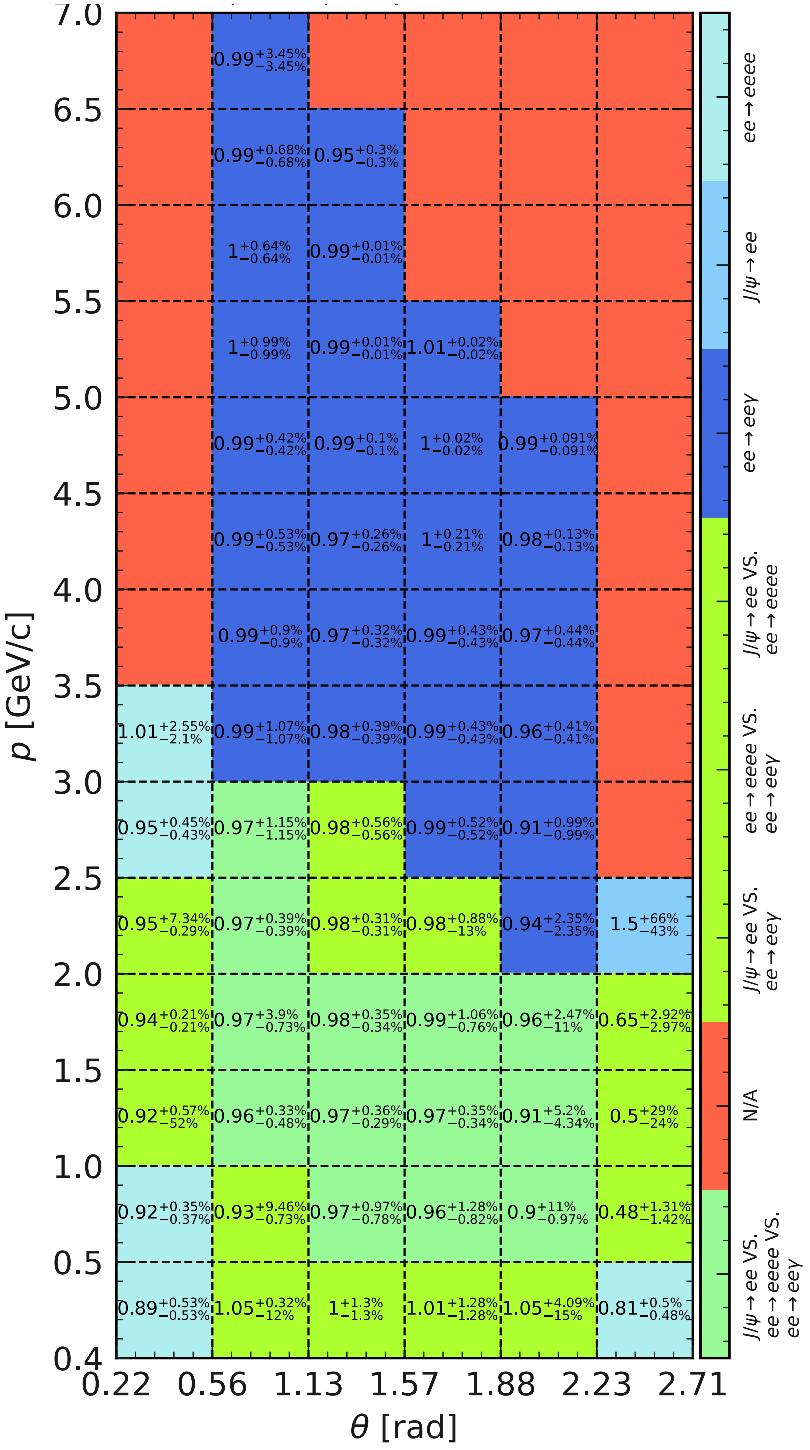
↑
Momentum in lab frame

Muons



Construct correction tables of efficiency ratios $\dfrac{\epsilon_{\text{Data}}}{\epsilon_{\text{MC}}}$

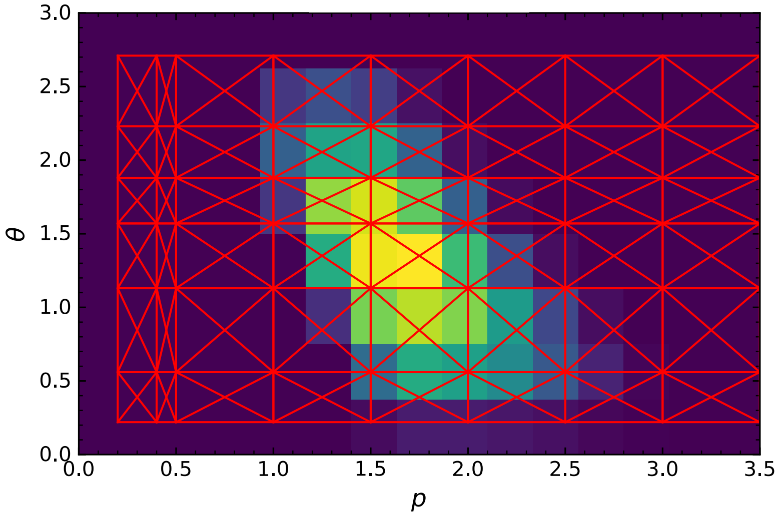
as a function of lab momentum and detector position (polar angle) to correct MC efficiencies



Precision limited by available control channel statistics (i.e. goes down by Lumi)

Non-closure between channels is added as extra uncertainty (limiting factor at very high luminosity)

Coverage of control channels and signal are different, i.e. not all control channels have same relevance)



→ Correlation model matters!

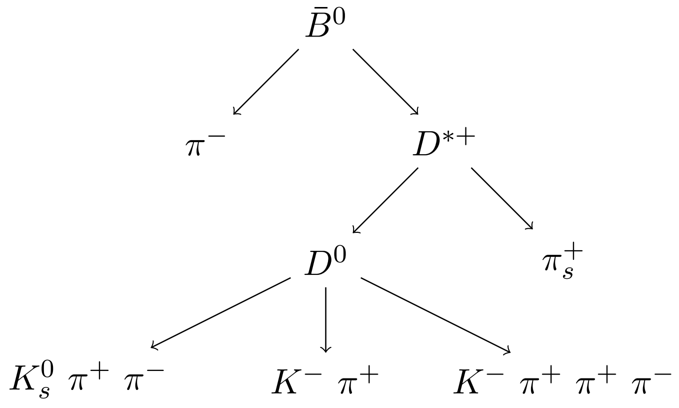
100% correlated errors
= maximal total eff. error, but no error on shapes

0% correlated errors
= minimal total eff. error, maximal error on shapes

Second example:

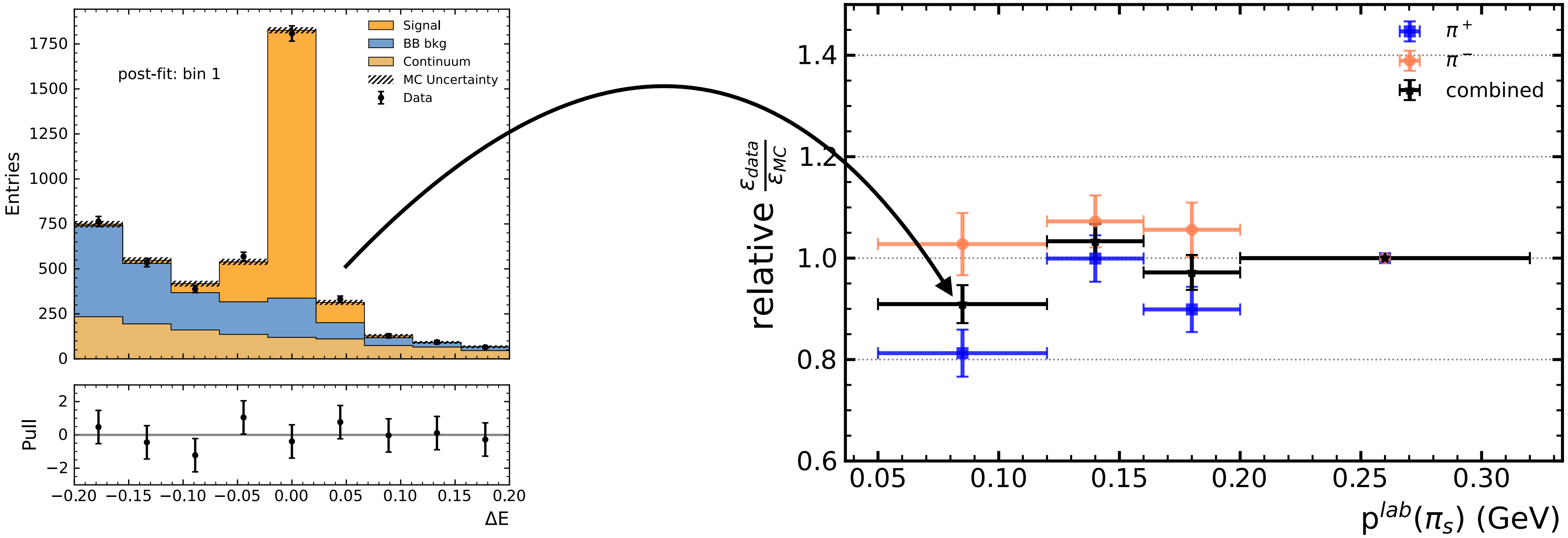
- Slow pion reconstruction efficiency

Also needs to be measured in data, e.g. via $B^0 \to D^{*+}\pi^-$ decays

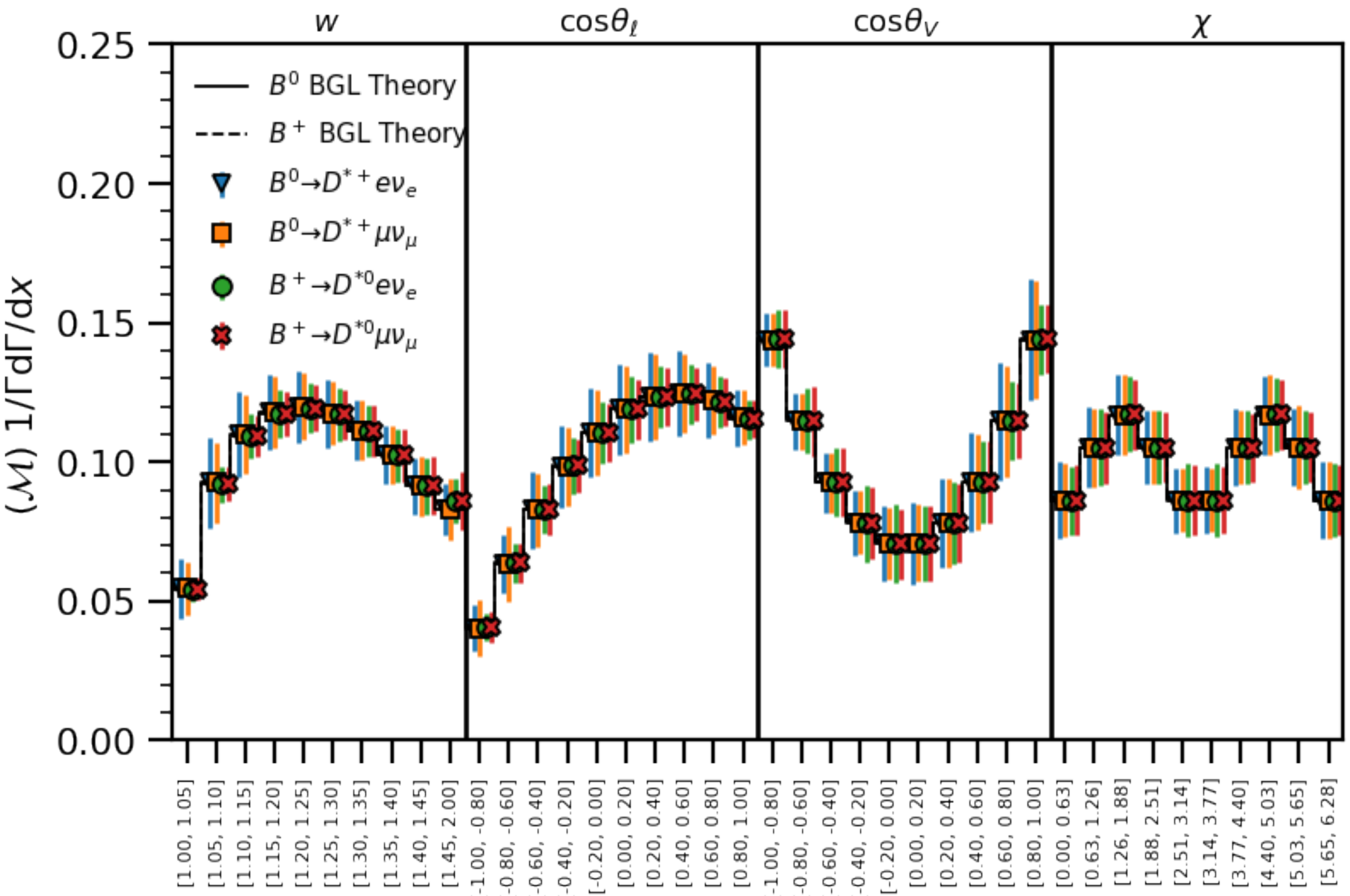


Extract signal in a fit to $\Delta E = \sqrt{s}/2 - E_B$ in bins of $p_{\pi_s}^{\text{lab}}$

Measure ratio efficiency ratio **relative** to high-momentum region of $p_{\pi_s}^{\text{lab}} > 200\,\text{MeV}$



The final result (MC)



Note how the different channels are complementary in different regions of phase-space

(e.g. B^+ has much better precision at low w than B^0, but both have equal precision at high w)

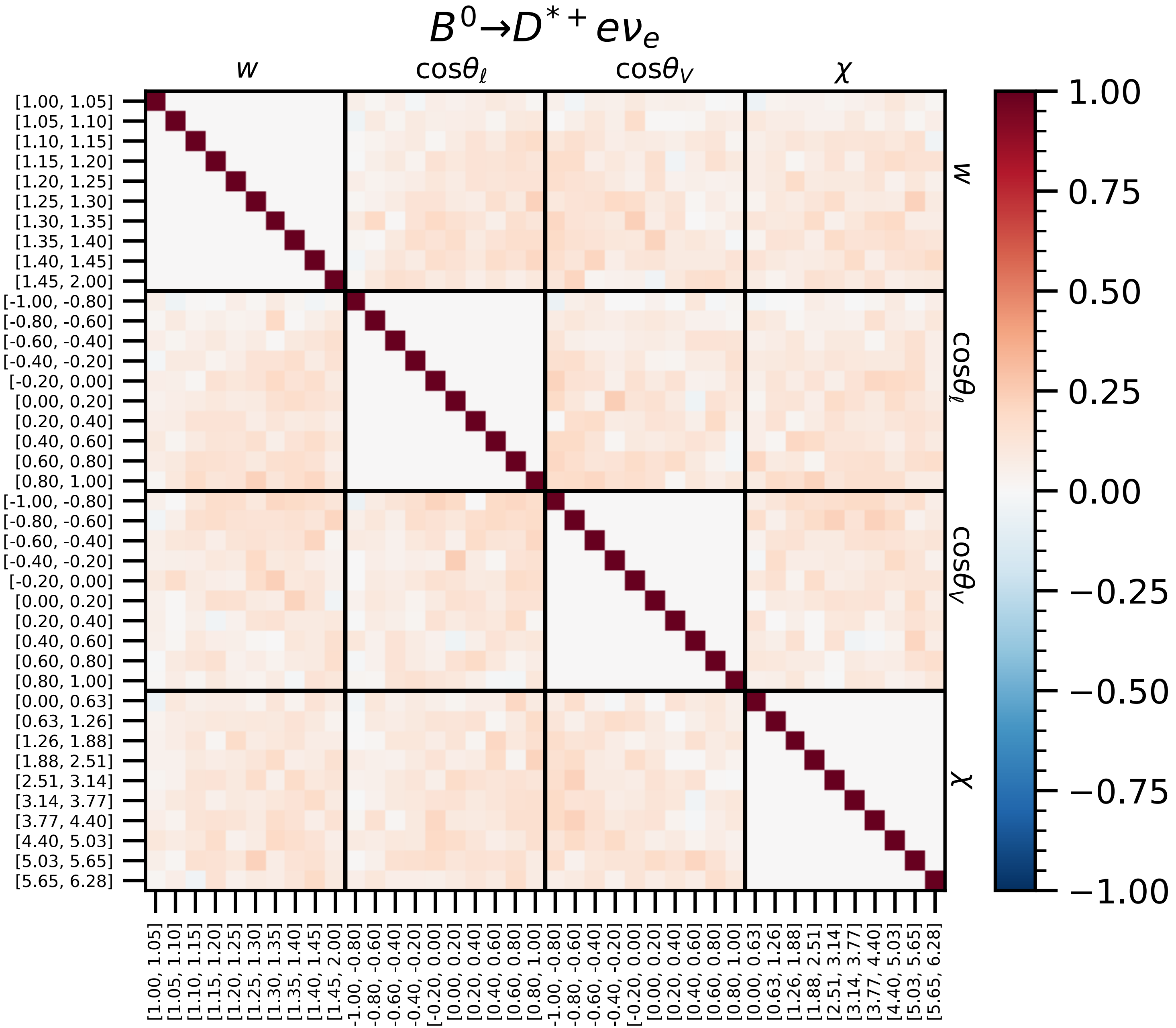
For a simultaneous analysis, need to determine correlations between different 1D projections → can be done using **boostrapping**

Very simple: create a replica of your data set by **sampling with replacement**

Repeat full analysis chain of 4 x1D measurement for **each replica**

Pearson correlator of replica sample provides estimator for statistical correlation between bins:

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



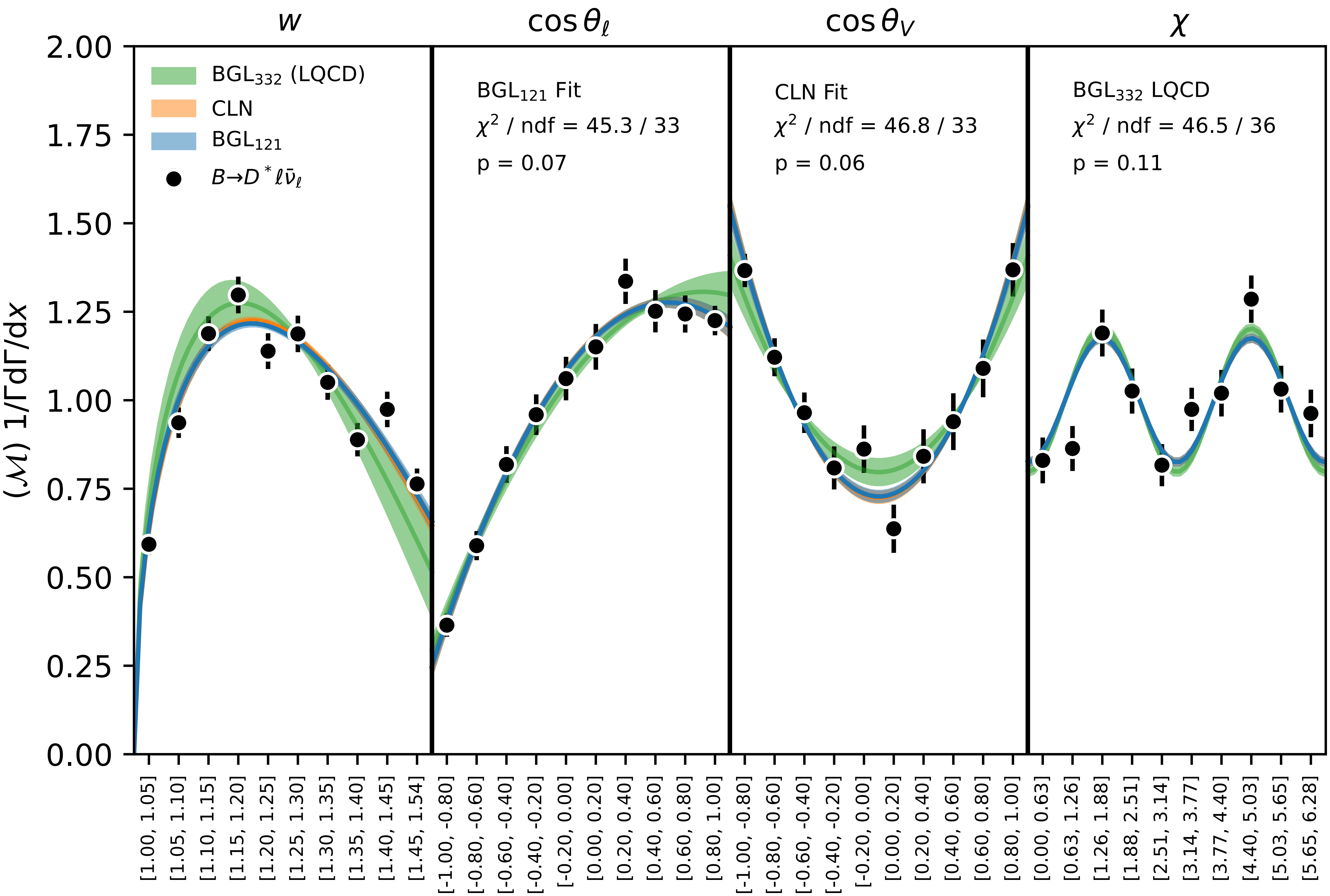
But since we measured projections of the same data, the effective **degrees of freedom** are not 40, but 37 (Jung, Van Dyk)

Best use of tagged data:

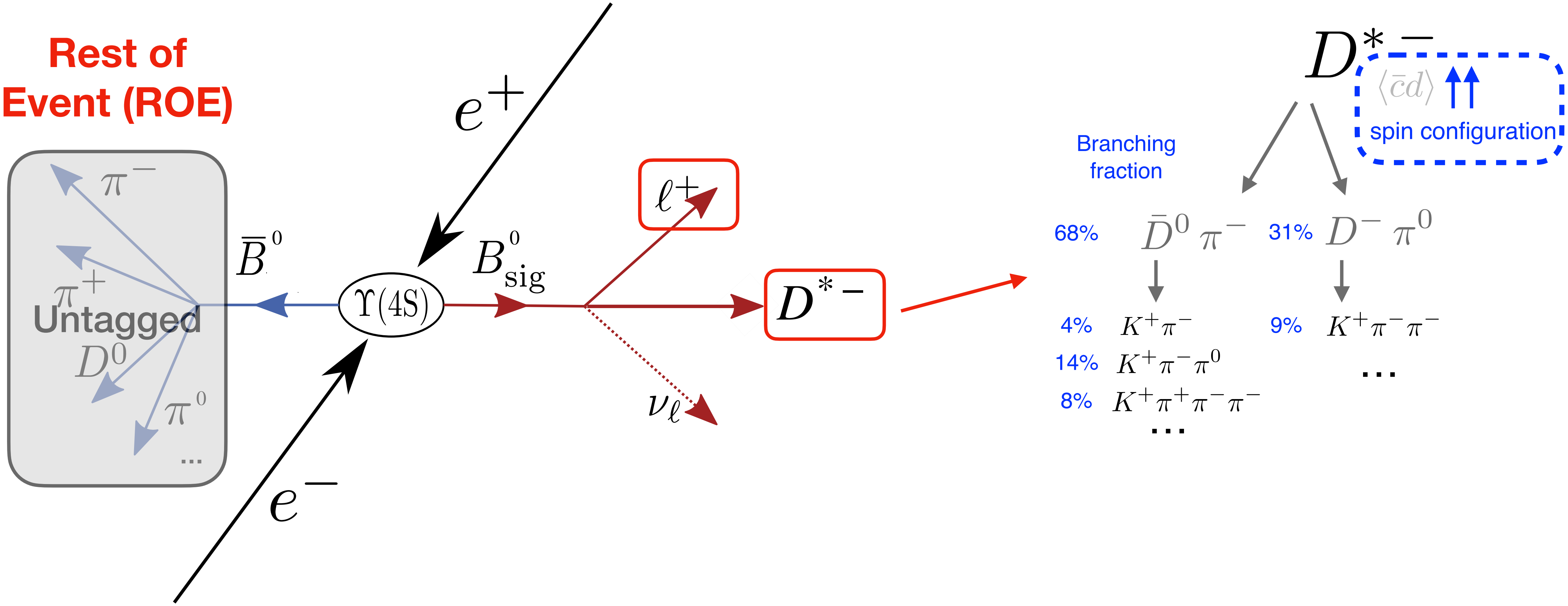
Fit normalized shapes (and if available total rate)

36 dof from shapes (4*9) and 1 from normalization

Final result :

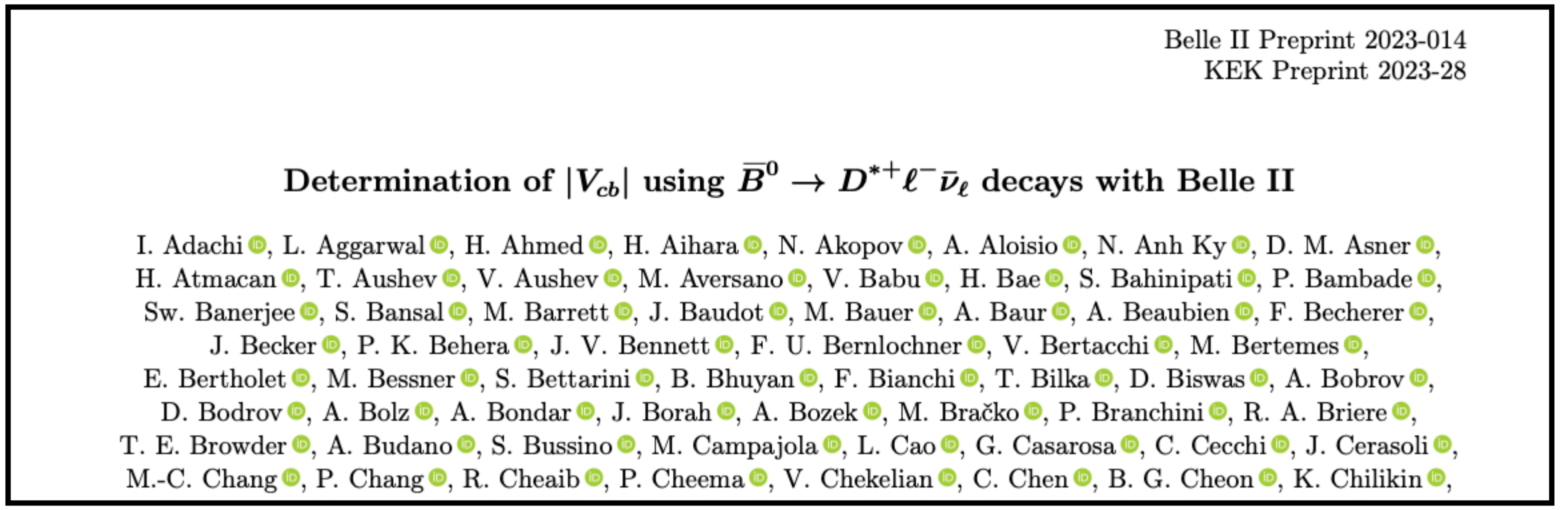


Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$



Recent Belle II result:

https://arxiv.org/abs/2310.01170
(accepted by PRD)

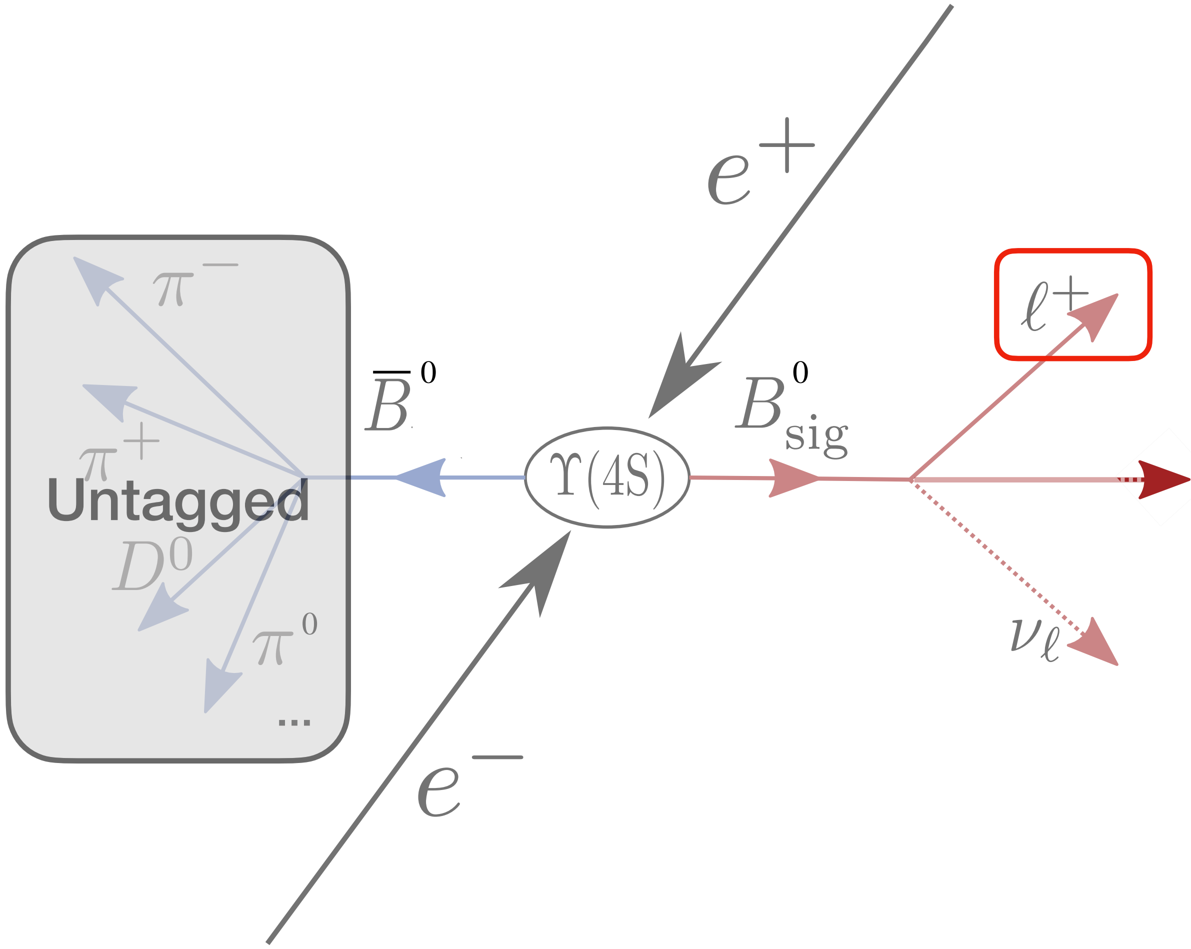


Belle II Preprint 2023-014
KEK Preprint 2023-28

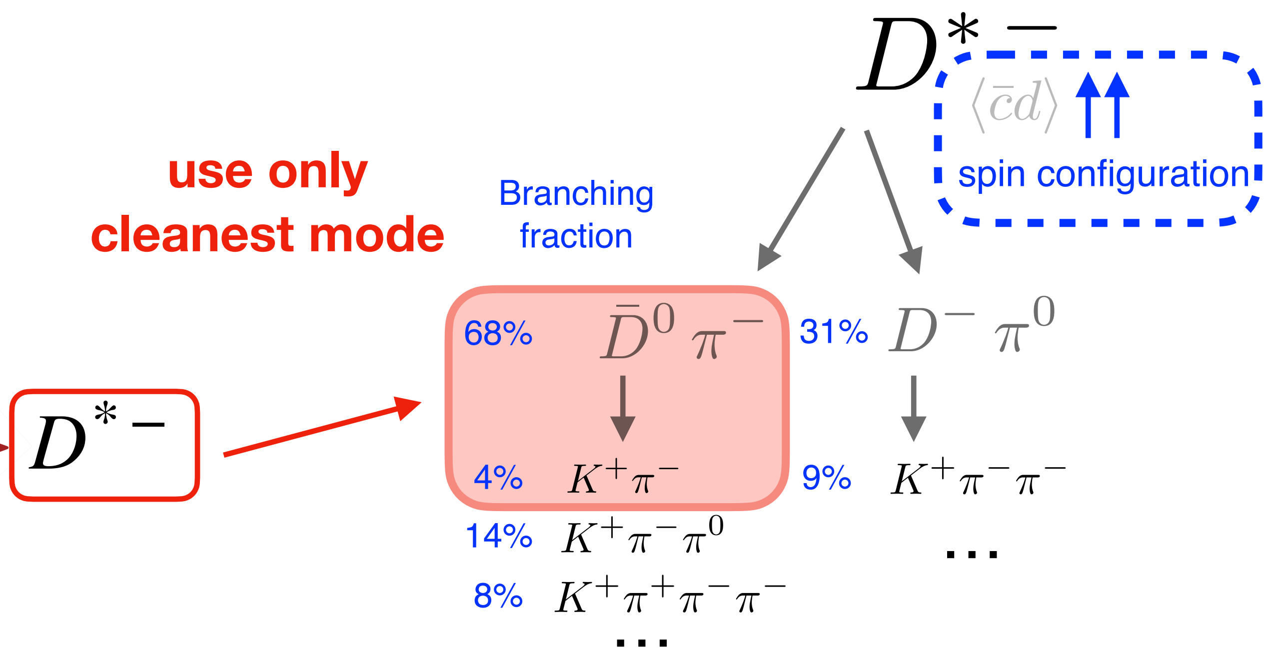
Determination of $|V_{cb}|$ using $\overline{B}^0 \to D^{*+}\ell^-\bar{\nu}_\ell$ decays with Belle II

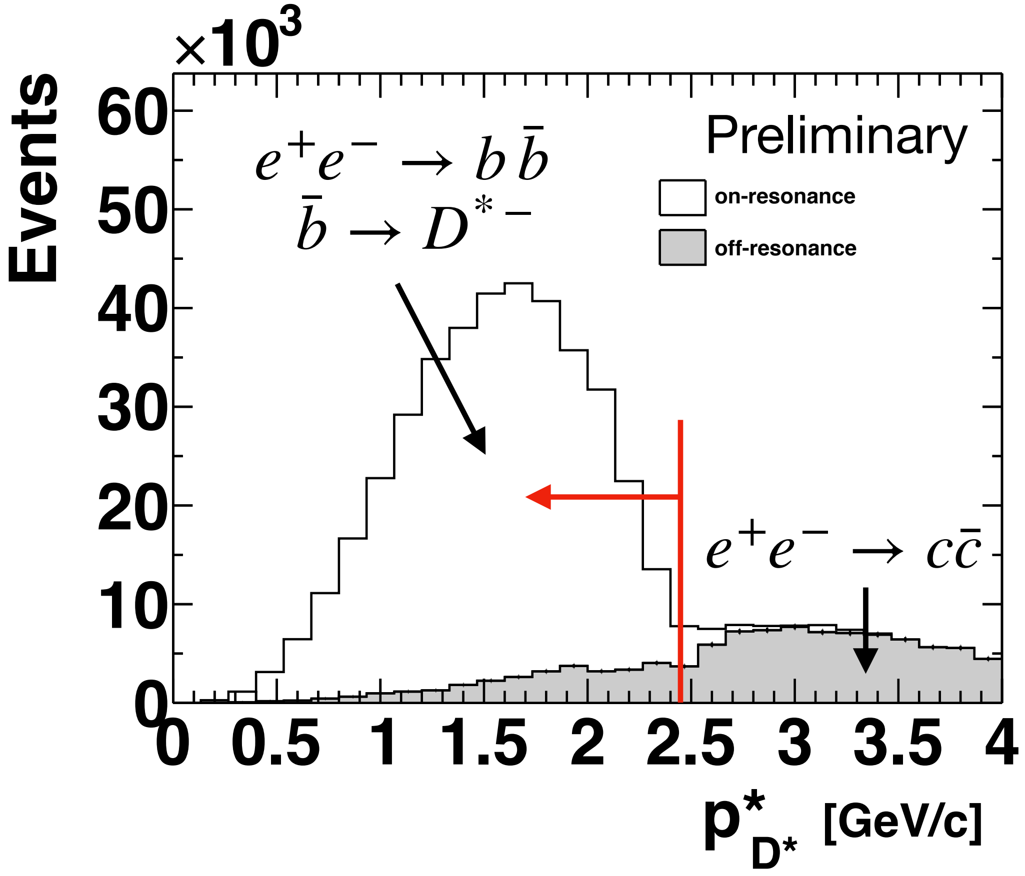
I. Adachi, L. Aggarwal, H. Ahmed, H. Aihara, N. Akopov, A. Aloisio, N. Anh Ky, D. M. Asner, H. Atmacan, T. Aushev, V. Aushev, M. Aversano, V. Babu, H. Bae, S. Bahinipati, P. Bambade, Sw. Banerjee, S. Bansal, M. Barrett, J. Baudot, M. Bauer, A. Baur, A. Beaubien, F. Becherer, J. Becker, P. K. Behera, J. V. Bennett, F. U. Bernlochner, V. Bertacchi, M. Bertemes, E. Bertholet, M. Bessner, S. Bettarini, B. Bhuyan, F. Bianchi, T. Bilka, D. Biswas, A. Bobrov, D. Bodrov, A. Bolz, A. Bondar, J. Borah, A. Bozek, M. Bračko, P. Branchini, R. A. Briere, T. E. Browder, A. Budano, S. Bussino, M. Campajola, L. Cao, G. Casarosa, C. Cecchi, J. Cerasoli, M.-C. Chang, P. Chang, R. Cheaib, P. Cheema, V. Chekelian, C. Chen, B. G. Cheon, K. Chilikin,

Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

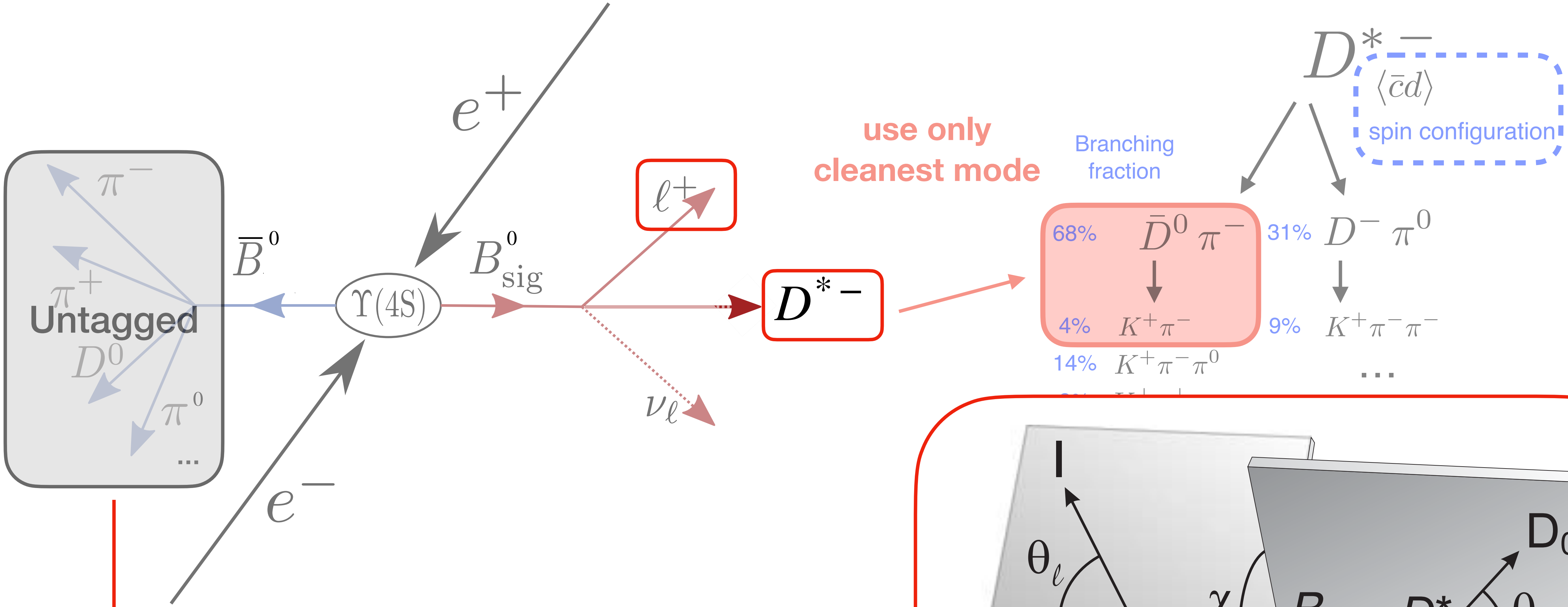


use only cleanest mode





Untagged measurements of $B \to D^{(*)}\ell\bar{\nu}_\ell$

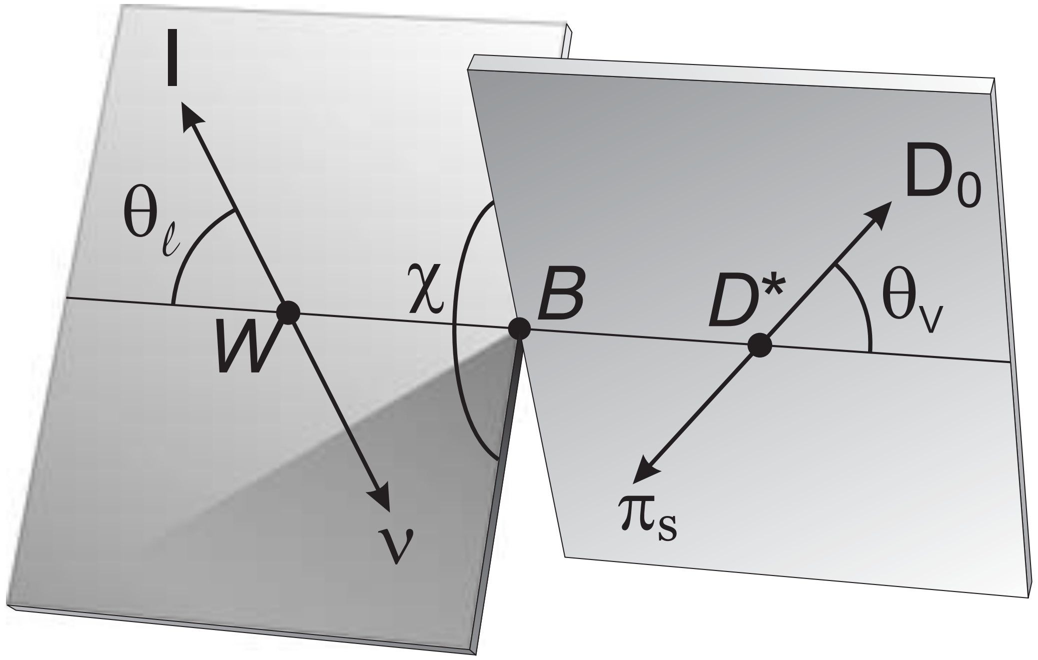


use only cleanest mode

$$\vec{p}_{\text{incl.}} = \sum_i \vec{p}_i \ ,$$

Reconstruct ROE to estimate $B_{\text{sig}}{}^0$ momentum

$$\vec{p}_{B_{\text{sig}}} = -\vec{p}_{\text{incl}}$$

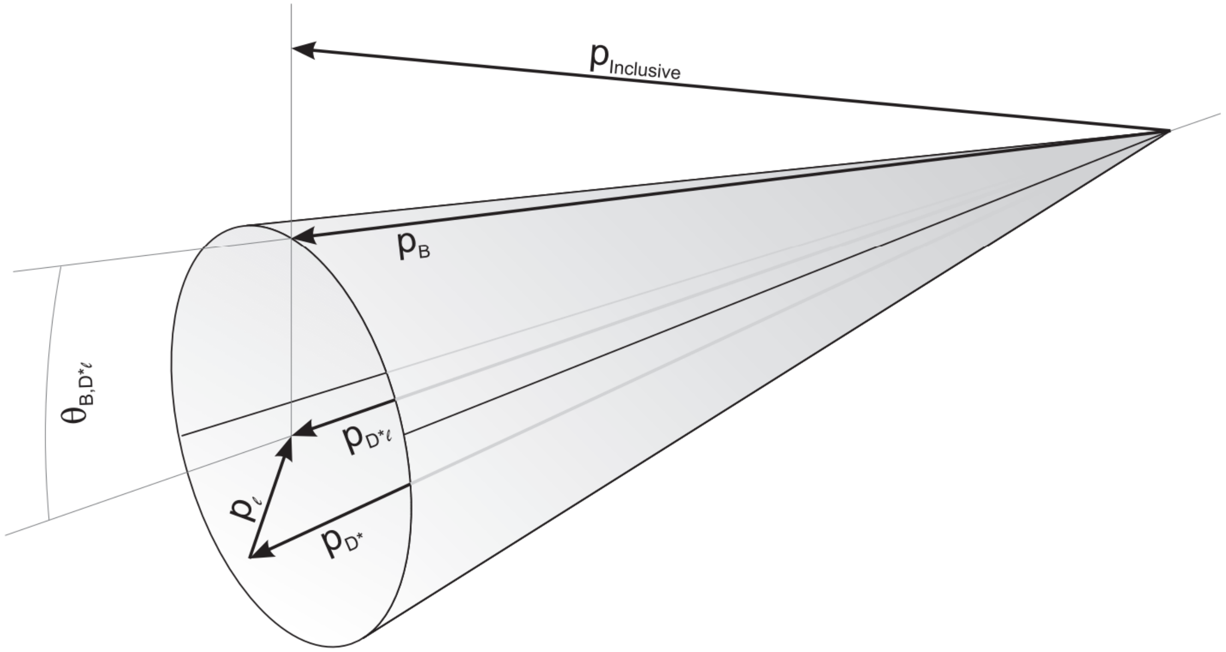


$$w \sim q^2 = (p_B - p_{D^*})^2$$

Improved Reconstruction Methods

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:

$$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}$$



Derivation :

$$0 = p_\nu^2 = \big(p_B - \underbrace{p_{D^*\ell}}_{p_{D^*}+p_\ell}\big)^2 \quad = p_B^2 + p_{D^*\ell}^2 - 2p_B p_{D^*\ell} = m_B^2 + m_{D^*\ell}^2 - 2E_B E_{D^*\ell} + 2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|\cos\theta_{B-D^*\ell}$$

$$\boxed{\rightarrow \quad \cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}}$$

Missing particles :

$$(p_\nu + p_{\text{miss}})^2 = m_B^2 + m_{D^*\ell}^2 - 2E_B E_{D^*\ell} + 2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|\cos\theta_{B-D^*\ell} \quad \rightarrow \quad \cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|} \boxed{+ \frac{\big(p_\nu + p_{\text{miss}}\big)^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}}$$

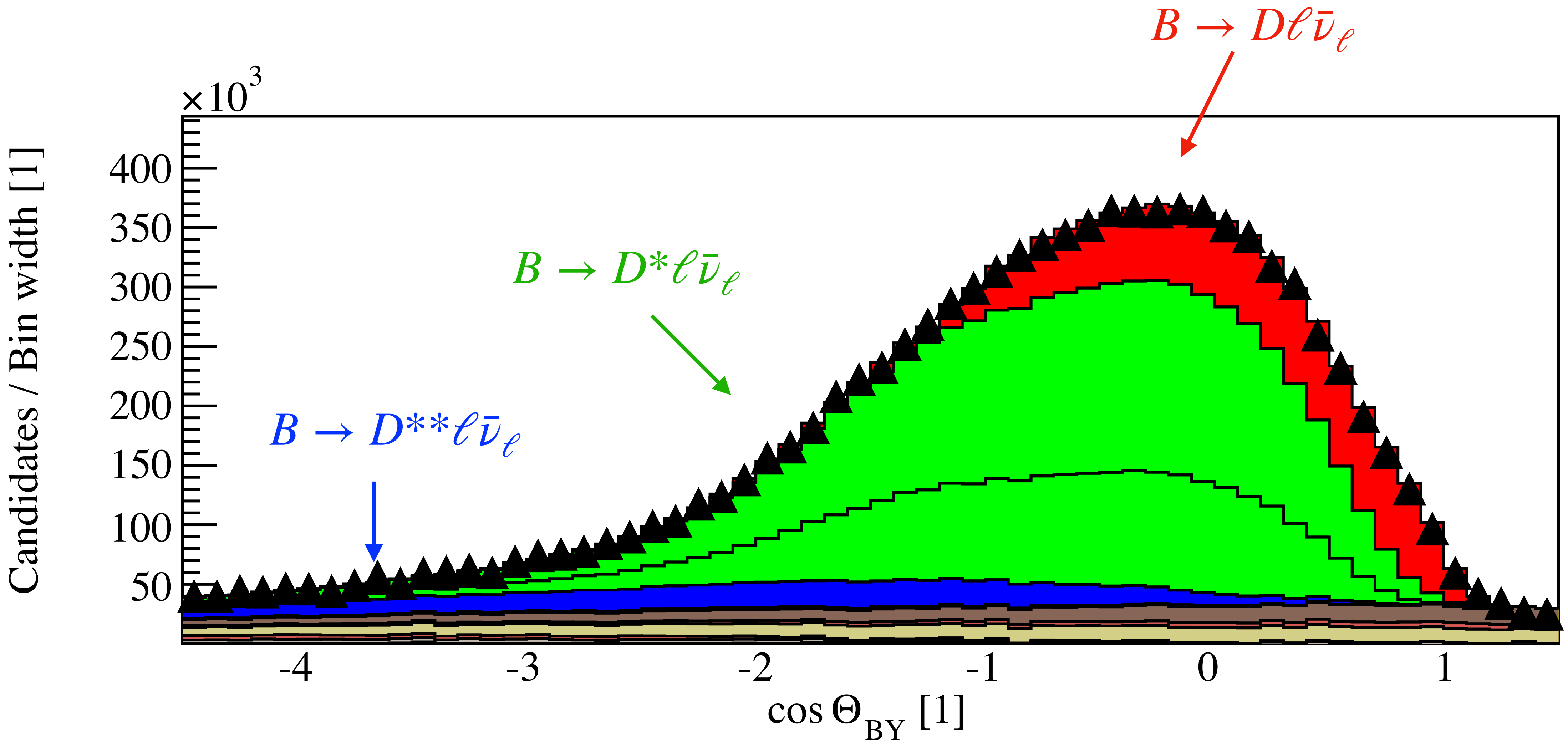
→ shifts $\cos\theta_{B,D^*\ell}$ to **negative** values if not included

Example: reconstruct $B \to D\ell\bar{\nu}_\ell$ (and allow for missing particles, i.e. untagged)

$$\downarrow$$

$$B \to DX\ell\bar{\nu}_\ell$$

D^0

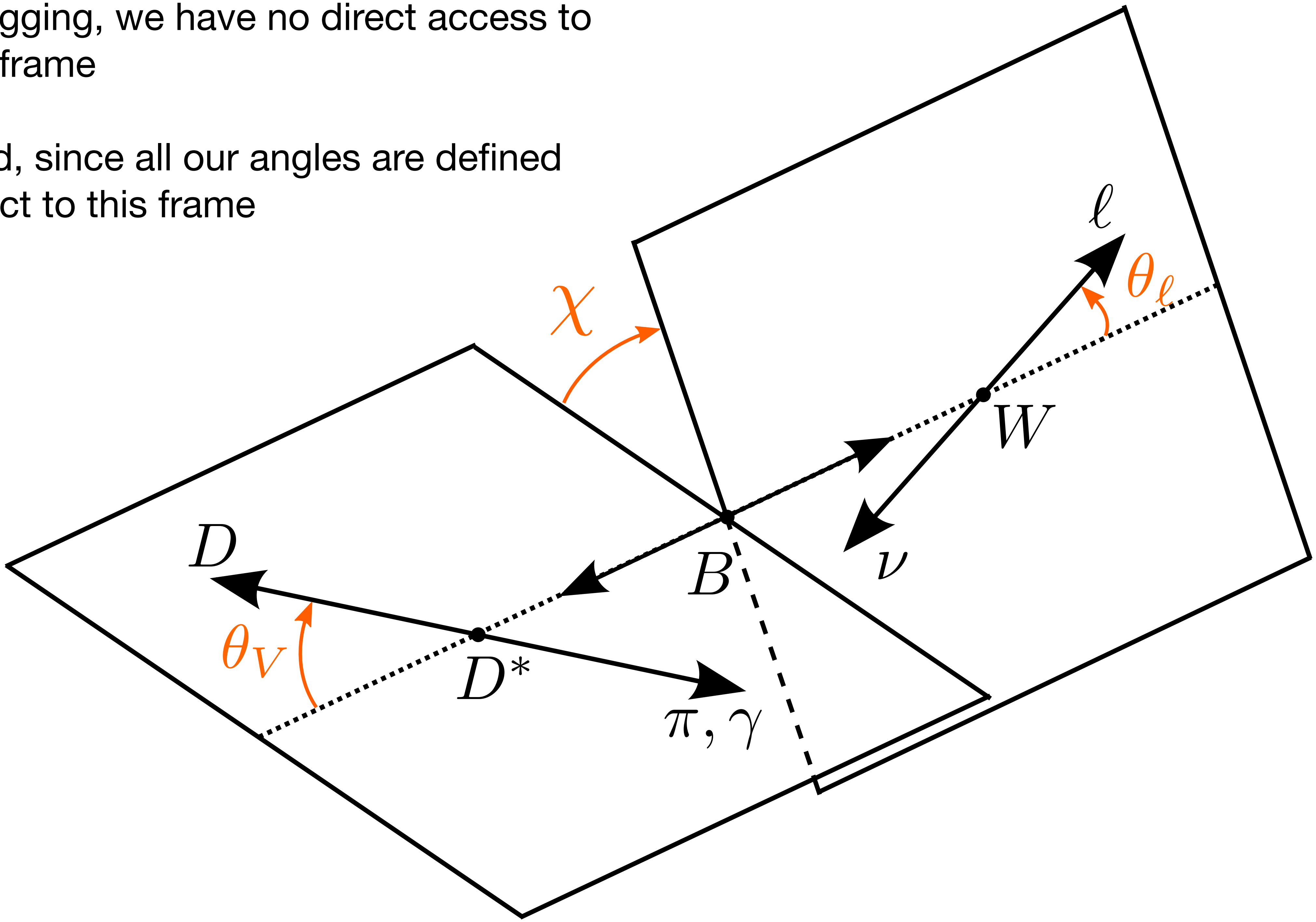


Good discriminating variable, so we will get back to using it.

Estimating the B Frame

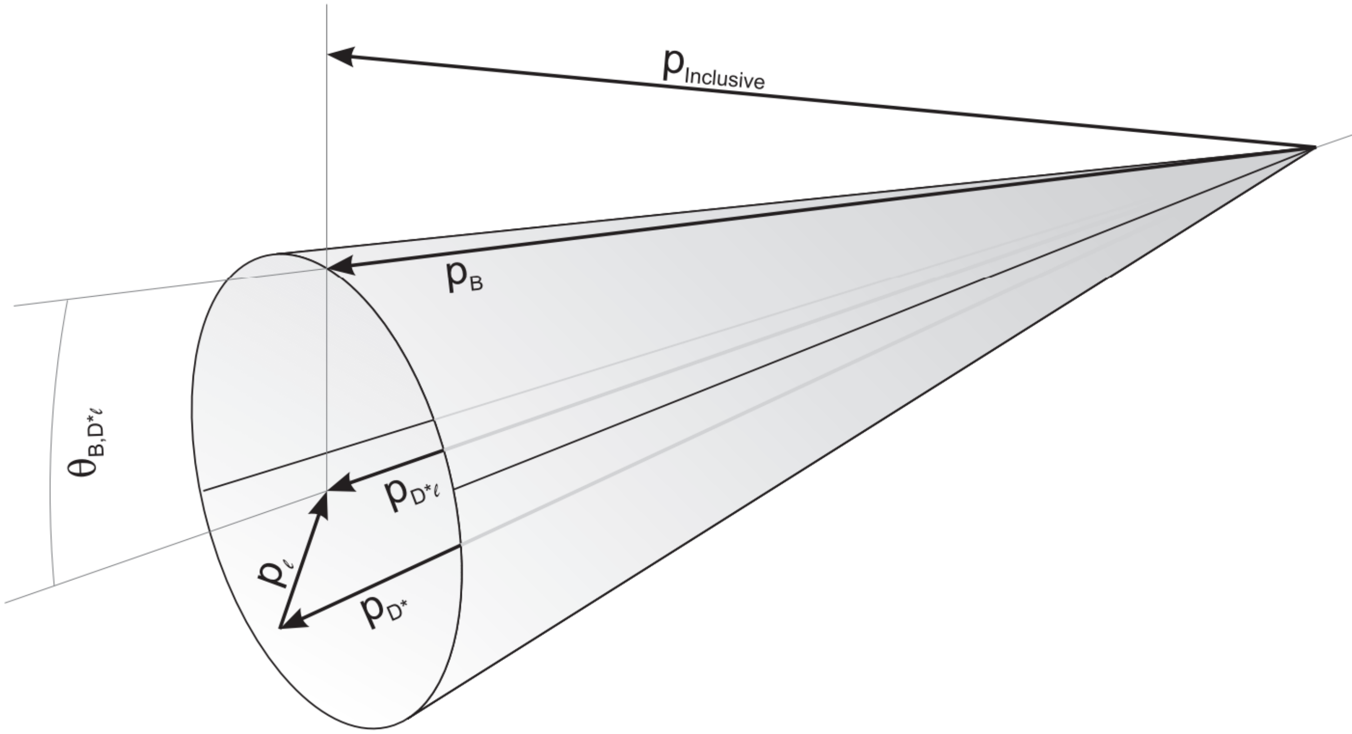
Without tagging, we have no direct access to the B rest frame

That is bad, since all our angles are defined with respect to this frame



Estimating the B Frame

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:



$$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2\,|\mathbf{p}_B|\,|\mathbf{p}_{D^*\ell}|}$$

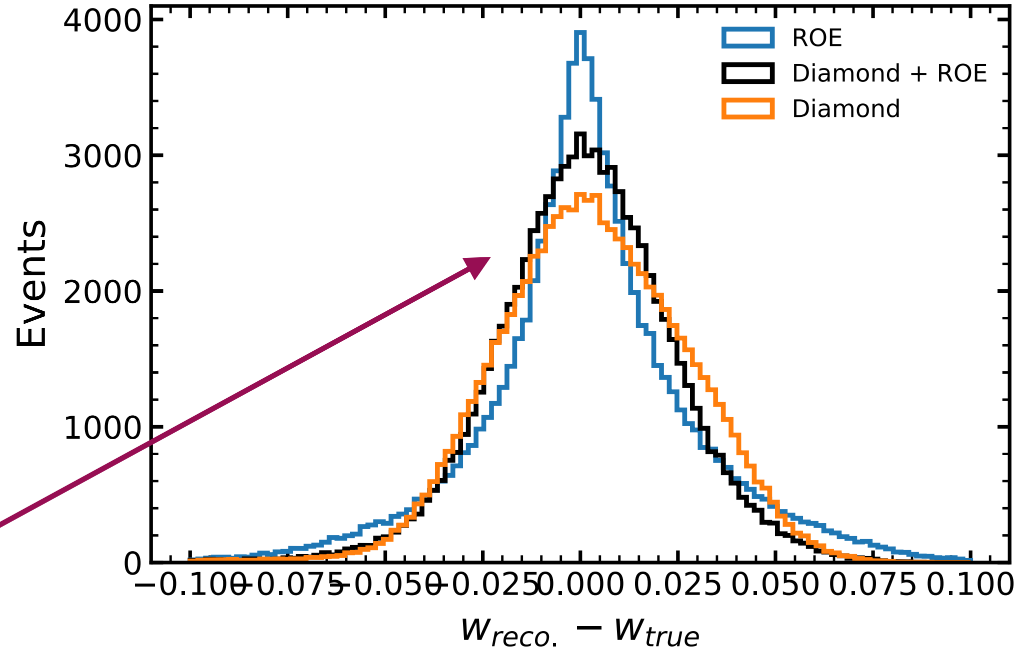
Can use this to estimate B meson direction building a weighted average on the cone

$$(E^B, p_B^x, p_B^y, p_B^z) = (\sqrt{s}/2, |\mathbf{p}_B| \sin\theta_{BY}\cos\phi, |\mathbf{p}_B| \sin\theta_{BY}\sin\phi, |\mathbf{p}_B|\cos\theta_{BY})$$

with weights according to $w_i = \sin^2\theta_i$ with θ denoting the polar angle

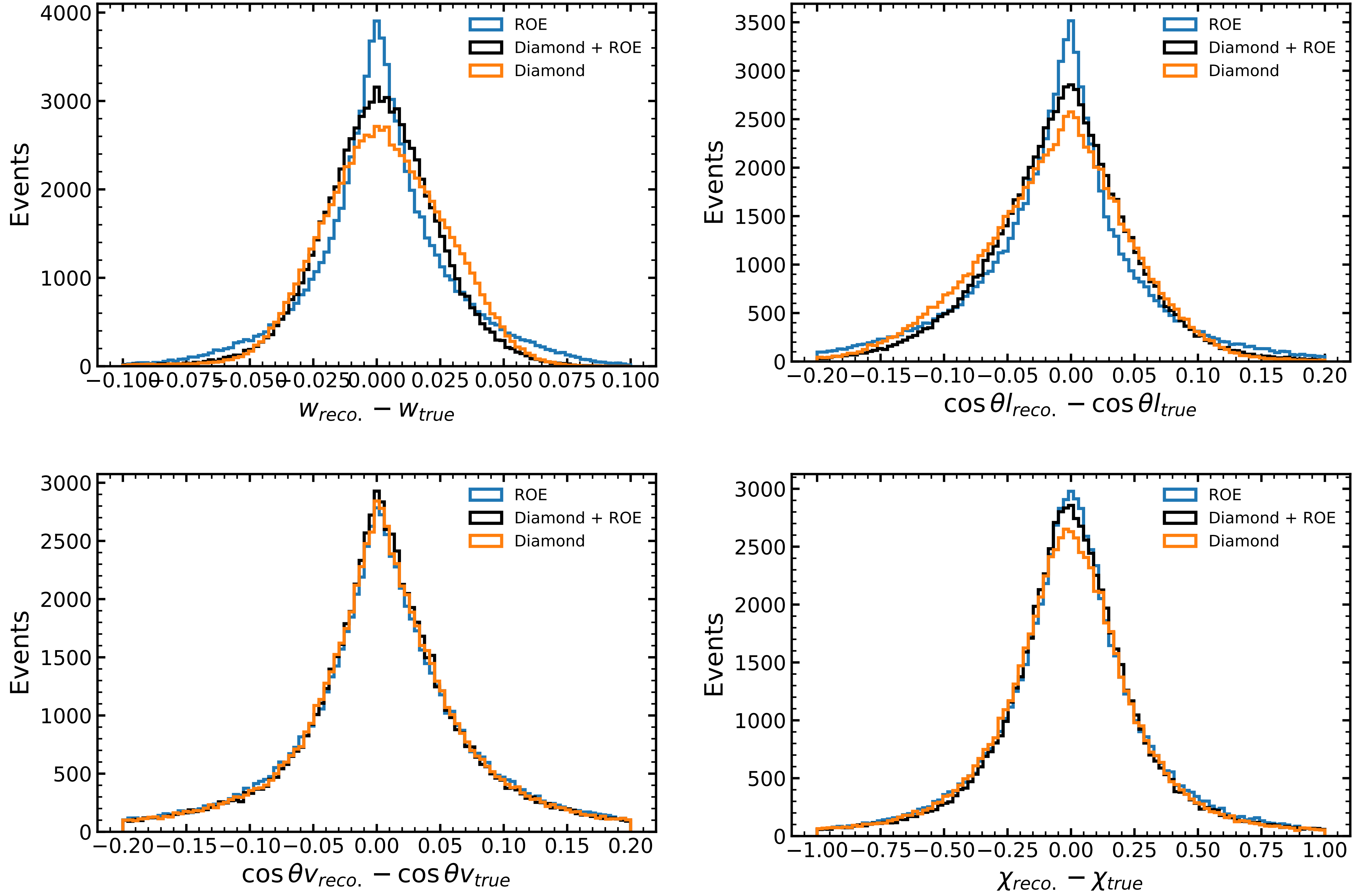
(following the angular distribution of $\Upsilon(4S) \to B\bar{B}$)

One can also **combine** both estimates



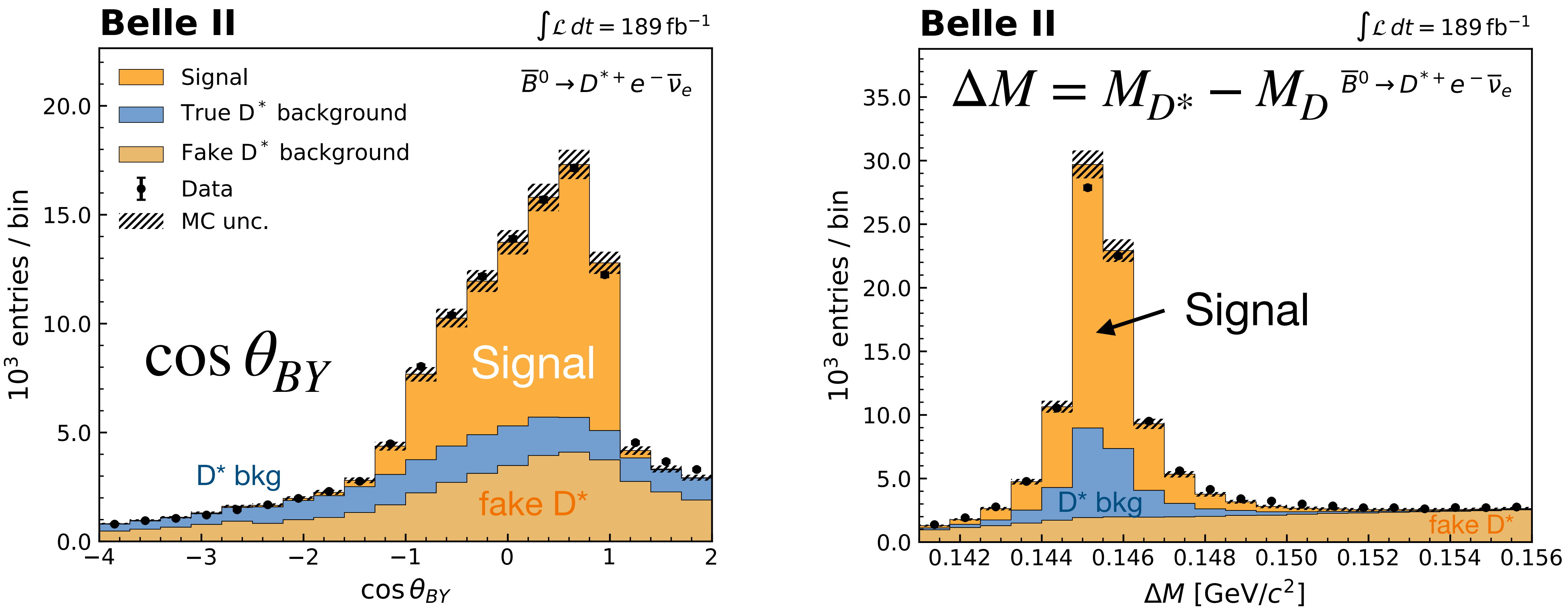
$$\tilde{w}_i = (1 - \widehat{p}_{\text{ROE}} \cdot \widehat{p}_{B_i}) \sin^2\theta_{B_i}$$

Estimating the B Frame



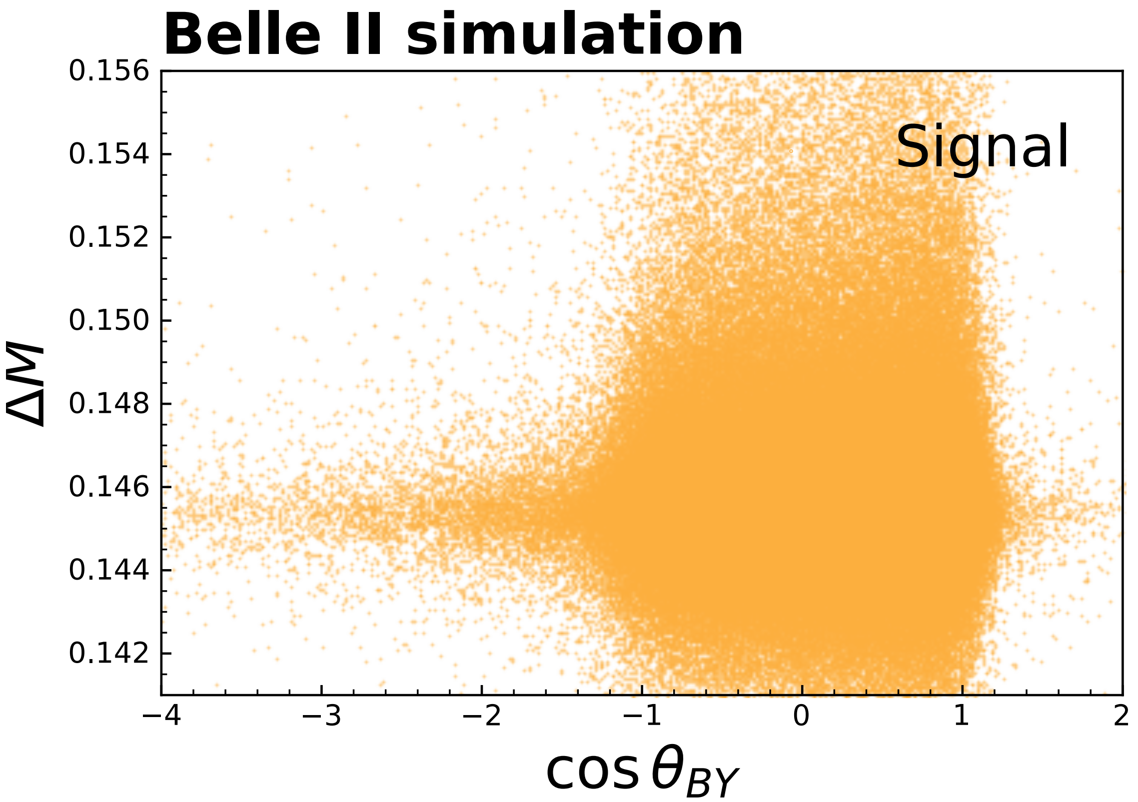
Background Subtraction

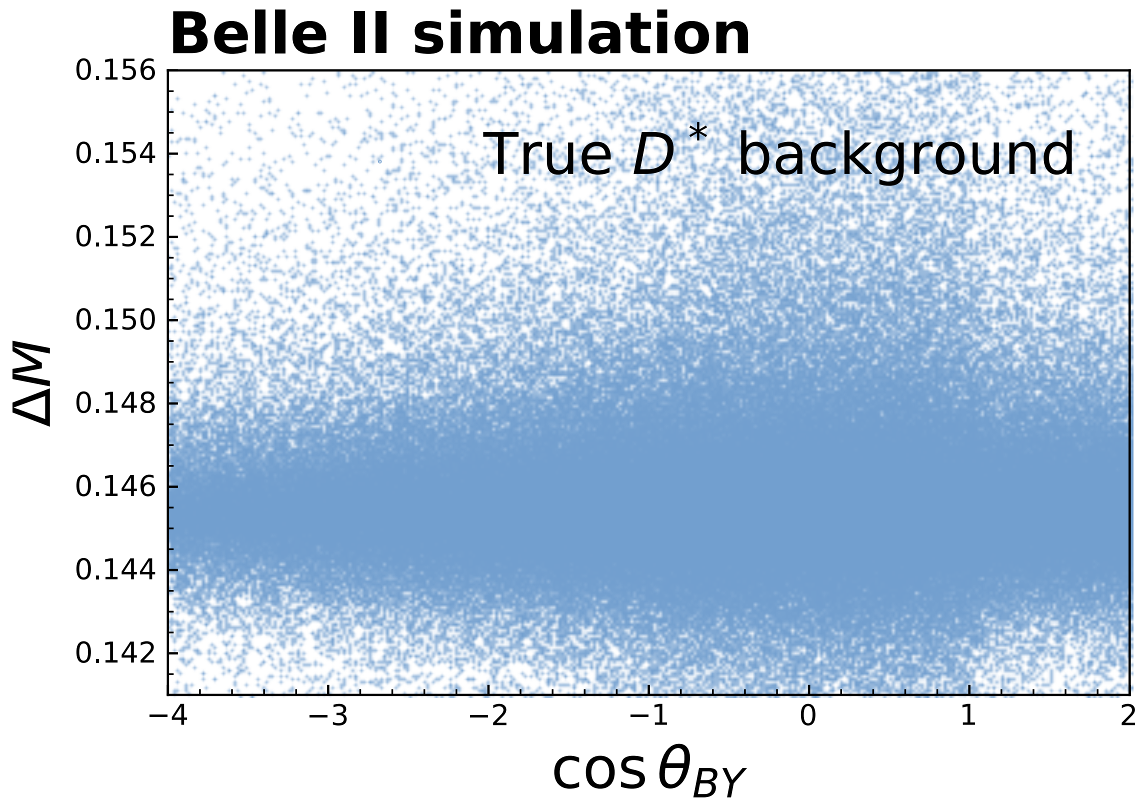
2D Fit of $$\cos\theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\vec{p}_B||\vec{p}_{D^*\ell}|}$$ $$\Delta M = m_{D*} - m_D$$

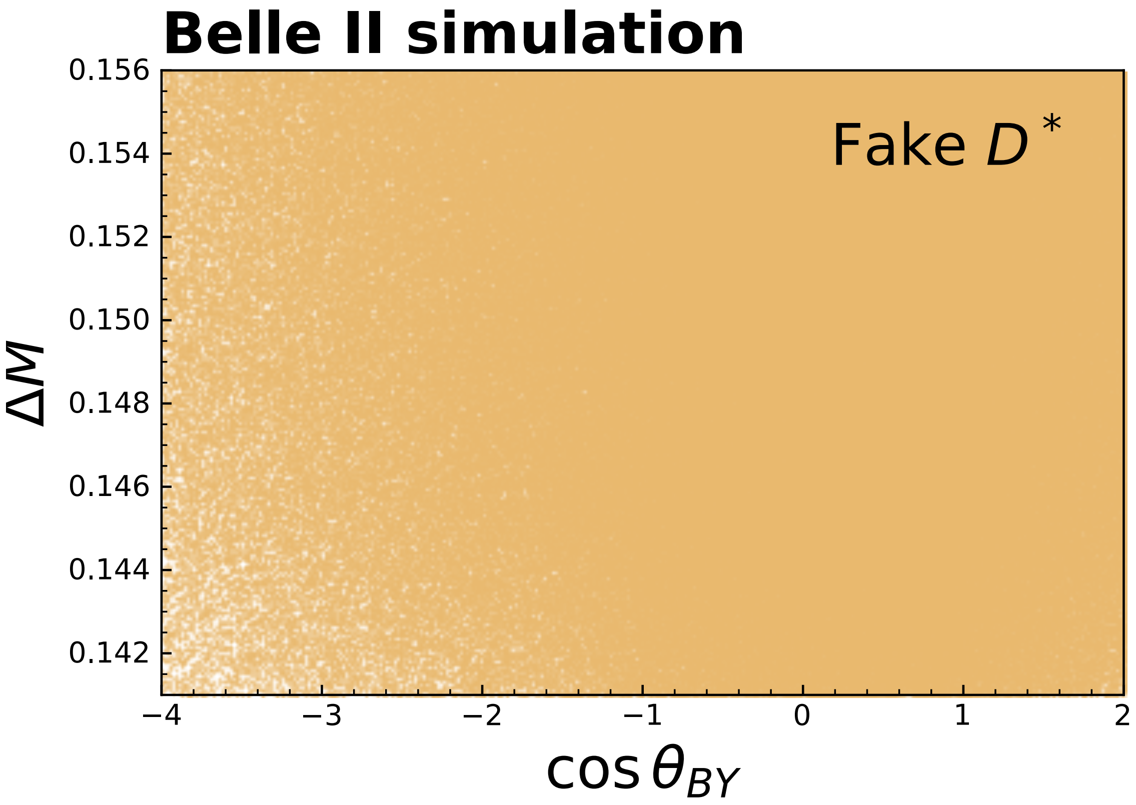


Fit **each bin** of the **kinematic variable**, unfold and correct for selection eff.

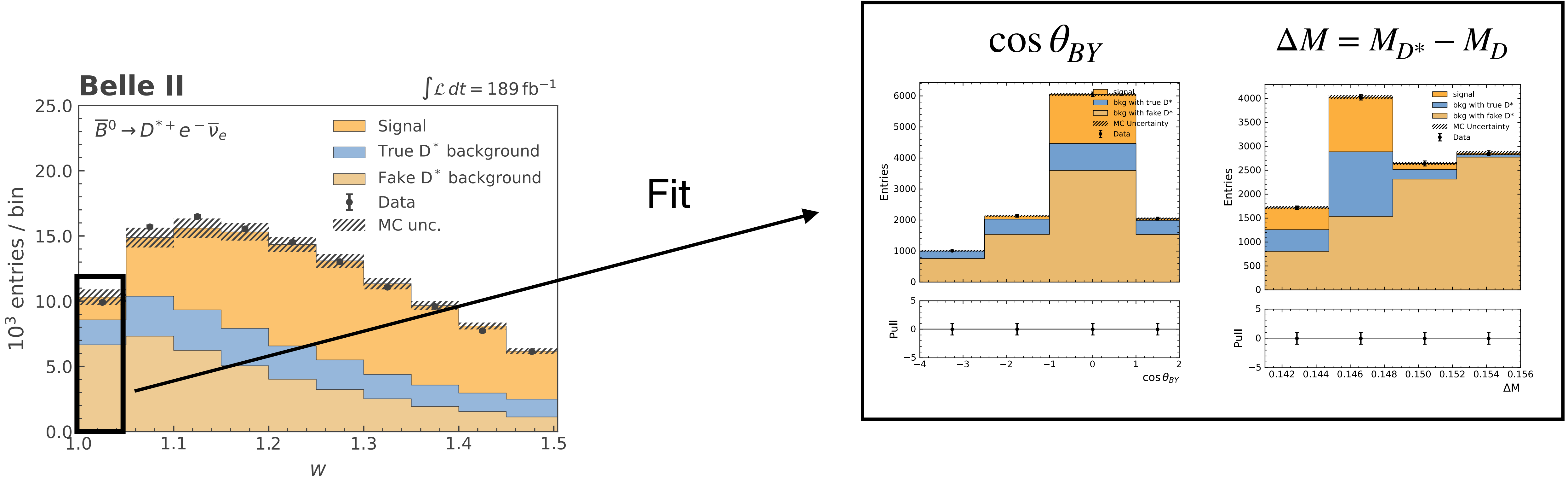
2D separation power :







Also focus initially on **1D projections**:

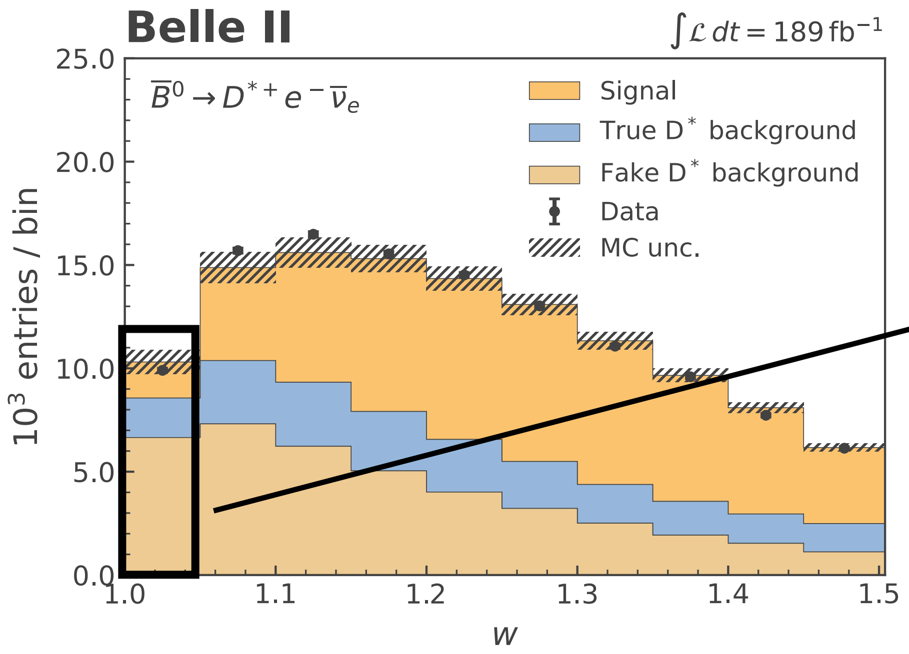


Fit

$\cos\theta_{BY}$

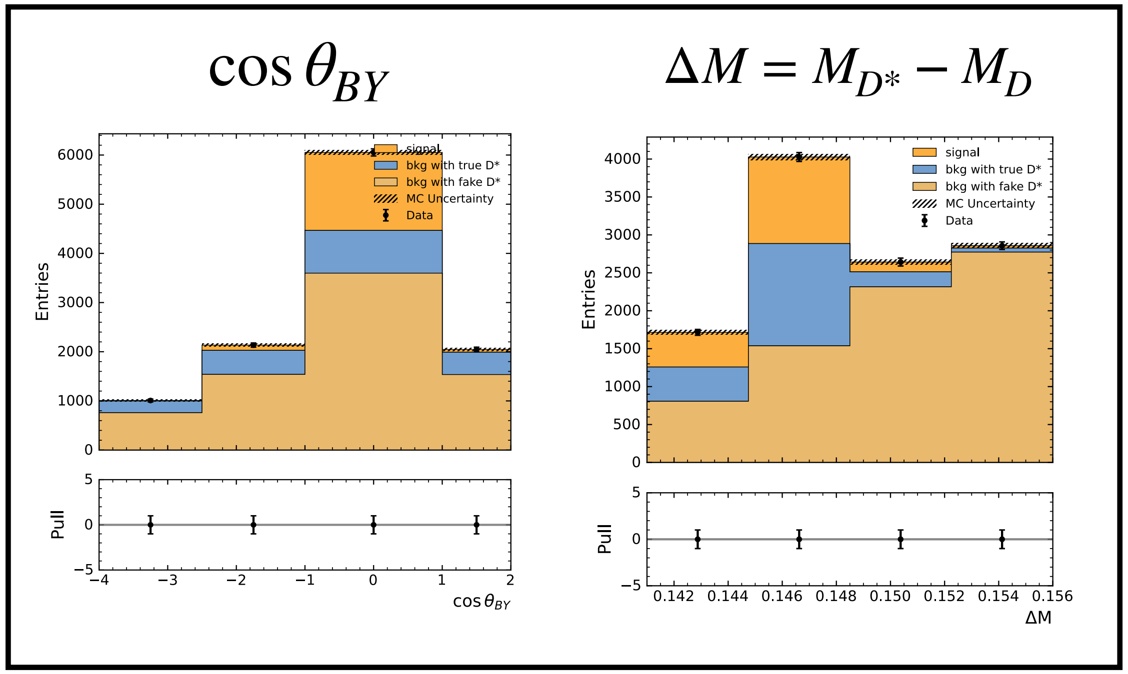
$\Delta M = M_{D*} - M_D$

Also focus initially on **1D projections**:

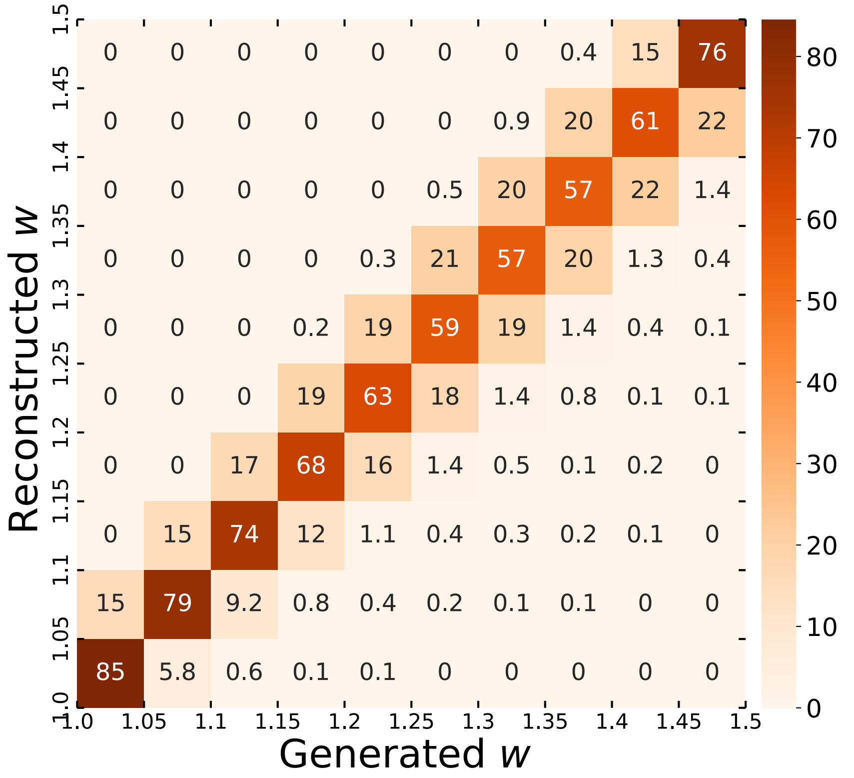


Fit

$\cos\theta_{BY}$ $\Delta M = M_{D*} - M_D$



Correct for migration effects:

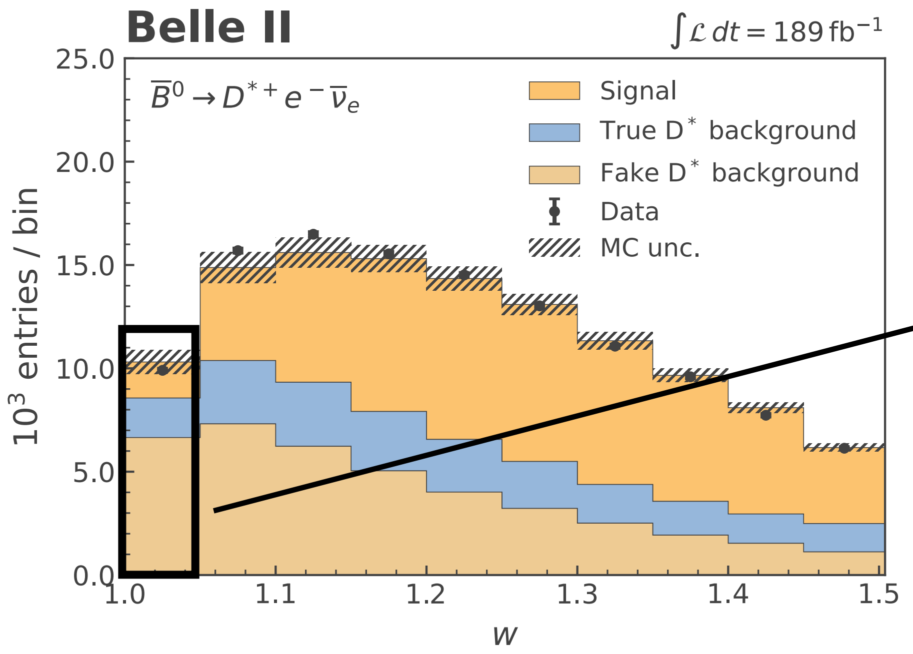


0	0	0	0	0	0	0	0.4	15	76
0	0	0	0	0	0	0.9	20	61	22
0	0	0	0	0	0.5	20	57	22	1.4
0	0	0	0	0.3	21	57	20	1.3	0.4
0	0	0	0.2	19	59	19	1.4	0.4	0.1
0	0	0	19	63	18	1.4	0.8	0.1	0.1
0	0	17	68	16	1.4	0.5	0.1	0.2	0
0	15	74	12	1.1	0.4	0.3	0.2	0.1	0
15	79	9.2	0.8	0.4	0.2	0.1	0.1	0	0
85	5.8	0.6	0.1	0.1	0	0	0	0	0

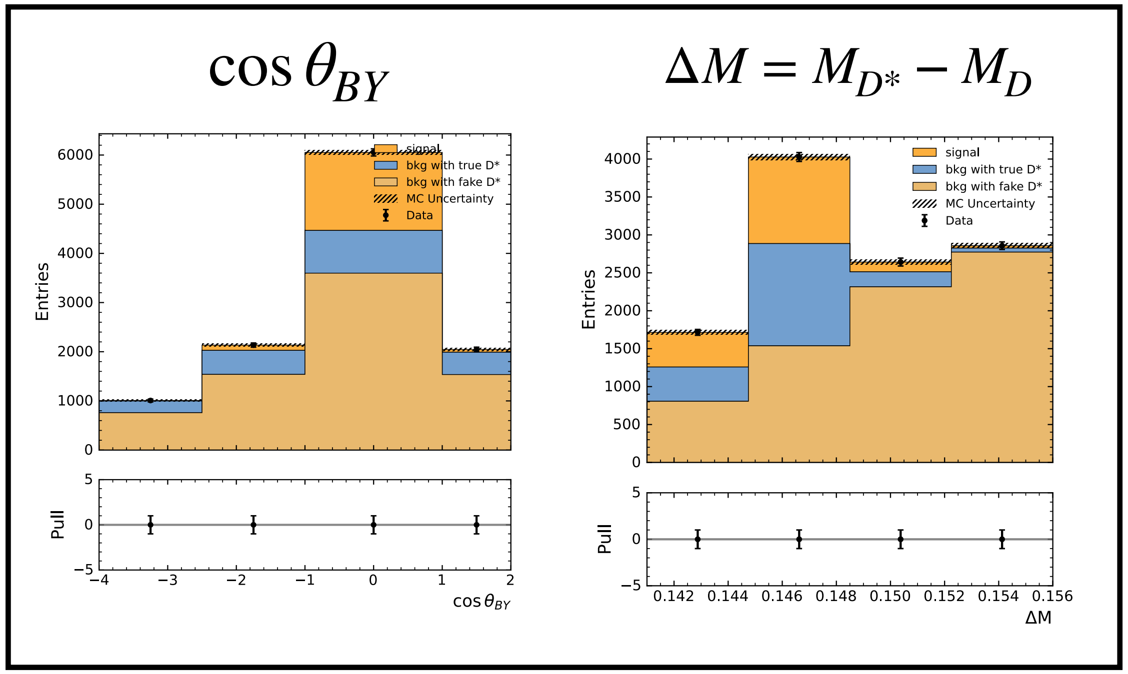
"Reco"

"True"

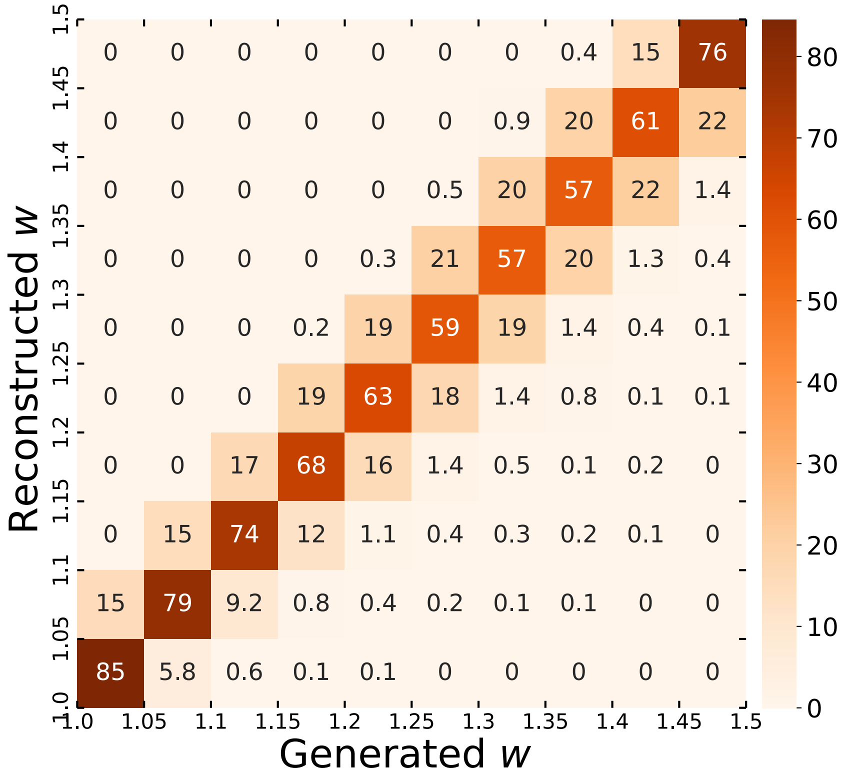
Also focus initially on **1D projections**:



Fit



Correct for migration effects:

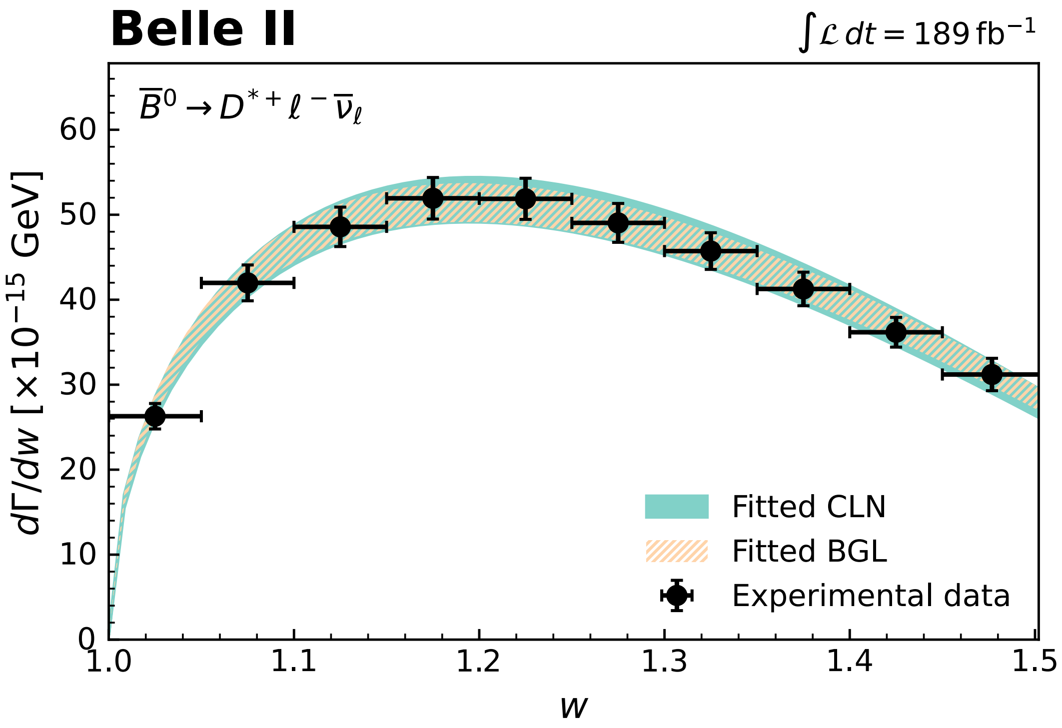


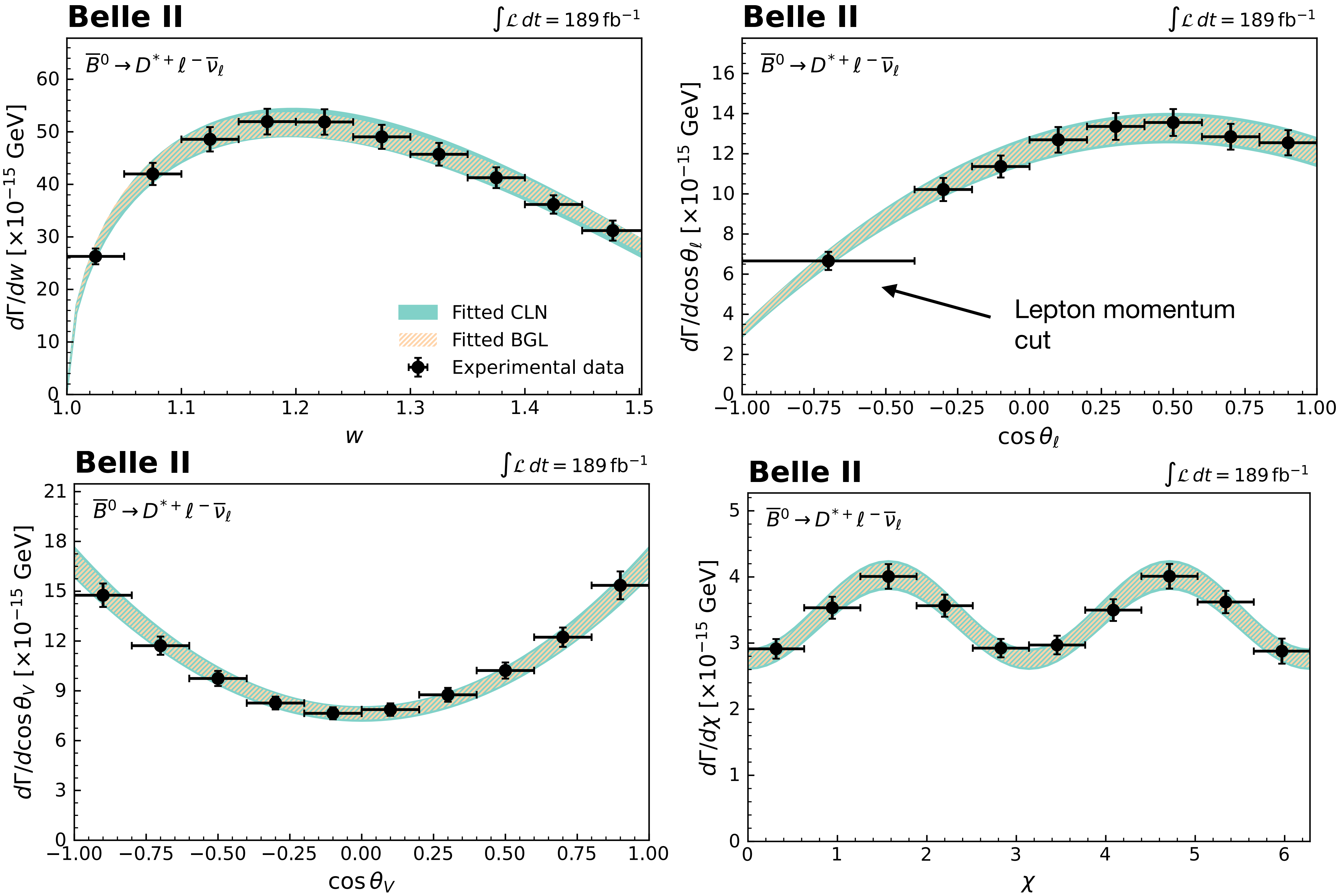
0	0	0	0	0	0	0	0.4	15	76
0	0	0	0	0	0	0.9	20	61	22
0	0	0	0	0	0.5	20	57	22	1.4
0	0	0	0	0.3	21	57	20	1.3	0.4
0	0	0	0.2	19	59	19	1.4	0.4	0.1
0	0	0	19	63	18	1.4	0.8	0.1	0.1
0	0	17	68	16	1.4	0.5	0.1	0.2	0
0	15	74	12	1.1	0.4	0.3	0.2	0.1	0
15	79	9.2	0.8	0.4	0.2	0.1	0.1	0	0
85	5.8	0.6	0.1	0.1	0	0	0	0	0

“Reco”

“True”

Correct for acceptance & efficiency





Lepton momentum cut

$$|V_{cb}|_{\mathrm{CLN}} = (40.2 \pm 0.3 \pm 0.9 \pm 0.6) \times 10^{-3}\,,$$

$$|V_{cb}|_{\mathrm{BGL}} = (40.6 \pm 0.3 \pm 1.0 \pm 0.6) \times 10^{-3}.$$

BGL truncation order determined using **Nested Hypothesis Test**

(n_a, n_b, n_c)	$\lvert V_{cb}\rvert \times 10^3$	$\rho_{\max}$	χ^2	Ndf	p-value
$(1,1,2)$	40.2 ± 1.1	0.28	40.5	32	14%
$(2,1,2)$	40.1 ± 1.1	0.97	38.6	31	16%
$\mathbf{(1,2,2)}$	$\mathbf{40.6 \pm 1.2}$	**0.57**	**39.1**	**31**	**15%**
$(1,1,3)$	40.1 ± 1.1	0.97	40	31	13%
$(2,2,2)$	40.2 ± 1.3	0.99	38.6	30	13%
$(1,3,2)$	39.8 ± 1.3	0.98	37.6	30	16%
$(1,2,3)$	40.5 ± 1.2	0.97	39	30	13%

Inclusive |Vub |

$\bar{B} \to X_u\, \ell\, \bar{\nu}_\ell$

+ Fermi Motion / Shape Function

Inclusive |Vcb |

$\bar{B} \to X_c\, \ell\, \bar{\nu}_\ell$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q\, \ell\, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive |Vub |

$\bar{B} \to \pi\, \ell\, \bar{\nu}_\ell, \Lambda_b \to p\, \mu\, \bar{\nu}_\mu$

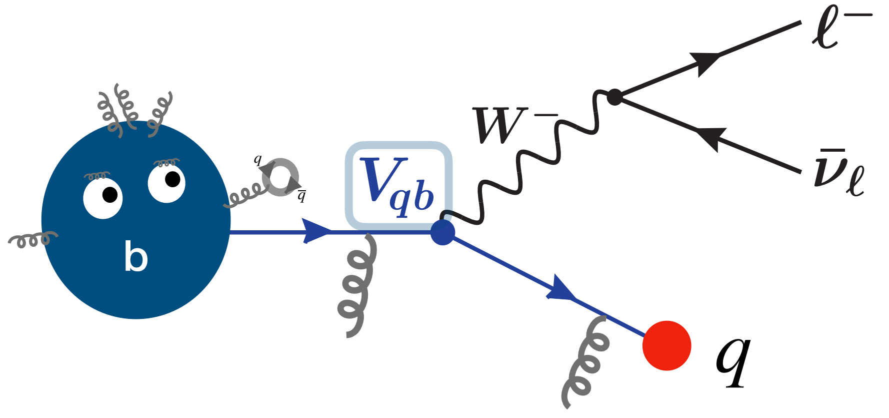
Exclusive |Vcb |

$\bar{B} \to D\, \ell\, \bar{\nu}_\ell, \bar{B} \to D^*\, \ell\, \bar{\nu}_\ell$

$$\mathcal{B} \propto |V_{cb}|^2\, f^2$$

Form Factors

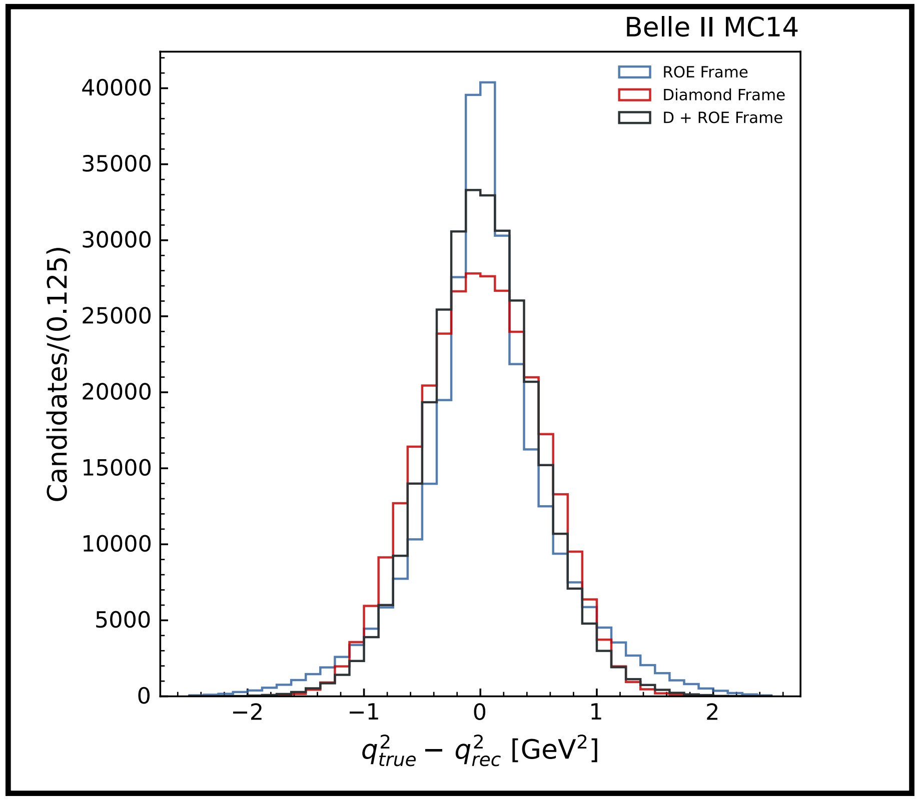
$$\langle B | H_\mu | P \rangle = (p + p')_\mu\, f_+$$



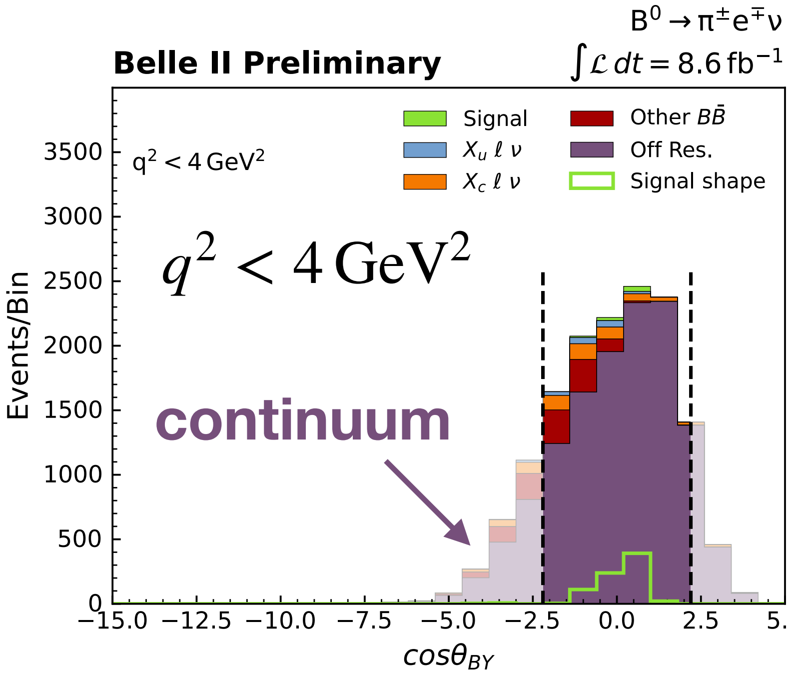
Exclusive measurements of $b \to u \ell \bar{\nu}_\ell$

Tagged strategy very similar, but **cross feed** from different modes (e.g. $B \to \rho \ell \bar{\nu}_\ell$) and **large** backgrounds from $B \to D^{(*)} \ell \bar{\nu}_\ell$ (+ other B decays) and **continuum**

Can reconstruct q^2 with the **same method** as for $B \to D^* \ell \bar{\nu}_\ell$ →

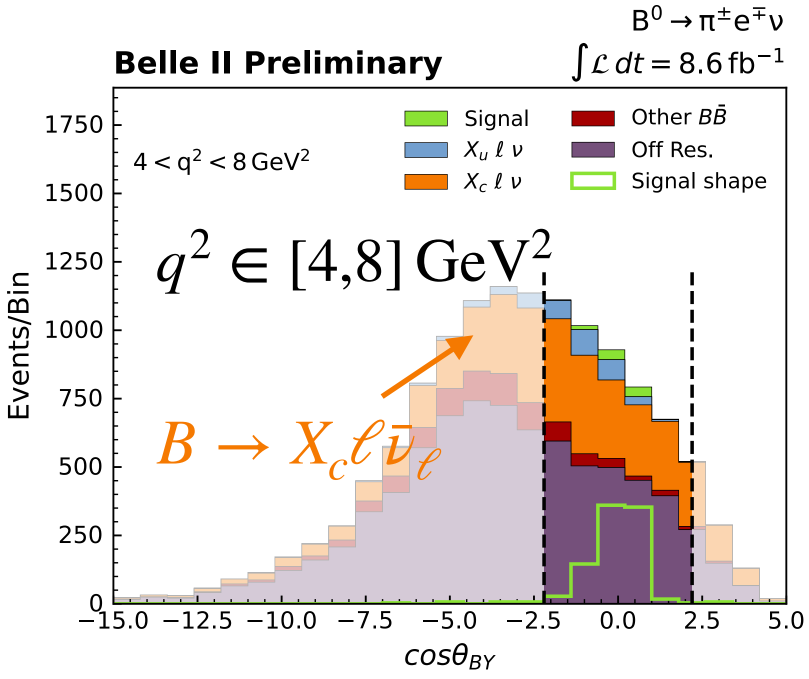


Amount of **background strongly changes** as a function of q^2



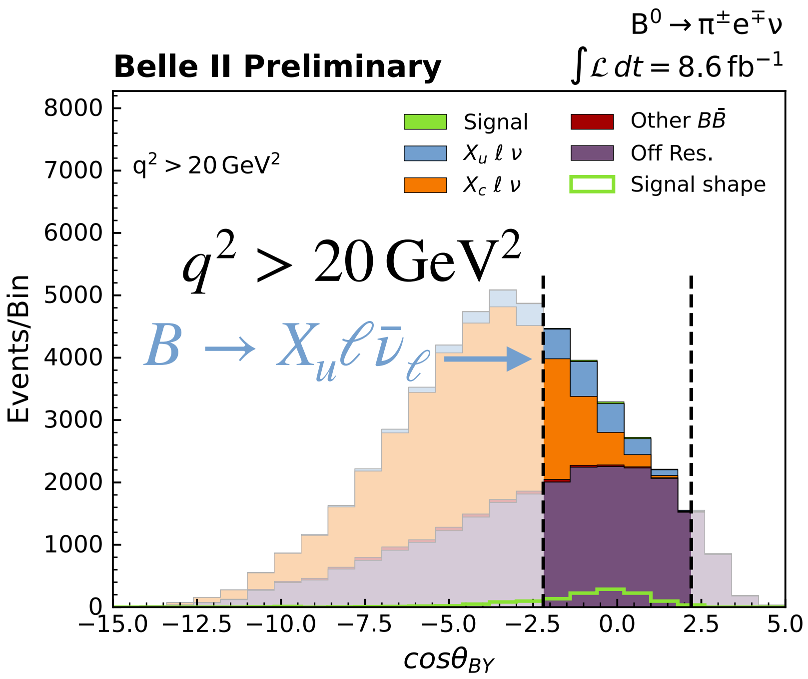
$q^2 < 4\,\mathrm{GeV}^2$

continuum



$q^2 \in [4,8]\,\mathrm{GeV}^2$

$B \to X_c \ell \bar{\nu}_\ell$



$q^2 > 20\,\mathrm{GeV}^2$

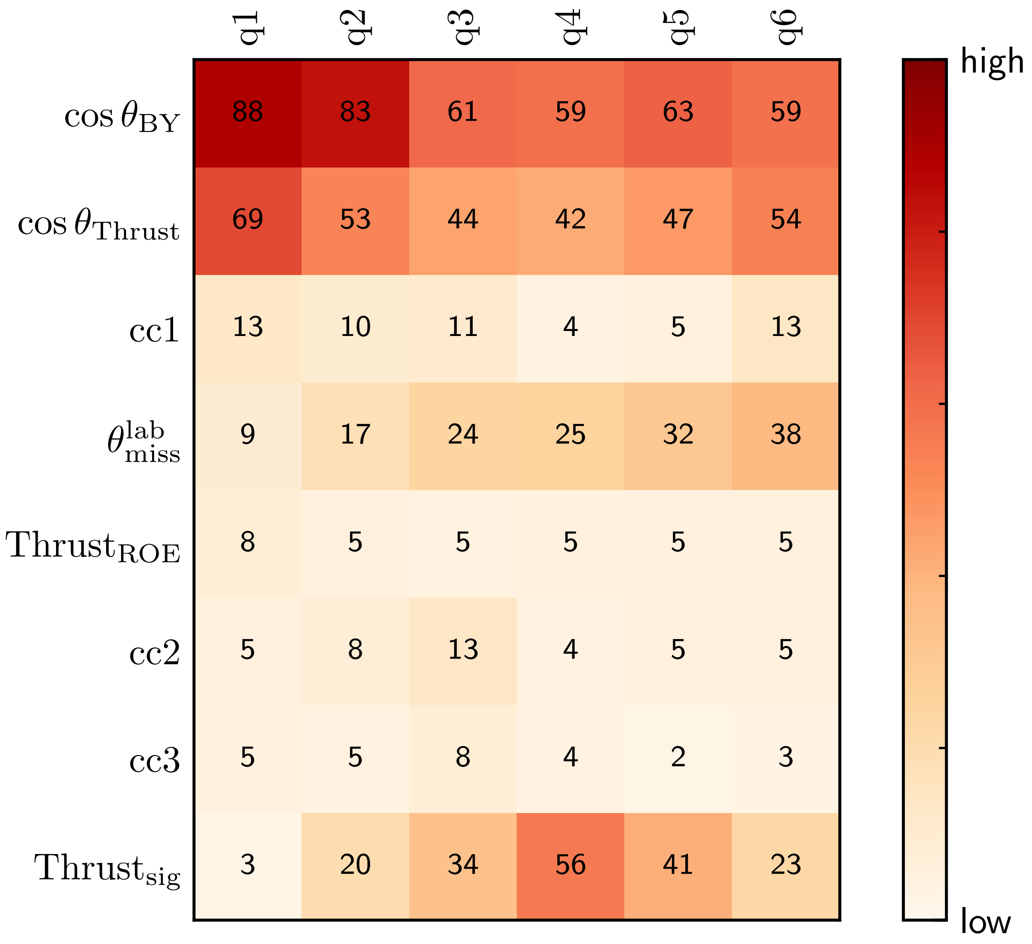
$B \to X_u \ell \bar{\nu}_\ell$

Exclusive measurements of $b \to u \ell \bar{\nu}_\ell$

Need **strong multivariate** suppression to carry out analysis :

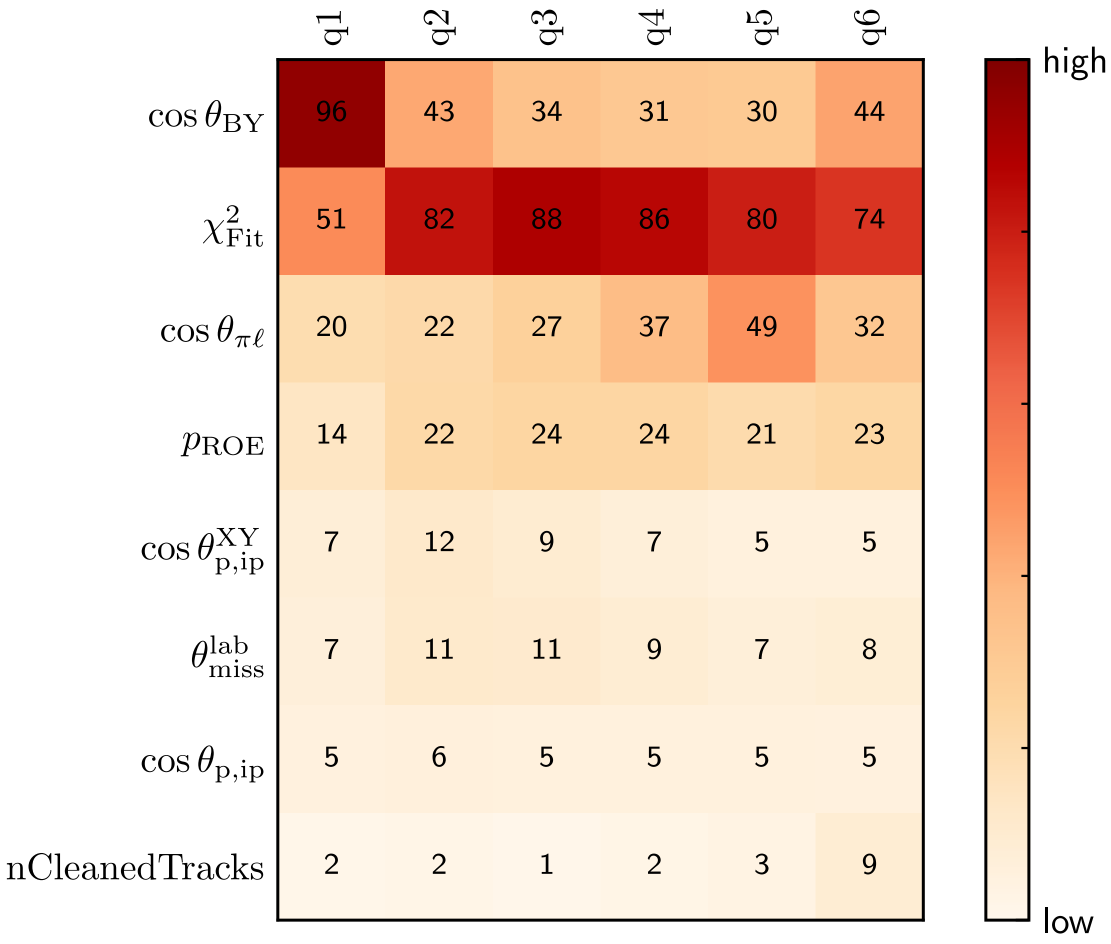
Due to different S/B and shapes, train separate one for each BDT bin

Continuum



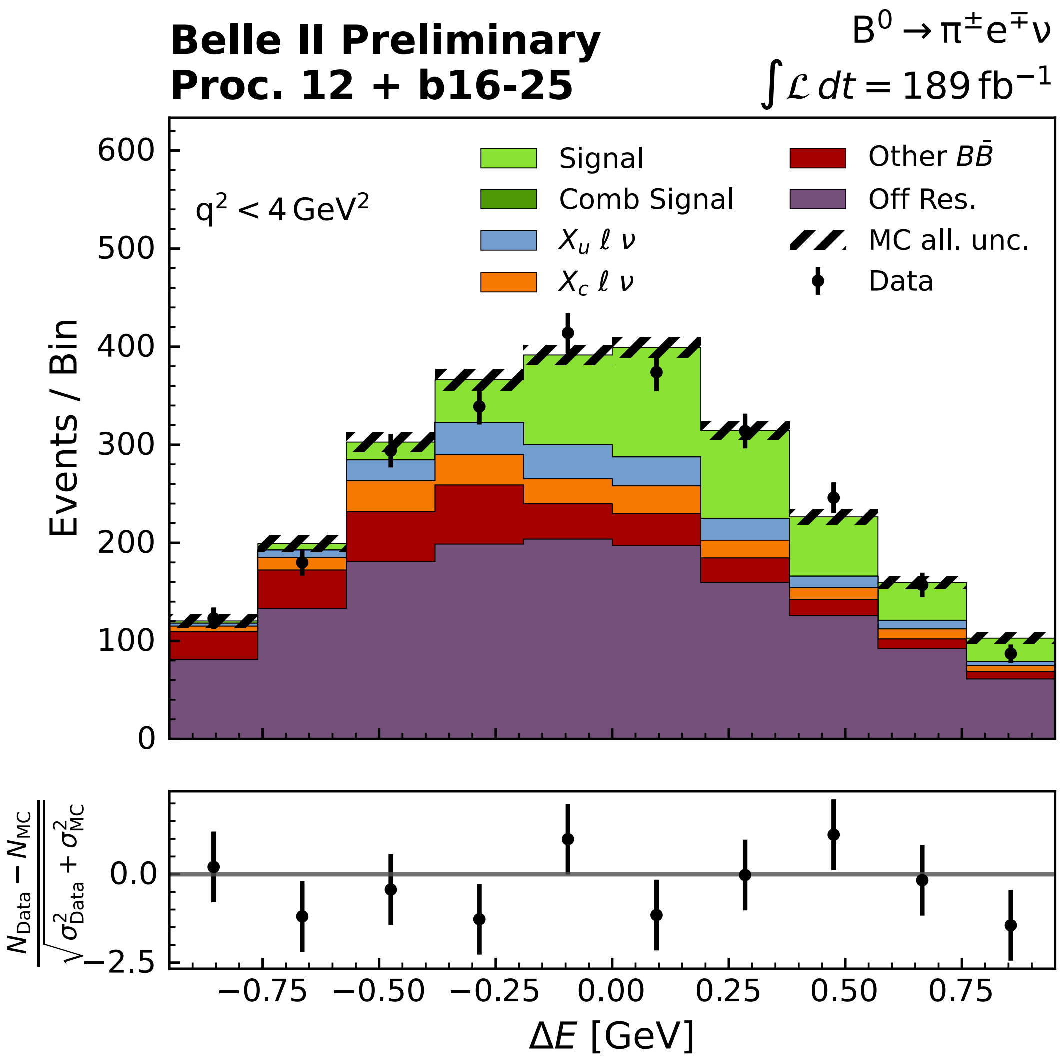
	q1	q2	q3	q4	q5	q6
$\cos\theta_{\mathrm{BY}}$	88	83	61	59	63	59
$\cos\theta_{\mathrm{Thrust}}$	69	53	44	42	47	54
cc1	13	10	11	4	5	13
$\theta^{\mathrm{lab}}_{\mathrm{miss}}$	9	17	24	25	32	38
$\mathrm{Thrust}_{\mathrm{ROE}}$	8	5	5	5	5	5
cc2	5	8	13	4	5	5
cc3	5	5	8	4	2	3
$\mathrm{Thrust}_{\mathrm{sig}}$	3	20	34	56	41	23

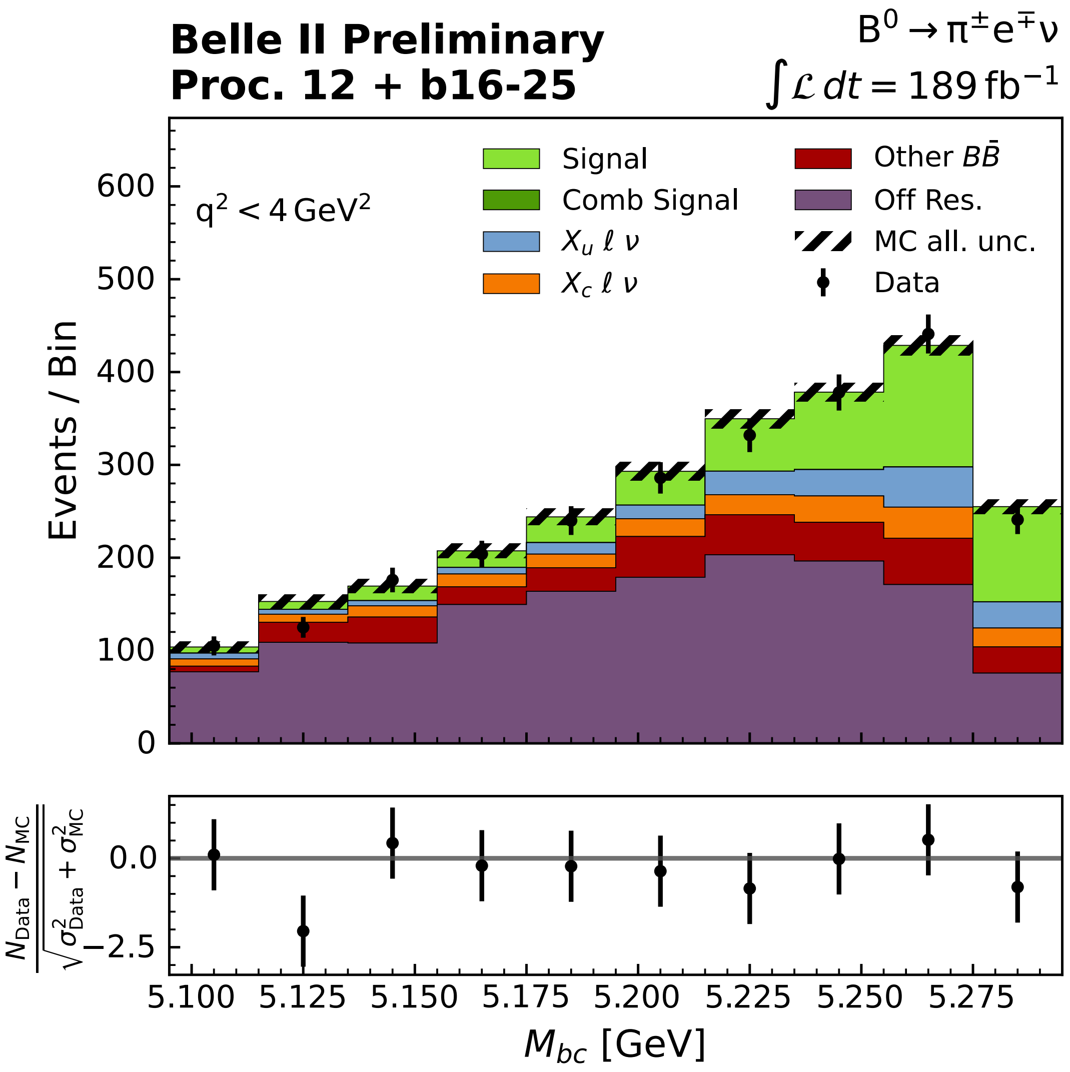
BB



	q1	q2	q3	q4	q5	q6
$\cos\theta_{\mathrm{BY}}$	96	43	34	31	30	44
χ^2_{Fit}	51	82	88	86	80	74
$\cos\theta_{\pi\ell}$	20	22	27	37	49	32
p_{ROE}	14	22	24	24	21	23
$\cos\theta^{\mathrm{XY}}_{\mathrm{p,ip}}$	7	12	9	7	5	5
$\theta^{\mathrm{lab}}_{\mathrm{miss}}$	7	11	11	9	7	8
$\cos\theta_{\mathrm{p,ip}}$	5	6	5	5	5	5
nCleanedTracks	2	2	1	2	3	9

After BDT selection :





$$\boxed{\Delta E = E_B^* - E_{\mathrm{beam}}^* = E_B^* - \sqrt{s}/2}$$

$$\boxed{M_{\mathrm{bc}} = \sqrt{E_{\mathrm{beam}}^{*\,2} - \mathbf{p}_\mathbf{B}^{*}} = \sqrt{s/4 - \mathbf{p}_\mathbf{B}^{*}}}$$

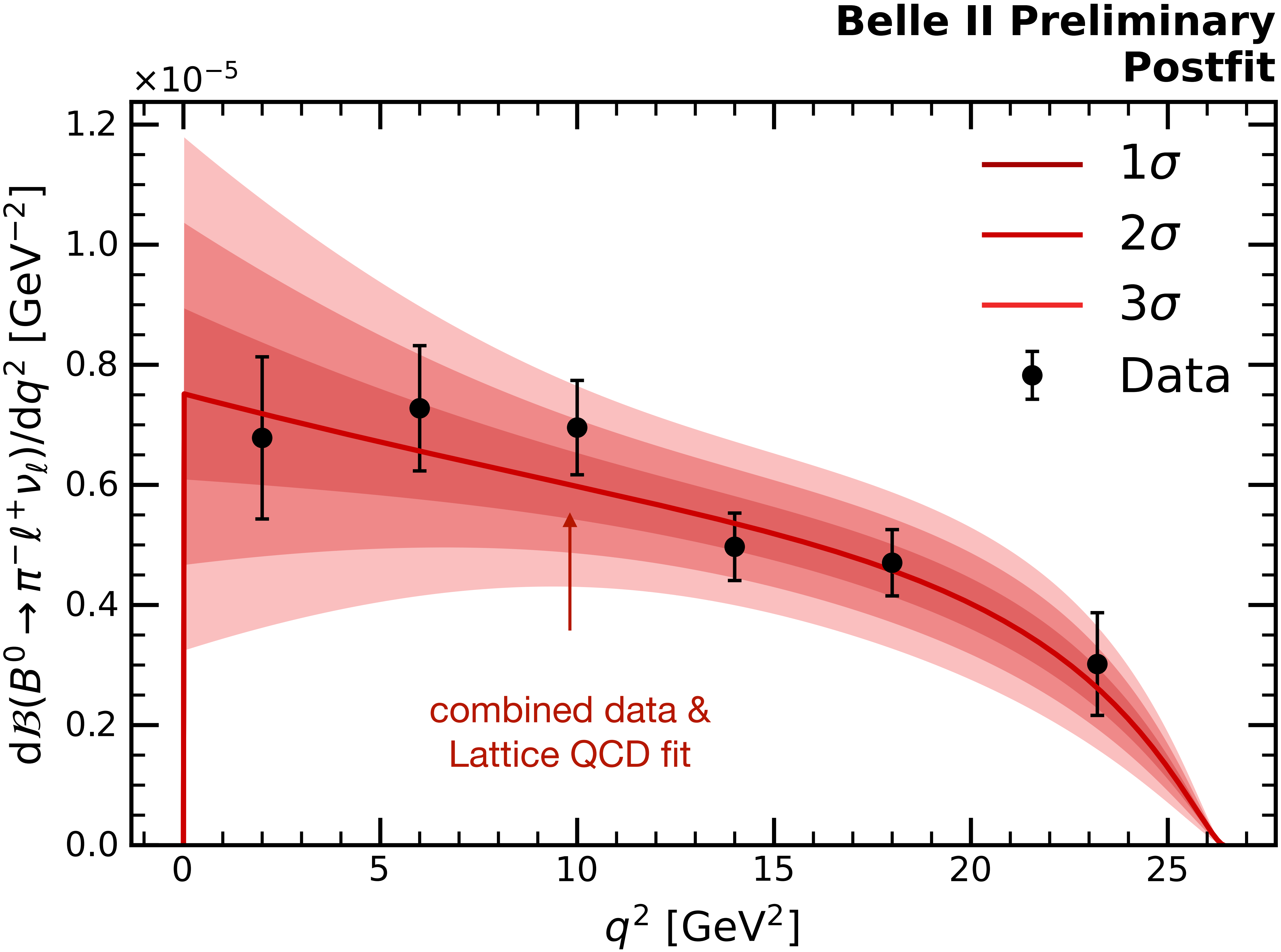
$$E_B^* = E_\pi + E_\ell + E_{\mathrm{miss}}$$

$$\mathbf{p}_\mathbf{B}^* = \mathbf{p}_\pi + \mathbf{p}_\ell + \mathbf{p}_{\mathrm{miss}}$$

Built from clusters
& tracks of ROE

$$p_{\mathrm{miss}} = (E_{\mathrm{miss}}, \mathbf{p}_{\mathrm{miss}}) = -\,p_{\mathrm{ROE}}$$

Final Spectrum :



Inclusive |V$_{ub}$ |

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive |V$_{cb}$ |

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive |V$_{ub}$ |

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

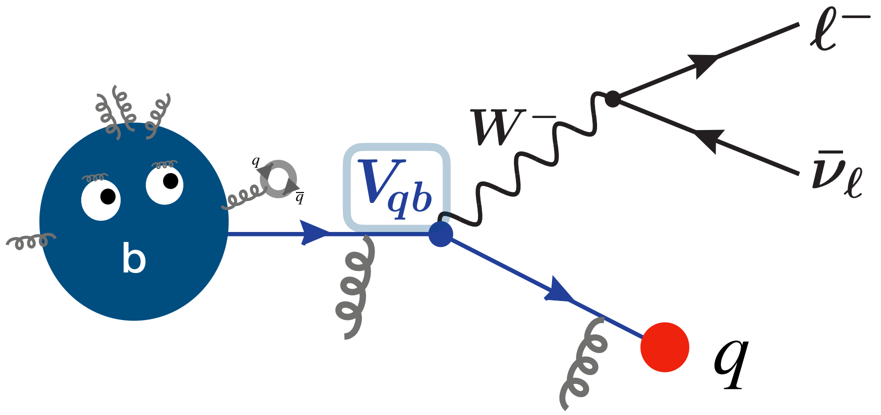
Exclusive |V$_{cb}$ |

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

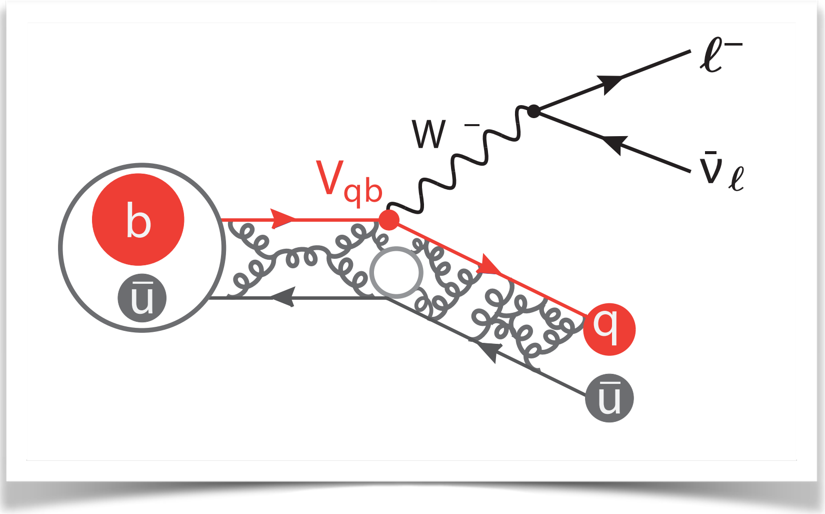
$$\mathcal{B} \propto |V_{cb}|^2 \, f^2$$

Form Factors

$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$



Overview $B \to X_c \ell \bar{\nu}_\ell$



Inclusive $|V_{cb}|$

$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$

Operator Product Expansion (OPE)

$$\boxed{\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]}$$

Established approach: Use **spectral moments** (hadronic mass moments, lepton energy moments etc.) to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \mathcal{O}(1/m_b^4)$$

$d\Gamma$ are calculated perturbatively

↳ Available at $\mathcal{O}(\alpha_s^3)$

Fael, Schönwald, Steinhauser
Phys. Rev. D 104, 016003 (2021)

$\mu_\pi, \mu_G, \rho_D, \rho_{LS}$ encapsulate non-perturbative dynamics

↳ HQE parameters must be extracted from data

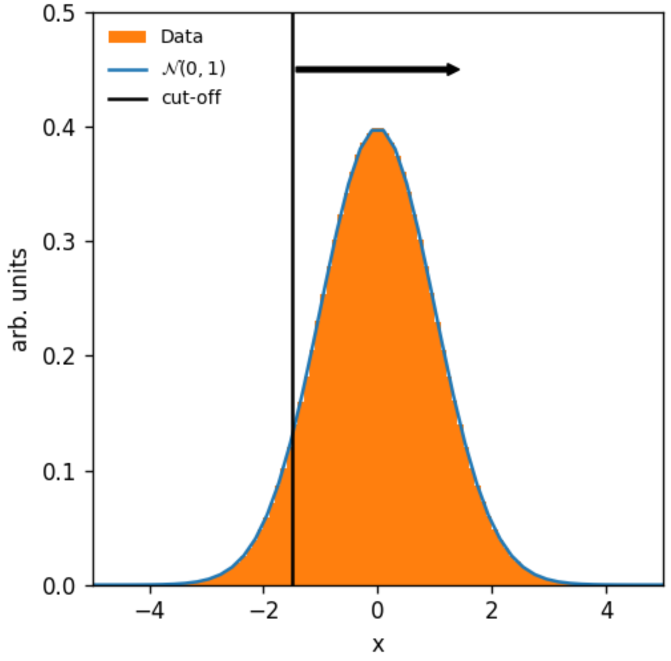
↳ requires the spectral moments of $B \to X_c \ell \nu$

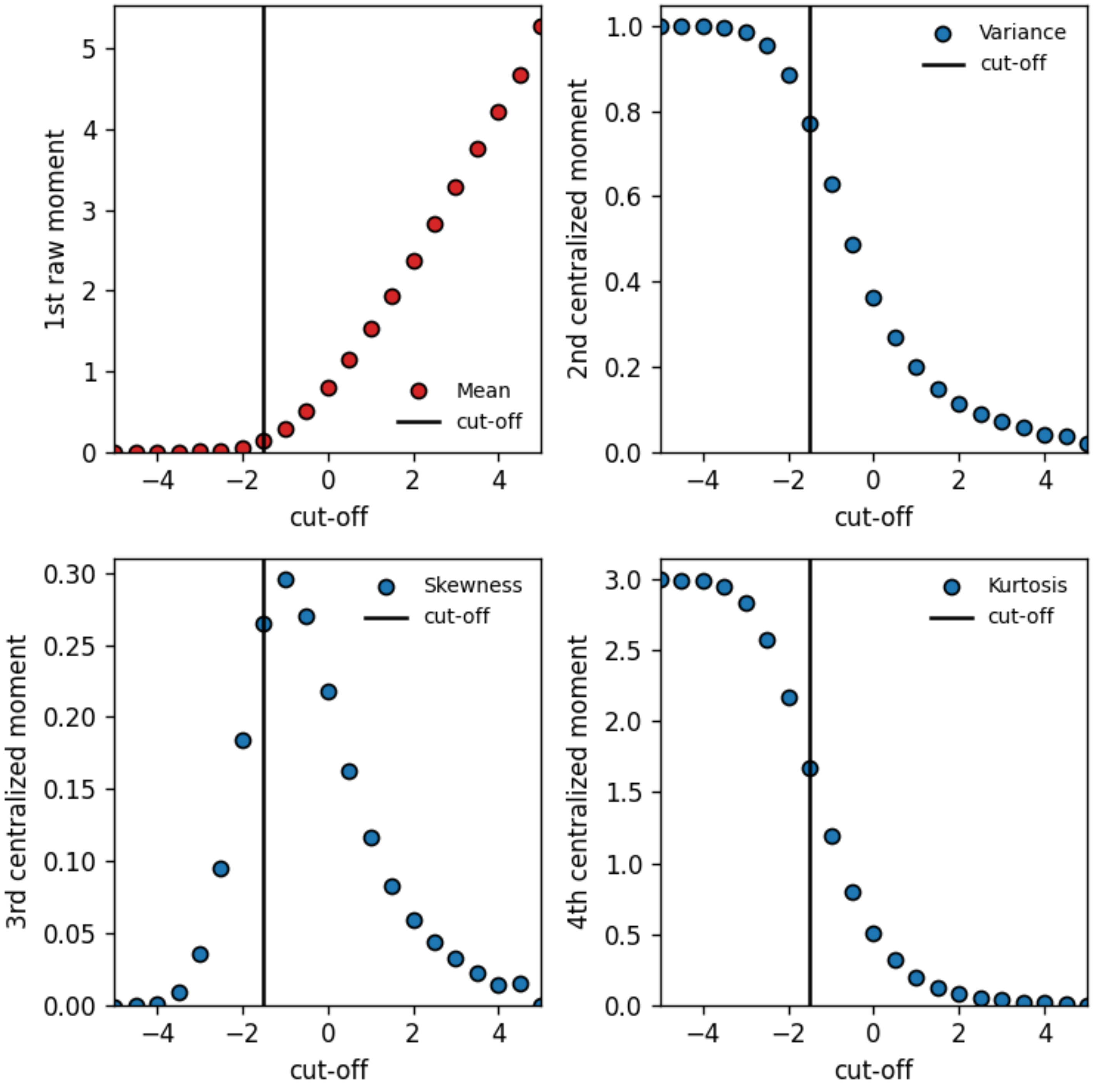
Challenge: Proliferation of HQE parameters at higher order

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Let’s take a moment or two

Illustrations by Markus Prim





$$\mu_n = \int_{-\infty}^{-\infty} (x-c)^n f(x) dx$$

Raw moment: $c = 0$

Central moment: $c = \text{Mean}$

First raw moment: Mean

Measures the location

Second central moment: Variance

Measures the spread

Third central moment: Skewness

Measures asymmetry

Fourth central moment: Kurtosis

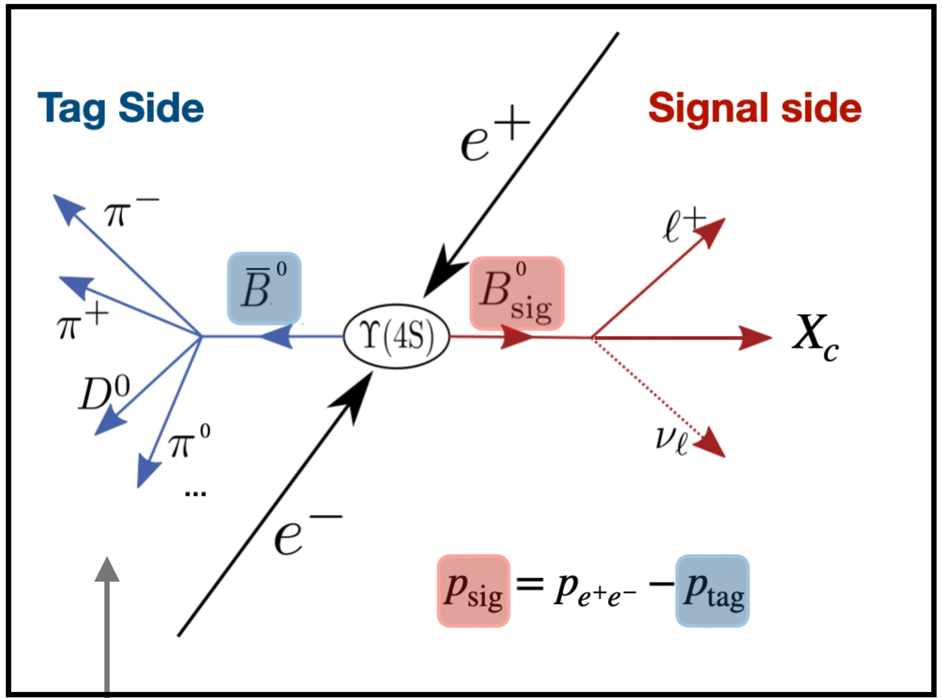
Measures “tailedness”

Moments are measured with progressive cuts in the distribution

→ **highly correlated measurements**

How to measure spectral moments

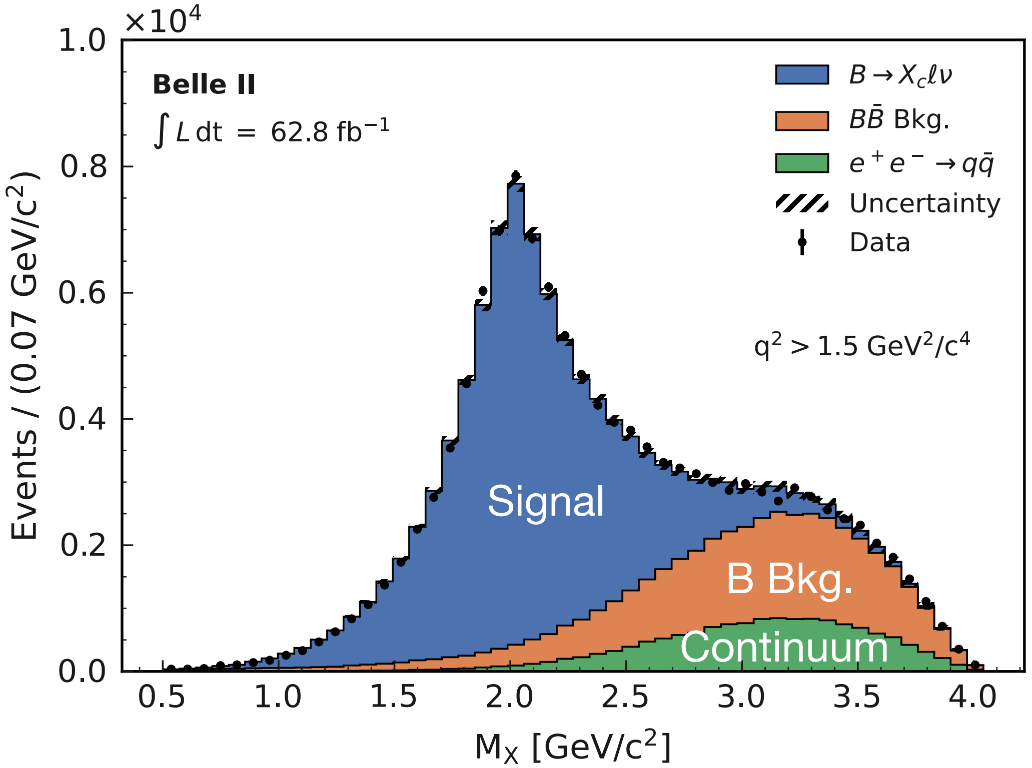
Key-technique: hadronic tagging

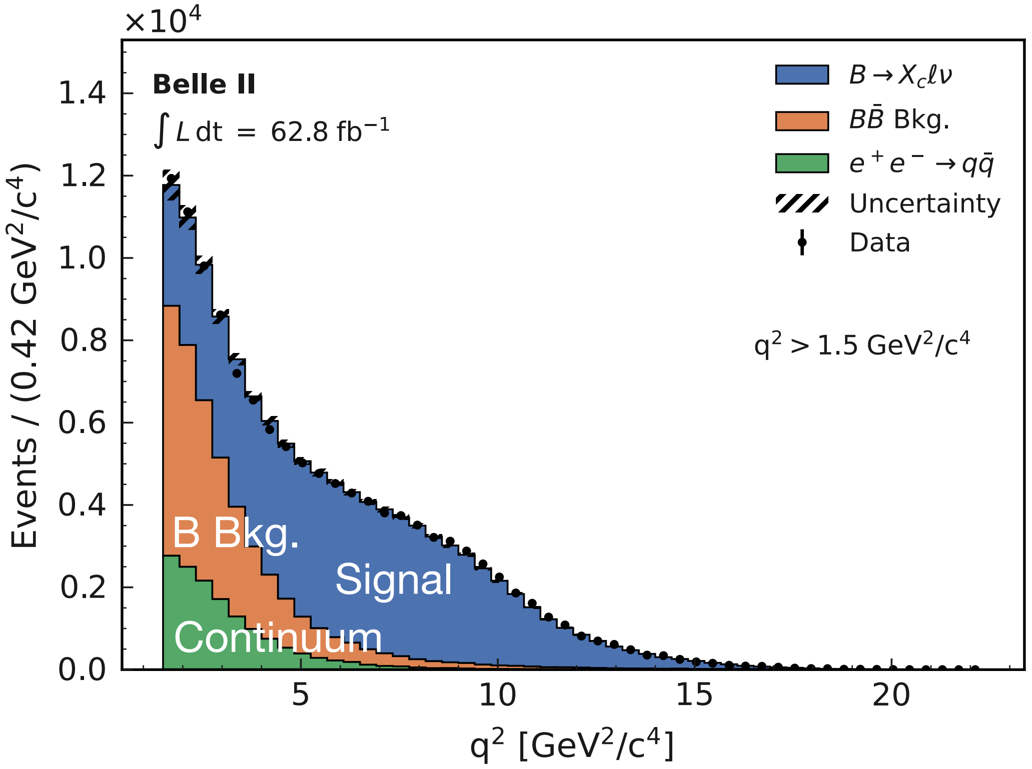


Can identify **X_c** **constituents**

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = \left(p_{\mathrm{sig}} - p_{X_c}\right)^2$$



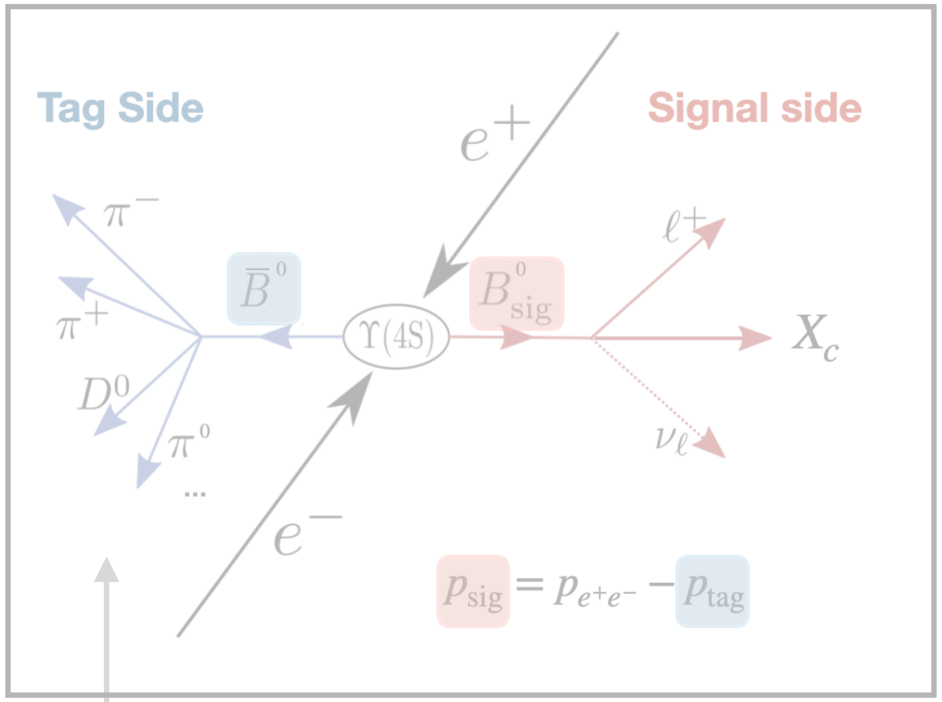


Hadronic Tagging
with **Belle II** algorithm (FEI)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

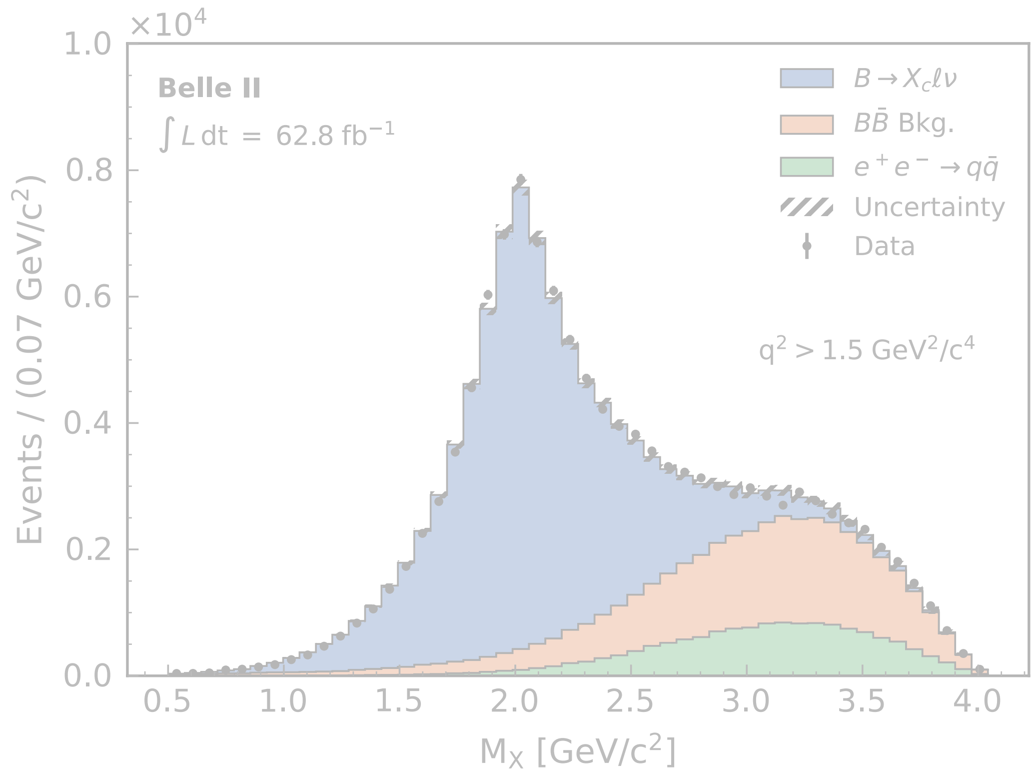
How to measure spectral moments

Key-technique: hadronic tagging



Can identify X_c constituents

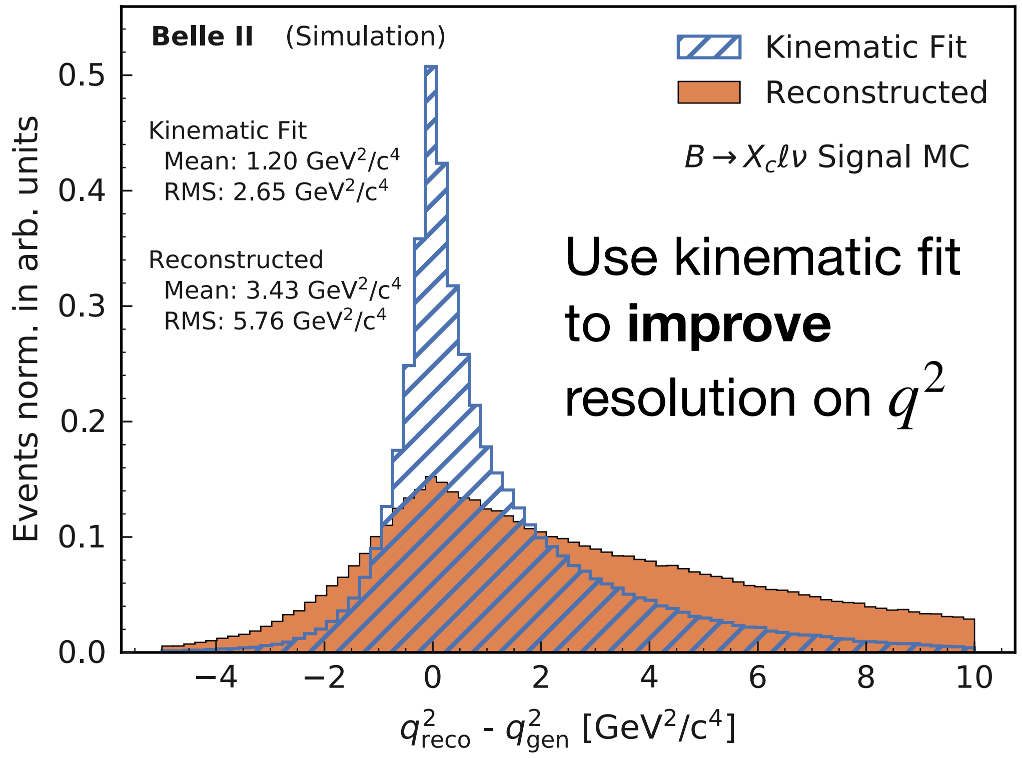
$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$



Improved Hadronic Tagging
using **Belle II** algorithm
(ca. 2 times more efficient)

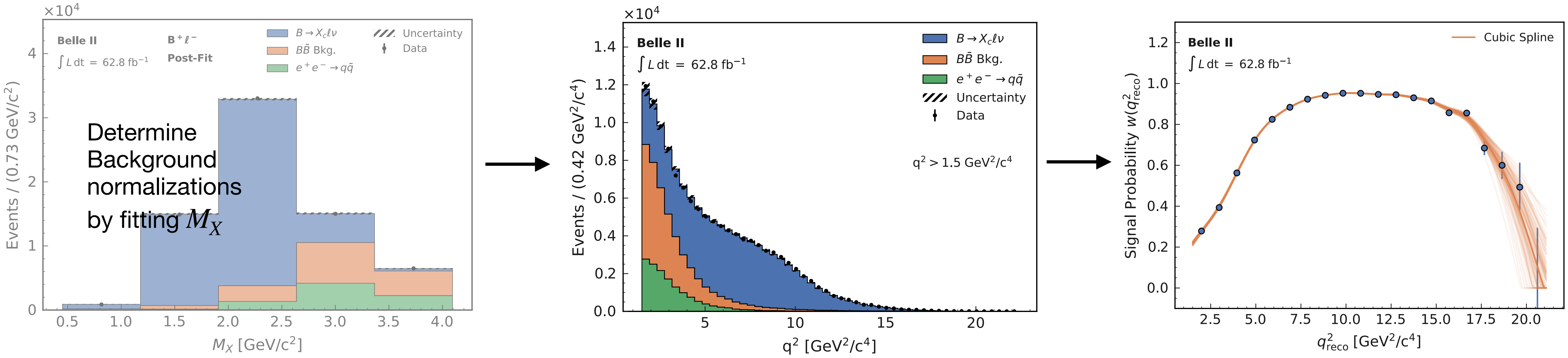
[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]

$$q^2 = \left(p_{\mathrm{sig}} - p_{X_c}\right)^2$$



Use kinematic fit to **improve** resolution on q^2

Measurement in a nutshell



Determine Background normalizations by fitting M_X

Step #1: Subtract Background

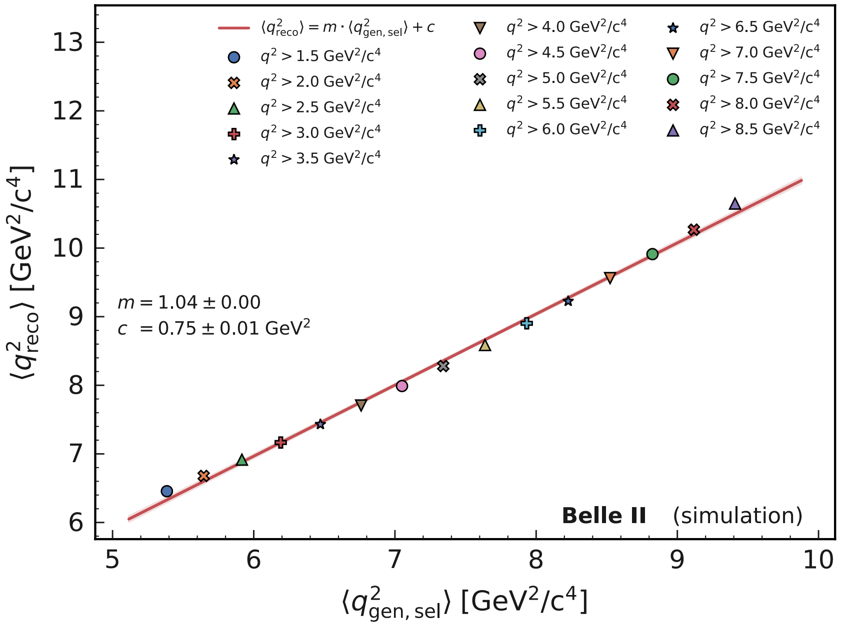
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_\mathrm{data}} w(q^2_\mathrm{reco,i}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_\mathrm{data}} w(q^2_\mathrm{reco,j})} \times \mathcal{C}_\mathrm{calib} \times \mathcal{C}_\mathrm{gen} \,,$$

Measurement in a nutshell

Exploit linear dependence between rec. & true moments

$$q^{2m}_{\text{cal}\,i} = \left(q^{2m}_{\text{reco}\,i} - c\right)/m$$



Step #1: Subtract Background

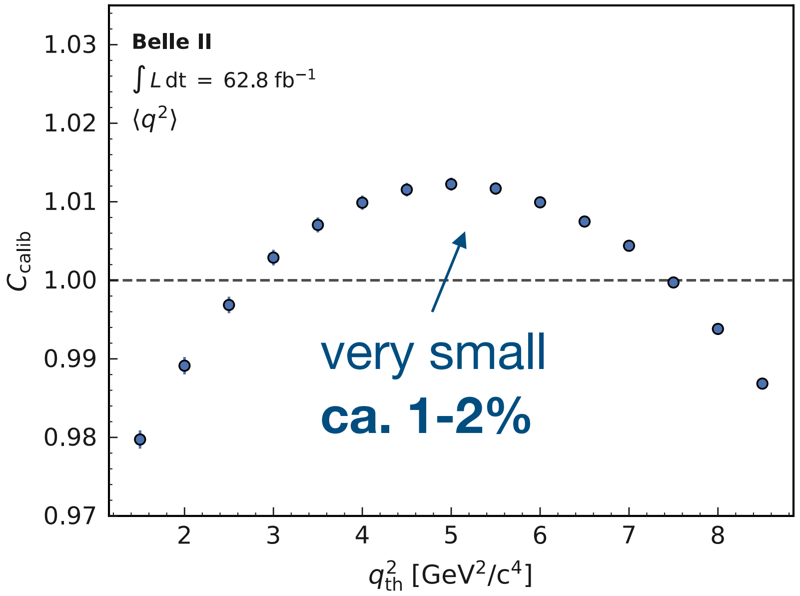
Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q^2_{\text{reco,i}}) \times \boxed{q^{2n}_{\text{calib},i}}}{\sum_j^{N_{\text{data}}} w(q^2_{\text{reco,j}})} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}}\,,$$

Measurement in a nutshell

Very small deviation from linear behavior between reconstruct and true q^2



very small

ca. 1-2%

Step #1: Subtract Background

Step #2: Calibrate moment

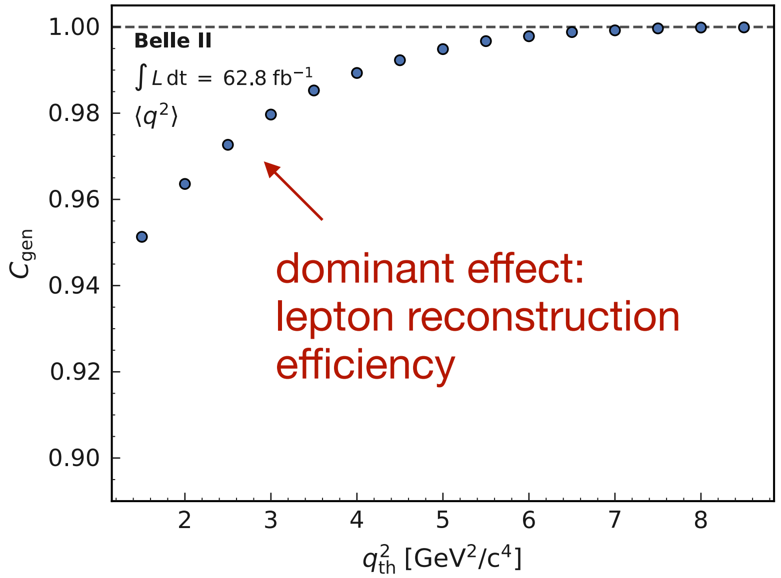
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_\mathrm{data}} w(q^2_\mathrm{reco,i}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_\mathrm{data}} w(q^2_\mathrm{reco,j})} \times \boxed{\mathcal{C}_\mathrm{calib}} \times \mathcal{C}_\mathrm{gen}\ ,$$

Step #3: If you fail, try again

Measurement in a nutshell

Account for efficiency & acceptance effects



dominant effect: lepton reconstruction efficiency

Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

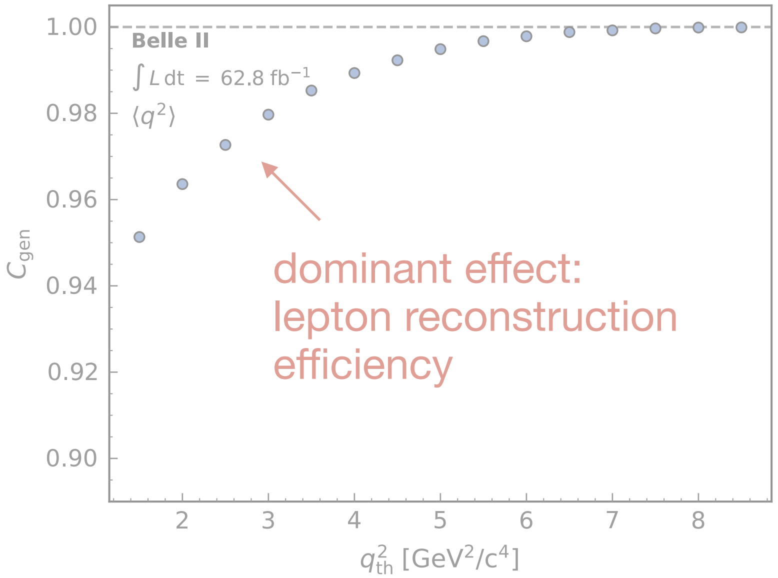
$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,i}}) \times q^{2n}_{\mathrm{calib},i}}{\sum_j^{N_{\mathrm{data}}} w(q^2_{\mathrm{reco,j}})} \times \mathcal{C}_{\mathrm{calib}} \times \boxed{\mathcal{C}_{\mathrm{gen}}},$$

Step #3: If you fail, try again

Step #4: Correct for selection effects

Measurement in a nutshell

Account for efficiency & acceptance effects



dominant effect: lepton reconstruction efficiency

Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

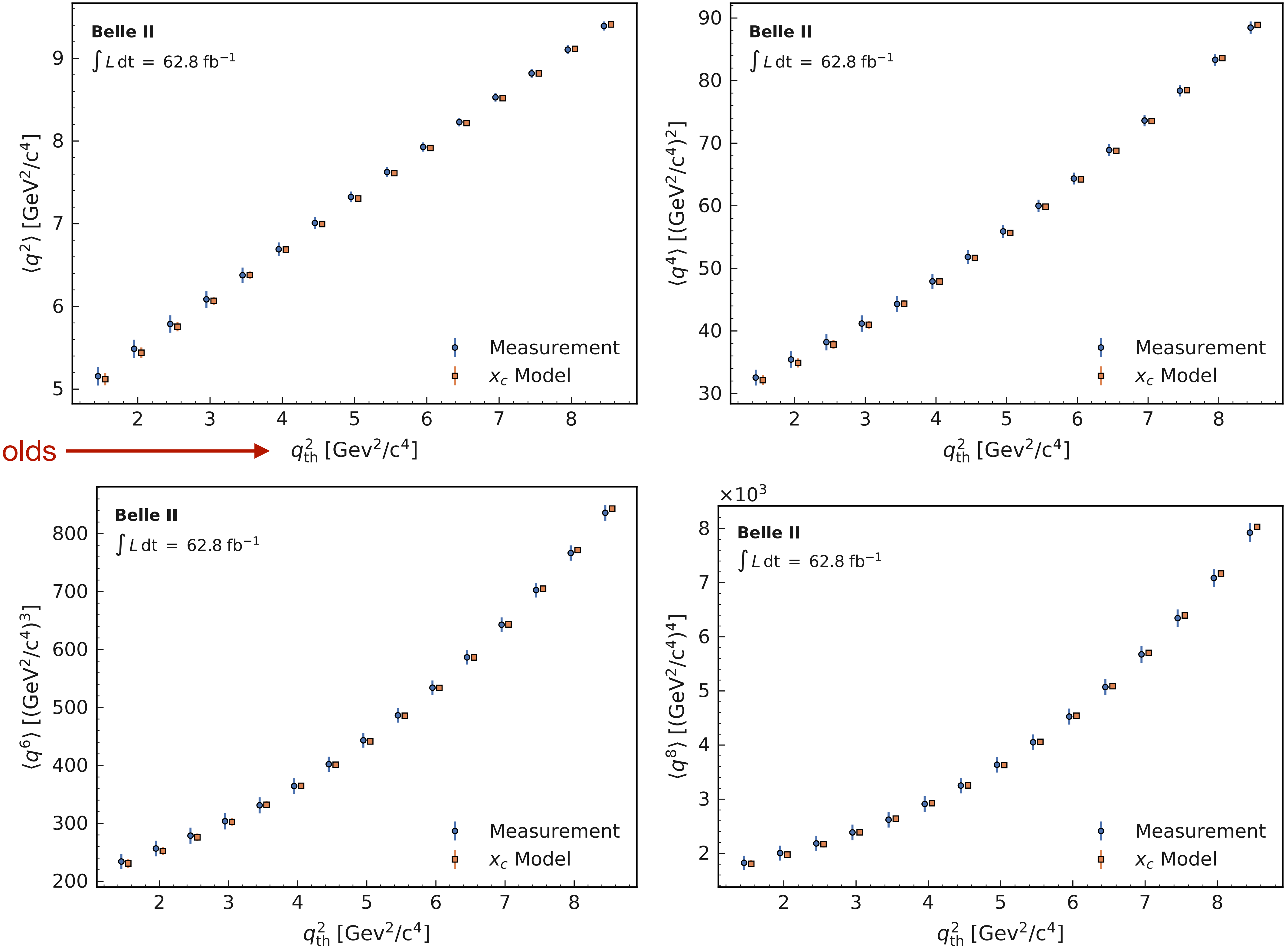
$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q^2_{\text{reco,i}}) \times q^{2n}_{\text{calib},i}}{\sum_j^{N_{\text{data}}} w(q^2_{\text{reco,j}})} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}} \,,$$

Step #3: If you fail, try again

Step #4: Correct for selection effects

→ Repeat this for many different thresholds cuts q^2_{th}

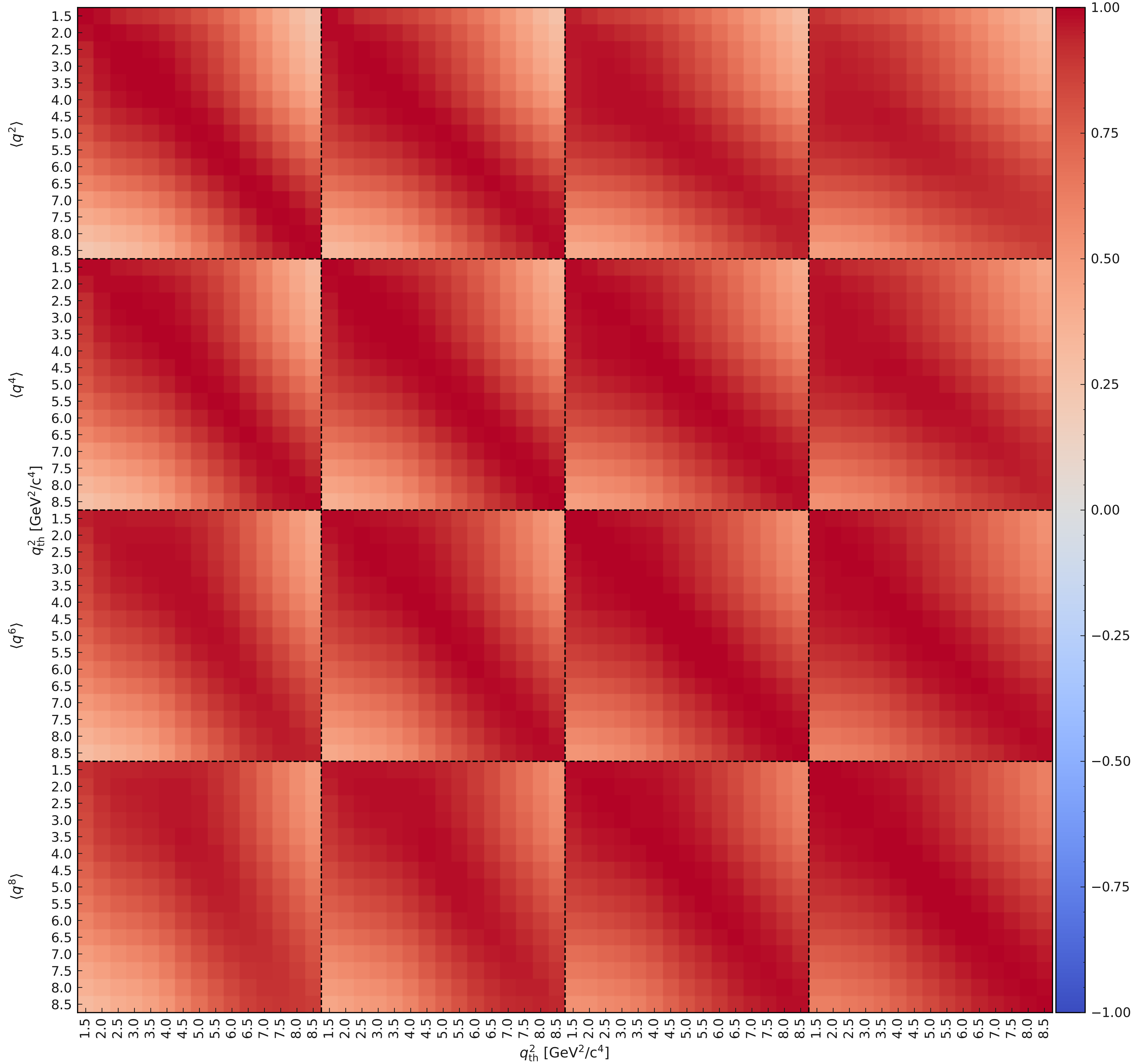
Belle II q^2 spectral moments



q^2 thresholds

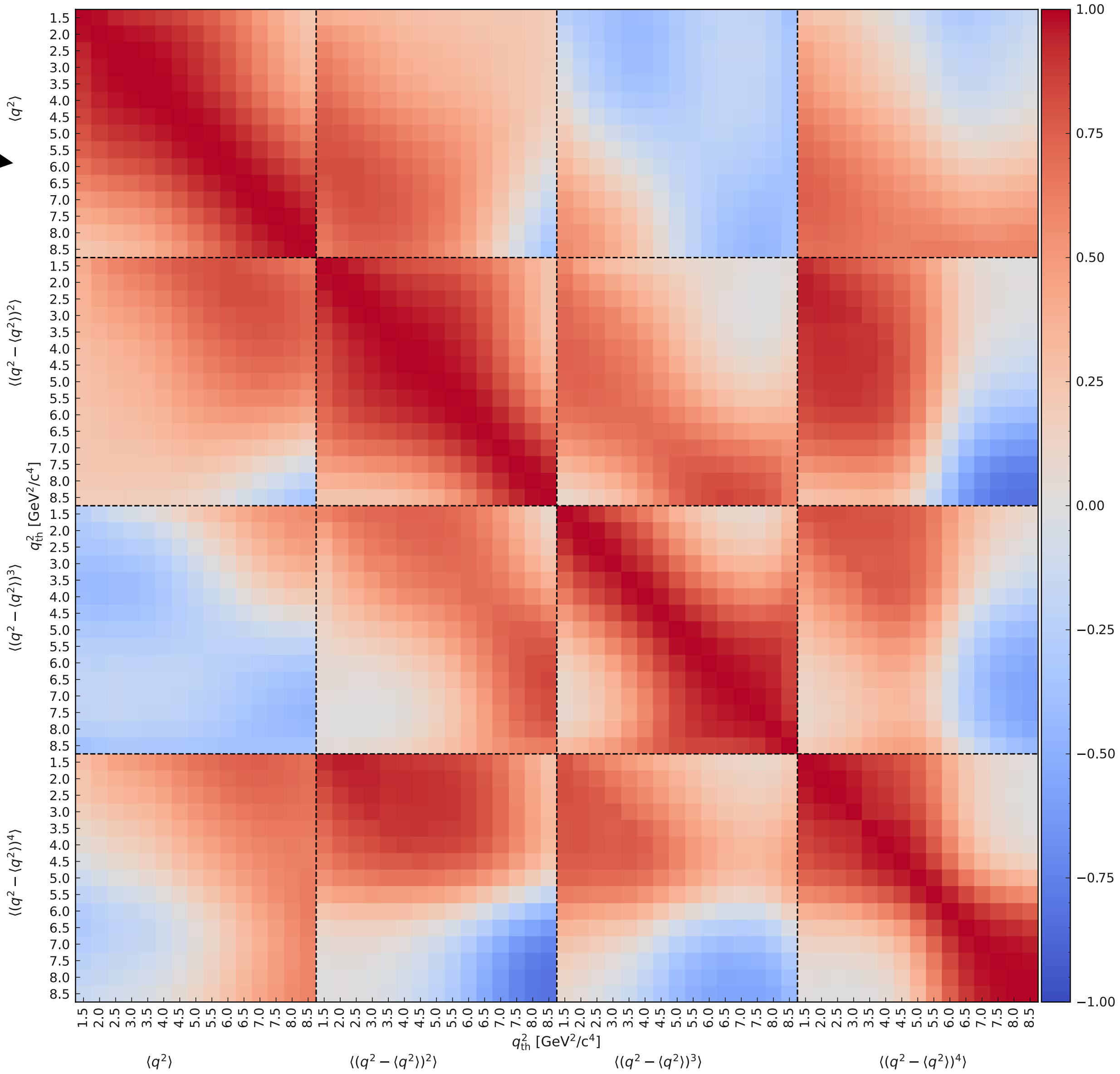
Statistical plus **systematic** correlations

strong correlations!

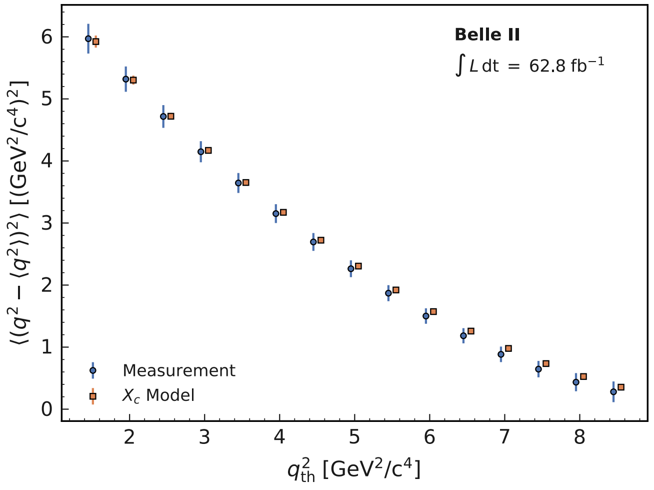


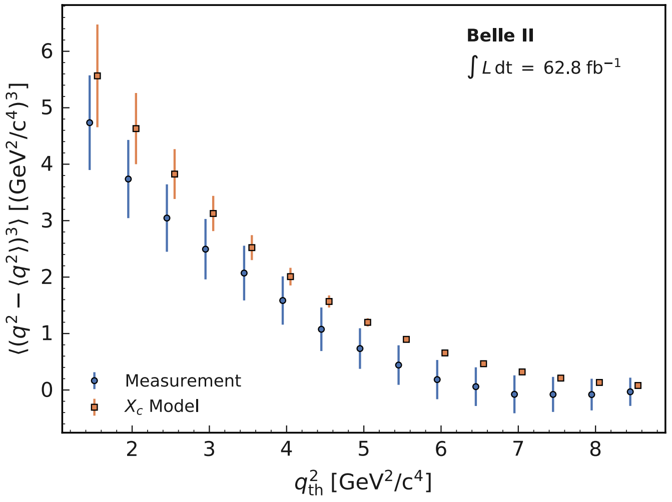
From moments to *central moments*

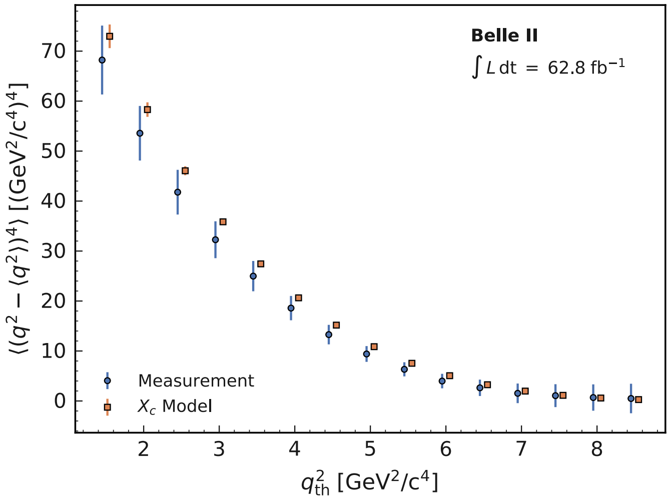
Central moments are **less** strongly correlated



$$\begin{pmatrix} \langle q^2 \rangle \\ \langle q^4 \rangle \\ \langle q^6 \rangle \\ \langle q^8 \rangle \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} \langle q^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^3 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^4 \rangle \end{pmatrix}$$







Inclusive $|V_{ub}|$

$\bar{B} \to X_u \ell \bar{\nu}_\ell$

+ Fermi Motion / Shape Function

Inclusive $|V_{cb}|$

$\bar{B} \to X_c \ell \bar{\nu}_\ell$

Operator Product Expansion

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]$$

Exclusive $|V_{ub}|$

$\bar{B} \to \pi \ell \bar{\nu}_\ell, \Lambda_b \to p \mu \bar{\nu}_\mu$

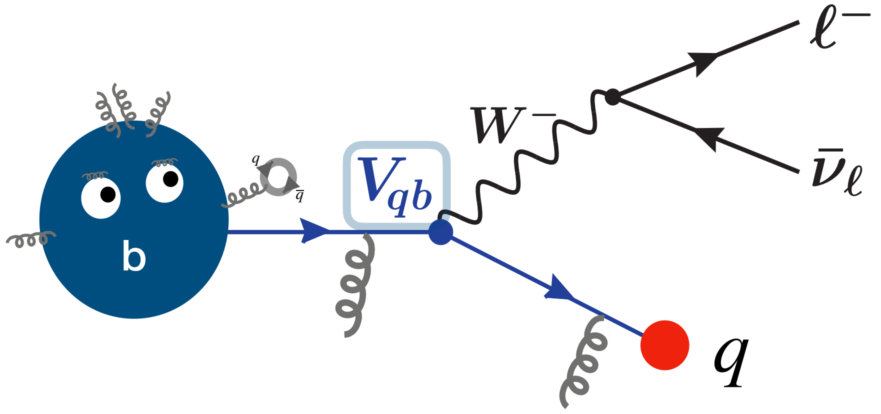
Exclusive $|V_{cb}|$

$\bar{B} \to D \ell \bar{\nu}_\ell, \bar{B} \to D^* \ell \bar{\nu}_\ell$

$$\mathcal{B} \propto |V_{cb}|^2 f^2$$

Form Factors

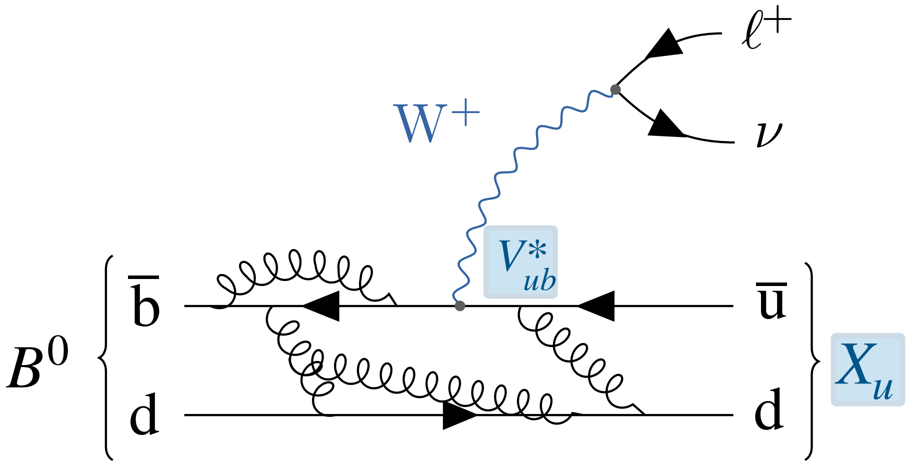
$\langle B|H_\mu|P\rangle = (p+p')_\mu f_+$

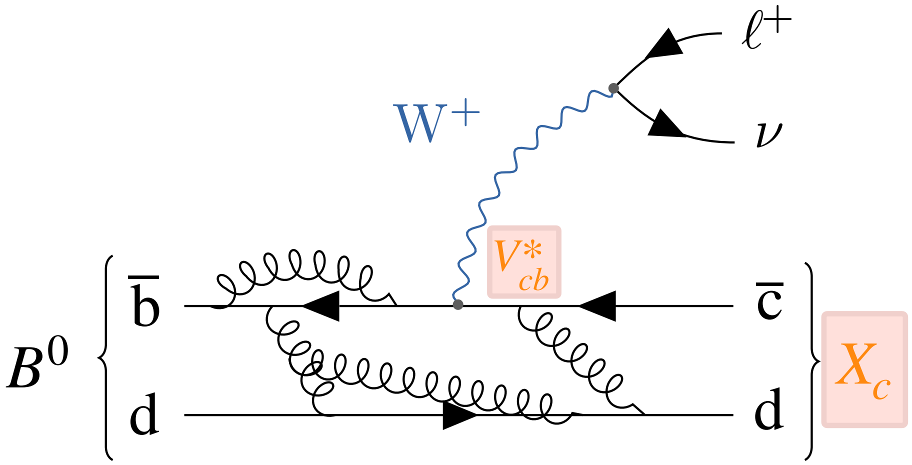


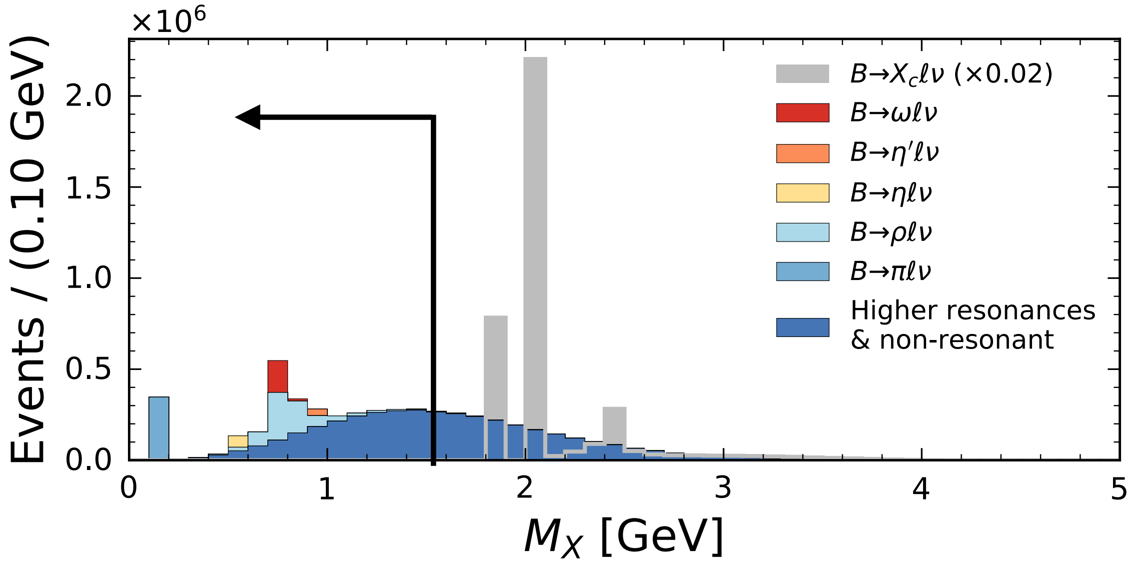
Overview $B \to X_u \ell \bar{\nu}_\ell$

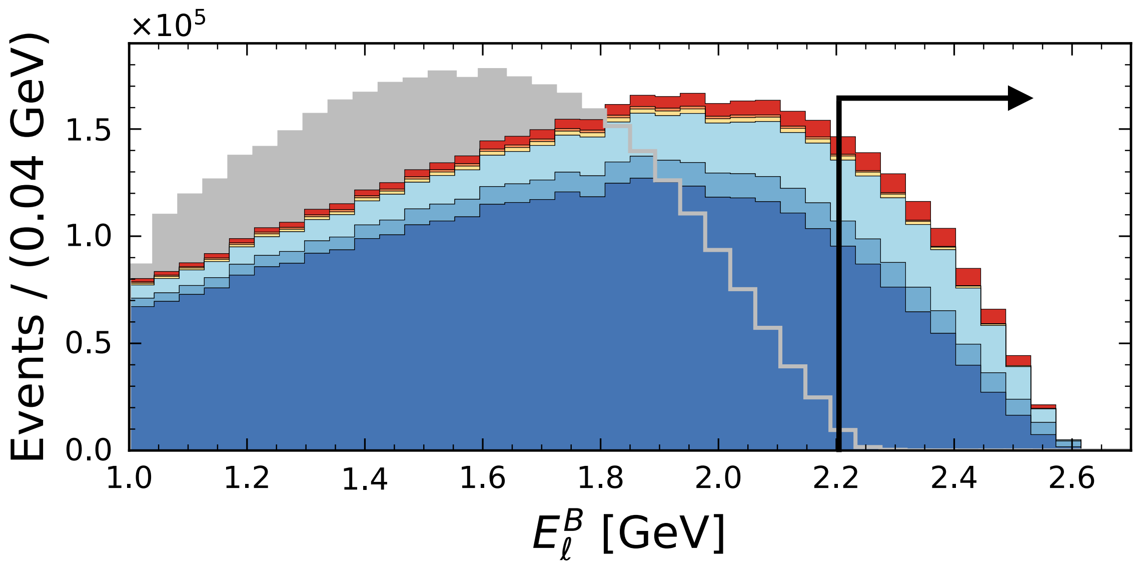
Measuring |V_{ub}| is **hard** due to $B \to X_c \ell \bar{\nu}_\ell$

- x $\mathcal{O}(100)$ more abundant
- Very similar **signature**:
  - high momentum lepton, hadronic system
- Clear separation only in corners of phase space
  - high E_ℓ, low M_X









Going Hybrid : MC for $B \to X_u \ell \bar{\nu}_\ell$

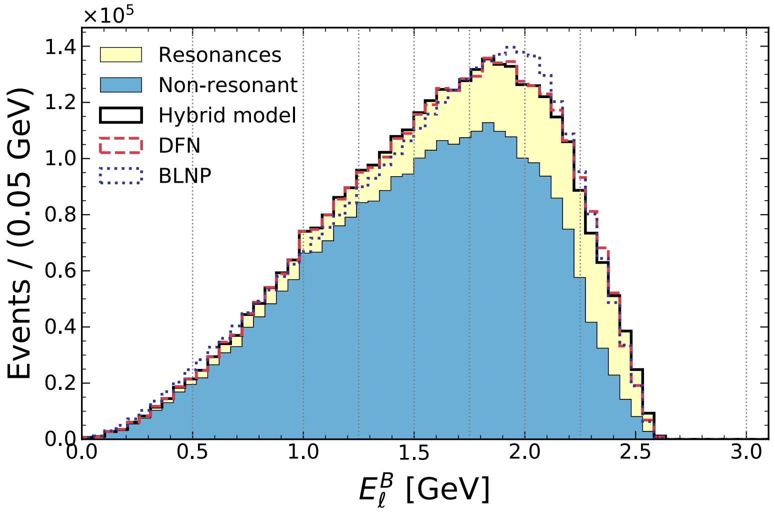
Exclusive make-up of $B \to X_u \ell \bar{\nu}_\ell$:

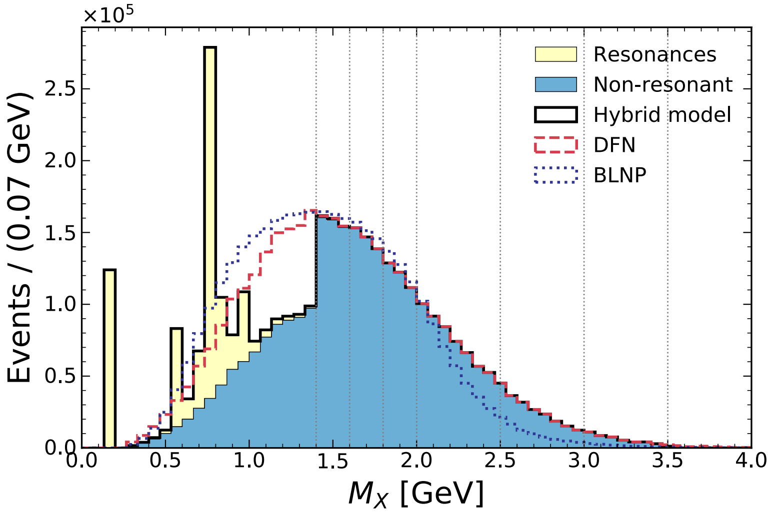
$\mathcal{B}$	Value B^+	Value B^0
$B \to \pi\, \ell^+\, \nu_\ell$ $^{\text{a,e}}$	$(7.8 \pm 0.3) \times 10^{-5}$	$(1.5 \pm 0.06) \times 10^{-4}$
$B \to \eta\, \ell^+\, \nu_\ell$ $^{\text{b,e}}$	$(3.9 \pm 0.5) \times 10^{-5}$	-
$B \to \eta'\, \ell^+\, \nu_\ell$ $^{\text{b,e}}$	$(2.3 \pm 0.8) \times 10^{-5}$	-
$B \to \omega\, \ell^+\, \nu_\ell$ $^{\text{c,e}}$	$(1.2 \pm 0.1) \times 10^{-4}$	-
$B \to \rho\, \ell^+\, \nu_\ell$ $^{\text{c,e}}$	$(1.6 \pm 0.1) \times 10^{-4}$	$(2.9 \pm 0.2) \times 10^{-4}$
$B \to X_u\, \ell^+\, \nu_\ell$ $^{\text{d,e}}$	$(2.2 \pm 0.3) \times 10^{-3}$	$(2.0 \pm 0.3) \times 10^{-3}$

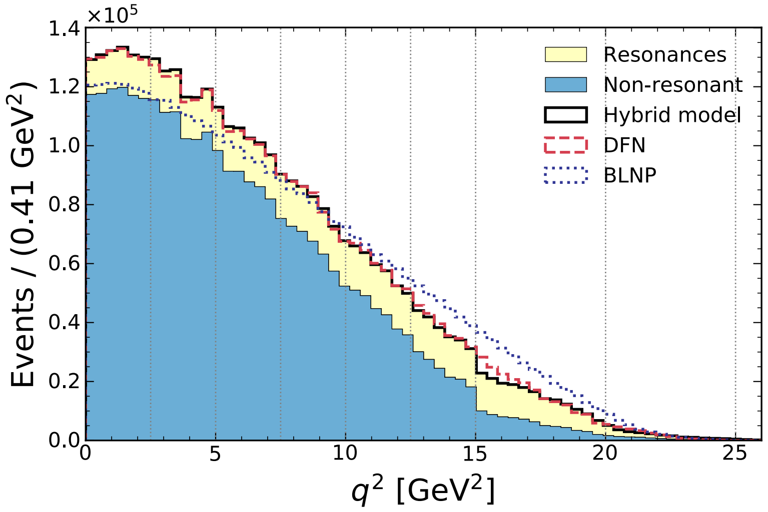
Hybrid = Combining exclusive & inclusive predictions

$$\Delta\mathcal{B}^{\text{incl}}_{ijk} = \Delta\mathcal{B}^{\text{excl}}_{ijk} + w_{ijk} \times \Delta\mathcal{B}^{\text{incl}}_{ijk} \,,$$

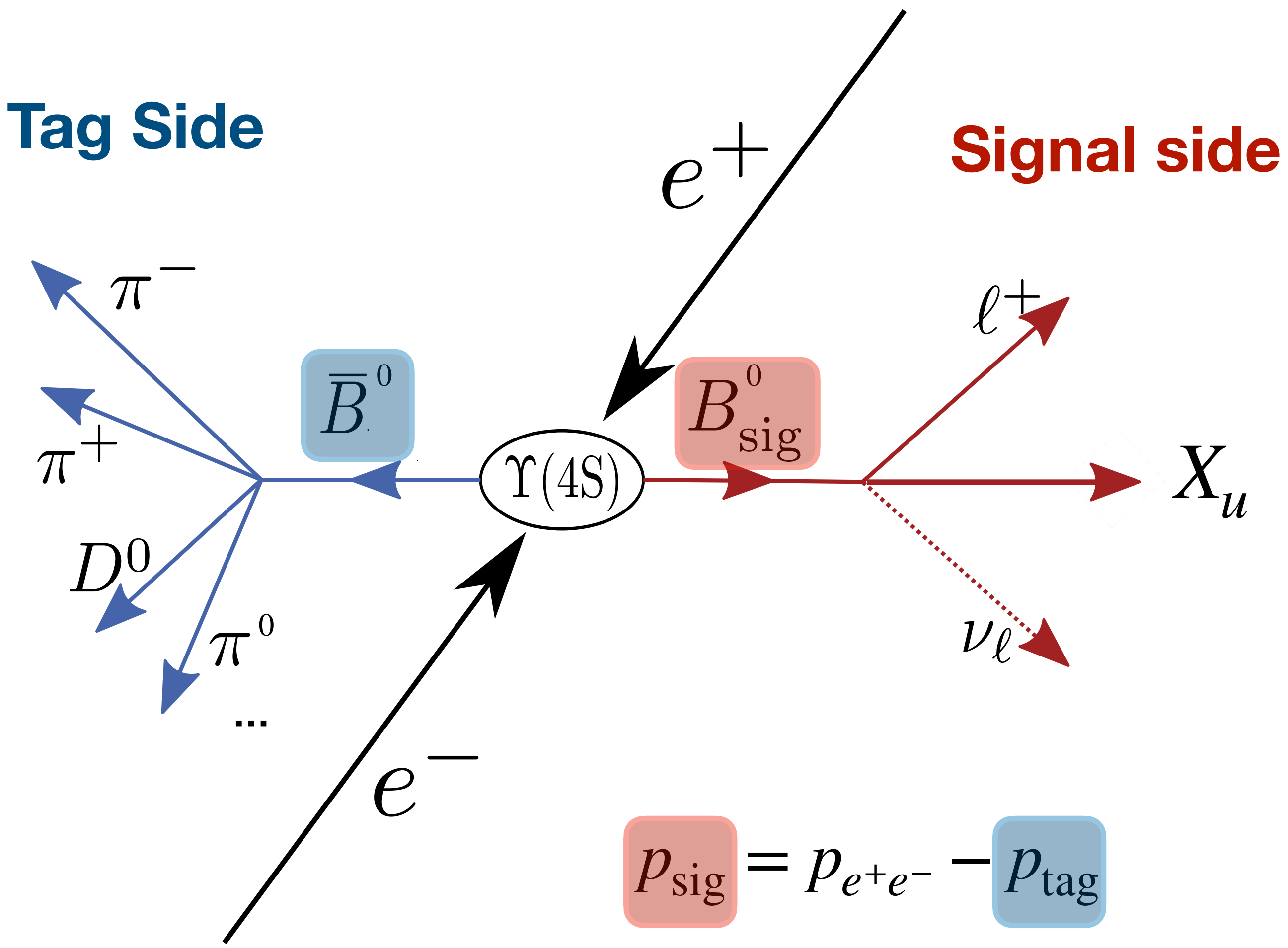
$$q^2 = [0, 2.5, 5, 7.5, 10, 12.5, 15, 20, 25]\, \text{GeV}^2 \,,$$
$$E_\ell^B = [0, 0.5, 1, 1.25, 1.5, 1.75, 2, 2.25, 3]\, \text{GeV} \,,$$
$$M_X = [0, 1.4, 1.6, 1.8, 2, 2.5, 3, 3.5]\, \text{GeV} \,.$$





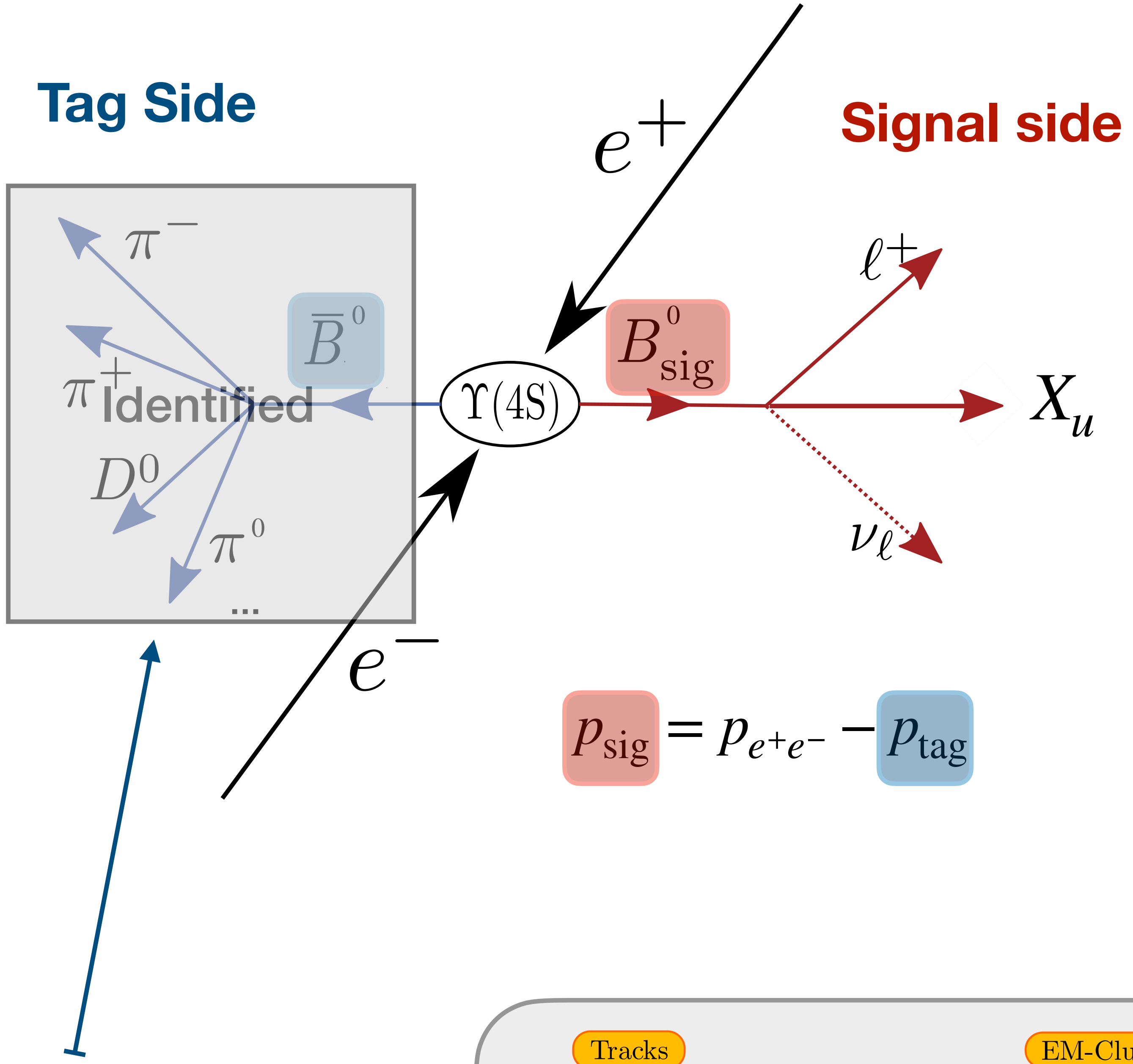


Analysis Strategy with hadronic Tagging



$$p_{\text{sig}} = p_{e^+e^-} - p_{\text{tag}}$$

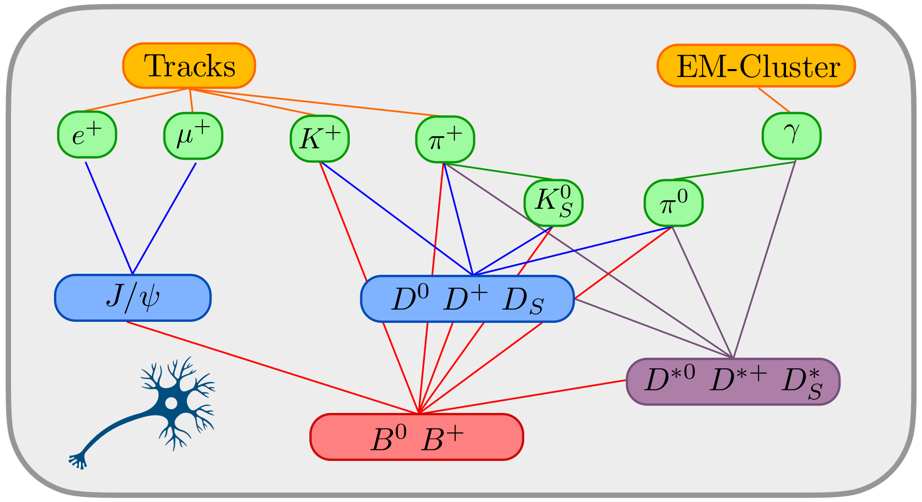
Analysis Strategy with hadronic Tagging



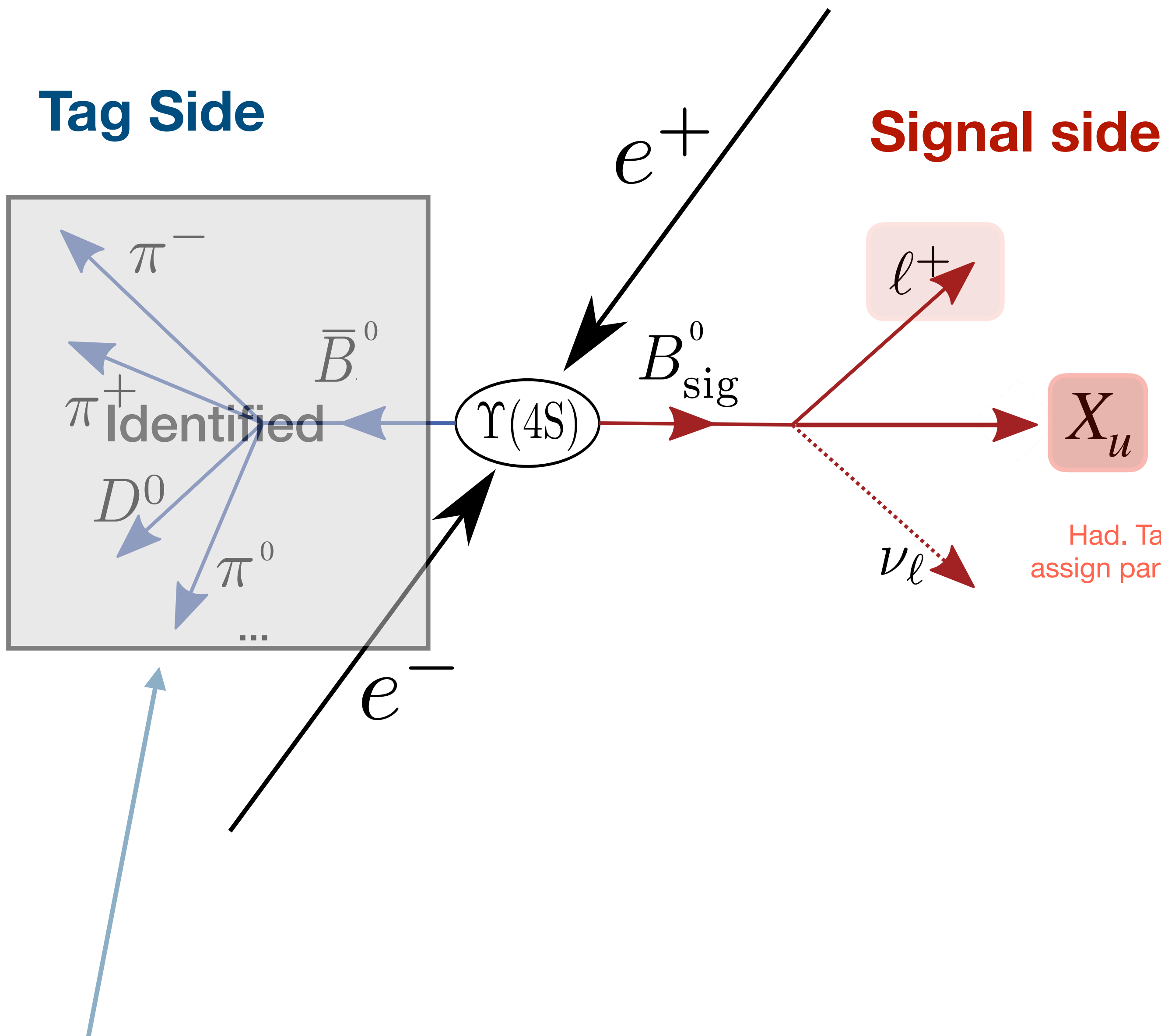
$$p_{\text{sig}} = p_{e^+e^-} - p_{\text{tag}}$$

Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

1104 decay cascades used with an **efficiency** of **0.28% / 0.18%** for $B^\pm$ and $B^0/\bar{B}^0$



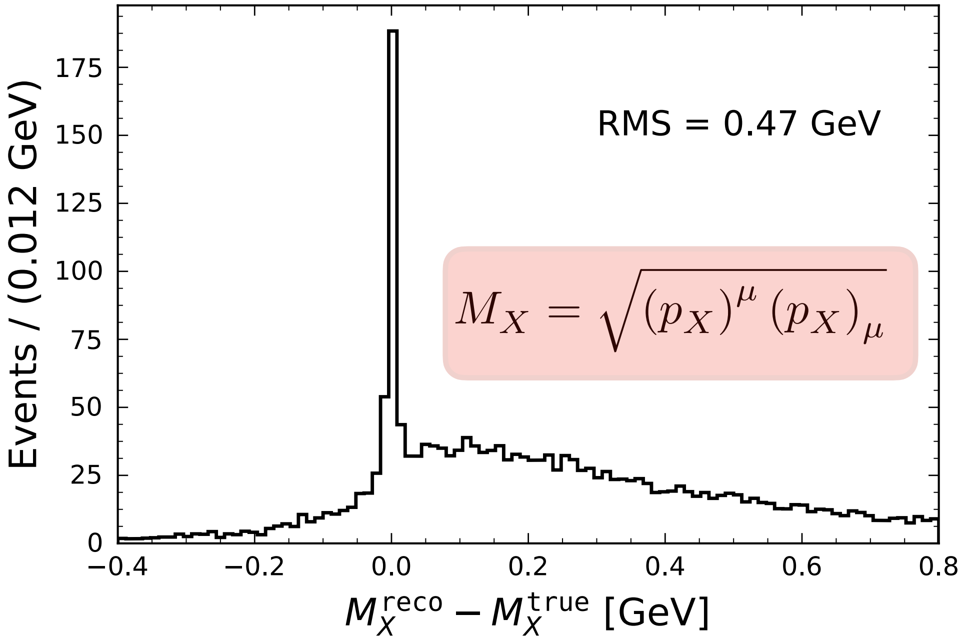
Analysis Strategy with hadronic Tagging



Had. Tagging allows to assign particles to X_u system

Charged Tracks   Neutral Clusters

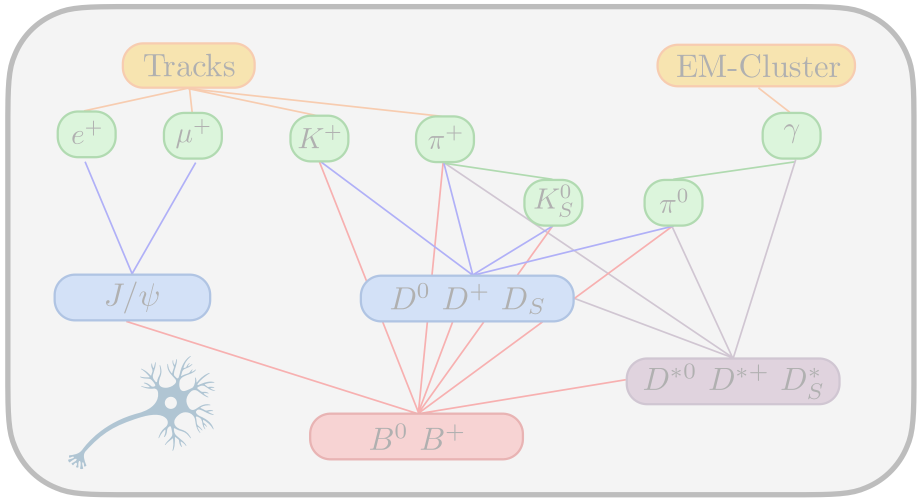
$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i\right) + \sum_j (E_j, \mathbf{k}_j)$$

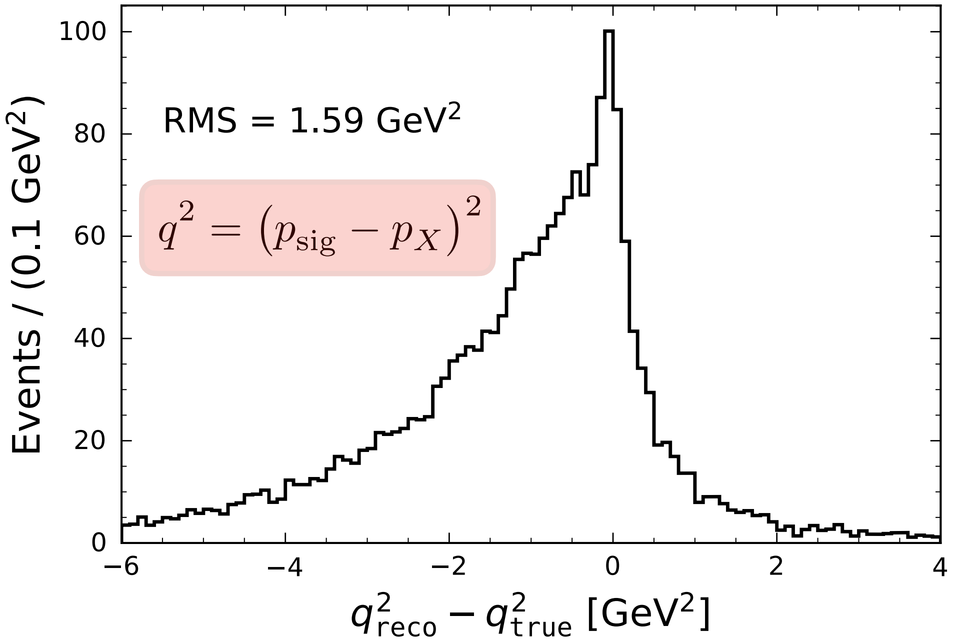


$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

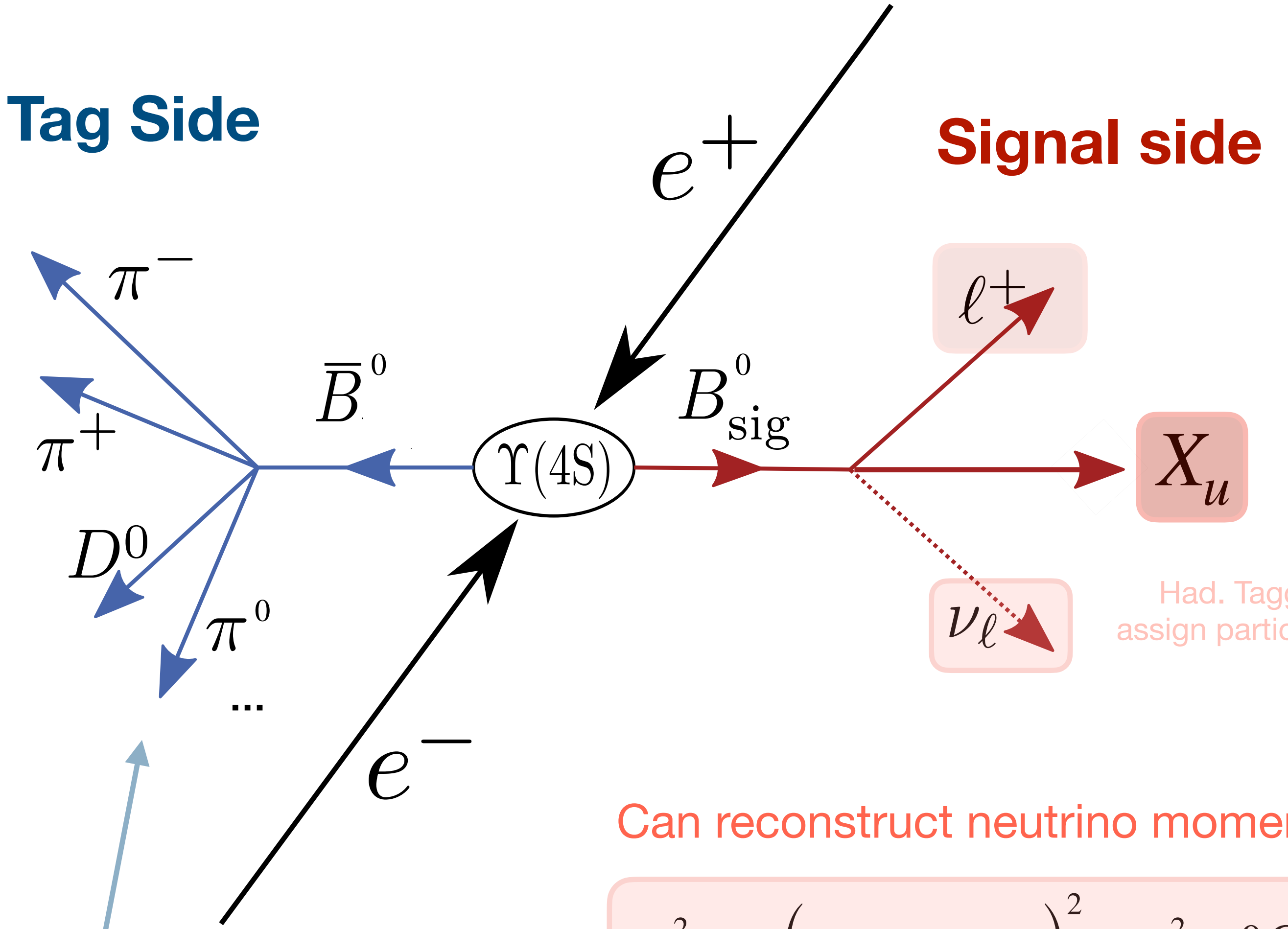
1104 decay cascades used with an **efficiency** of **0.28% / 0.18%** for $B^\pm$ and $B^0/\bar{B}^0$





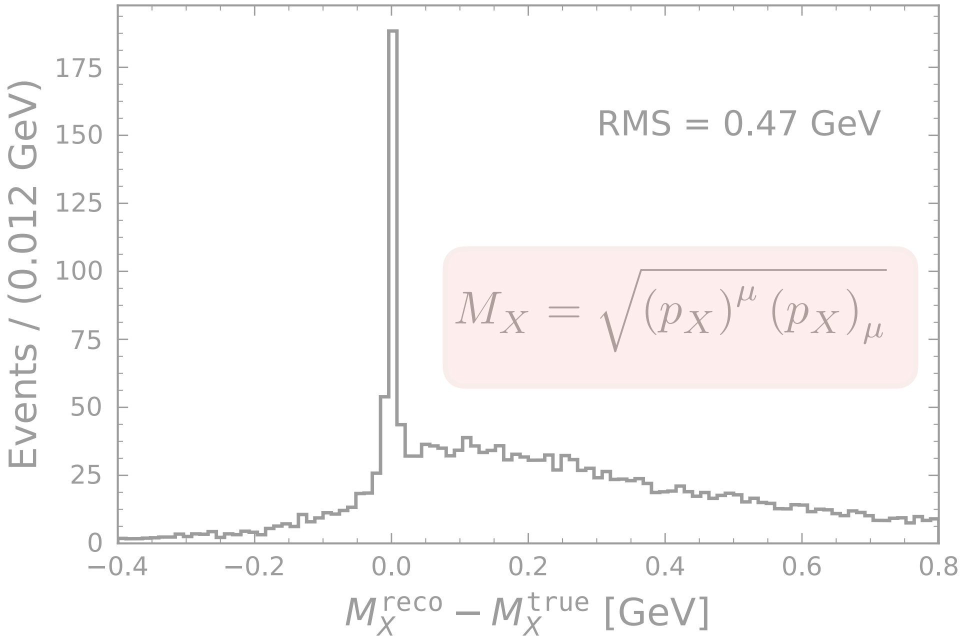
$$q^2 = (p_{\text{sig}} - p_X)^2$$

Analysis Strategy with hadronic Tagging



$$p_X = \sum_i \left(\sqrt{m_\pi^2 + |\mathbf{p}_i|^2}, \mathbf{p}_i\right) + \sum_j (E_j, \mathbf{k}_j)$$

Had. Tagging allows to assign particles to X_u system



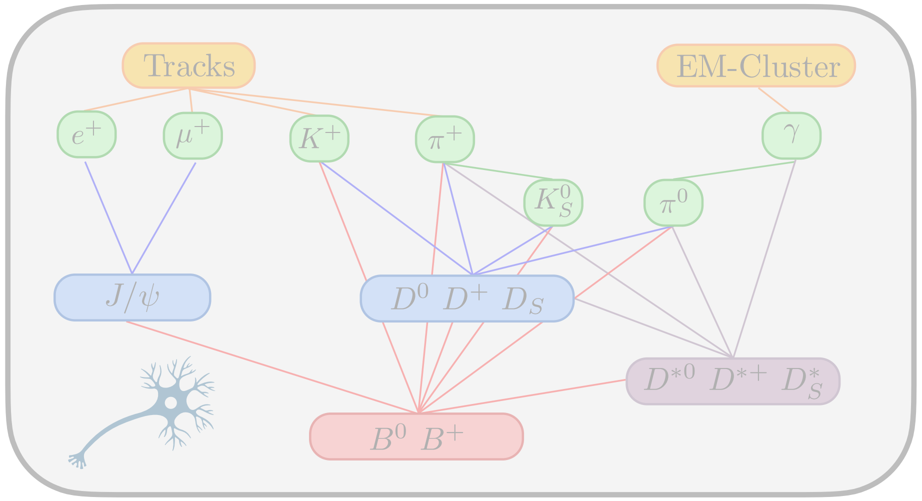
$$M_X = \sqrt{(p_X)^\mu (p_X)_\mu}$$

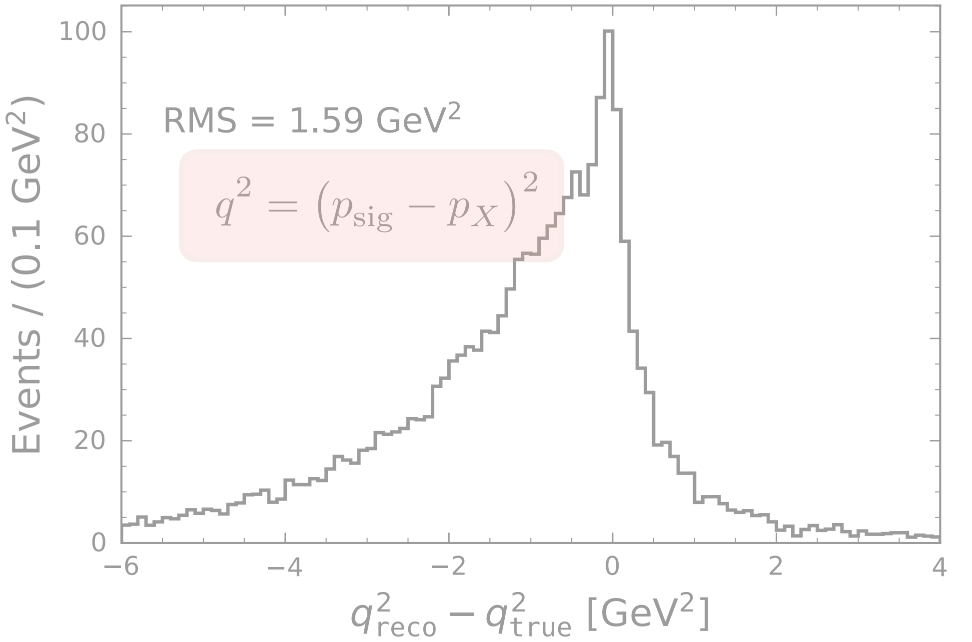
Can reconstruct neutrino momentum:

$$m_{\rm miss}^2 = \left(p_{\rm sig} - p_X - p_\ell\right)^2 \approx m_\nu^2 = 0\,\mathrm{GeV}^2$$

Candidates reconstructed with **hierarchical** approach & **neural networks** in **hadronic modes**

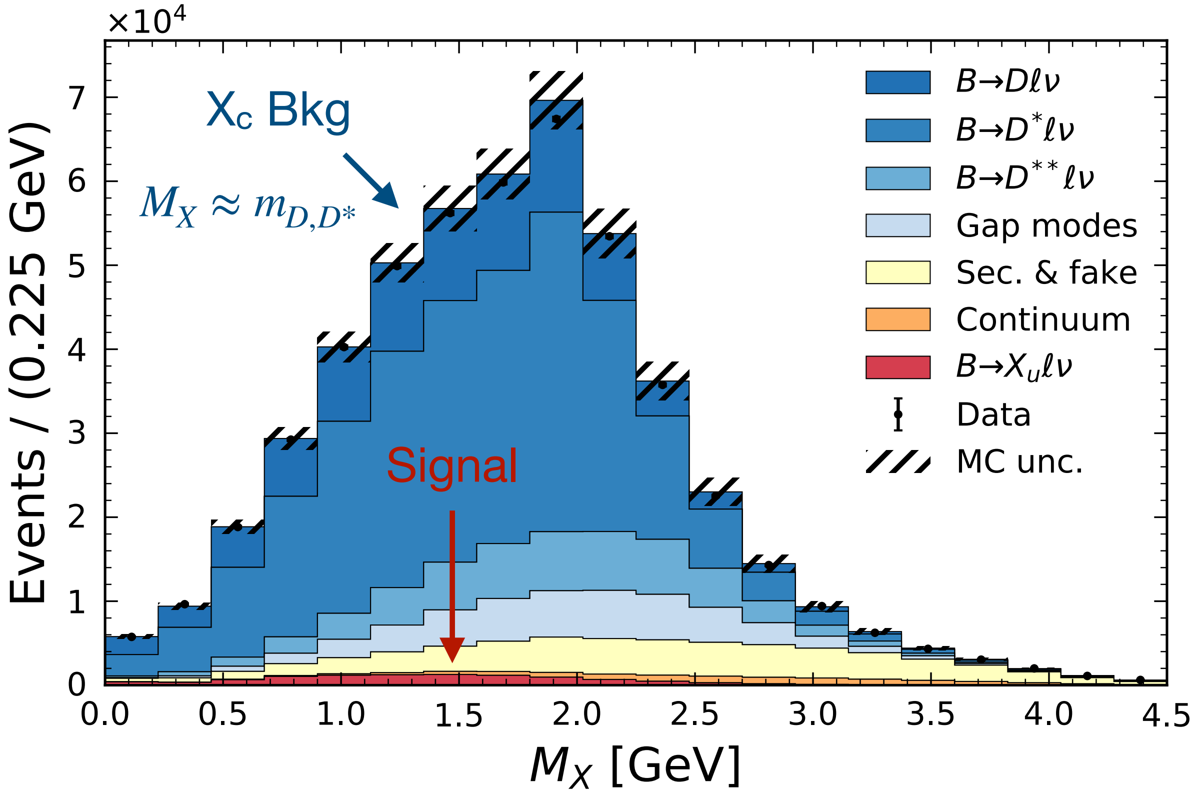
1104 decay cascades used with an **efficiency** of **0.28% / 0.18%** for $B^\pm$ and $B^0/\bar{B}^0$



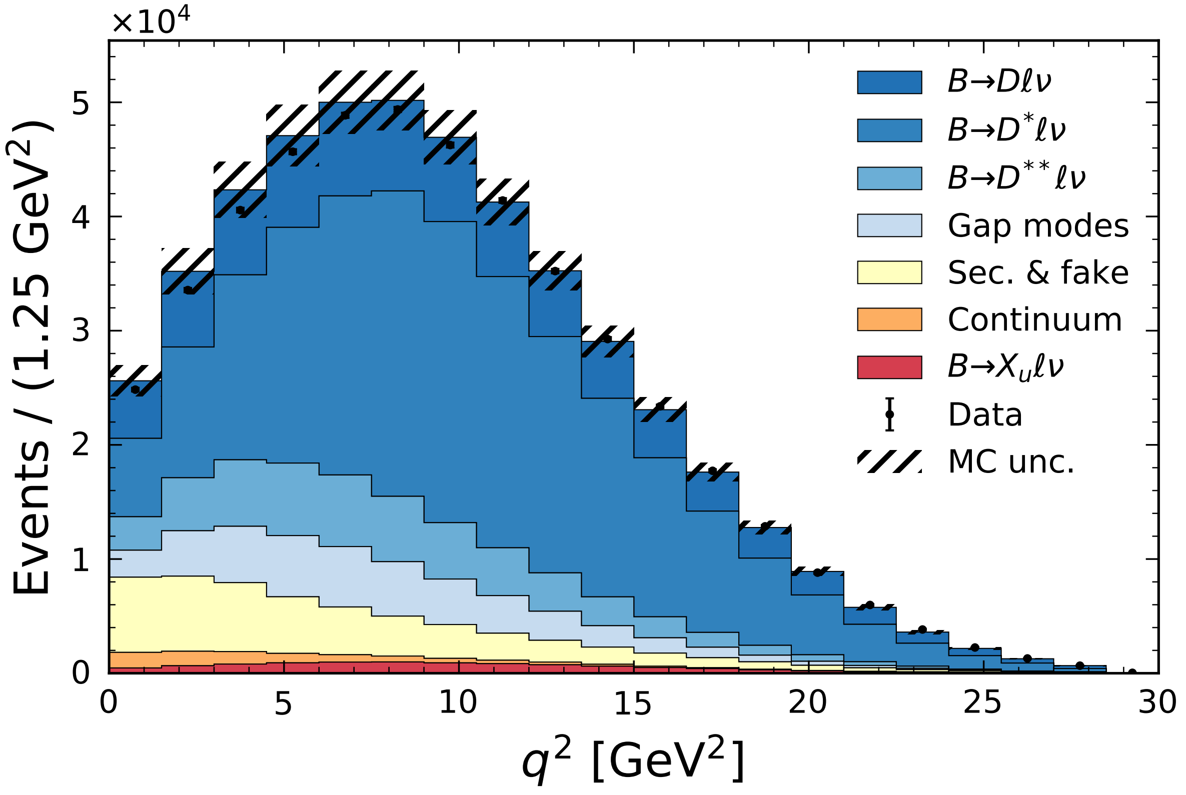


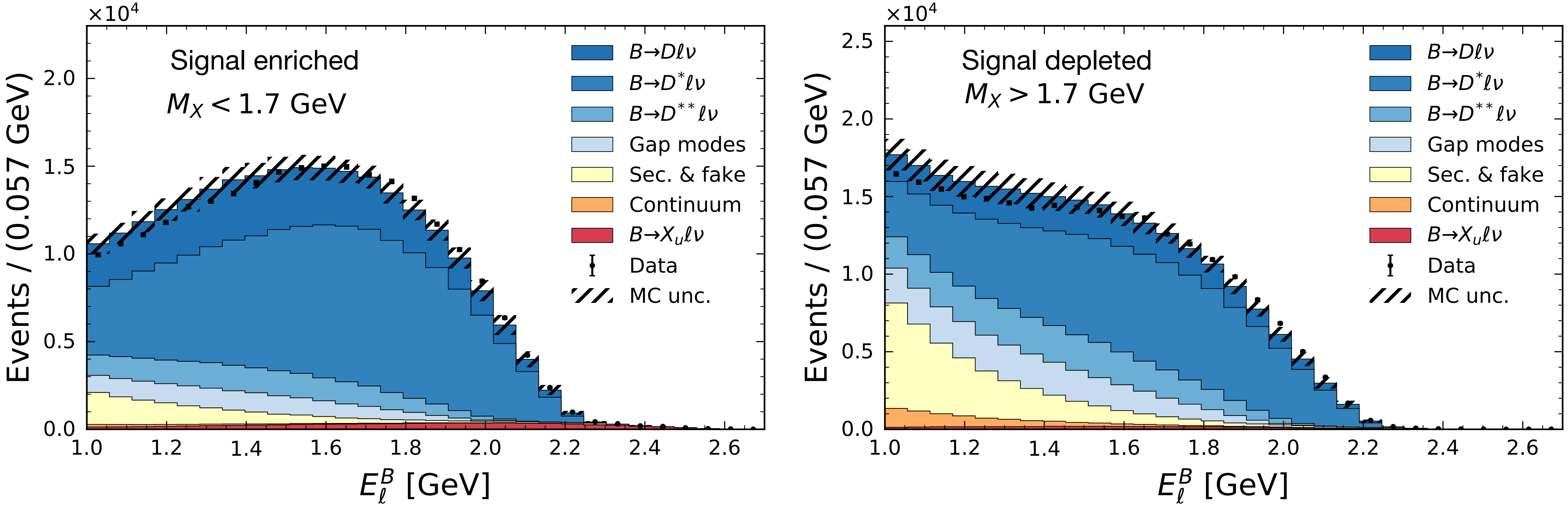
$$q^2 = \left(p_{\rm sig} - p_X\right)^2$$

Hadronic Mass $M_X = \sqrt{p_X^2}$



Four-momentum transfer squared $q^2 = (p_B - p_X)^2$





Lepton Energy in signal B rest frame E_ℓ^B

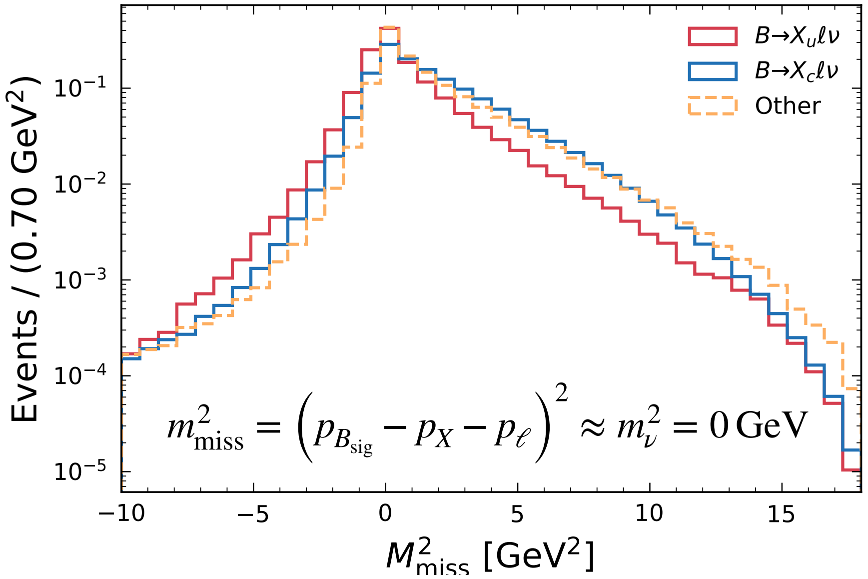
Multivariate Sledgehammer

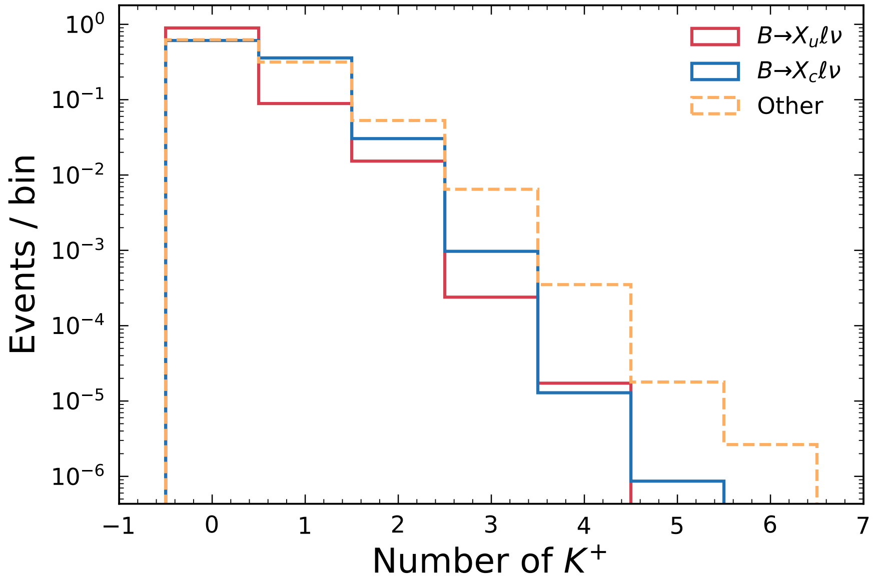
Can exploit that there are differences:

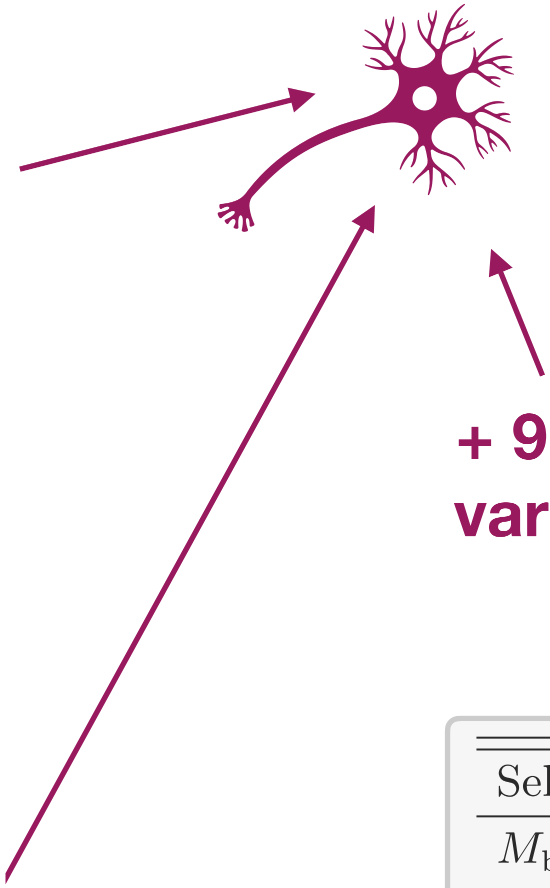
$X_u \longleftrightarrow X_c$

Direct cuts on m_X, E_ℓ **problematic**
(i.e. direct theory / shape-function dependence)

Higher multiplicity
Often come with charged and neutral **Kaons**
D* decays (**slow pions**)
(**Slightly lower** E_ℓ)

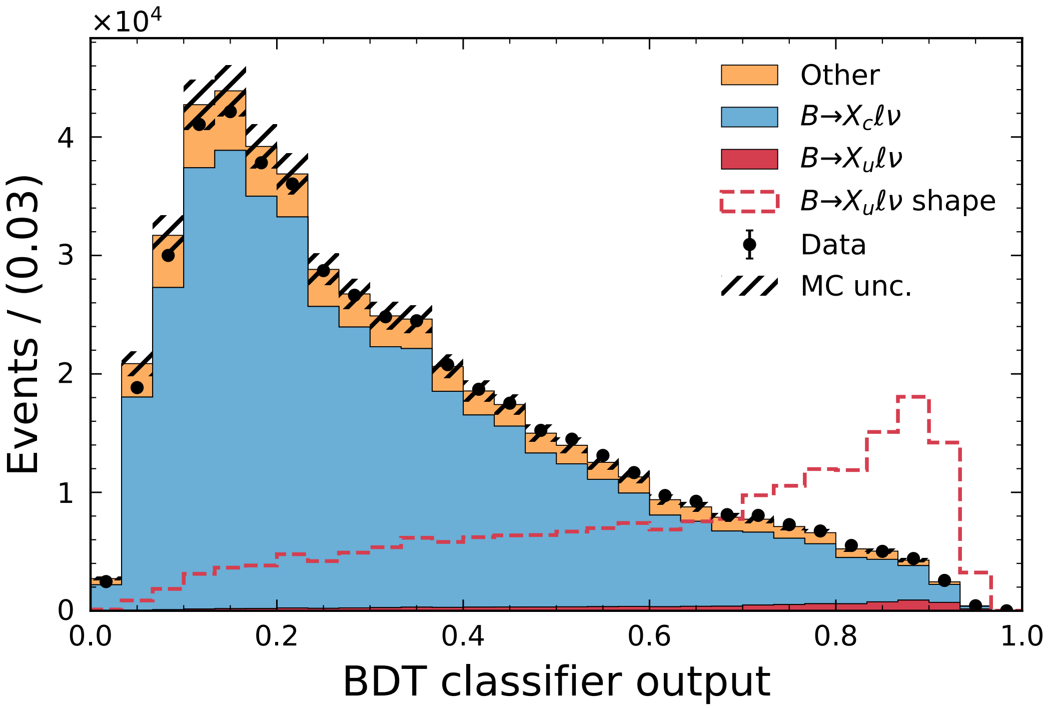






=

+ 9 other variables



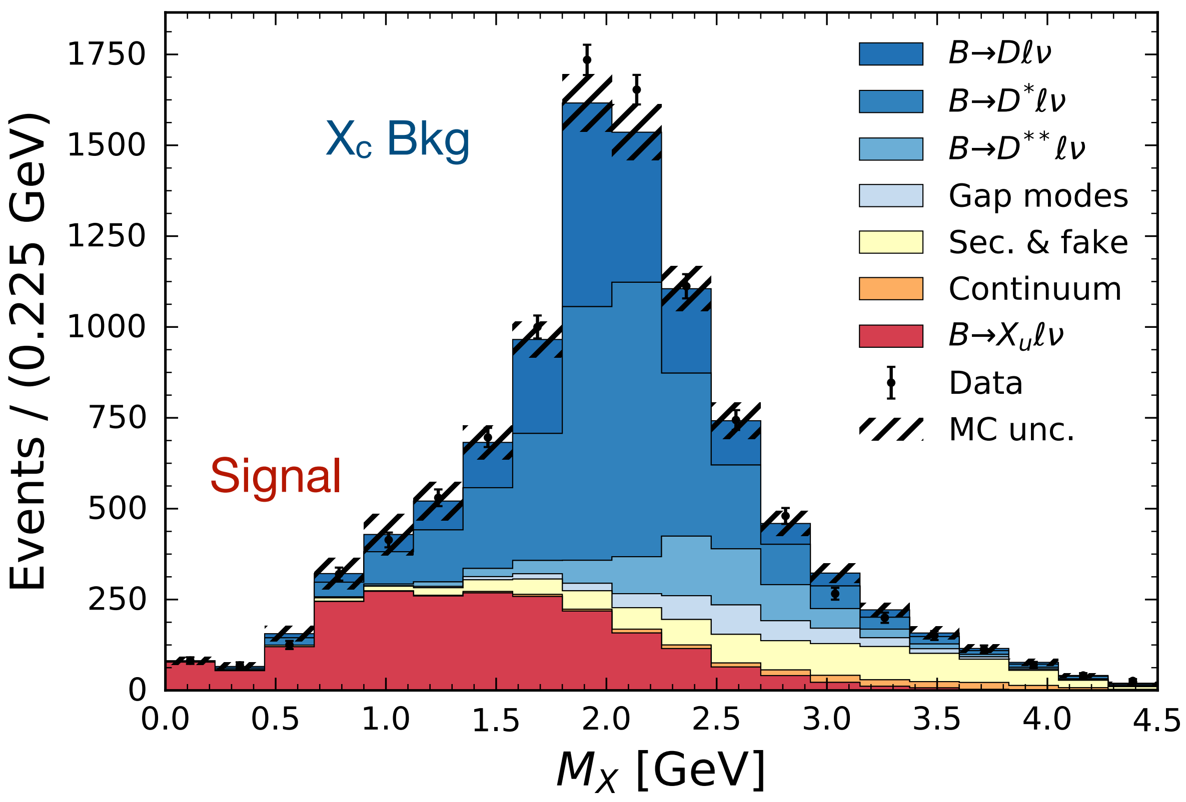
Can reject **98.7%** of X_c

Selection	$B \to X_u\,\ell^+\,\nu_\ell$	$B \to X_c\,\ell^+\,\nu_\ell$	Data
$M_{\rm bc} > 5.27\,\mathrm{GeV}$	84.8%	83.8%	80.2%
$\mathcal{O}_{\rm BDT} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\rm BDT} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\rm BDT} > 0.87$	14.5%	0.9%	1.1%

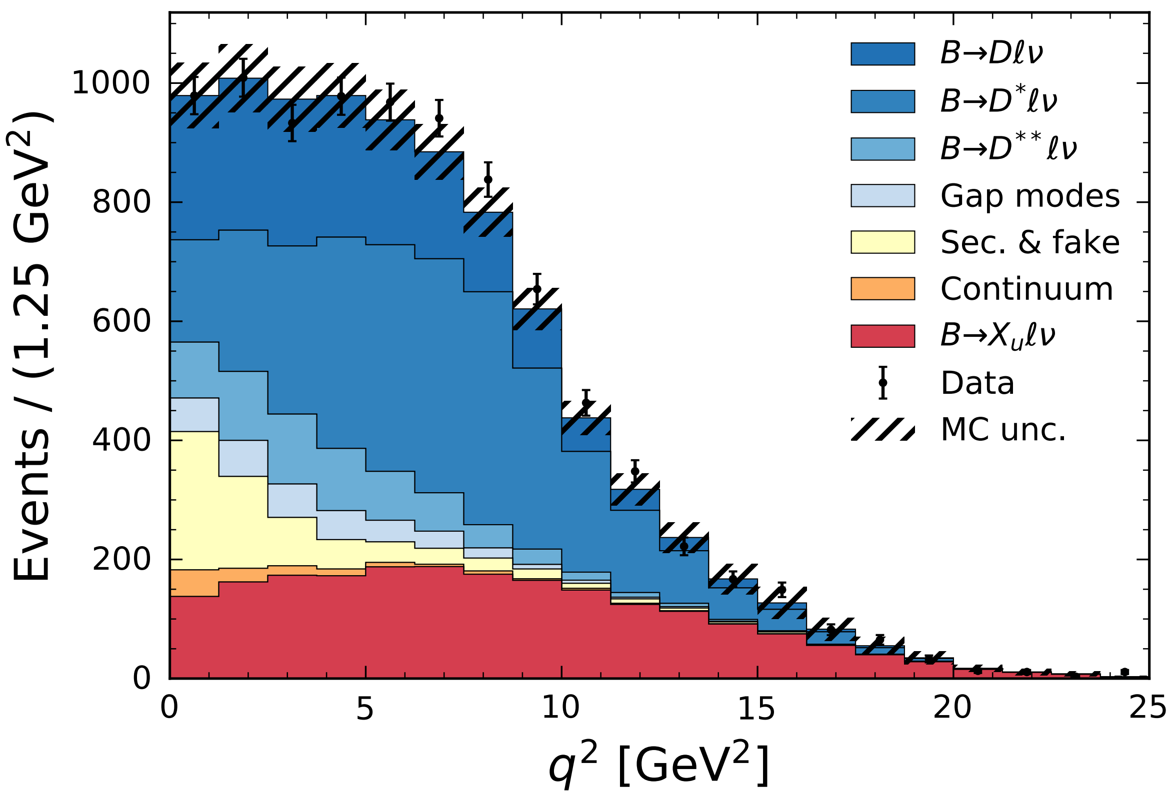
... and retain **18.5%** of X_u

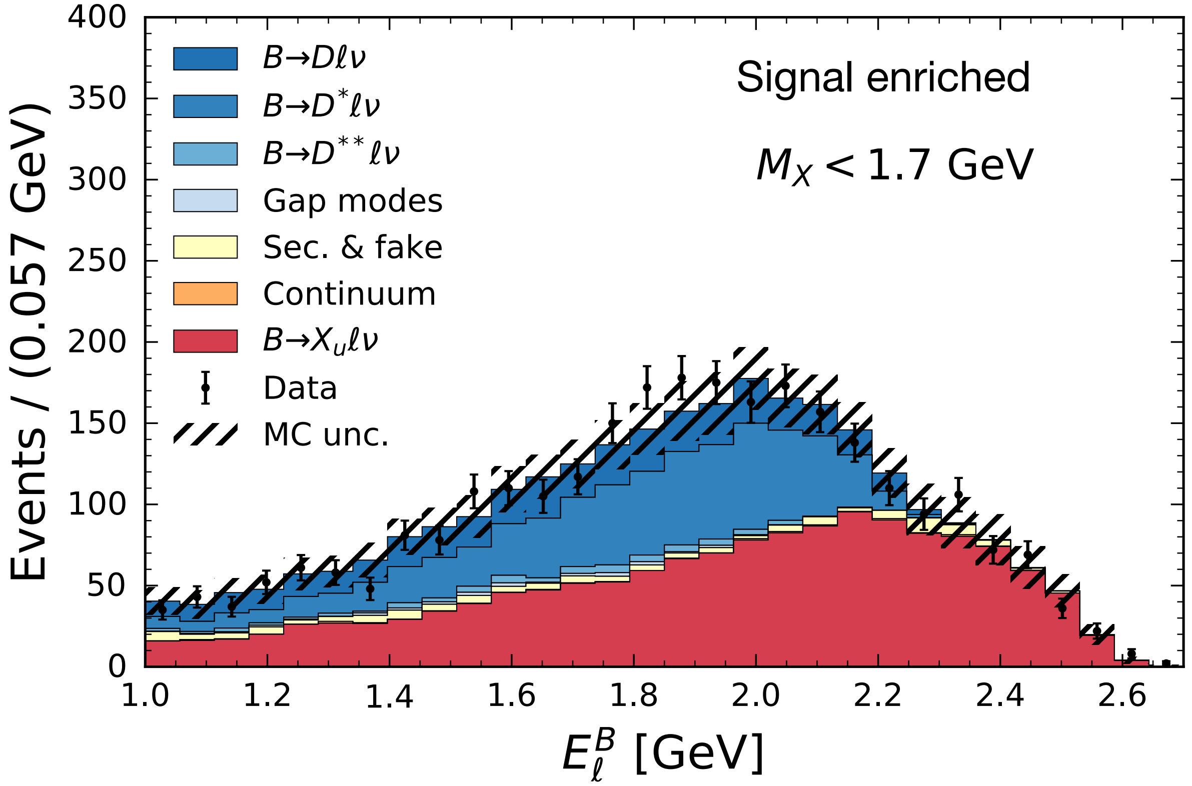
After BDT selection

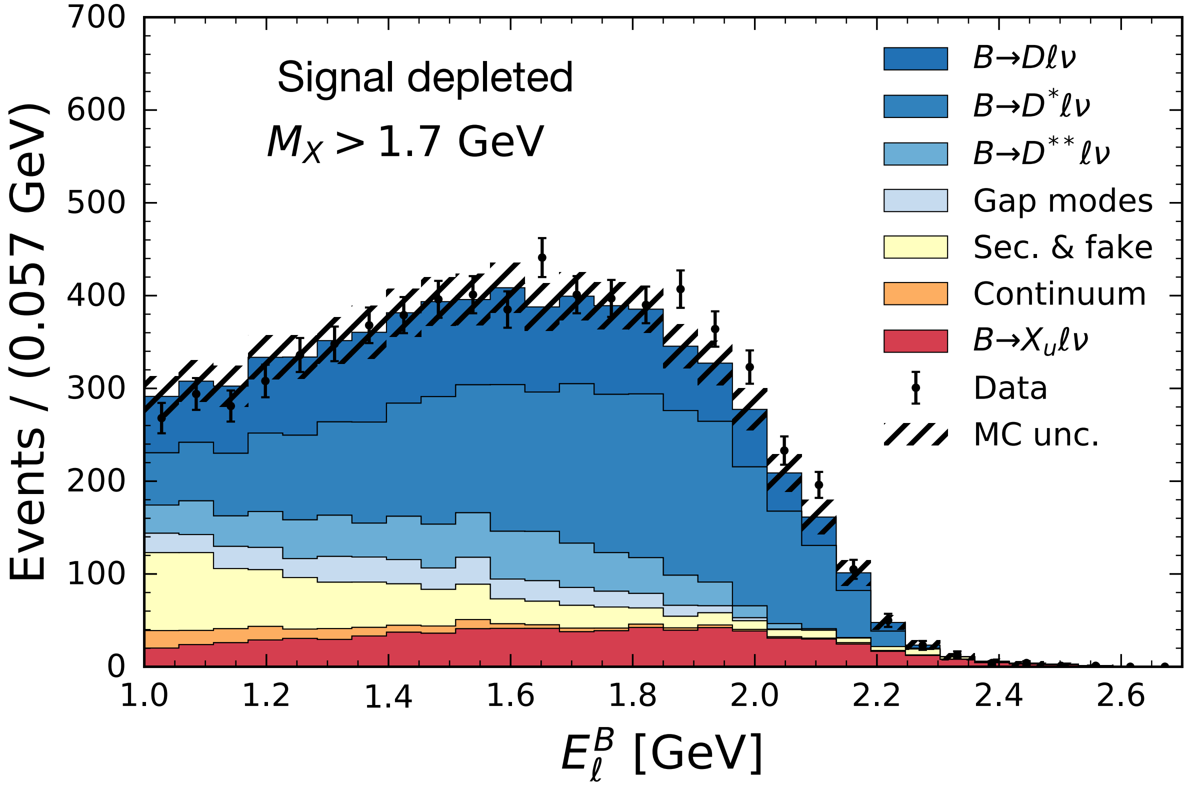
Hadronic Mass $M_X = \sqrt{p_X^2}$



Four-momentum transfer squared $q^2 = (p_B - p_X)^2$



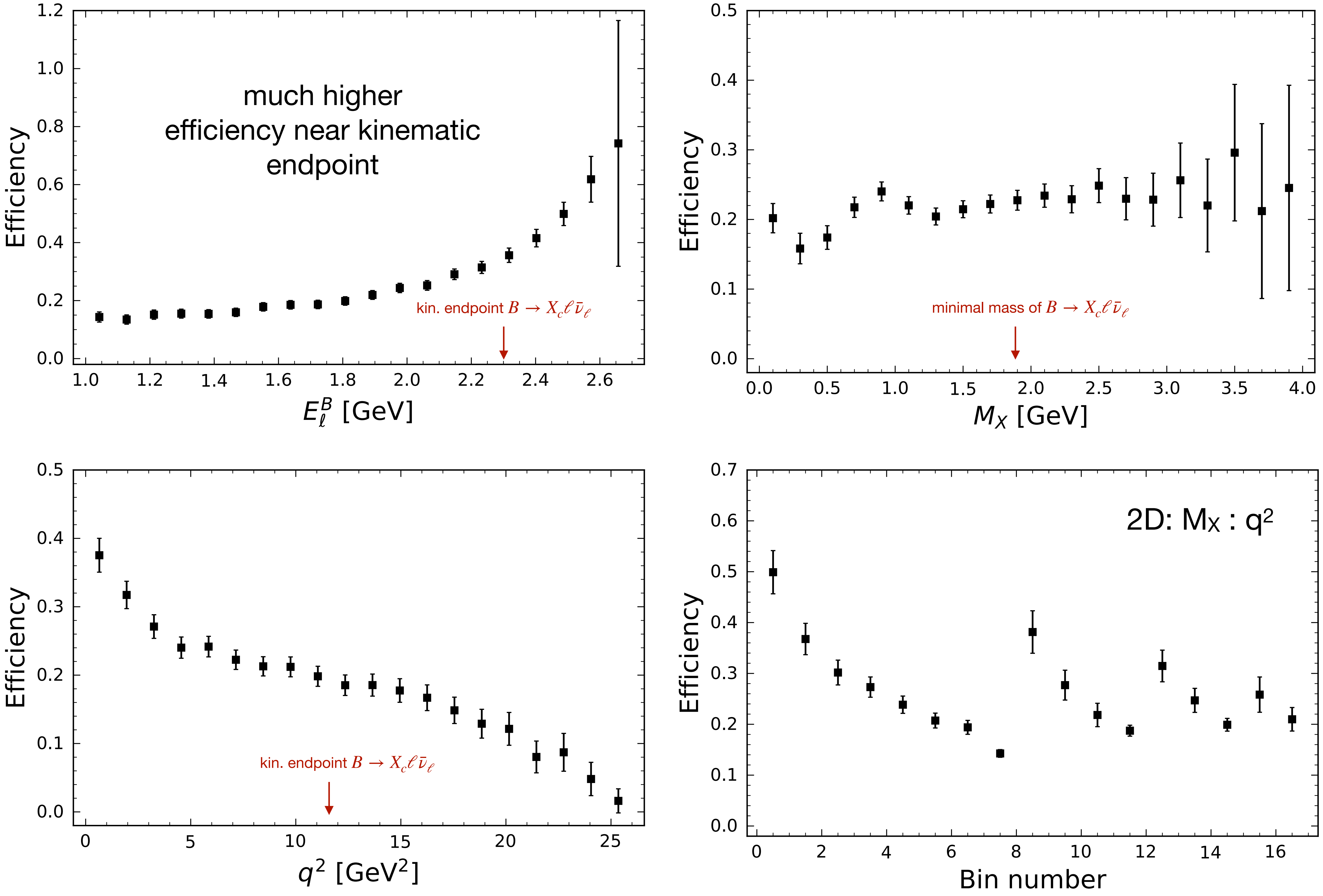




Lepton Energy in signal B restframe E_ℓ^B

The Cost

Shaping in efficiencies, so some model dependence



much higher efficiency near kinematic endpoint

2D: M_X : q^2

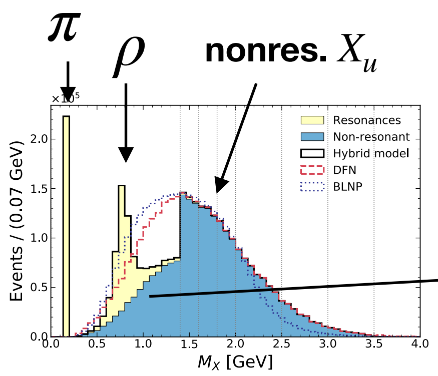
Fit for partial BFs

Subtraction of bkg in fit with coarse binning to minimize X_u modelling dependence

(low m_X, high q^2)

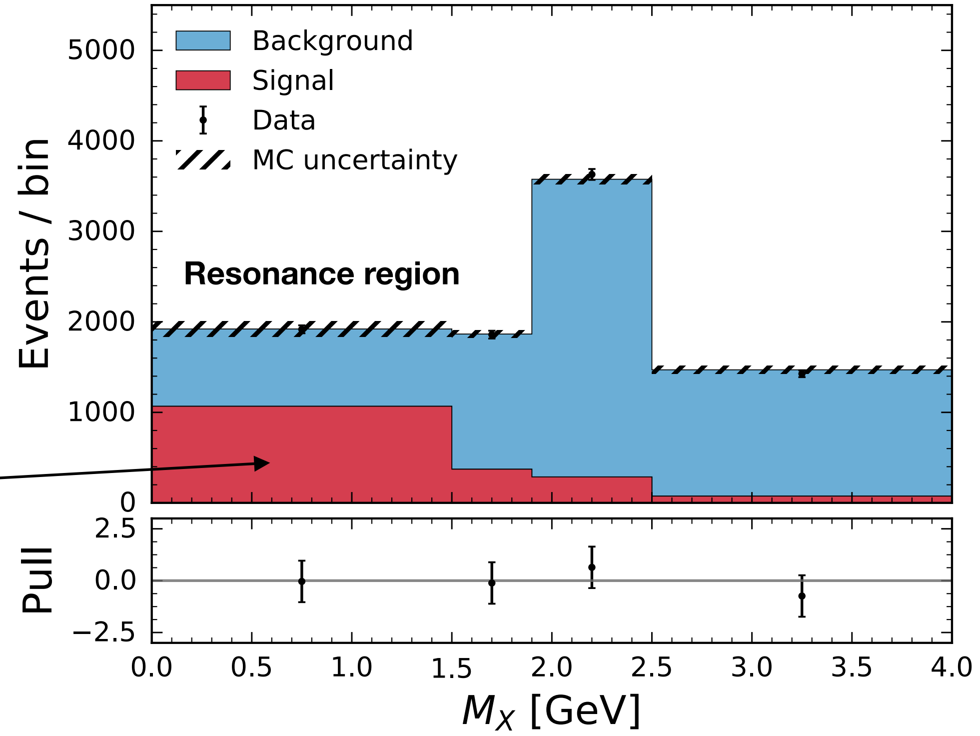
$$\mathcal{L} = \prod_i^{\text{bins}} \mathcal{P}\left(n_i; \nu_i\right) \times \prod_k \mathcal{G}_k \, ,$$

Signal and Bkg shape errors included in Fit via NPs



W/o detector smearing

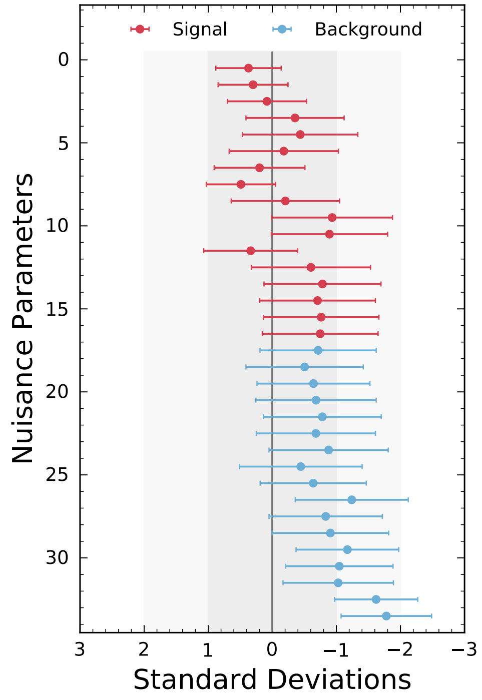
Projections of 2D fit in $m_X : q^2$

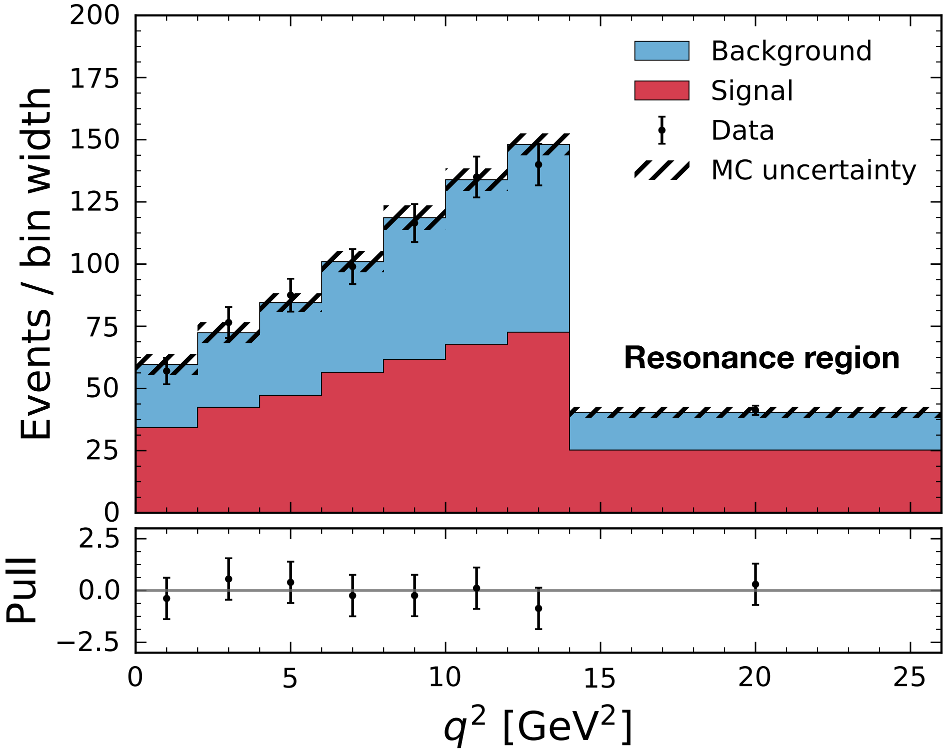


Unfold measured yields to **3 phase-space regions**:

Phase-space region
$M_X < 1.7\,\text{GeV}$
$M_X < 1.7\,\text{GeV},\ q^2 > 8\,\text{GeV}^2$
$E_\ell^B > 1\,\text{GeV}$

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\, \ell^+\, \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\, \ell^+\, \nu_\ell)}}$$

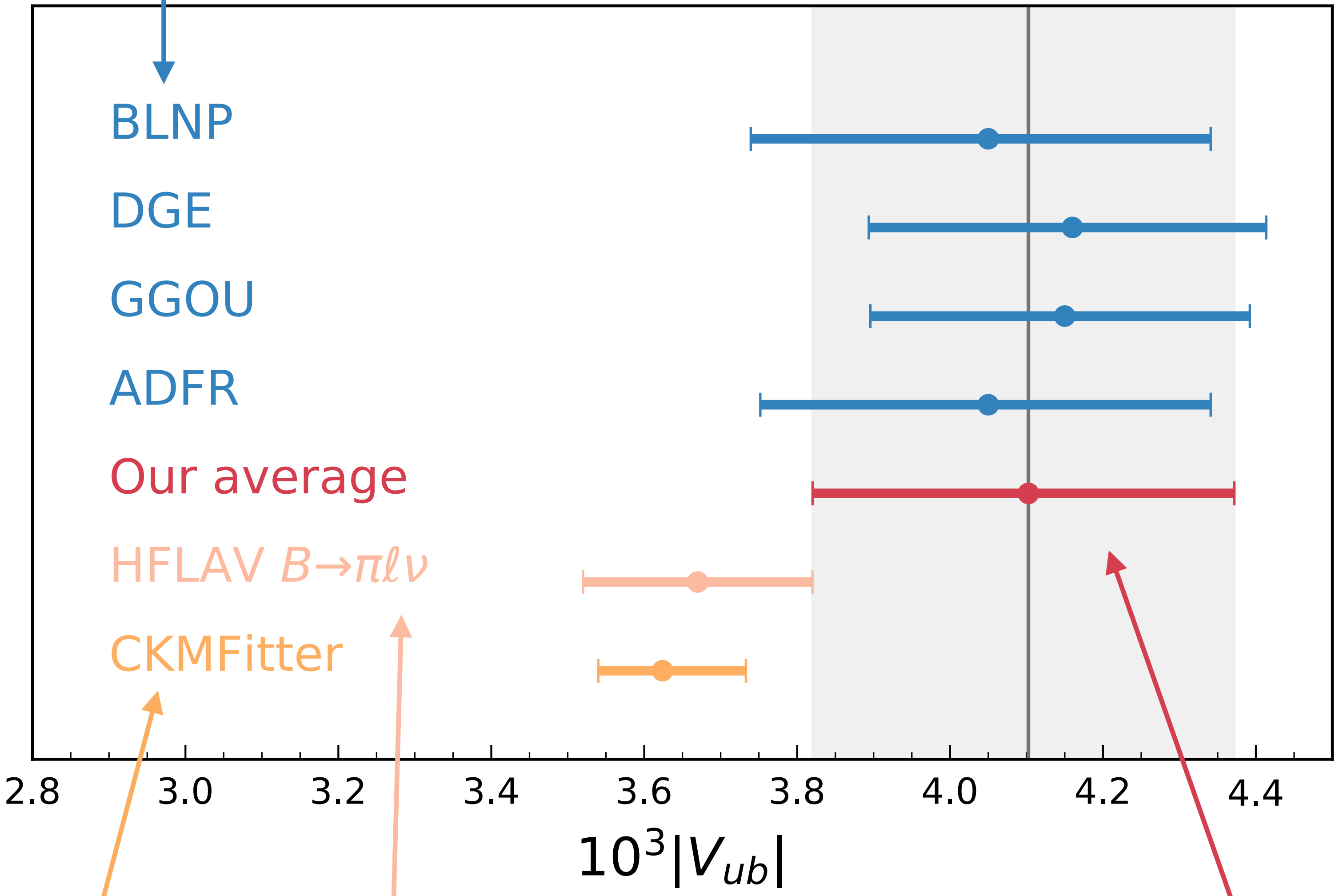




$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u\, \ell^+\, \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u\, \ell^+\, \nu_\ell)}}$$

Fit kinematic distributions and measure partial BF

4 predictions of the partial rate

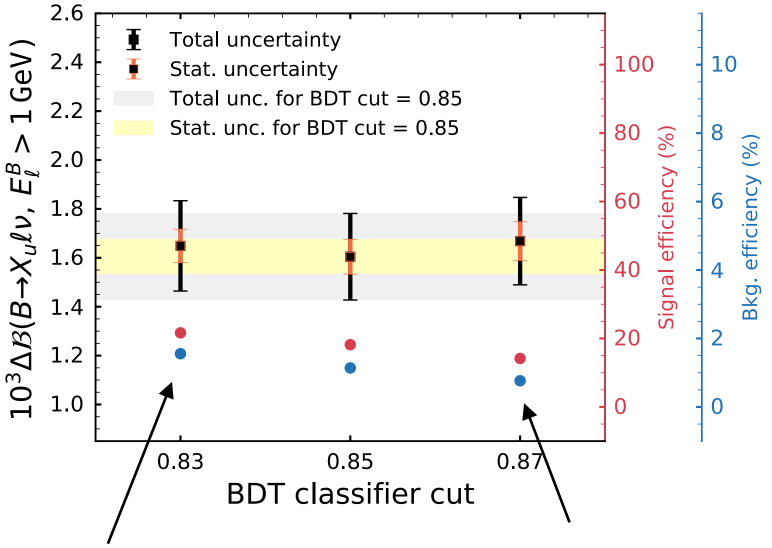


Exclusive Average for $B \to \pi\ell\bar{\nu}_\ell$:
$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$

Arithmetic average:
$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$

CKM Unitarity:
$|V_{ub}| = (3.62^{+0.11}_{-0.08}) \times 10^{-3}$

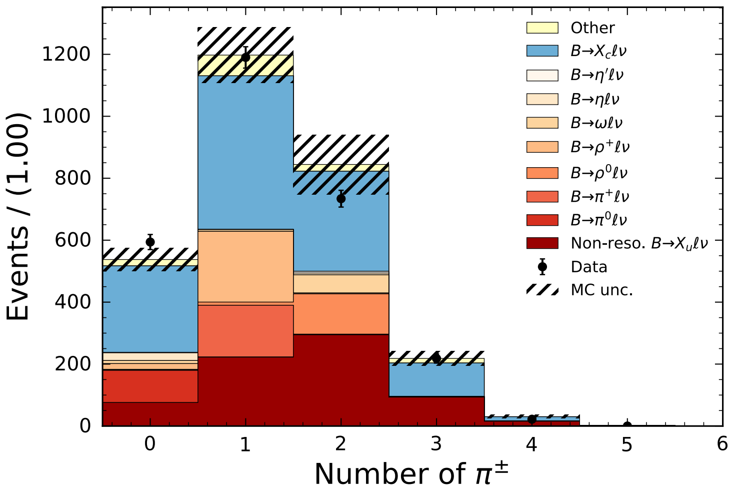
Stability as a function of BDT cut:



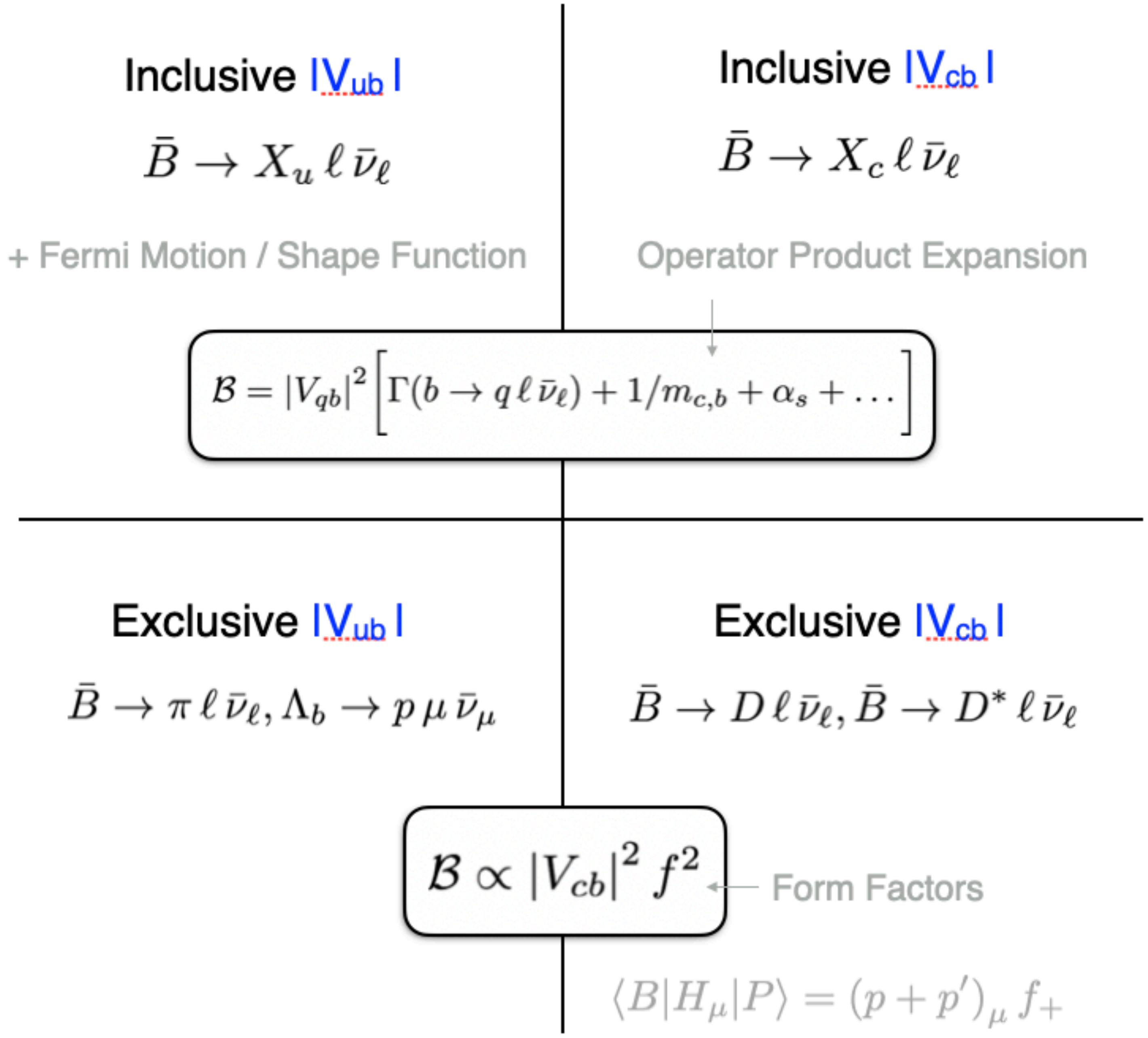
33%
more Bkg

37%
less Bkg

Post-fit N_{π^+} distribution:



Dr. Lu Cao



Inclusive |V$_{ub}$|

$$\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell$$

+ Fermi Motion / Shape Function

Inclusive |V$_{cb}$|

$$\bar{B} \to X_c \, \ell \, \bar{\nu}_\ell$$

Operator Product Expansion

$$\boxed{\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \to q \, \ell \, \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \ldots \right]}$$

Exclusive |V$_{ub}$|

$$\bar{B} \to \pi \, \ell \, \bar{\nu}_\ell, \Lambda_b \to p \, \mu \, \bar{\nu}_\mu$$

Exclusive |V$_{cb}$|

$$\bar{B} \to D \, \ell \, \bar{\nu}_\ell, \bar{B} \to D^* \, \ell \, \bar{\nu}_\ell$$

$$\boxed{\mathcal{B} \propto |V_{cb}|^2 \, f^2}$$

Form Factors

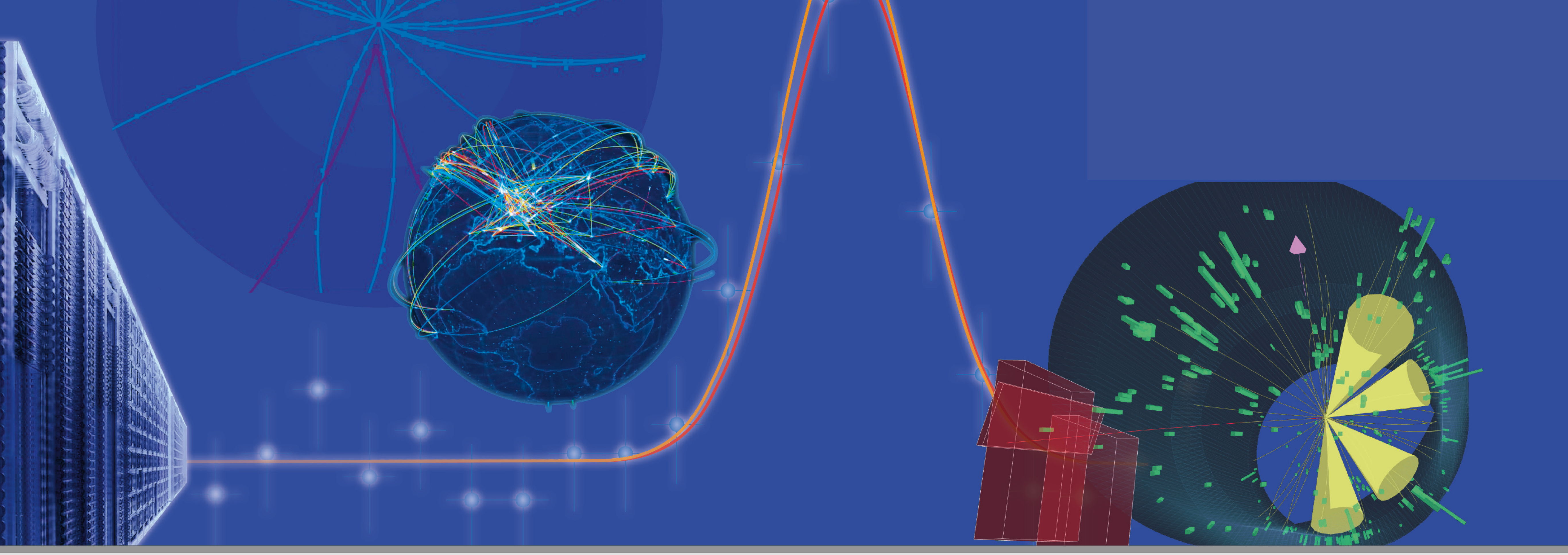
$$\langle B | H_\mu | P \rangle = (p + p')_\mu \, f_+$$

Time to stop — thank you for your attention!

Time flies if you are having fun :-)

Things we did not talk about (non-exhaustive list)

- Form Factor Expansions (i.e. what do you actually fit to your measurements)
- Experimental Questions that arise from this : Truncation uncertainties, etc.
- Measurements with τ (Additional introductory material is attached)
- Differential Measurements of inclusive $B \to X_u \ell \bar{\nu}_\ell$ and why they offer unique input to non-perturbative physics
- $B \to D^{**} \ell \bar{\nu}_\ell$ and the like
- NP Fits and Full angular measurements



A) Measurements with τ

or

let’s make this even harder :-)

Measurement Strategies

Signal

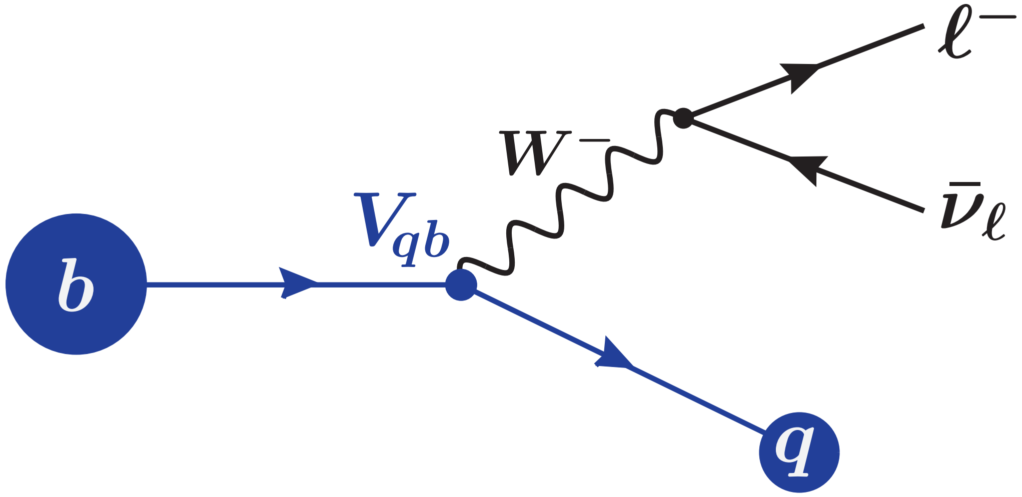
$$R = \frac{b \to q\,\tau\bar{\nu}_\tau}{b \to q\,\ell\bar{\nu}_\ell}$$

Normalization

$\ell = e, \mu$

1. Leptonic or Hadronic τ decays?

Some properties (e.g. τ **polarization**) readily **accessible** in **hadronic decays.**



2. Albeit not necessarily a rare decay of O(%) in BF, TRICKY to separate from normalisation and backgrounds

LHCb: **Isolation** criteria, **displacement** of τ, **kinematics**

B-Factories: Full reconstruction of event (**Tagging**), **matching topology, kinematics**

Measurement Strategies

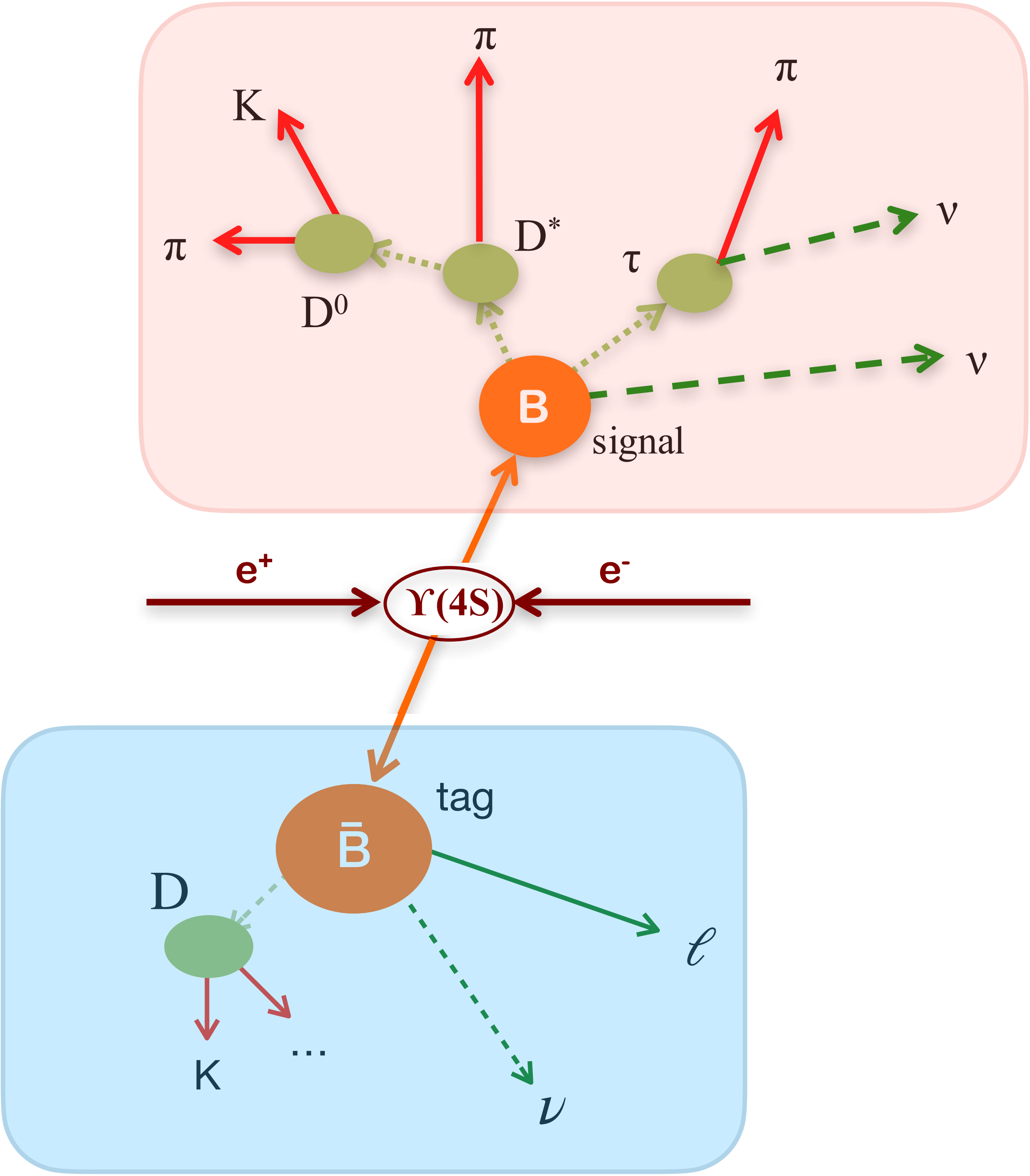
3. Semileptonic decays at B-Factories

- e^+/e^- collision produces ***Y(4S)* → *B*$\overline{B}$**
- Fully reconstruct one of the two B-mesons (‘tag’) → **possible** to **assign all particles** to either signal or tag B
- **Missing four-momentum (neutrinos)** can be reconstructed with high precision

$$p_{\text{miss}} = (p_{\text{beam}} - p_{B\text{tag}} - p_{D^{(*)}} - p_{\ell})$$

✓ **Small efficiency (~0.2-0.4%) compensated by large integrated luminosity**

Signal



Tag

Nice Illustration from C. Bozzi

Measurement Strategies

4. Semileptonic decays at LHCb

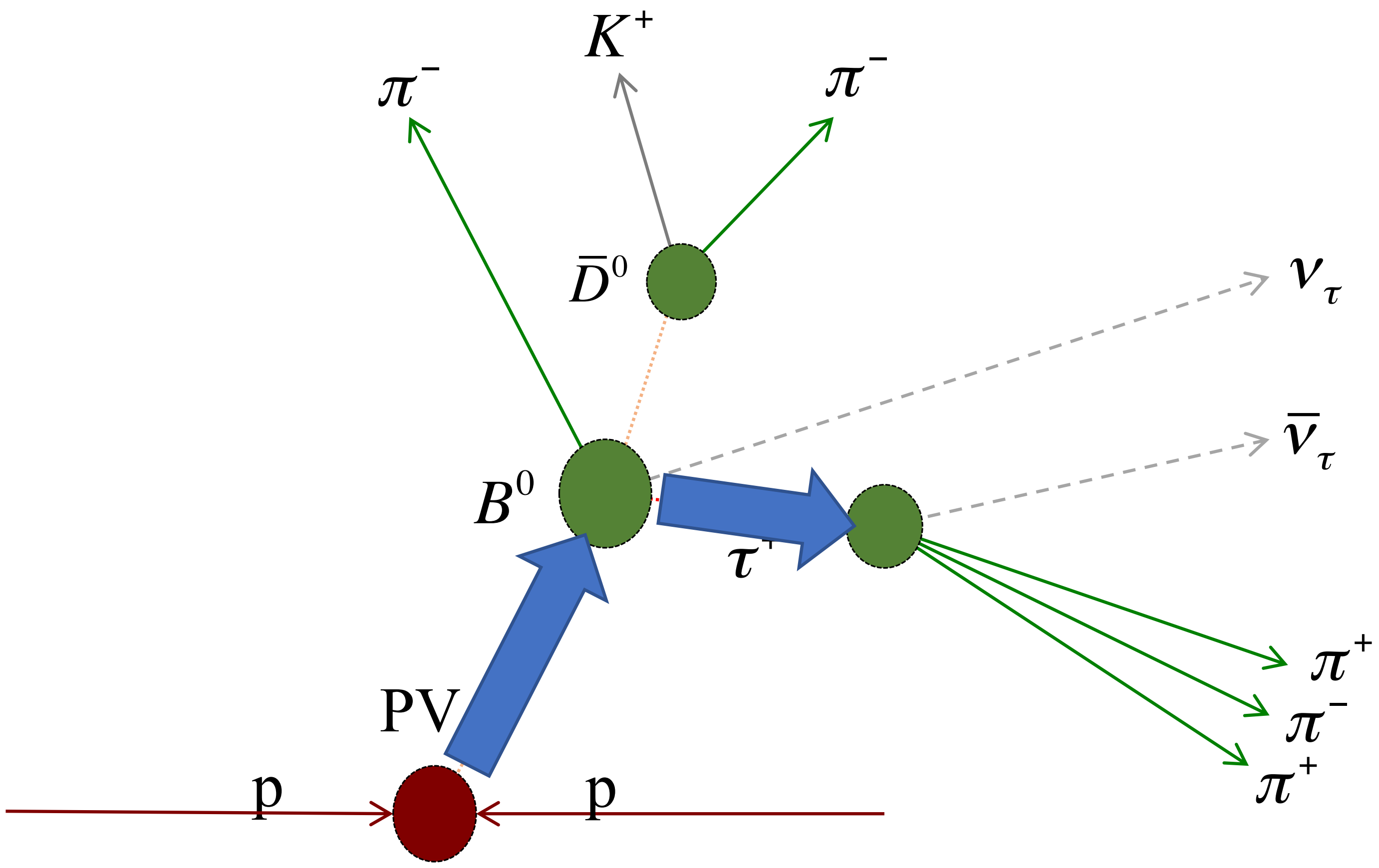
- No constraint from beam energy at a hadron machine, **but..**
- **Large Lorentz boost** with decay lengths in the range of **mm**

✓ **Well-separated decay vertices**

✓ **Momentum direction of decaying particle is well known**

- With known masses and other decay products can even **reconstruct four-momentum transfer squared q²** up to a two-fold ambiguity

$$q^2 = \left(p_{X_b} - p_{X_q}\right)^2$$



Nice Illustration from C. Bozzi

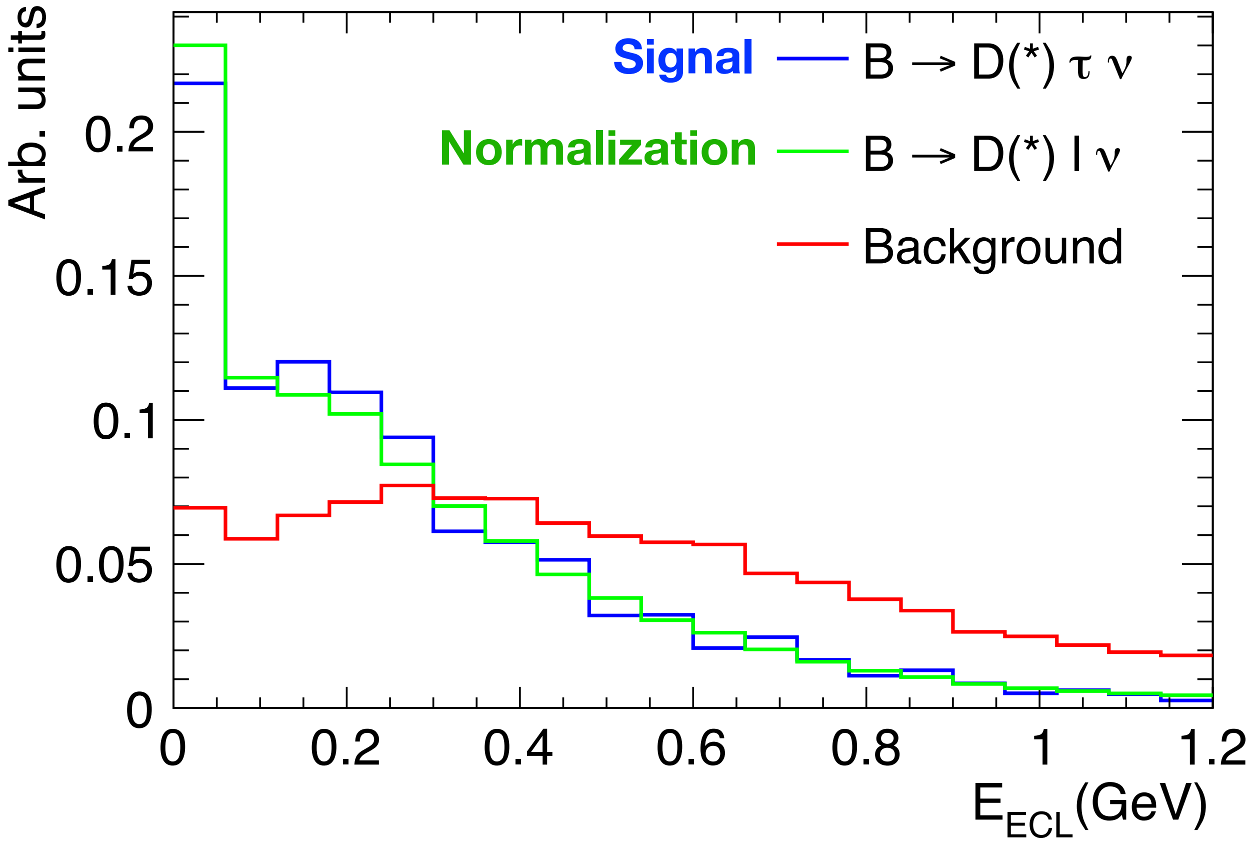
Even bit more complicated for leptonic tau decays

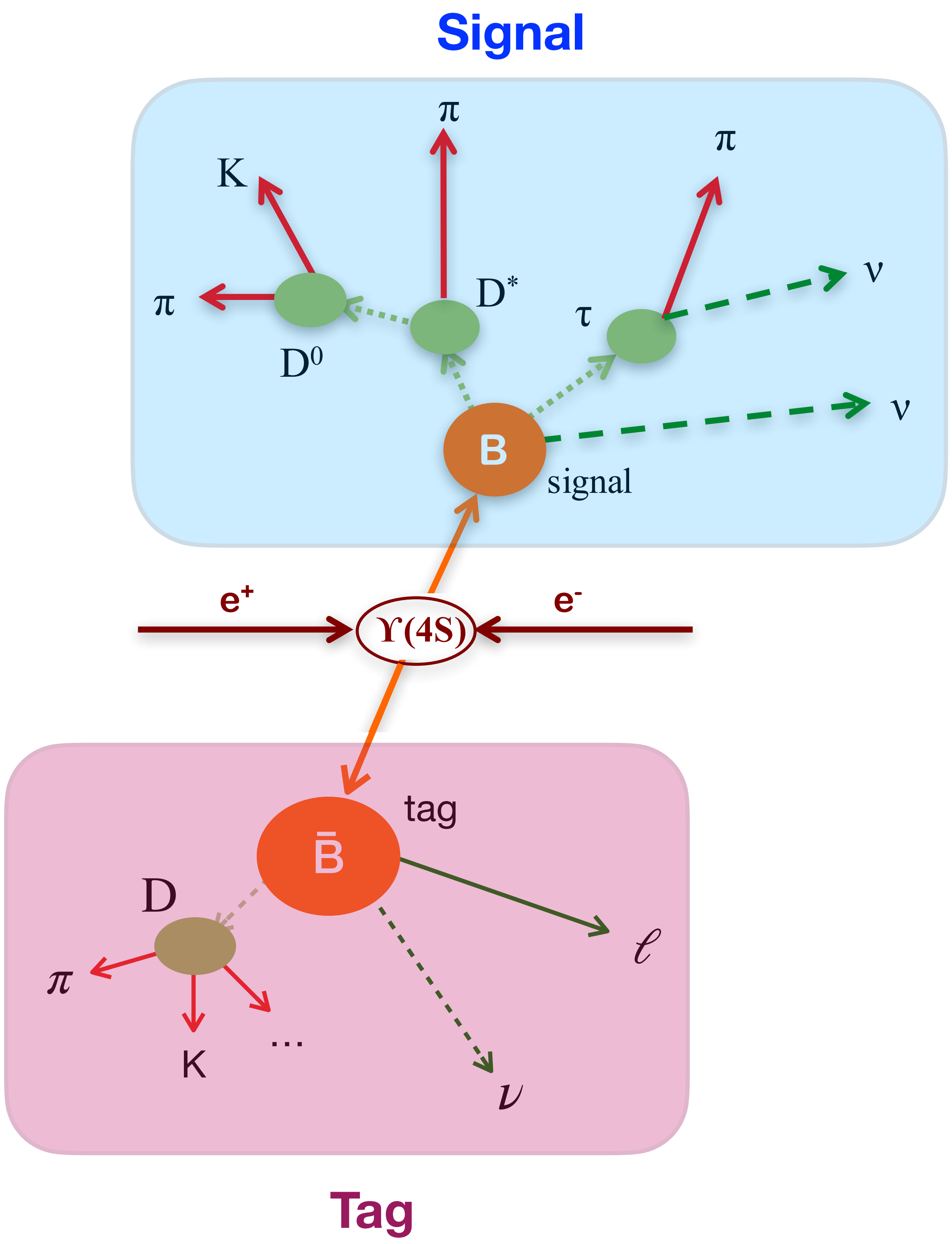
$R(D^{(*)})$ from Belle

G. Caria et al (Belle),
Phys. Rev. Lett. 124, 161803, April 2020
[arXiv:1904.08794]

- Reconstruct one of the two B-mesons (‘tag’) in **semileptonic modes → possible to assign all particles in detector** to tag- & signal-side
- Demand Matching topology + **unassigned** energy in the calorimeter E_{ECL} to discriminate background from signal

$$E_{\mathrm{extra}} = E_{\mathrm{ECL}} = \sum_i E_i^{\gamma}$$

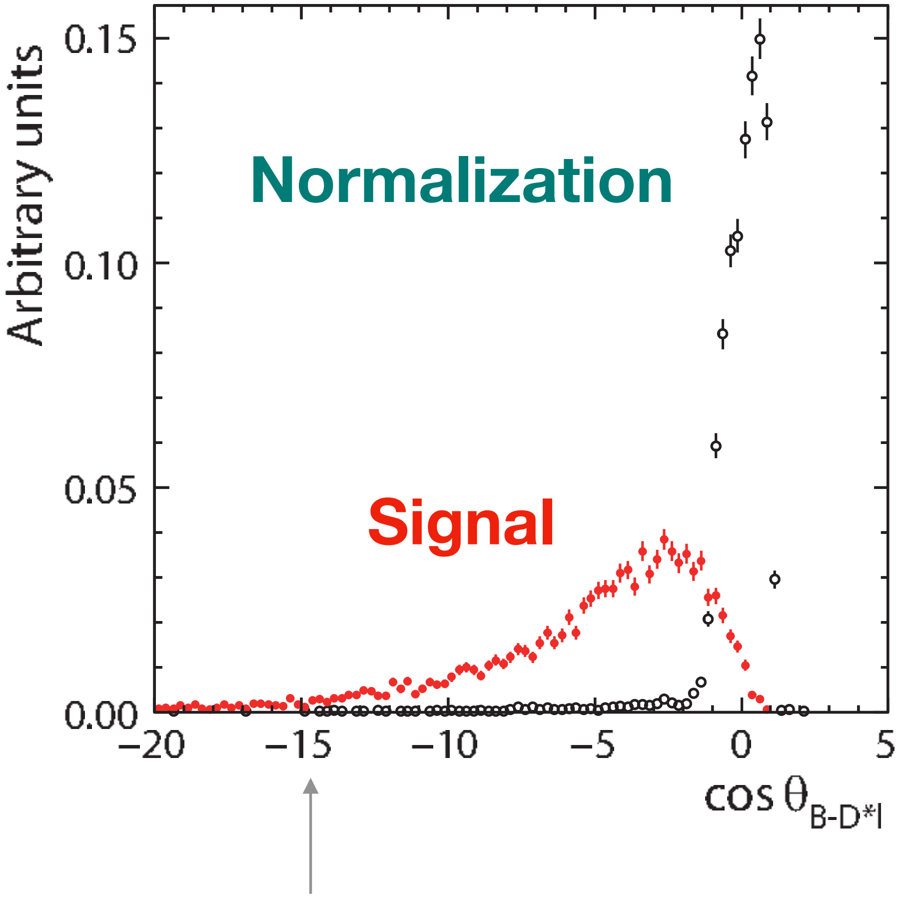


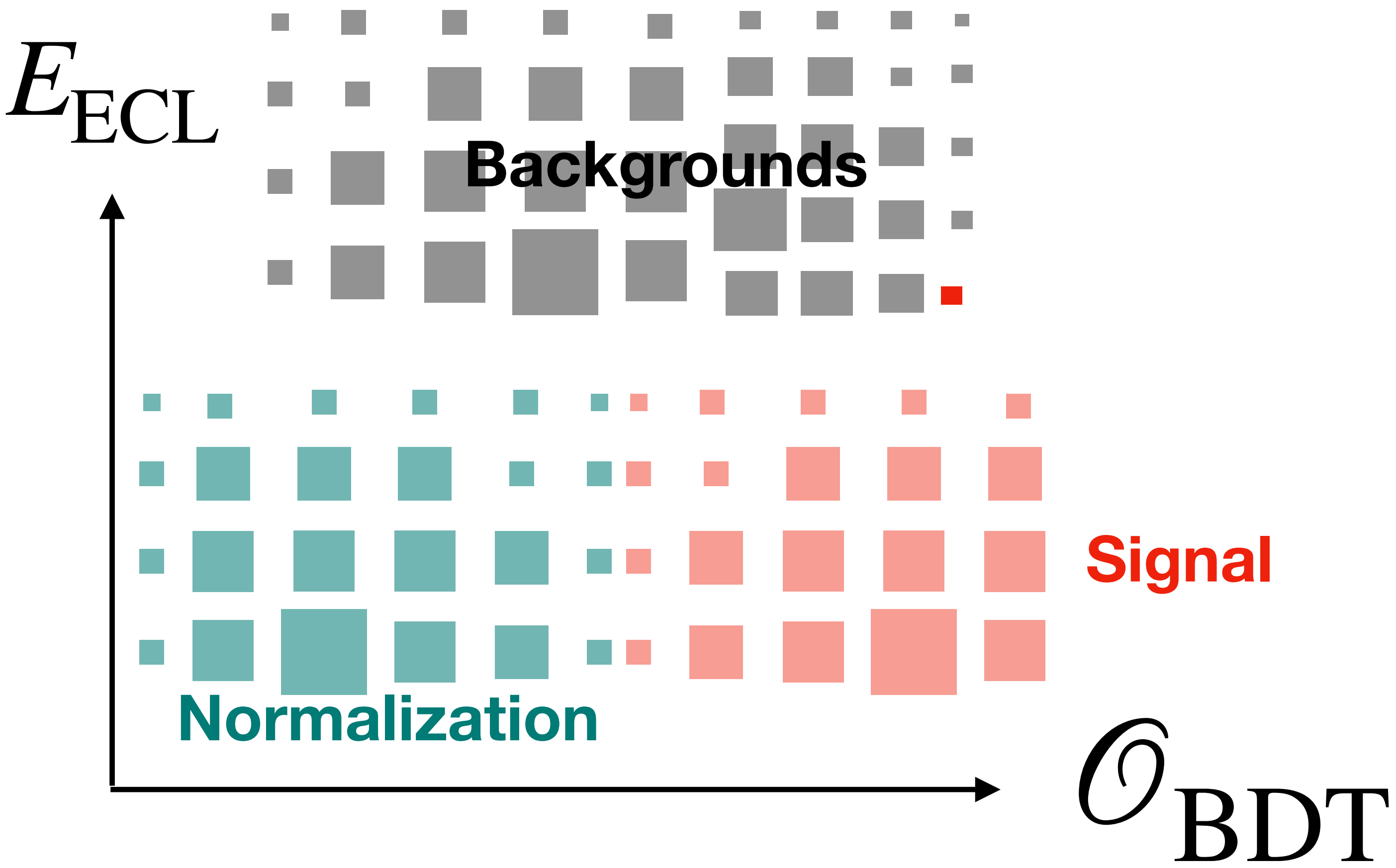


Separation of signal & normalization

- Use kinematic properties to separate $B \to D^{(*)}\tau\nu$ signal from $B \to D^{(*)}\ell\nu$ normalization

- Construct BDT with 3 variables: $\cos\theta_{B-D^{(*)}\ell}$, E_{vis}, $m^2_{\mathrm{miss}} = p^2_{\mathrm{miss}}$

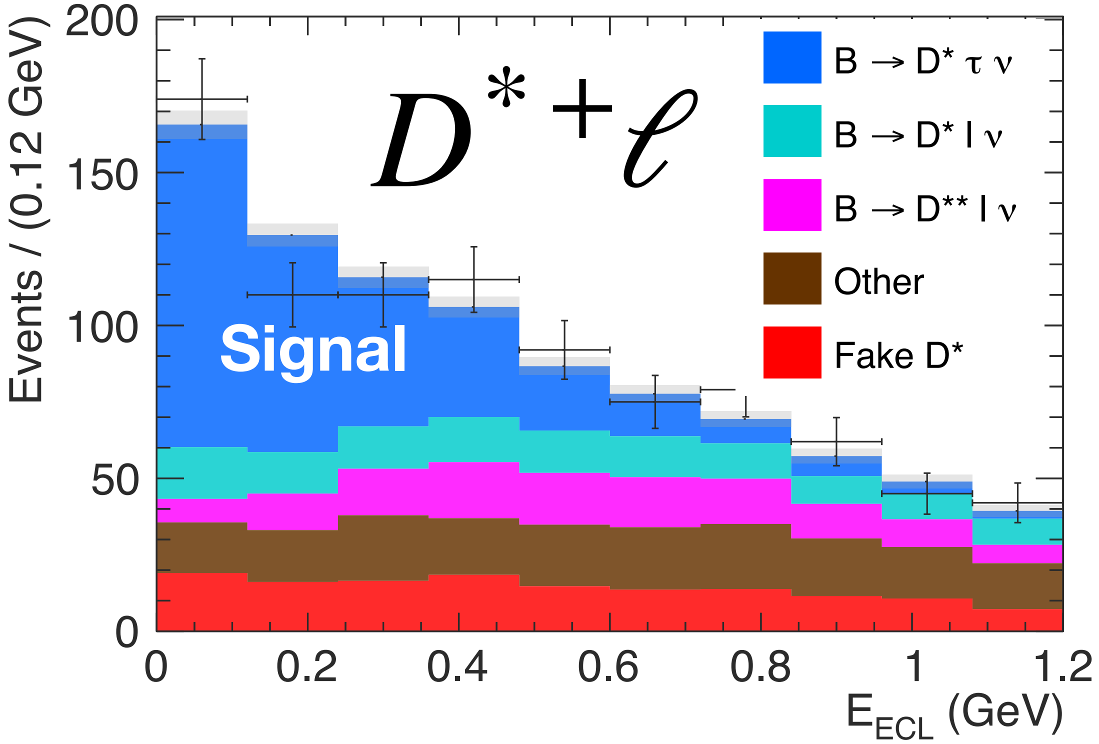




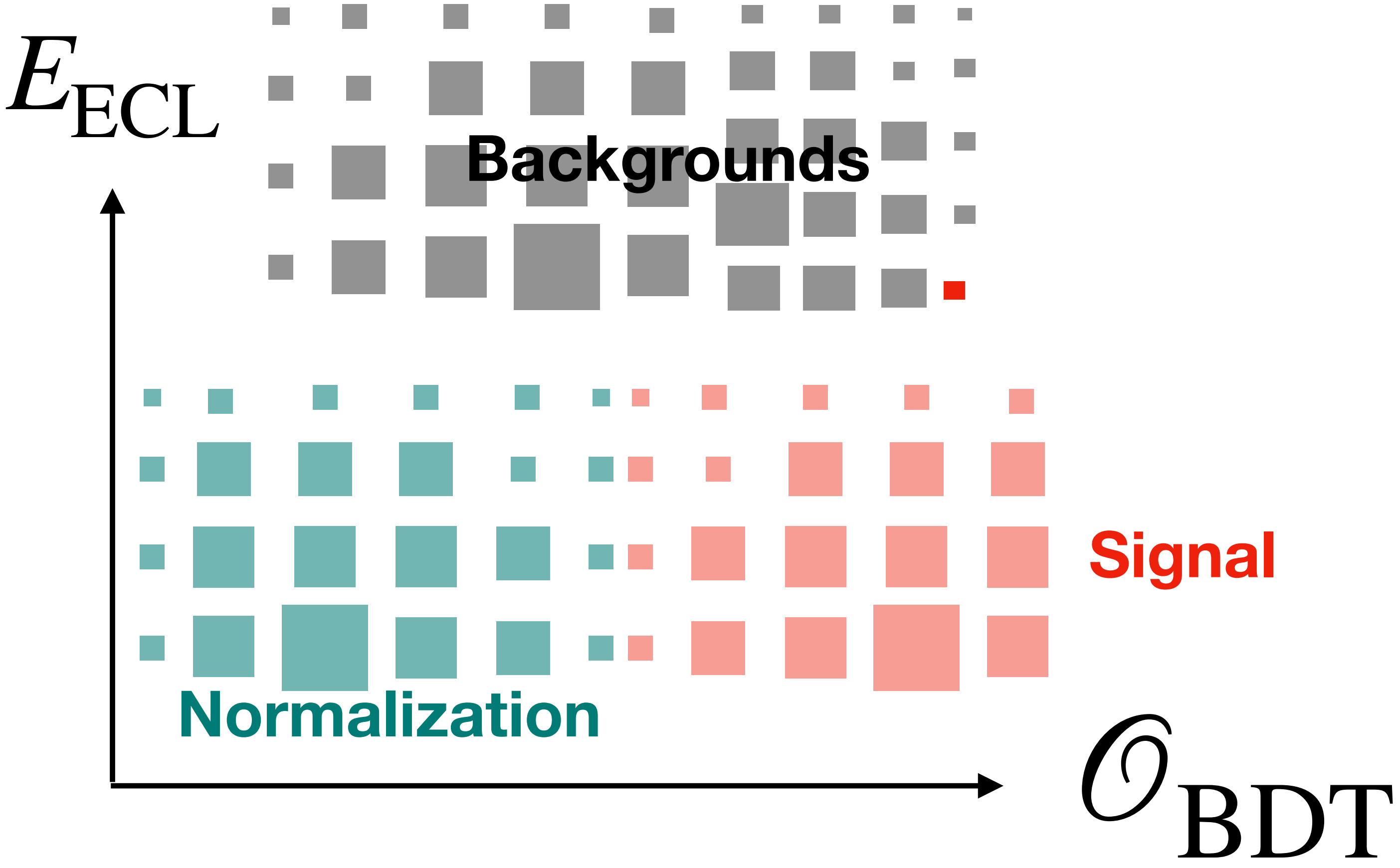
In case you are wondering how a cosine can be outside [-1,1]: it's because the reconstruction uses measured energies and the definition assumes only a single missing neutrino

Separation of signal & normalization

- Use kinematic properties to separate $B \to D^{(*)}\tau\nu$ signal from $B \to D^{(*)}\ell\nu$ normalization
- Construct BDT with 3 variables: $\cos\theta_{B-D^{(*)}\ell}, E_{\text{vis}}, m^2_{\text{miss}} = p^2_{\text{miss}}$



Signal-enriched selection with cut on $\mathcal{O}_{\text{BDT}}$



Separation of signal & normalization

- Use kinematic properties to separate $B \to D^{(*)}\tau\nu$ signal from $B \to D^{(*)}\ell\nu$ normalization

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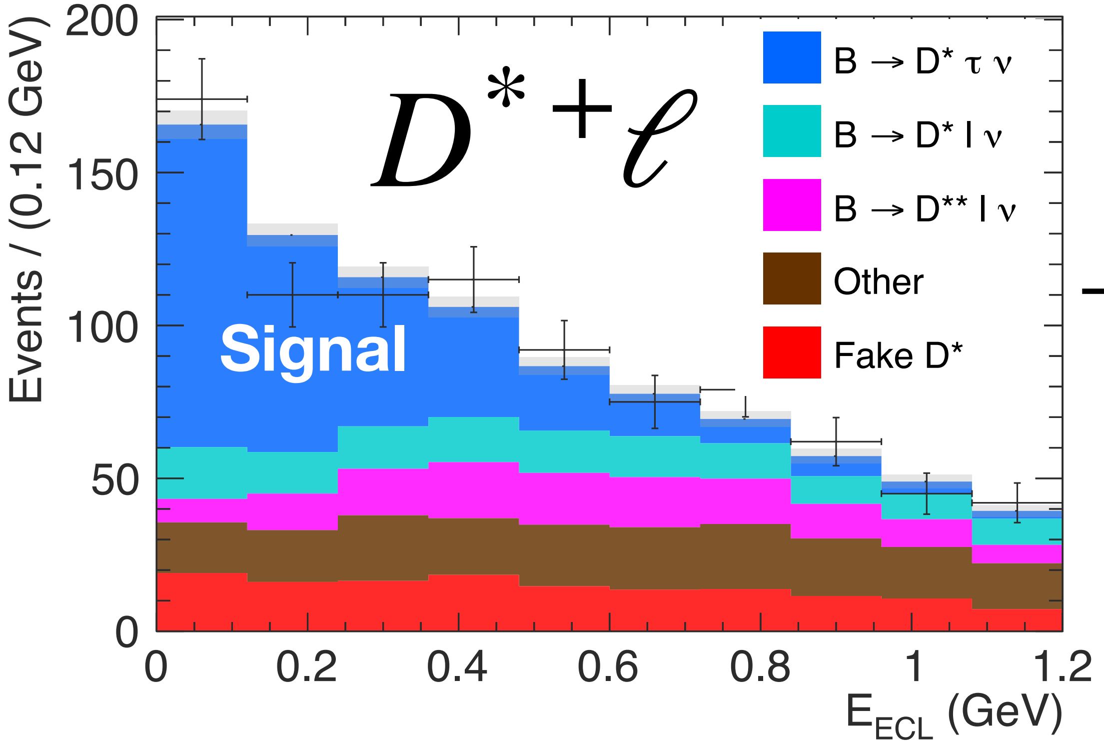


Signal enriched selection with cut on $\mathcal{O}_{\text{BDT}}$

→

$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

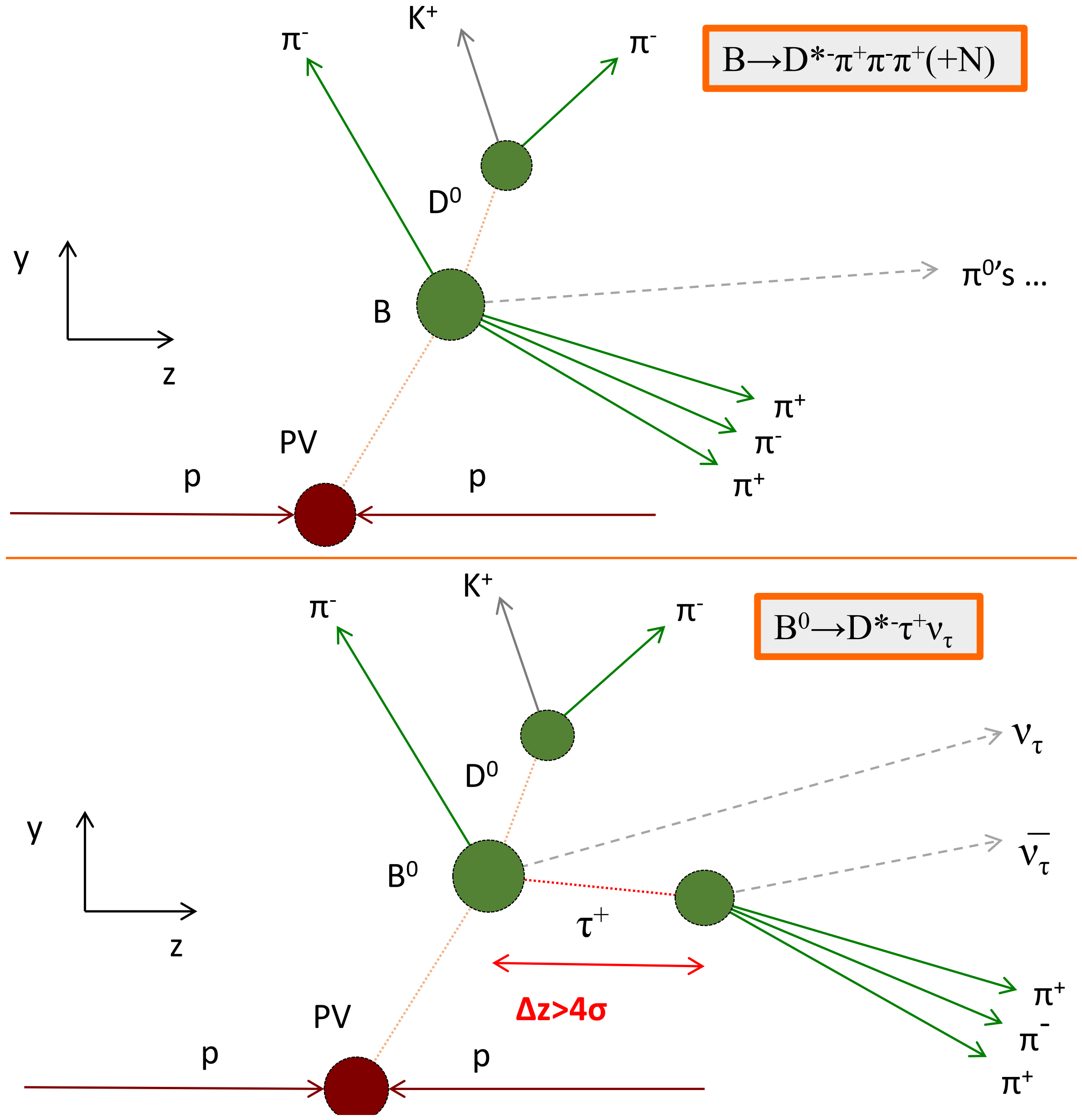
Most precise measurement to date

LHCb Measurement of $R(D^*)$

R. Aaij et al (LHCb),
Phys.Rev.Lett.120,171802 (2018) [arXiv:1708.08856]
Phys.Rev.D 97, 072013 (2018) [arXiv:1709.02505]

- Tau reconstructed via $\tau \rightarrow \pi^+\pi^+\pi^-(\pi^0)\nu$, only two neutrinos missing

Although a semileptonic decay is studied, nearly no background from $B \rightarrow D^* X \mu \nu$



- Main background: prompt **$X_b \rightarrow D^*\pi\pi\pi$ + neutrals**

BF ~ 100 times larger than signal, all pions are promptly produced

- Suppressed by requiring minimum distance between ***X_b & τ vertices (> 4 $\sigma_{\Delta z}$)***

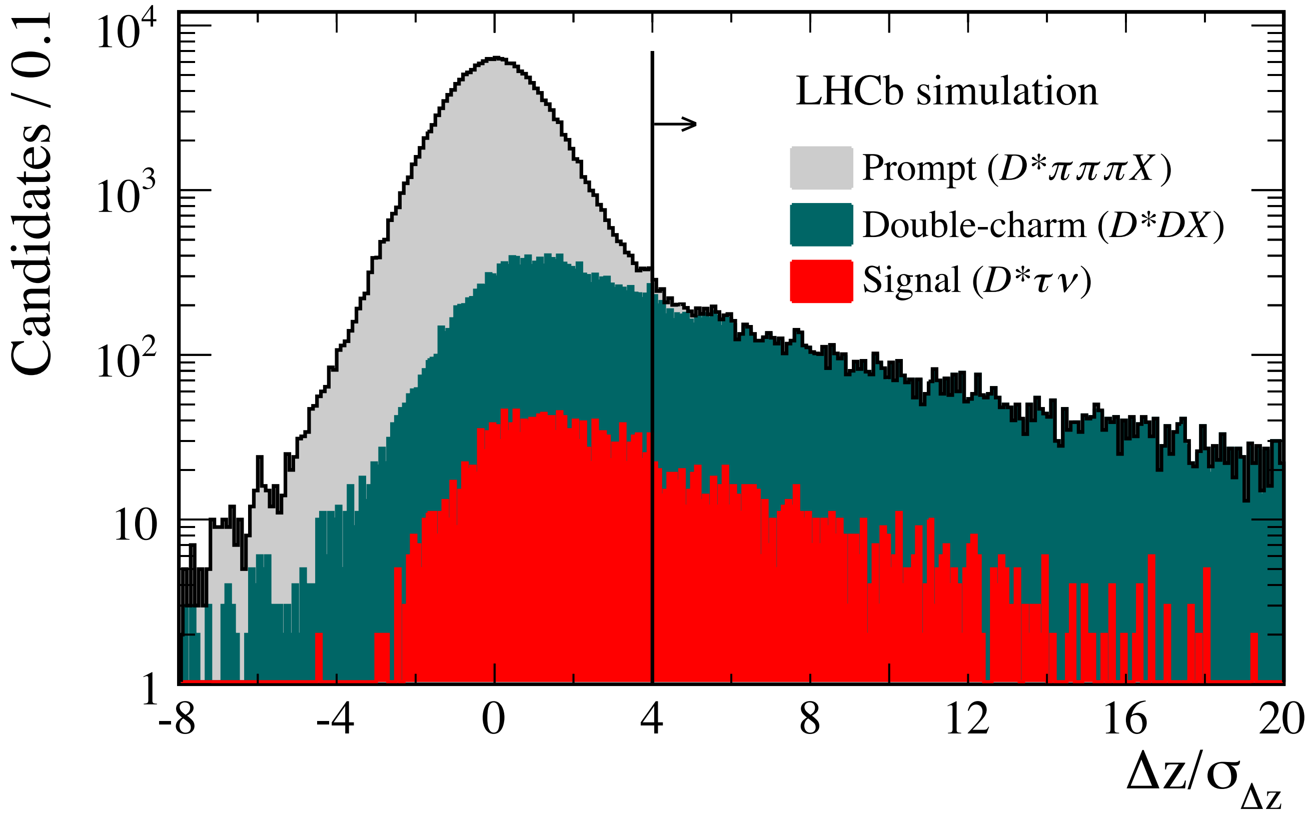
$\sigma_{\Delta z}$: resolution of vertices separation

- Reduces this background by three orders of magnitude

LHCb Measurement of $R(D^*)$

- Tau reconstructed via $\tau \to \pi^+\pi^+\pi^-(\pi^0)\nu$, only two neutrinos missing

Although a semileptonic decay is studied, nearly no background from $B \to D^* X\mu\, \nu$



- Remaining double charm bkgs:

$X_b \to D^{*-}D_s^+X$ ~ 10 x Signal

$X_b \to D^{*-}D^+X$ ~ 1 x Signal

$X_b \to D^{*-}D_{s0}^+X$~ 0.2 x Signal

- Main background: prompt **$X_b \to D^*\pi\pi\pi$ + neutrals**

BF ~ 100 times larger than signal, all pions are promptly produced

- Suppressed by requiring minimum distance between ***X_b & τ vertices (> 4 $\sigma_{\Delta z}$)***

$\sigma_{\Delta z}$: resolution of vertices separation

- Reduces this background by three orders of magnitude

LHCb Measurement of $R(D^*)$

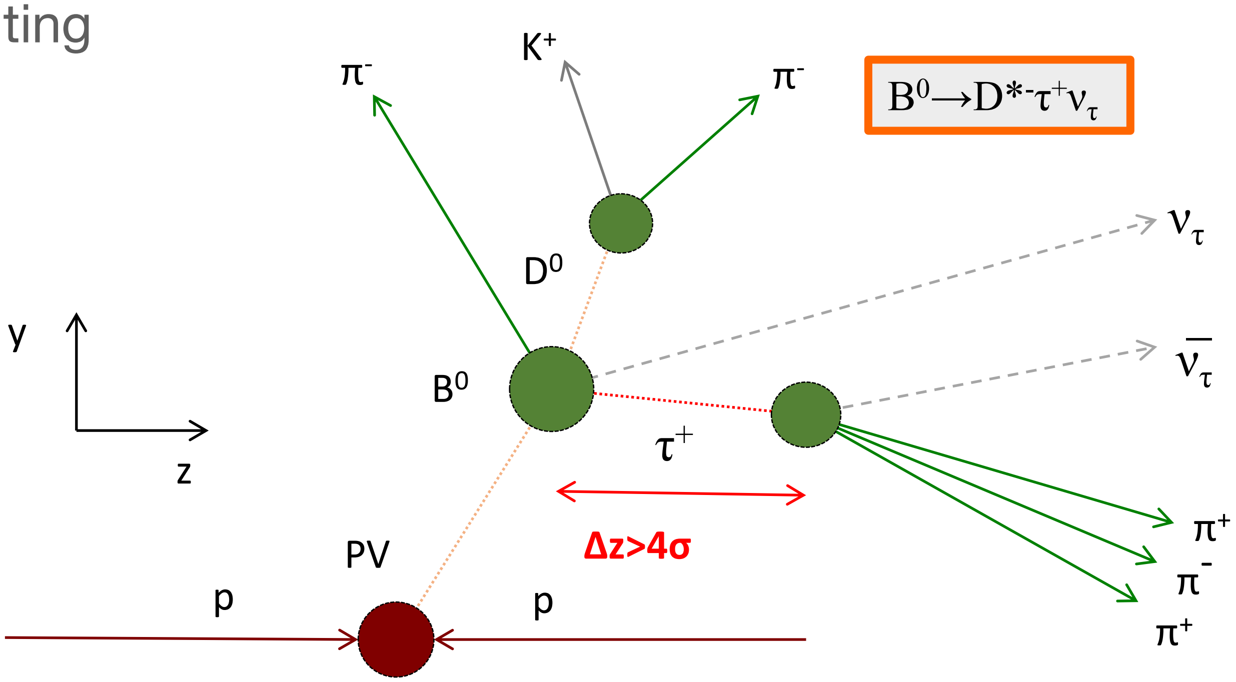
- Remaining backgrounds reduced via isolation & MVA

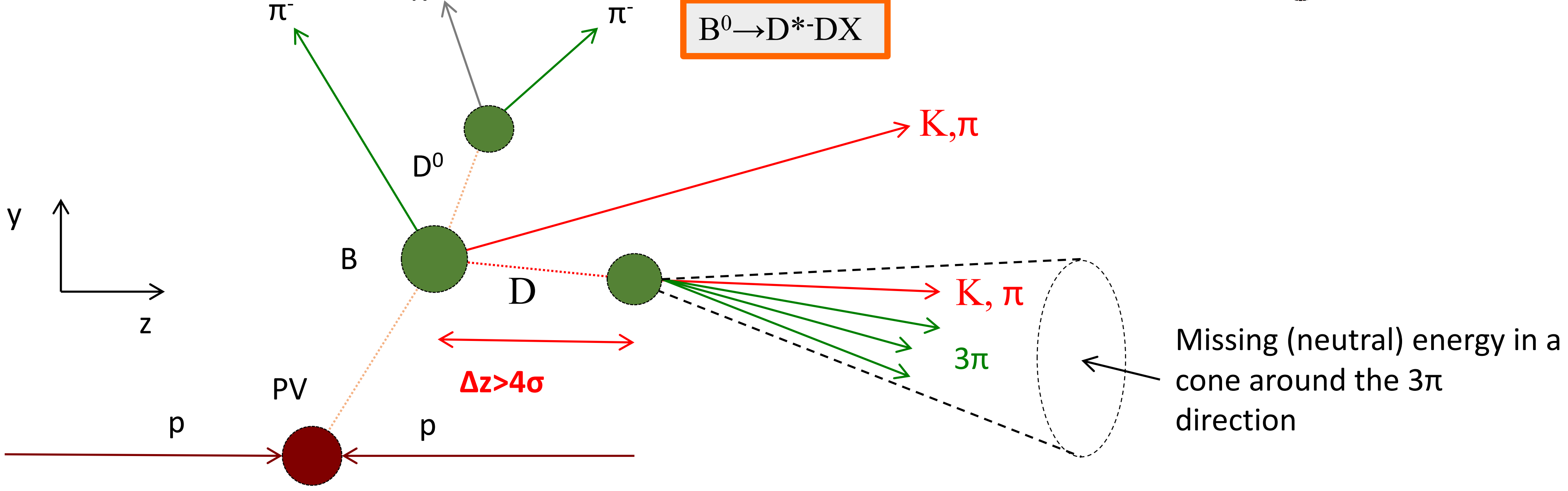
Require signal candidates to be well isolated

i.e. reject events with extra charged particles pointing to the B and/or τ

Events with additional neutral energy are suppressed with a MVA

More information about that in backup





LHCb Measurement of $R(D^*)$

- Extraction in **3D fit** to

MVA : q² : τ decay time

Invariant masses of 3π system
Invariant mass of $D^*3\pi$ system
Neutral isolation variables

q² reconstructed with some tricks (more in backup)

4 Bins 8 Bins 8 Bins

- Components:

1 Signal component for $\tau\rightarrow\pi^+\pi^+\pi^-(\pi^0)\nu$

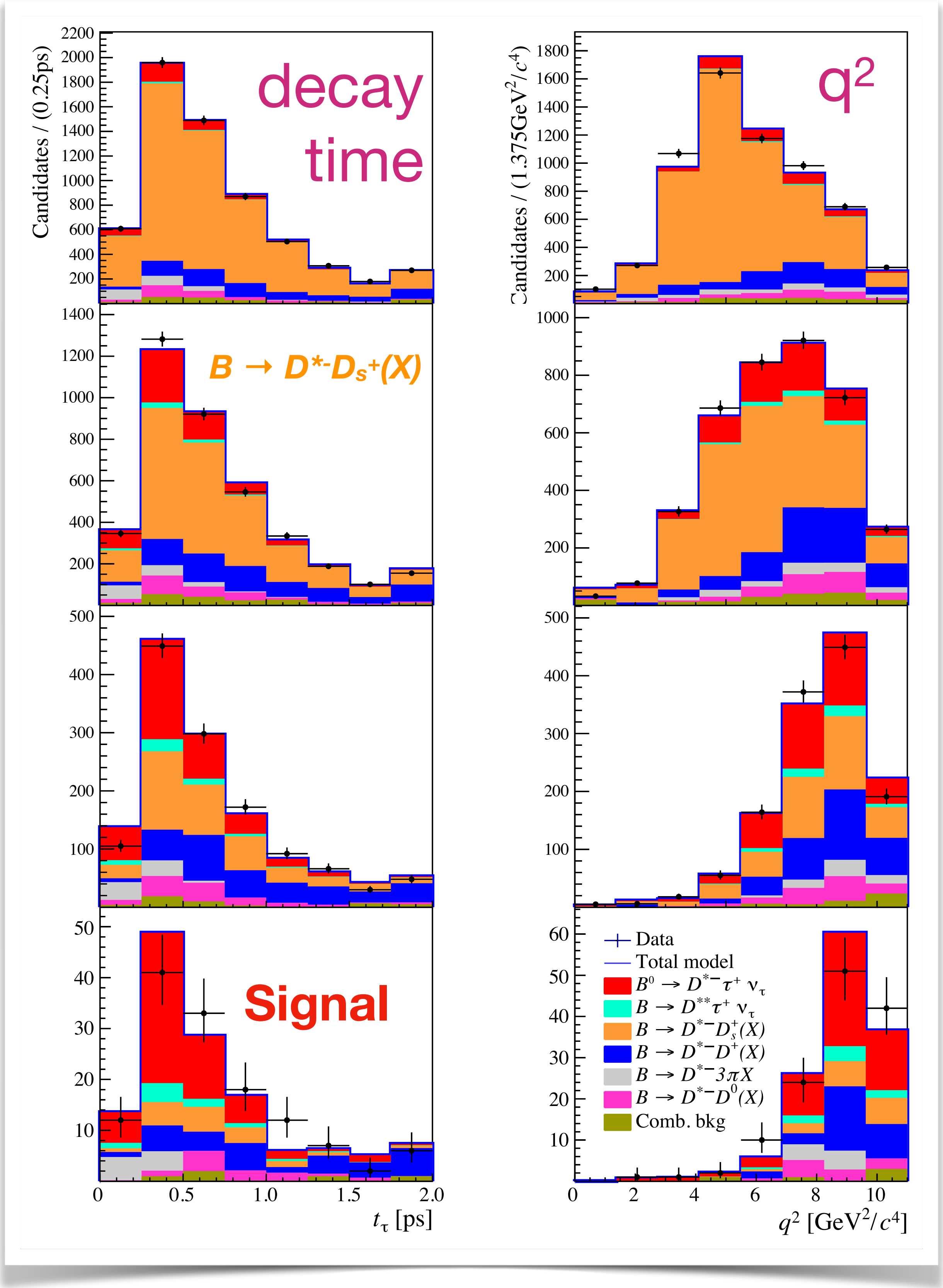
11 Background components

- ~ 1296 ± 86 Signal events
- Using normalization mode and light lepton BFs:

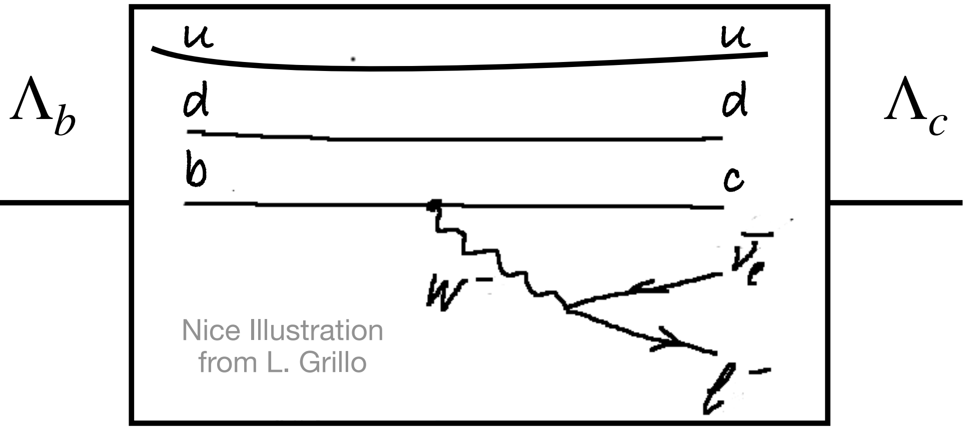
More information about normalization in backup

R(D*) = 0.286 ± 0.019 (stat) ± 0.025 (syst) ± 0.021 (norm)

Purer MVA Selection

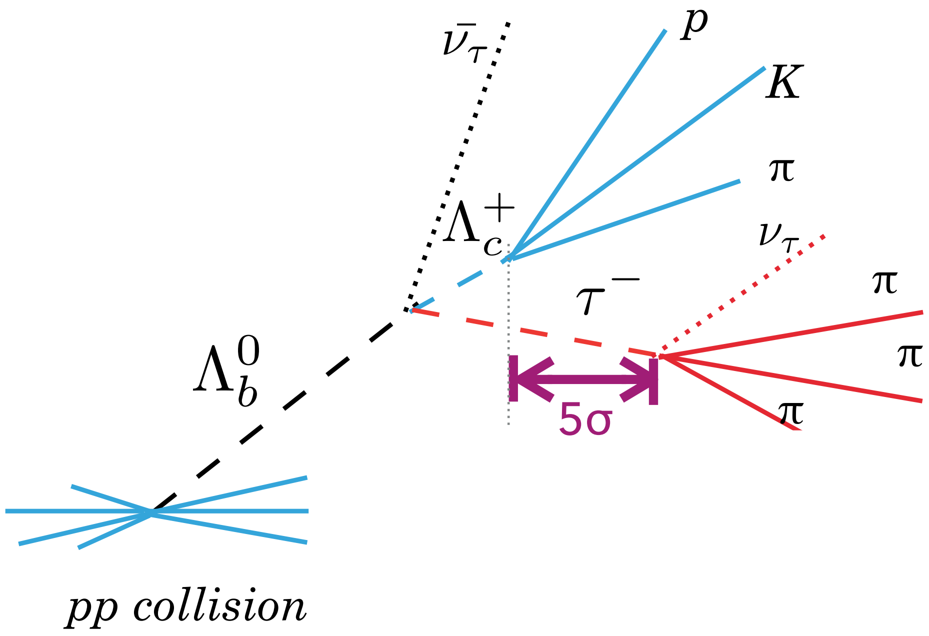


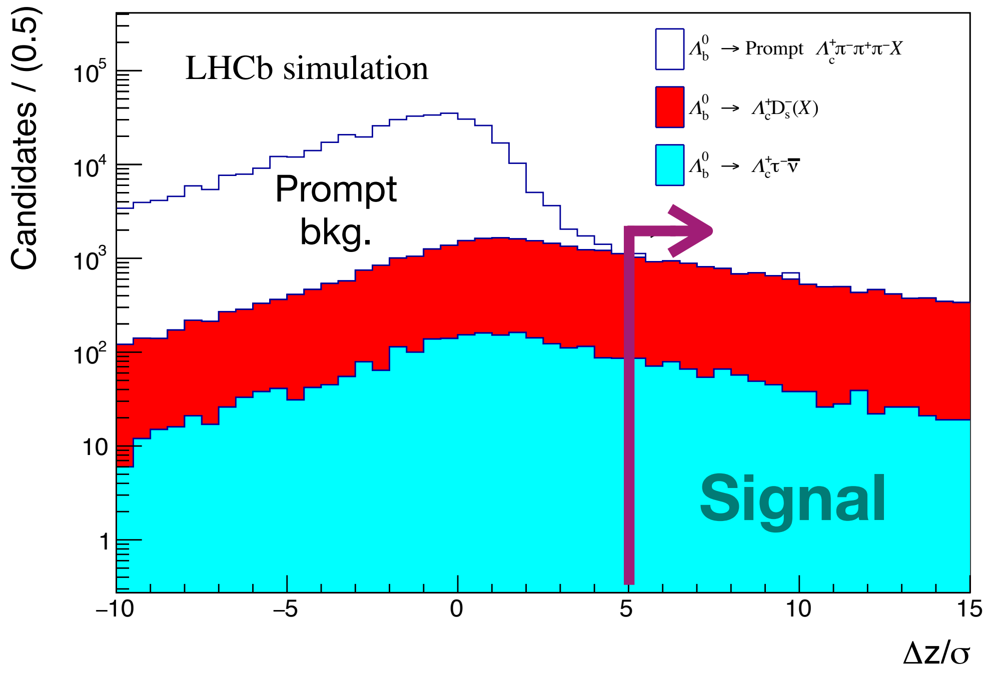
LHCb $R(\Lambda_c)$ Measurement



Nice Illustration from L. Grillo

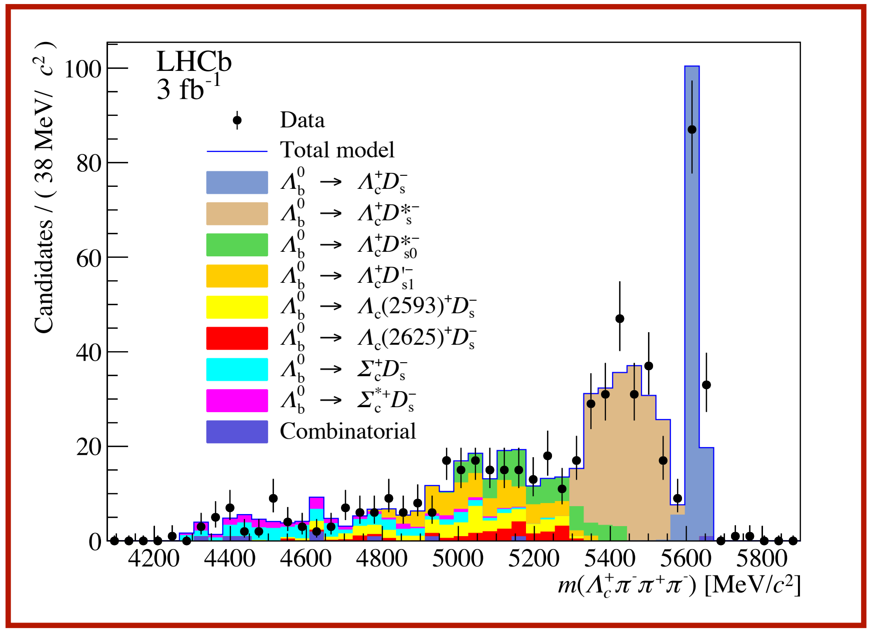
Same experimental Method: exploit vertex **separation**





Long-lived bkg. (D_s)

$m_{3\pi} \in [m_{D_s} - 45\,\mathrm{MeV}, m_{D_s} + 45\,\mathrm{MeV}]$



Bkg. composition constrained by fit to $m_{3\pi}$

Target **ratio**:

$$\mathcal{K}(\Lambda_c^+) = \frac{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+ 3\pi)} = \frac{N_{sig}}{N_{norm}} \times \frac{\epsilon_{norm}}{\epsilon_{sig}} \times \frac{1}{\mathcal{B}(\tau^- \to 3\pi(\pi^0)\nu_\tau)}$$

- Extraction in **3D fit** to
  MVA : q² : τ decay time

Kinematic and angular information of 3π system, neutral energy in cone around 3π direction

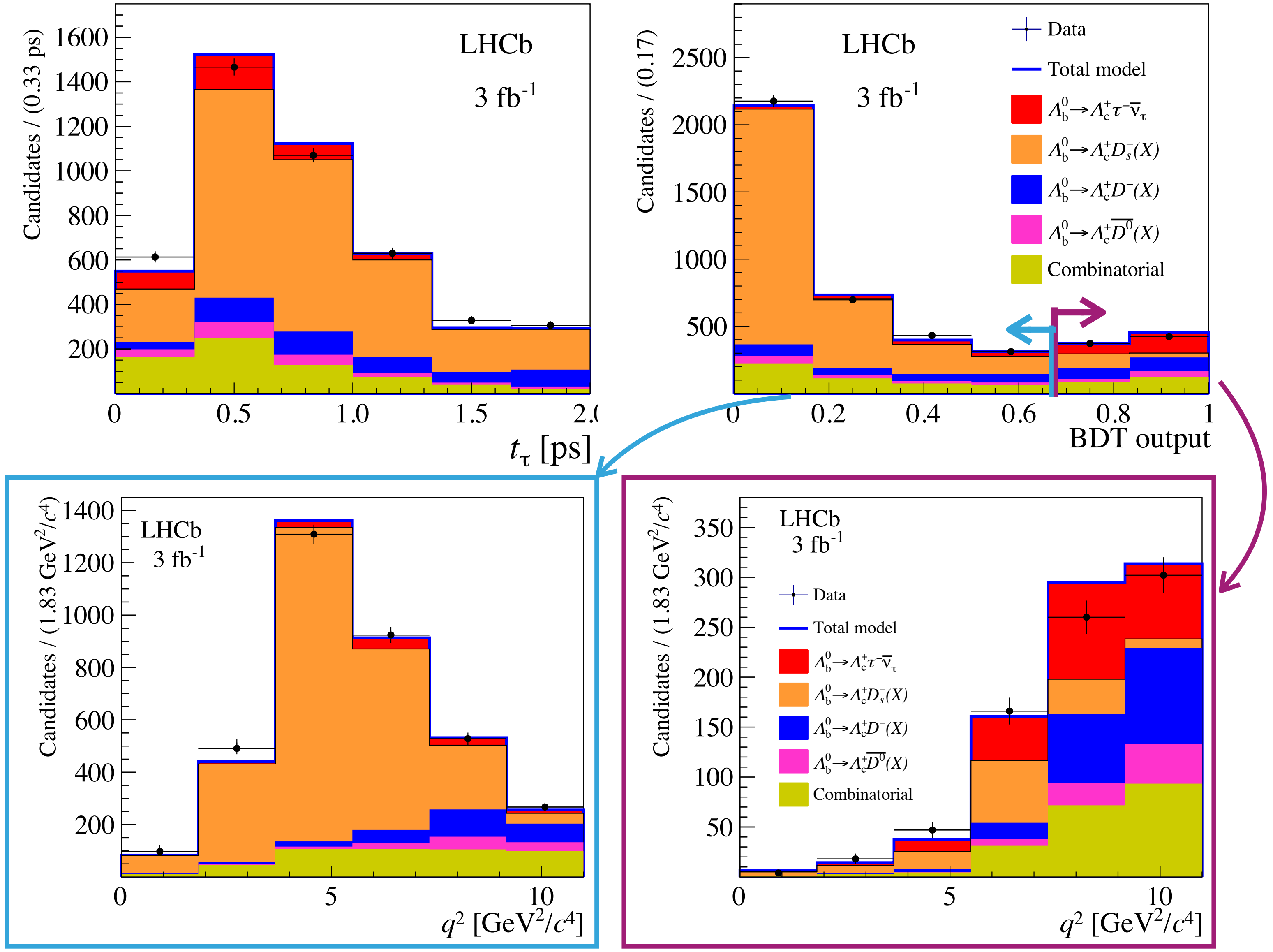
$$N(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau) = 349 \pm 40$$
$$N(\Lambda_b^0 \to \Lambda_c^+ D_s^-(X)) = 2757 \pm 80$$

External input:

$$\mathscr{B}(\Lambda_b^0 \to \Lambda_c^+ 3\pi) = (6.14 \pm 0.94) \times 10^{-3}$$

$$\mathscr{B}(\Lambda_b^0 \to \Lambda_c^+ \tau \bar{\nu}_\tau) = (1.50 \pm 0.16 \pm 0.25 \pm 0.23)\,\%$$

First observation with 6.1 σ !



More external input:

$$\mathscr{B}(\Lambda_b^0 \to \Lambda_c^+ \mu \bar{\nu}_\mu) = (6.2 \pm 1.4)\,\%$$

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\rm stat} \pm 0.040_{\rm syst} \pm 0.059_{\rm ext}$$

Compatible with SM

$$R(\Lambda_c^+)_{\rm SM} = 0.340 \pm 0.004$$

F. Bernlochner, Zoltan Ligeti, Dean J. Robinson, William L. Sutcliffe, [arXiv:1808.09464], [arXiv:1812.07593]

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\rm stat} \pm 0.040_{\rm syst} \pm 0.059_{\rm ext}$$

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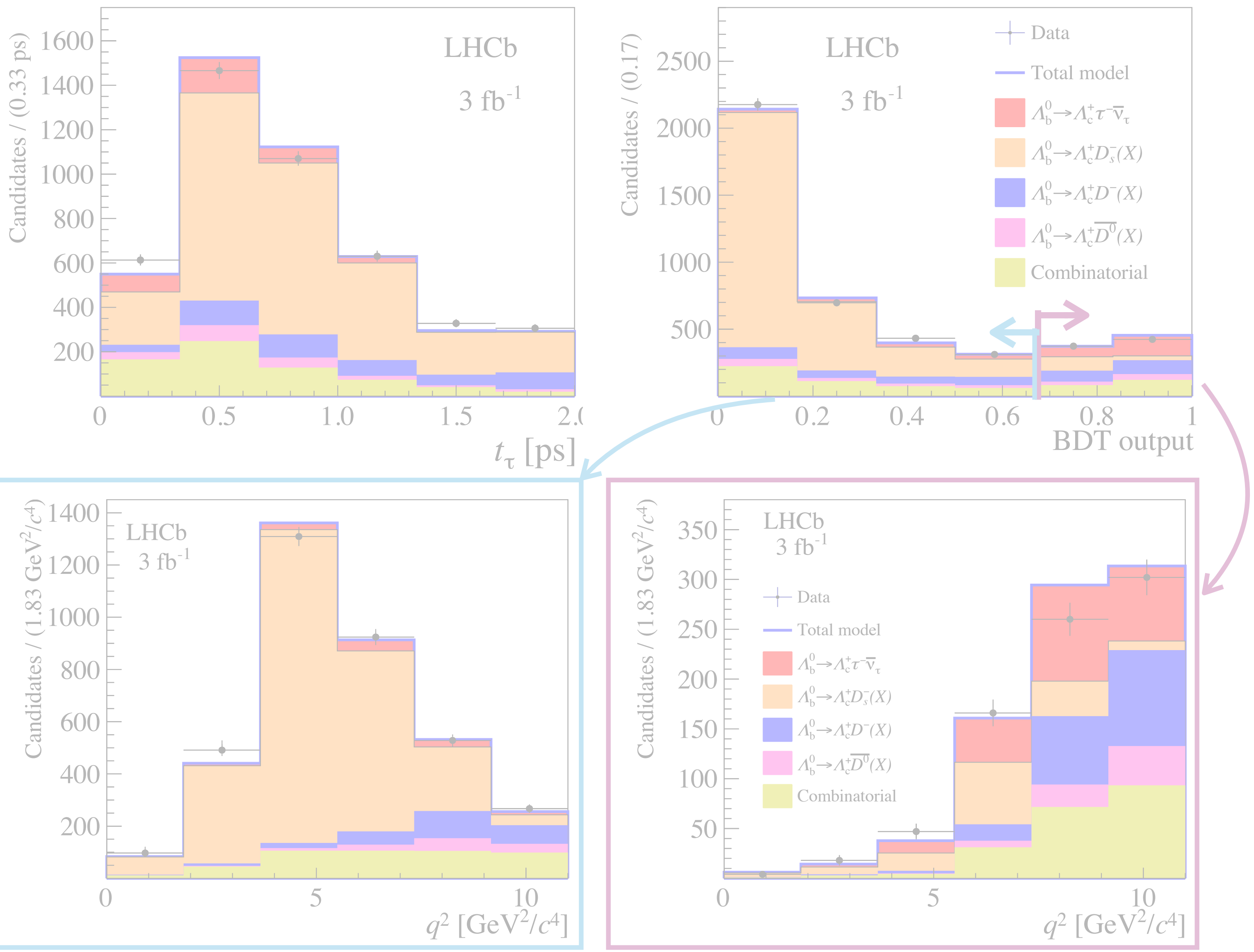
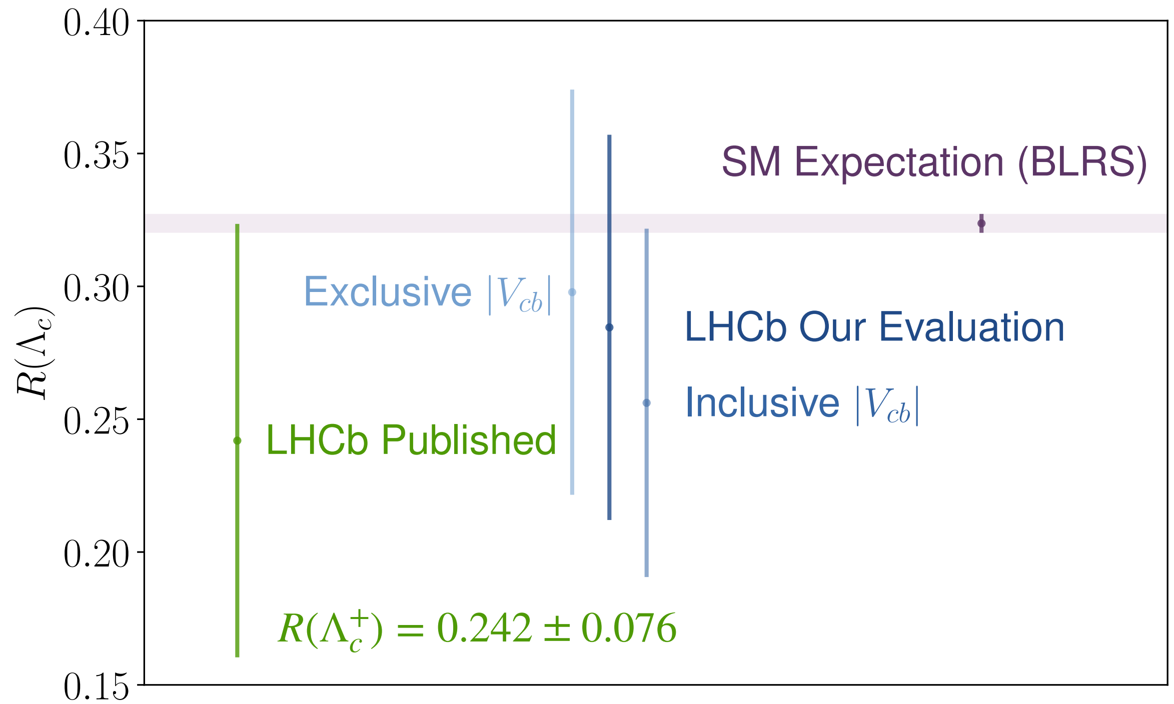


Can also use SM prediction for $\mathscr{B}(\Lambda_b^0 \to \Lambda_c^+ \mu \bar{\nu}_\mu)$ instead of LEP measurement

FB, Zoltan Ligeti, Michele Papucci, Dean Robinson, [arXiv:2206.11282 [hep-ph]]

$$R(\Lambda_c^+) = 0.285 \pm 0.073$$



$$\frac{\mathscr{R}(D^{(*)})}{\mathscr{R}(D^{(*)})_{\mathrm{SM}}}$$

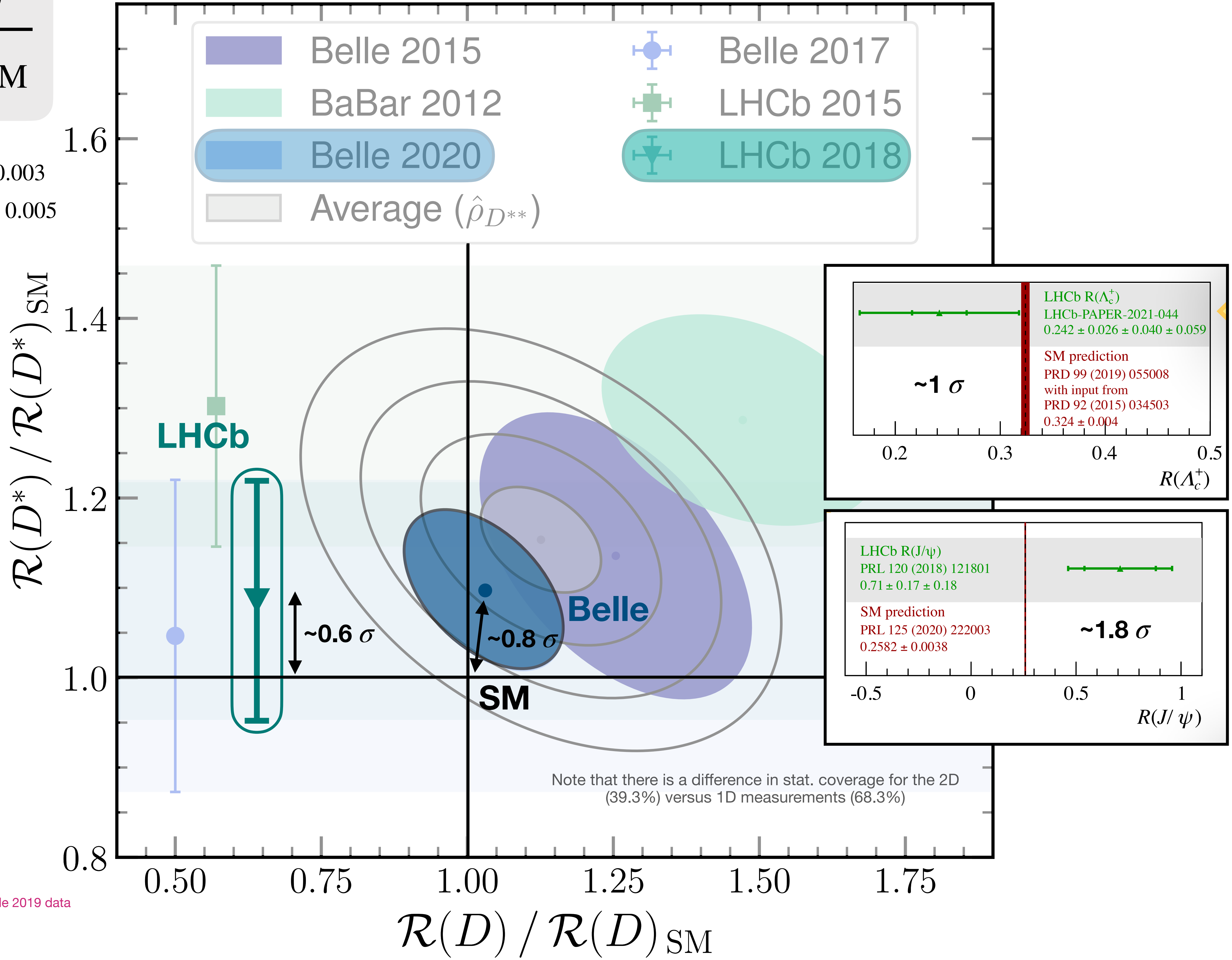
FB, M. Sevilla, D. Robinson, G. Wormser
[Rev. Mod. Phys. **94**, 015003,arXiv:2101.08326]

$\mathscr{R}(D)_{\mathrm{SM}} = 0.299 \pm 0.003$

$\mathscr{R}(D^*)_{\mathrm{SM}} = 0.258 \pm 0.005$

↑

HFLAV arithmetic average
of SM Calculations



Note that there is a difference in stat. coverage for the 2D
(39.3%) versus 1D measurements (68.3%)

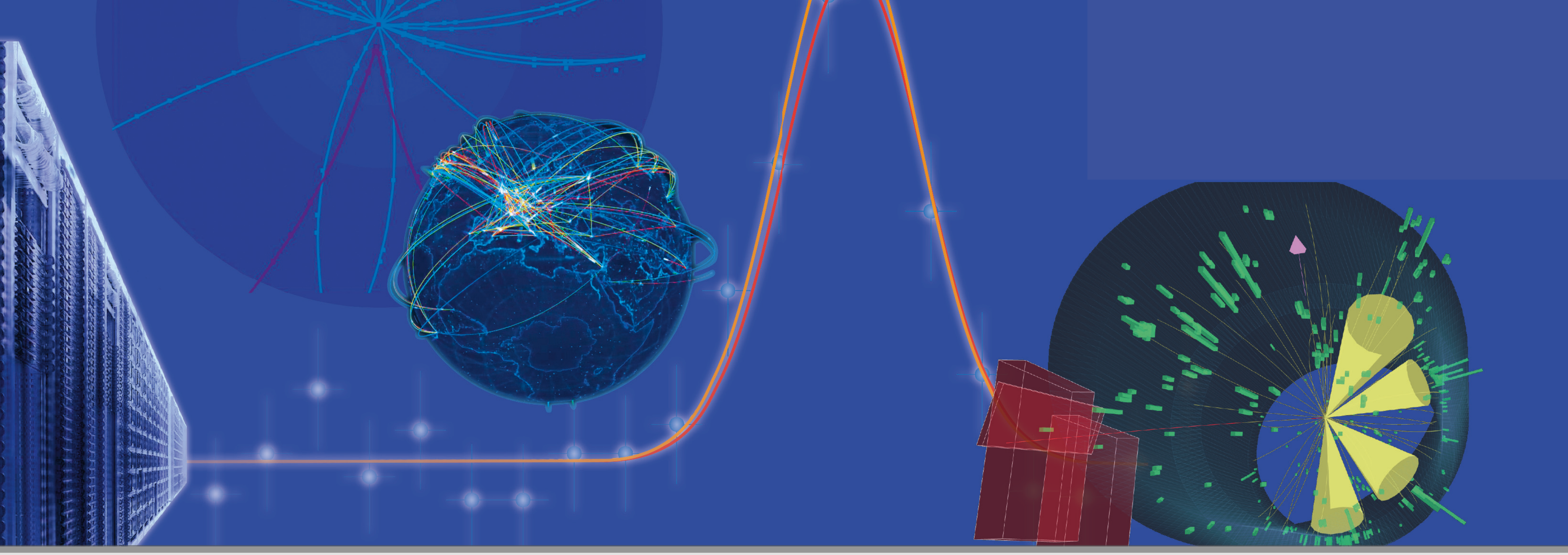
More Recent SM Calculations:

BaBar B->D*
https://arxiv.org/abs/1903.10002
- R(D*)=0.253+-0.005

Gambino, Jung, Schacht using Belle 2019 data
https://arxiv.org/abs/1905.08209
- R(D*)=0.254 +0.007 -0.006

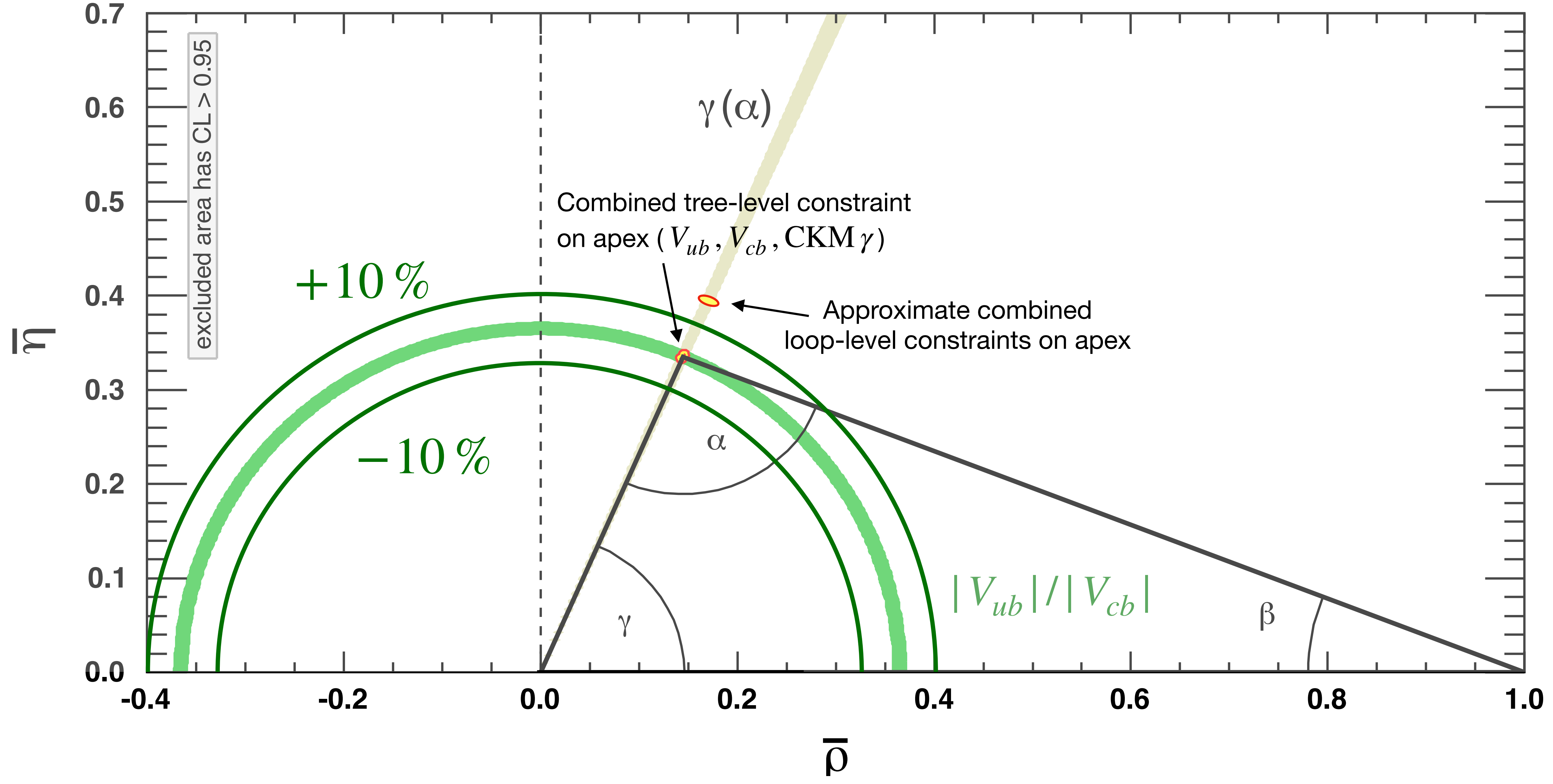
Bordone, Jung, van Dyk using Belle 2019 data
https://arxiv.org/abs/1908.09398
- RD=297+-0.003, RD*=0.250+-0.003

See also: https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html



B) More on some selected Topics

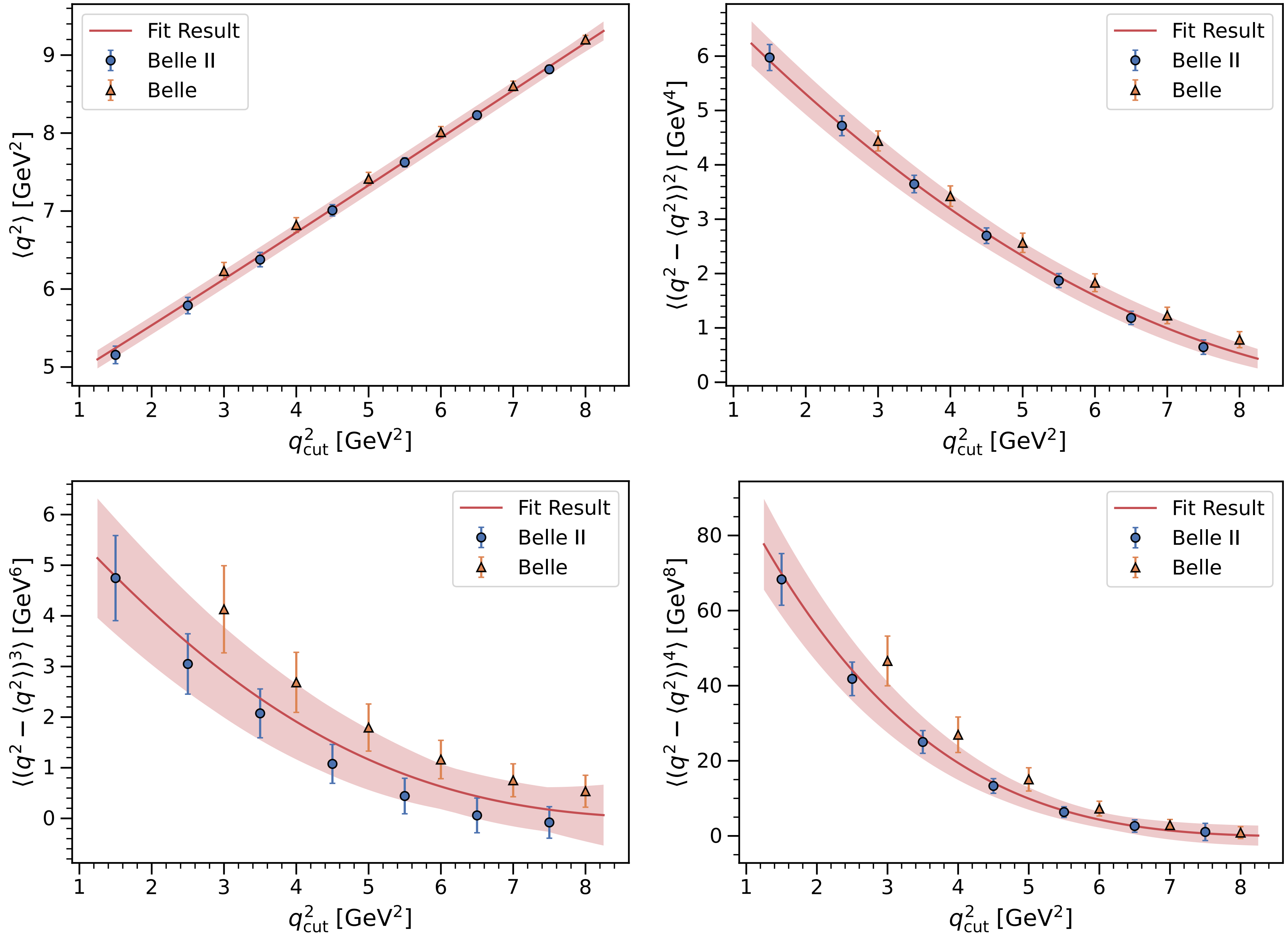
Future sensitivity with **full Belle II** and **LHCb** data sets + some improvements on LQCD / incl. calculations



$|V_{cb}|$ from q^2 mom.

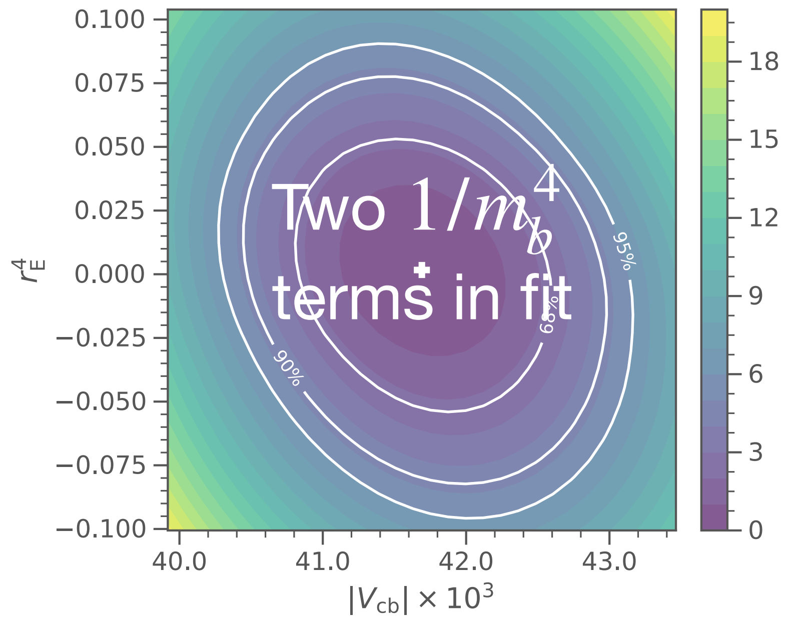
F. Bernlochner, M. Fael, K. Olschwesky, E. Persson, R. Van Tonder, K. Vos, M. Welsch [arXiv:2205.10274]

Extraction of $|V_{cb}|$ from q^2 moments:



Included corrections on the mom. predictions

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			



Two $1/m_b^4$ terms in fit

$$\rightarrow \quad |V_{cb}| = (41.69 \pm 0.59|_{\mathrm{fit}} \pm 0.23|_{\mathrm{h.o.}}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$$

Chiral Fermion states

Reminder: **Helicity** = Projection of spin on momentum of particle

spin

$$\hat{h} = \vec{s} \cdot \frac{\vec{p}}{|\vec{p}|}$$

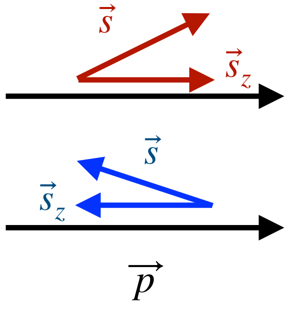
momentum

Eigenvalues of $\hat{h}$:

$$\lambda = \pm \frac{1}{2}$$

$$\lambda = +\frac{1}{2}$$

$$\lambda = -\frac{1}{2}$$



"right-handed" particle $\vec{s}_z \uparrow \uparrow \vec{p}$

"left-handed" particle $\vec{s}_z \uparrow \downarrow \vec{p}$

Chirality = "Handedness", no observable

BUT property of particle state

u_L, u_R [u : Dirac spinor]

<u>Define:</u>

$$\boxed{u_L = \frac{1}{2}\left(1 - \gamma^5\right) u \qquad u_R = \frac{1}{2}\left(1 + \gamma^5\right) u}$$

"left-chiral" **"right-chiral"**

Weak charged Currents

Couple to LH Chiral particles & RH Chiral anti-particles

$$\Big\downarrow \; m = 0$$

Couple to LH Helicity particles & RH Helicty anti-particles

Introduce **chiral projection operators**

$$P_L = \frac{1}{2}\left(1 - \gamma^5\right) \qquad P_R = \frac{1}{2}\left(1 + \gamma^5\right) \longrightarrow$$

"select" L,R component
of Dirac spinor

with properties: $P^2_{L,R} = P_{L,R}$ $\qquad P_L + P_R = 1 \qquad P_L P_R = P_R P_L = 0$

$$u_{\mathrm{RH}} = P_R\, u_{\mathrm{RH}} + P_L\, u_{\mathrm{RH}} = \frac{1}{2}\left(1 + \frac{p}{E+m}\right) u_R + \frac{1}{2}\left(1 - \frac{p}{E+m}\right) u_L$$

right-handed ($\lambda = +\frac{1}{2}$)

right-chiral

left-chiral

$$u_{\mathrm{LH}} = P_R\, u_{\mathrm{LH}} + P_L\, u_{\mathrm{LH}} = \frac{1}{2}\left(1 - \frac{p}{E+m}\right) u_R + \frac{1}{2}\left(1 + \frac{p}{E+m}\right) u_L$$

left-handed ($\lambda = -\frac{1}{2}$)

right-chiral

left-chiral

Although only LH chiral particles participate in the Weak interaction, the contribution from RH Helicity states is not necessarily zero!

Derivation of this important result on next few slides (not covered in this lecture, cf. particle physics lecture)

In matrix form (**Dirac representation**):

$$P_L = \frac{1}{2}\left(1 - \gamma^5\right) = \frac{1}{2}\left[\begin{pmatrix}1 & 0\\ 0 & \underset{\substack{\uparrow\\ = I_{2\times2}}}{1}\end{pmatrix} - \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix}\overset{\substack{= I_{2\times2}\\ \downarrow}}{1} & -1\\ -1 & 1\end{pmatrix}$$

Note: for discussion of chiral states, the **Weyl representation** (also “chiral representation”) is often used, since then

$$\gamma^5 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix} \quad \Rightarrow \quad P_L = \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} \qquad P_R = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$P_L \begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix} = \begin{pmatrix}0\\ \psi_L\end{pmatrix} \qquad P_R \begin{pmatrix}\psi_R\\ \psi_L\end{pmatrix} = \begin{pmatrix}\psi_R\\ 0\end{pmatrix}$$

Look at Dirac spinors for fermions with $\overrightarrow{p} = \begin{pmatrix} 0 \\ 0 \\ p \end{pmatrix}$

Case 1: $m = 0$ or $E \gg m$ (e.g. neutrinos or highly relativistic particles)

$$u_{\mathrm{RH}} = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ \dfrac{p}{E+m} \\ 0 \end{pmatrix} \xrightarrow[E \gg m]{} \sqrt{E} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

has $\lambda = +\frac{1}{2}$ (RH)

$$u_{\mathrm{LH}} = \sqrt{E+m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\dfrac{p}{E+m} \end{pmatrix} \xrightarrow[E \gg m]{} \sqrt{E} \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

has $\lambda = -\frac{1}{2}$ (LH)

$$\longrightarrow \quad P_L \, u_{\mathrm{RH}} = \frac{1}{2}\sqrt{E} \left(\begin{array}{cc:cc} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ \hdashline -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right) \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\longrightarrow \quad P_L\, u_{\mathrm{LH}} = \frac{1}{2}\sqrt{E}\left(\begin{array}{cc:cc} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ \hdashline -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = \frac{1}{2}\sqrt{E}\begin{pmatrix} 0 \\ 2 \\ 0 \\ -2 \end{pmatrix} = \sqrt{E}\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} = u_L$$

$$P_R\, u_{\mathrm{RH}} = u_{\mathrm{RH}} = u_R$$

$$P_R\, u_{\mathrm{LH}} = 0$$

⟶ For $m = 0$: helicity = chirality

For arbitrary spinors u, we can define chirality components via

$$u = (P_R + P_L)u = P_R u + P_L u = u_R + u_L$$

Note:

$$\bar{u}_L = u_L^\dagger \gamma^0 \qquad (\gamma^5)^\dagger = \gamma^5 \downarrow$$

$$= u^\dagger \frac{1}{2}\left(1 - \gamma^5\right)\gamma^0$$

$$= u^\dagger \frac{1}{2}\left(\gamma^0 - \gamma^5\gamma^0\right)$$

$$= u^\dagger \frac{1}{2}\left(\gamma^0 + \gamma^0\gamma^5\right) \leftarrow \gamma^5\gamma^0 = -\gamma^0\gamma^5$$

$$= u^\dagger \gamma^0 \frac{1}{2}\left(1 + \gamma^5\right) = \bar{u}\frac{1}{2}\left(1 + \gamma^5\right)$$

Particles	Anti-Particles
$u_L = \frac{1}{2}\left(1 - \gamma^5\right) u$	$v_L = \frac{1}{2}\left(1 + \gamma^5\right) v$
$u_R = \frac{1}{2}\left(1 + \gamma^5\right) u$	$v_R = \frac{1}{2}\left(1 - \gamma^5\right) v$
$\bar{u}_L = \bar{u}\frac{1}{2}\left(1 + \gamma^5\right)$	$\bar{v}_L = \bar{v}\frac{1}{2}\left(1 - \gamma^5\right)$
$\bar{u}_R = \bar{u}\frac{1}{2}\left(1 - \gamma^5\right)$	$\bar{v}_R = \bar{v}\frac{1}{2}\left(1 + \gamma^5\right)$

Case 2: $m \neq 0$

$\lambda = +\frac{1}{2}$:

$$\frac{p}{E} = \beta \approx 1$$

$$E \gg m \ \Big\uparrow$$

$$P_L u_{\text{RH}} = \frac{1}{2}\sqrt{E+m}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ \frac{p}{E+m} \\ 0 \end{pmatrix} = \frac{1}{2}\sqrt{E+m}\begin{pmatrix} 1 - \frac{p}{E+m} \\ 0 \\ -1 + \frac{p}{E+m} \\ 0 \end{pmatrix} = \frac{1}{2}\sqrt{E+m}\left(1 - \frac{p}{E+m}\right)\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \xrightarrow{E \gg m} 0$$

$$P_R u_{\text{RH}} = \frac{1}{2}\sqrt{E+m}\left(1 + \frac{p}{E+m}\right)\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \xrightarrow{E \gg m} u_R$$

$\lambda = -\frac{1}{2}$: $$P_L u_{\text{LH}} = \frac{1}{2}\sqrt{E+m}\left(1 + \frac{p}{E+m}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \xrightarrow{E \gg m} u_L$$

$$P_R u_{\text{LH}} = \frac{1}{2}\sqrt{E+m}\left(1 - \frac{p}{E+m}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \xrightarrow{E \gg m} 0$$

Probability

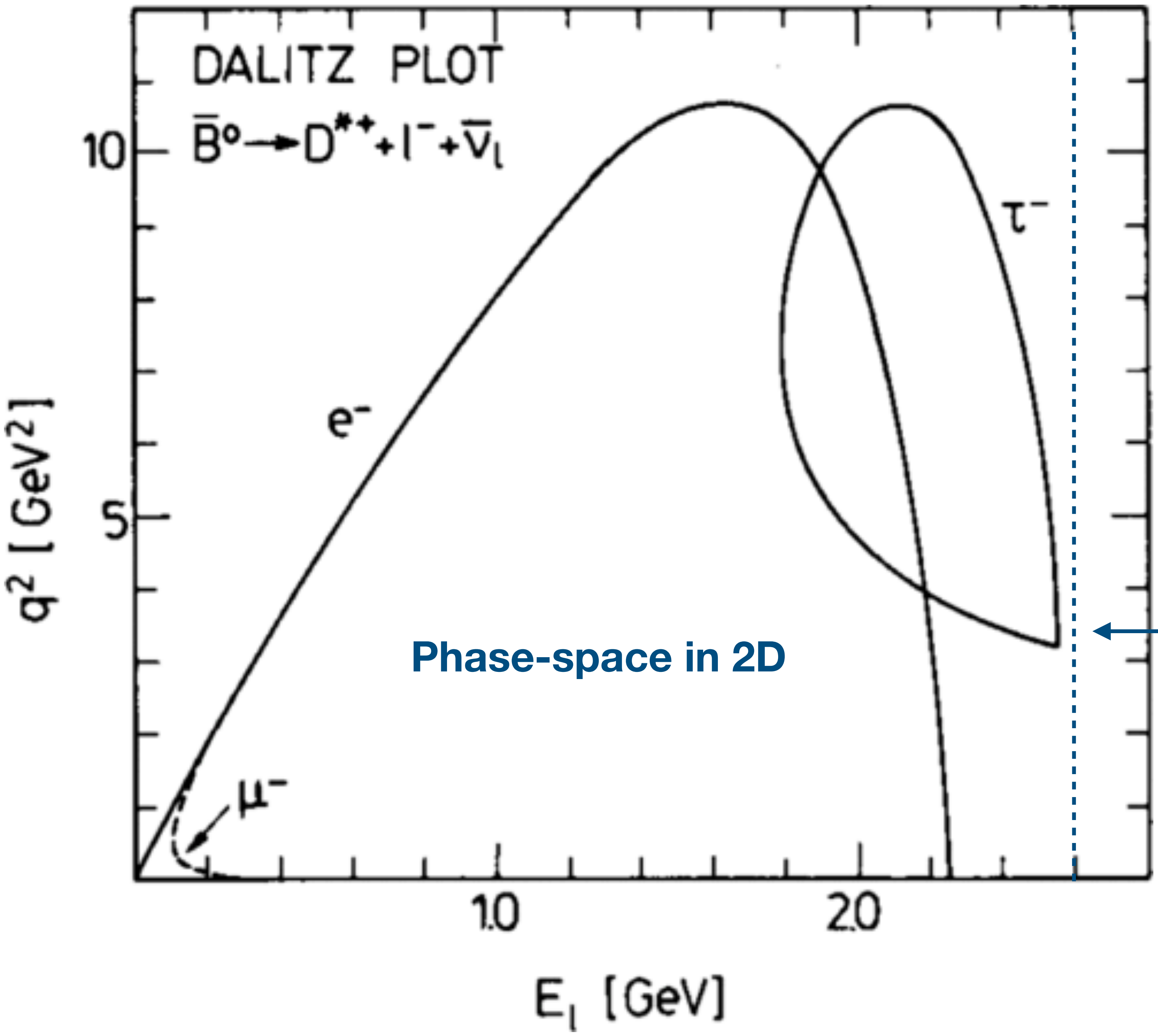
Degree of "polarization": $$\frac{P(\lambda = +\frac{1}{2}) - P(\lambda = -\frac{1}{2})}{P(\lambda = +\frac{1}{2}) + P(\lambda = -\frac{1}{2})} = \frac{(1 - \frac{p}{E+m})^2 - (1 + \frac{p}{E+m})^2}{(1 - \frac{p}{E+m})^2 + (1 + \frac{p}{E+m})^2} \xrightarrow{E \gg m} -\frac{p}{E} = -\beta = \boxed{-\frac{v}{c}}$$

⇒ Leptons that couple to W bosons have **negative helicity** with a probability of β (close to a 100% for relativistic particles). The probability for **positive helicity** is $1 - \beta$

For anti-leptons it's the other way around!

We note: Electron and muon are both relativistic ($\beta \approx 0.99$), tau carries a lot less momentum with respect to its mass

$$(\langle\beta\rangle_{\max} \approx \frac{\langle p\rangle_{\max}}{\langle E\rangle_{\max}} = \frac{1.9\,\mathrm{GeV}}{\sqrt{(1.9)^2 + 1.77^2}\mathrm{GeV})} \approx 0.73$$



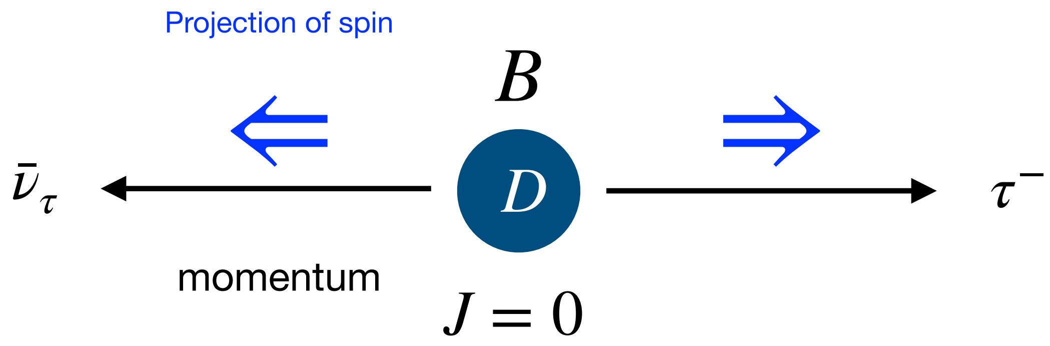
Phase-space in 2D

$\leftarrow p \approx 1.9\,\mathrm{GeV}$

Spin situation: D-meson is a pseudo-scalar and has spin 0:

Let us consider the situation for a B decaying into D, tau, anti-tau neutrino **(i.e. particles) at zero recoil**

$$w = 1,\ q^2 = q^2_{\max}$$



Weak force couples
only to RH chirale antiparticles
(and LH chirale particles)

↓

Anti-Neutrinos have almost no mass, i.e. have RH helicity

↓

To conserve angular momentum
tau needs to possess RH helicity

RH Helicity = Sum of RH chiral and LH chiral contributions

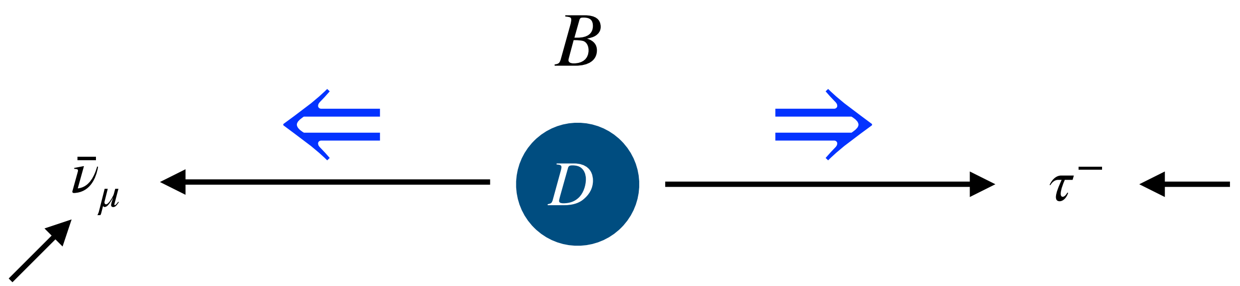
$$\underbrace{u_{\rm RH}}_{\text{right-handed } (\lambda = +\frac{1}{2})} = P_R\, u_{\rm RH} + P_L\, u_{\rm RH} = \frac{1}{2}\left(1 + \frac{p}{E+m}\right)\underbrace{u_R}_{\text{right-chiral}} + \frac{1}{2}\left(1 - \frac{p}{E+m}\right)\underbrace{u_L}_{\text{left-chiral}}$$

$m \to 0:$ RH helicity = RH chirality → Particles: no coupling to Weak force

Anti-particles: Coupling to Weak force

Zero-recoil is strongly helicity suppressed for light leptons with $\dfrac{p}{E+m} \approx 1$

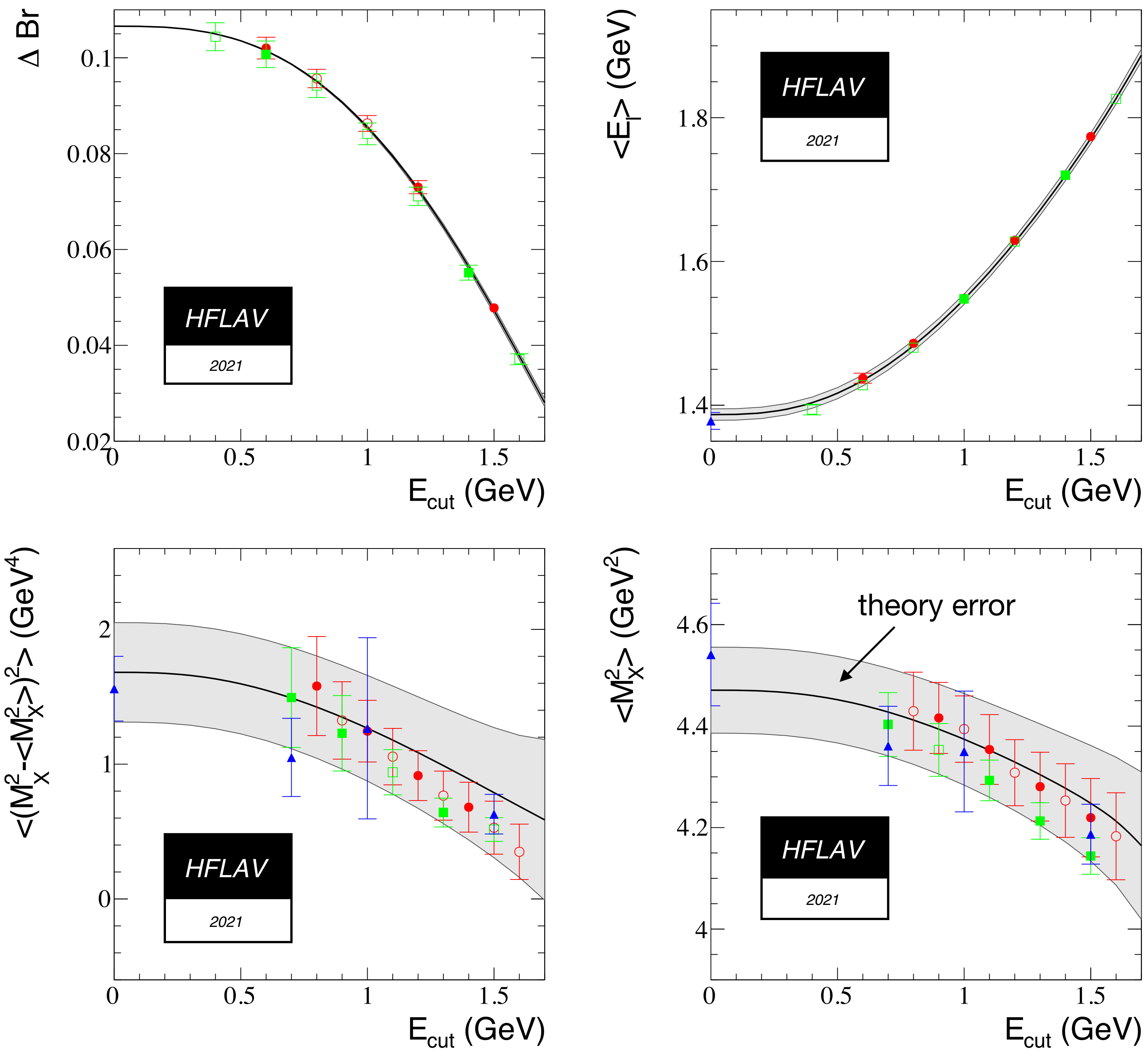
But for τ leptons this is not the case:



$\beta \approx 0.1 - 0.7$ RH helicity state has LH chiral contributions

$m_\nu \approx 0:$ RH helicity state = RH chiral state

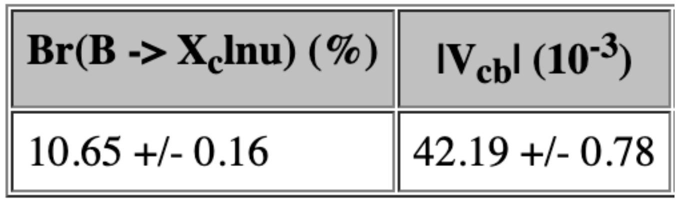
Spectral Moment Fit from HFLAV (Kinetic scheme)



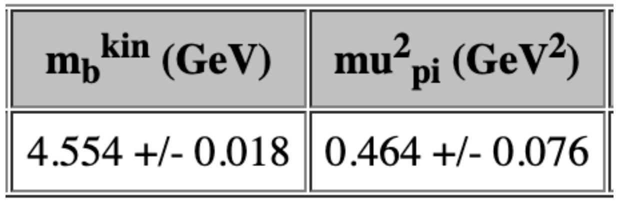
χ^2-fit of the spectral moments, which **includes theory uncertainties and correlations** based on a fixed correlation model

Constrains $m_c^{1S} = 0.986 \pm 0.013\,\mathrm{GeV}$

(Phys. Rev. D80 074010, 2009)



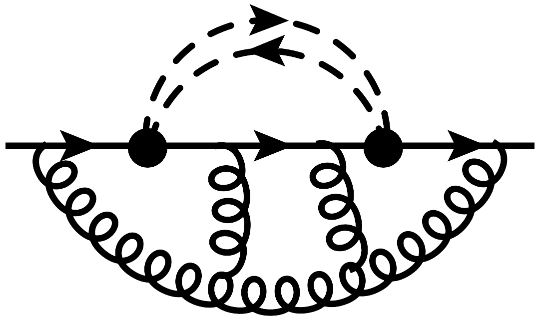
Br(B -> $X_c$lnu) (%)	$\lvert V_{cb}\rvert$ (10^{-3})
10.65 +/- 0.16	42.19 +/- 0.78



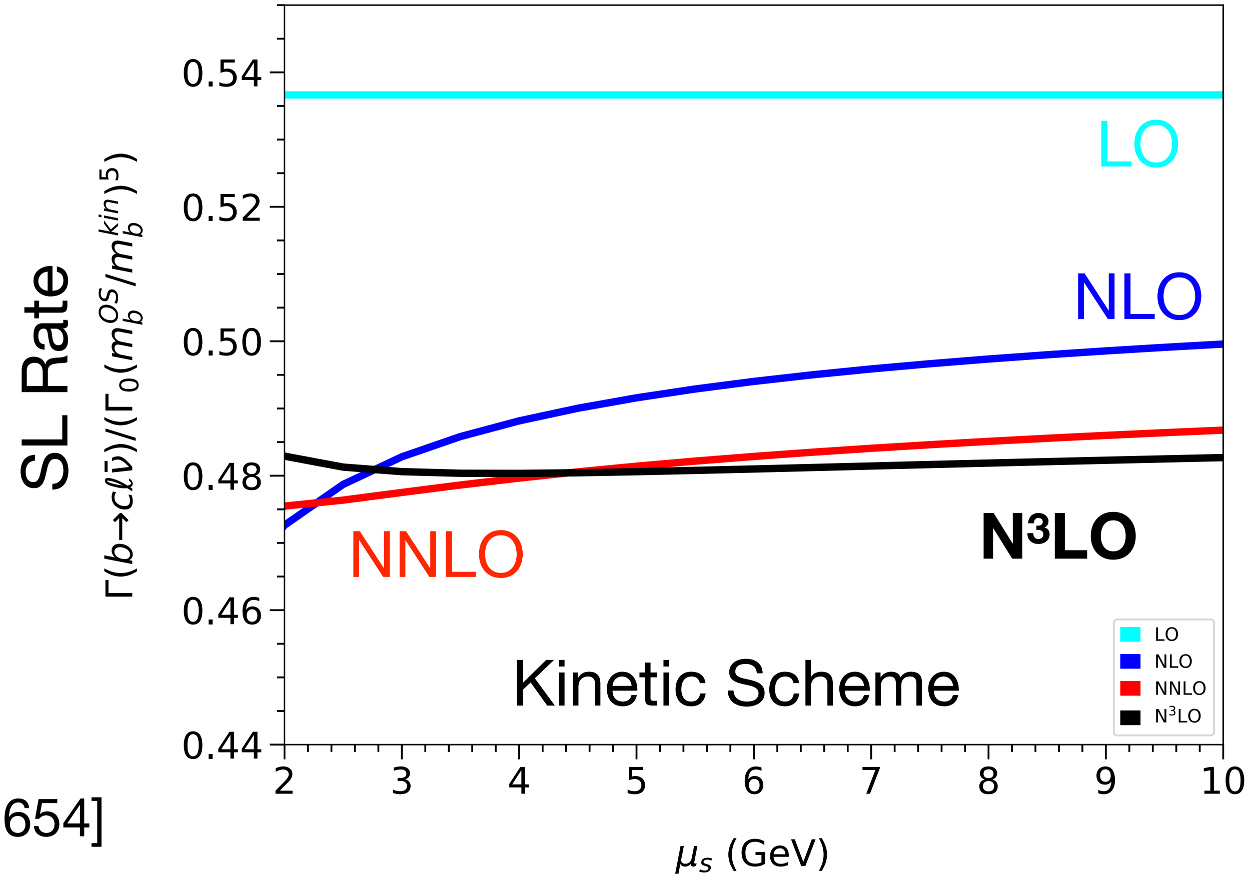
m_b^{kin} (GeV)	mu^2_{pi} (GeV^2)
4.554 +/- 0.018	0.464 +/- 0.076

State-of-the-art

Fantastic progress on the theory side:
semileptonic rate @ **N³LO**!



M. Fael, K. Schönwald, M. Steinhauser
[Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]



Renormalization scale

Updated inclusive fit to $\langle E_\ell \rangle, \langle M_X \rangle$ moments:

$$|V_{cb}| = 42.16(30)_{th}(32)_{exp}(25)_{\Gamma}\ 10^{-3}$$

↘ $\Delta |V_{cb}| / |V_{cb}| = 1.2\,\%$!

m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\|V_{cb}\|$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51
1	0.307	-0.141	0.047	0.612	-0.196	-0.064	-0.420
	1	0.018	-0.010	-0.162	0.048	0.028	0.061
		1	0.735	-0.054	0.067	0.172	0.429
			1	-0.157	-0.149	0.091	0.299
				1	0.001	0.013	-0.225
					1	-0.033	-0.005
						1	0.684
							1

M. Bordone, B. Capdevila, P. Gambino
[Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

See also [Phys.Lett.B 829 (2022) 137068, 2202.01434] for very recent 1S fit finding $|V_{cb}| = (42.5 \pm 1.1) \times 10^{-3}$

$$\boxed{\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots}$$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472]
(M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting **reparametrization invariance**, but **not true for every observable**

Spectral moments :

$v = p_B/m_B$

$$\langle M^n[w]\rangle = \int \mathrm{d}\Phi\, w^n(v, p_\ell, p_\nu)\, W^{\mu\nu}\, L_{\mu\nu}$$

$w = (m_B v - q)^2 \Rightarrow \langle M_X^n\rangle$ Moments	not RPI (depends on v)
$w = v \cdot p_\ell \Rightarrow \langle E_\ell^n\rangle$ Moments	not RPI (depends on v)
$w = q^2 \Rightarrow \langle (q^2)^n\rangle$ Moments	RPI! (does not depend on v)

$$\mathrm{d}\Gamma = \mathrm{d}\Gamma_0 + \mathrm{d}\Gamma_{\mu_\pi}\frac{\mu_\pi^2}{m_b^2} + \mathrm{d}\Gamma_{\mu_G}\frac{\mu_G^2}{m_b^2} + \mathrm{d}\Gamma_{\rho_D}\frac{\rho_D^3}{m_b^3} + \mathrm{d}\Gamma_{\rho_{LS}}\frac{\rho_{LS}^3}{m_b^3} + \ldots$$

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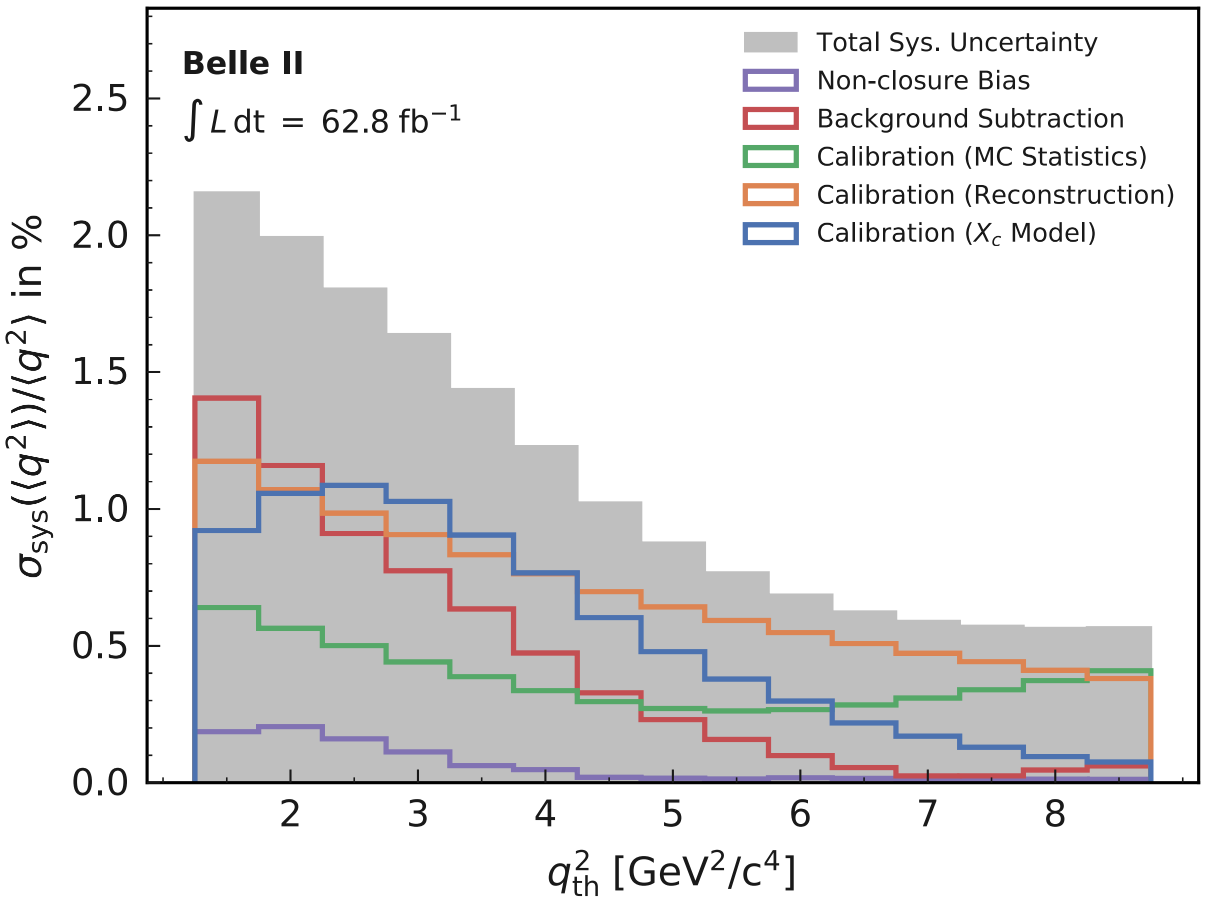
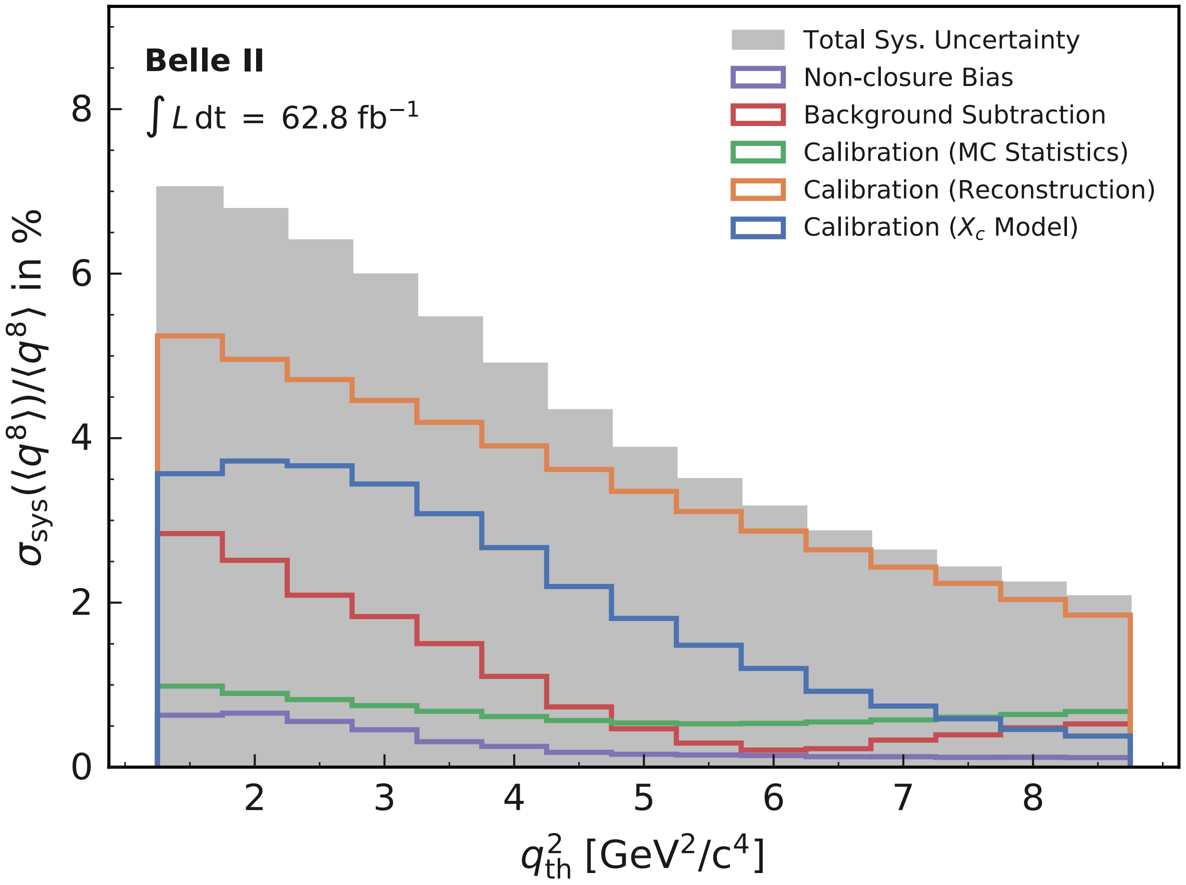
Measurements of q^2 **moments** of inclusive $B \to X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

Measurements of Lepton **Mass squared moments** in inclusive $B \to X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment
[Under review by PRD, arXiv:2205.06372]

Belle II q^2 spectral moments

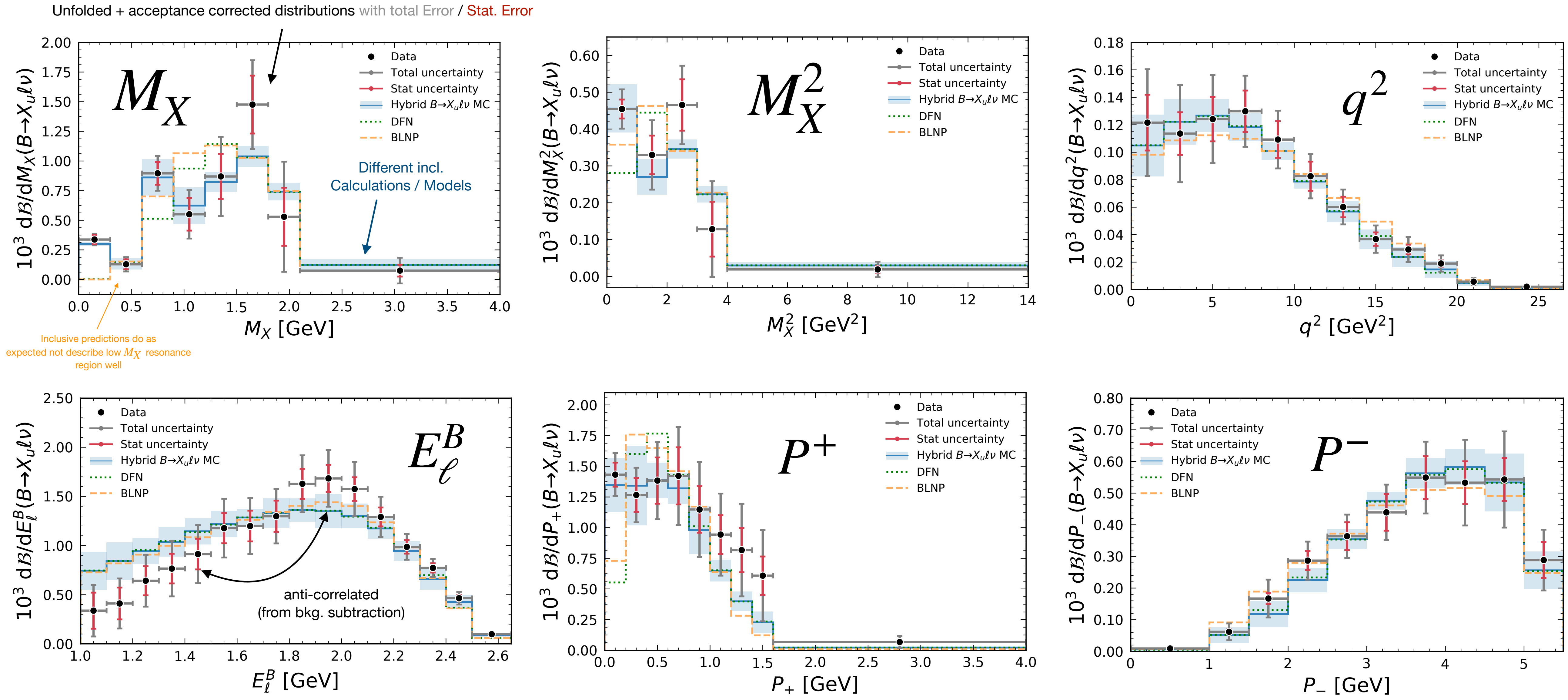
Largest uncertainty from reconstruction, background subtraction, X_c model





Belle II sensitivity similar to Belle already.

Going differential :



Unfolded + acceptance corrected distributions with total Error / Stat. Error

Different incl. Calculations / Models

Inclusive predictions do as expected not describe low M_X resonance region well

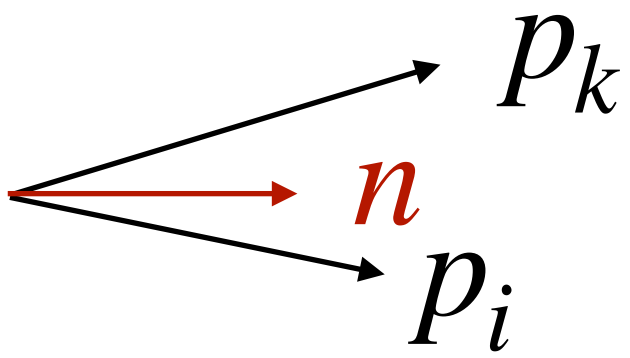
anti-correlated (from bkg. subtraction)

Agreement
(w/o theory uncertainties)

χ^2	E_ℓ^B	M_X	M_X^2	q^2	P_+	P_-
n.d.f.	16	8	5	12	9	10
Hybrid	13.5	2.5	2.6	4.5	1.7	5.2
DFN	16.2	63.2	13.1	18.5	29.3	6.1
BLNP	16.5	61.0	6.3	20.6	23.6	13.7

Simultaneous Determination of Inclusive and Exclusive $|V_{ub}|$

Accepted by PRL ▯



New Idea: Exploit that exclusive X_u final states can be separated using the # of charged pions

X_u

$n_{\pi^+} = 0$: $B \to \pi^0 \ell \bar{\nu}_\ell$

$n_{\pi^+} = 1$: $B \to \pi^+ \ell \bar{\nu}_\ell$

$n_{\pi^+} = 2$:

$n_{\pi^+} \geq 3$:

other $B \to X_u \ell \bar{\nu}_\ell$

Use **‘thrust’**,
expect more collimated system
for $B \to \pi^0 \ell \bar{\nu}_\ell$ and $B \to \pi^+ \ell \bar{\nu}_\ell$
than for other proceses

$$\boxed{\max_{|\mathbf{n}|=1} \left(\sum_i |\mathbf{p_i} \cdot \mathbf{n}| / \sum_i |\mathbf{p_i}| \right)}$$

q^2

Extraction of **BFs** and $B \to \pi$ **form factors**, in 2D fit of $q^2 : n_{\pi^+}$

M_X

Use high M_X to constrain $B \to X_c \ell \bar{\nu}_\ell$

3. First **Simultaneous** Determination of Inclusive and Exclusive $|V_{ub}|$

[Submitted to PRL, arXiv:2205.06372]

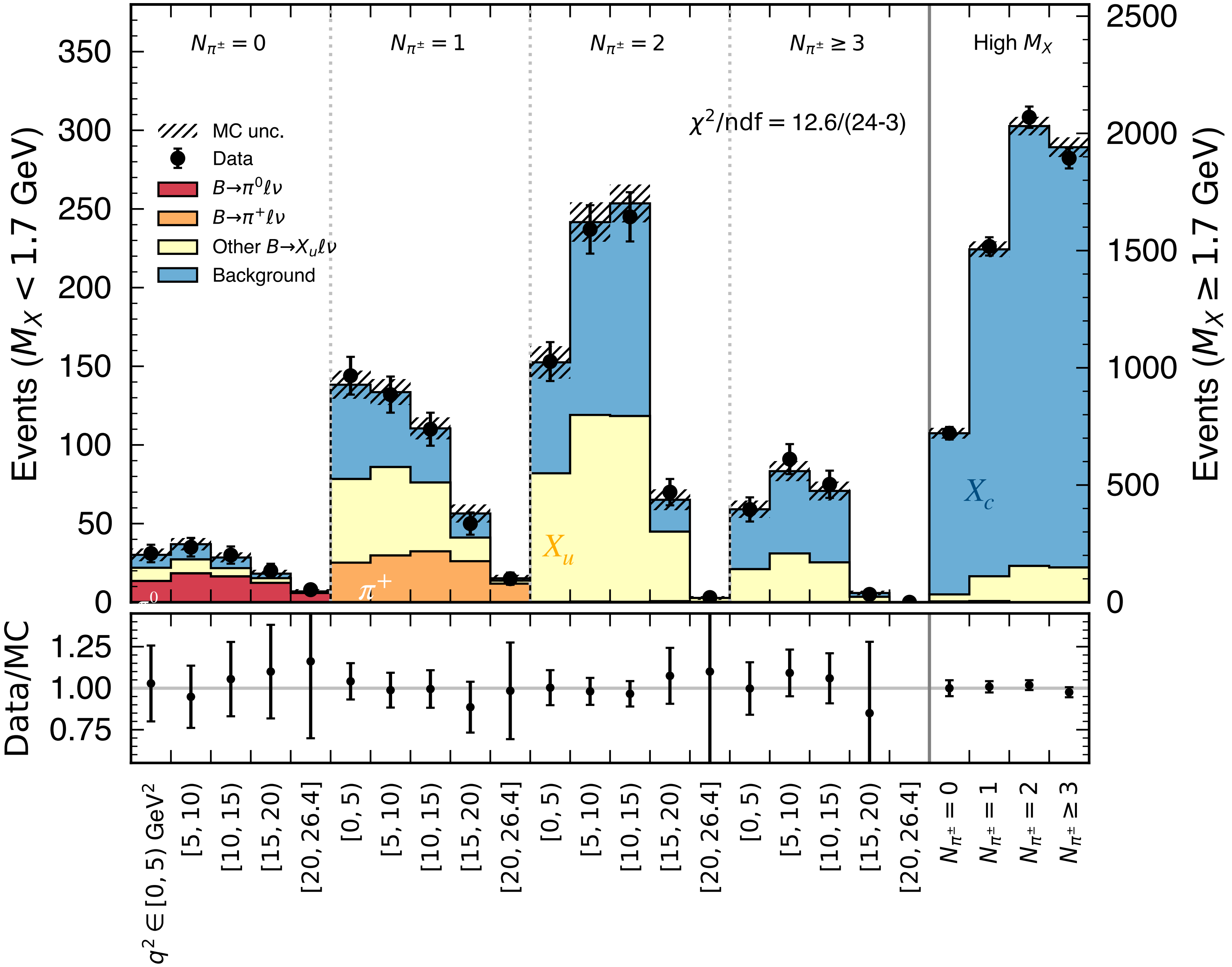
2D Categories :

For fit link

$$B \to \pi^0 \ell \bar{\nu}_\ell$$

$$B \to \pi^+ \ell \bar{\nu}_\ell$$

assuming **isospin**



Float BCL $B \to \pi$ FF constrained to **FLAG 2022** WA [Eur.Phys.J.C 82 (2022) 10, 869]

$$f_+(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N^+-1} a_n^+ \left[z^n - (-1)^{n-N^+} \frac{n}{N^+} z^{N^+} \right]$$

$$f_0(q^2) = \sum_{n=0}^{N^0-1} a_n^0 z^n \,, \qquad (3)$$

→

$$\boxed{\begin{aligned} \mathcal{B}(\overline{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell) &= (1.43 \pm 0.19 \pm 0.13) \times 10^{-4} \,, \\ \Delta\mathcal{B}(B \to X_u \ell \bar{\nu}_\ell) &= (1.40 \pm 0.14 \pm 0.23) \times 10^{-3} \,, \end{aligned}}$$

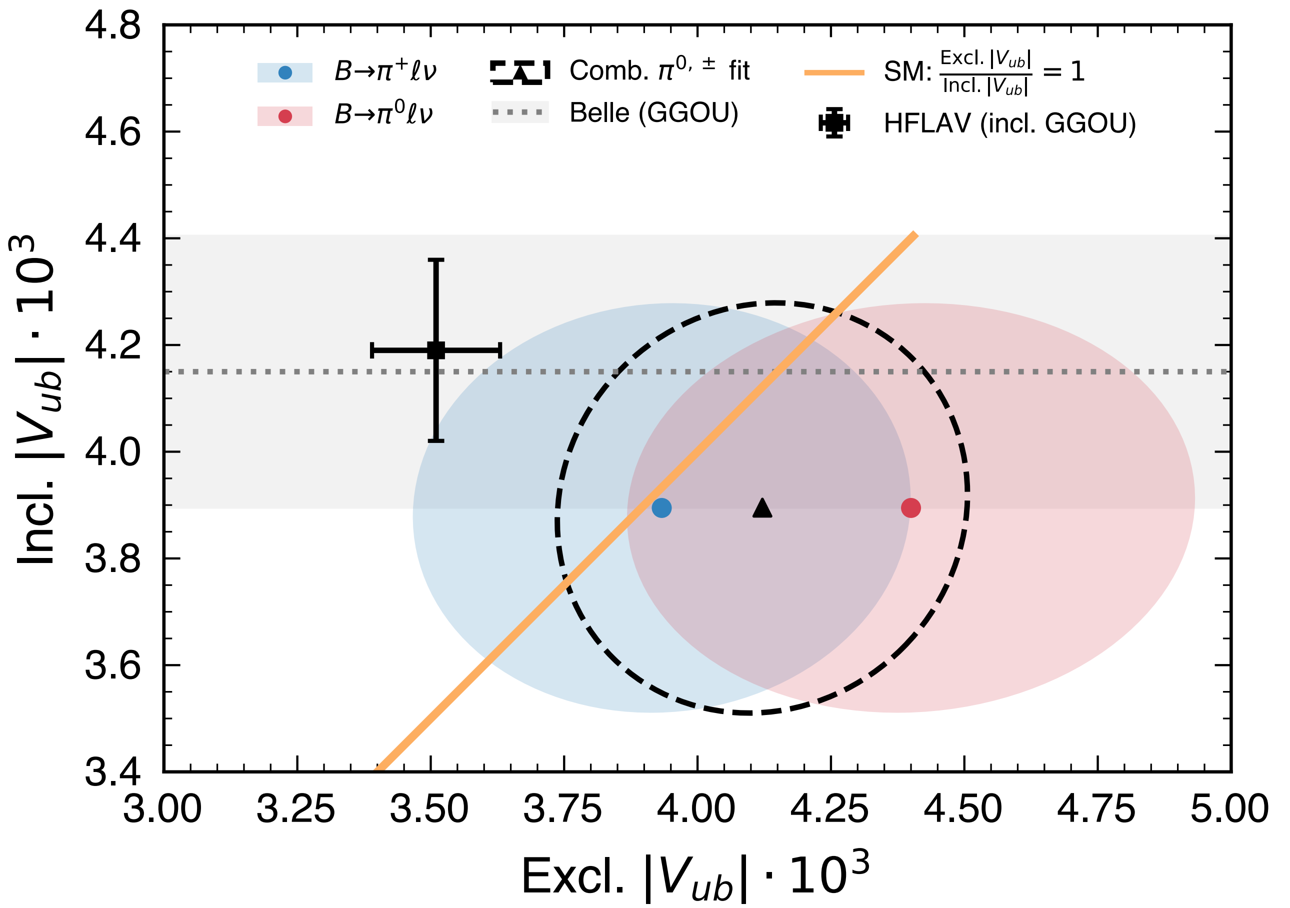
$\rho = 0.10$

(Note that $B \to X_u \ell \bar{\nu}_\ell$ of course contains $B \to \pi \ell \bar{\nu}_\ell$)

Two sets of results:

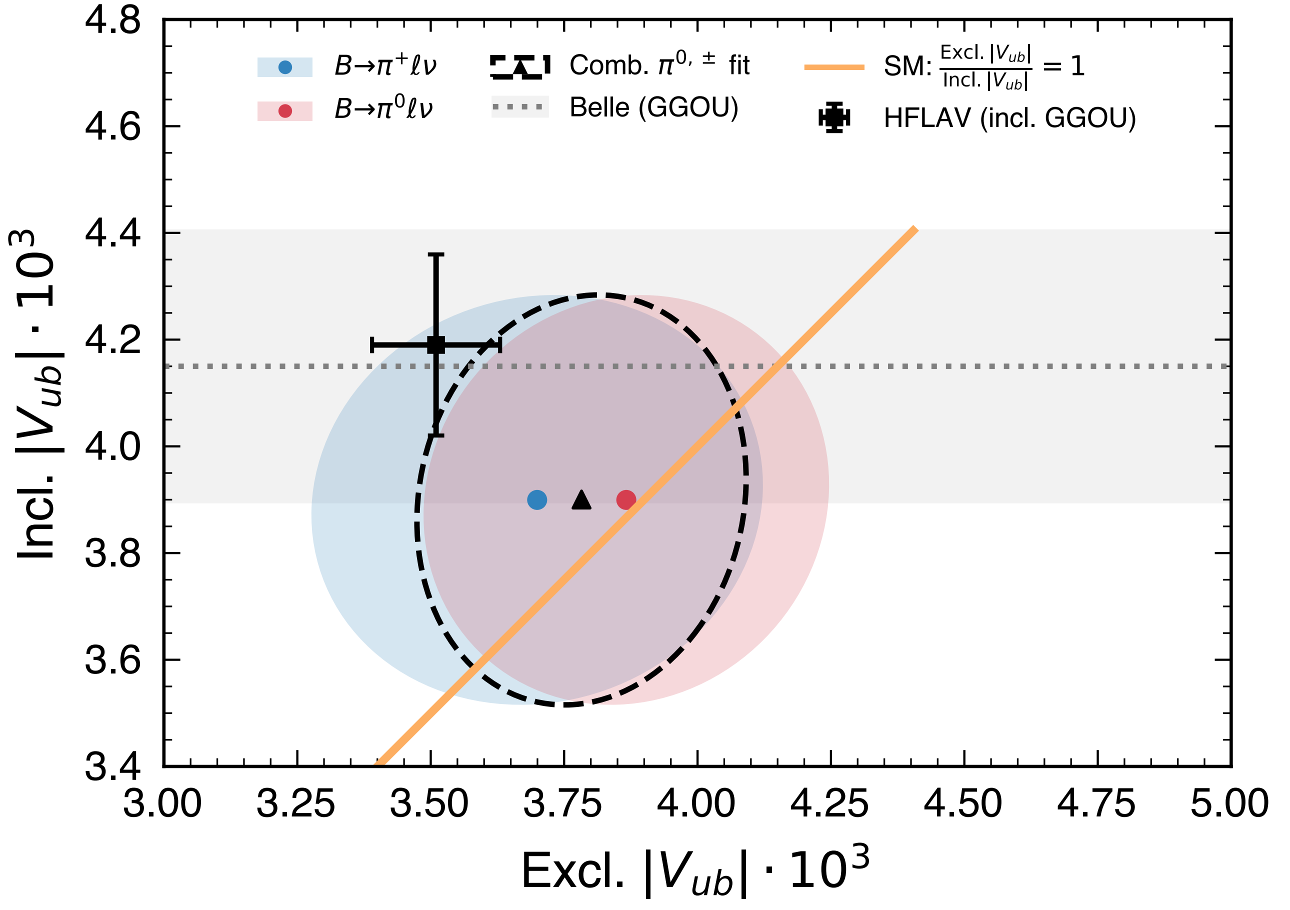
1) FLAG 2022

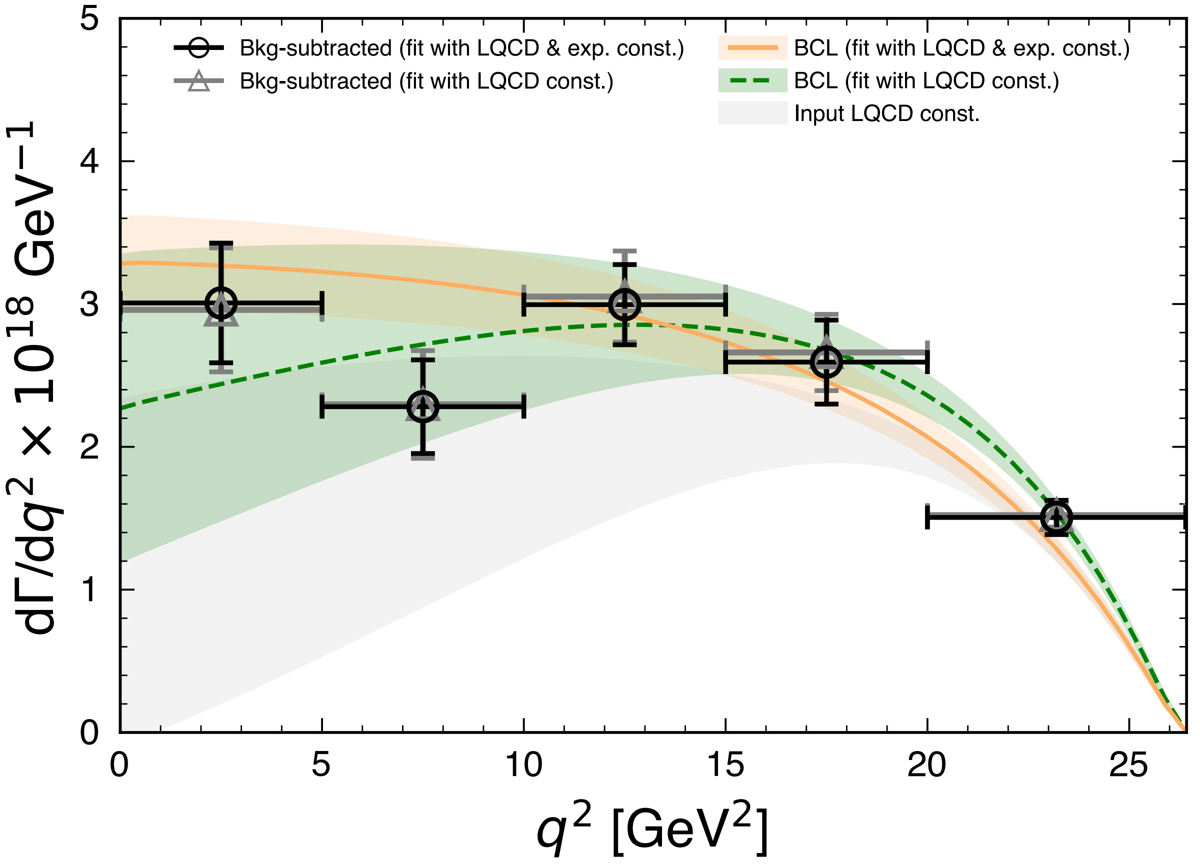
$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 1.06 \pm 0.14\,,$$



2) FLAG 2022 + all experimental information on $B \to \pi$ FF

$$\left|V_{ub}^{\text{excl.}}\right| / \left|V_{ub}^{\text{incl.}}\right| = 0.97 \pm 0.12\,,$$





Truncation Order in FF Fits

Martin will tell us more about form factors (FF) and how to determine from these distributions $|V_{cb}|$

One model independent way to parametrize FFs is the **BGL** parametrization (Boyd-Grinstein-Lebed, [arXiv:hep-ph/9705252])

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^{N} a_n z^n, \qquad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^{N} b_n z^n, \qquad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{N} c_n z^n,$$

One Problem you face as an experimentalist: where do you truncate?

Truncate too soon:

- Model dependence in extracted result for $|V_{cb}|$?

Truncate too late:

- Unnecessarily increase variance on $|V_{cb}|$?

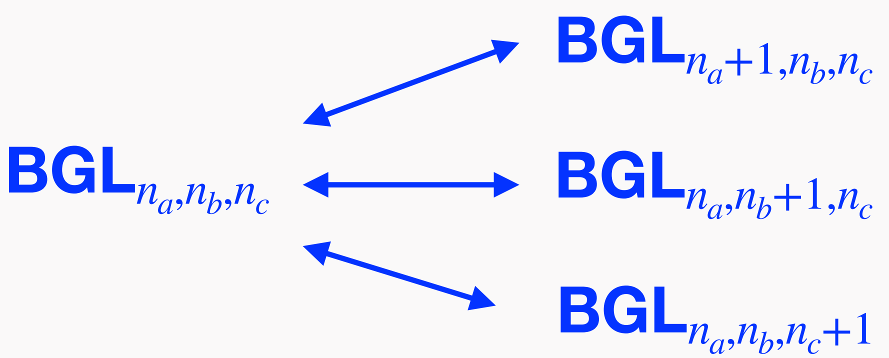
Is there an **ideal** truncation order?

Nested Hypothesis Tests or Saturation Constraints

This work
[arXiv:1902.09553, PRD100,013005 (2019)]

Use a **nested hypothesis test** to determine optimal truncation order

Challenge nested fits



Test statistics & Decision boundary

$$\Delta\chi^2 = \chi^2_N - \chi^2_{N+1} \qquad \Delta\chi^2 > 1$$

Distributed like a χ^2-distribution with 1 dof
(Wilk's theorem)

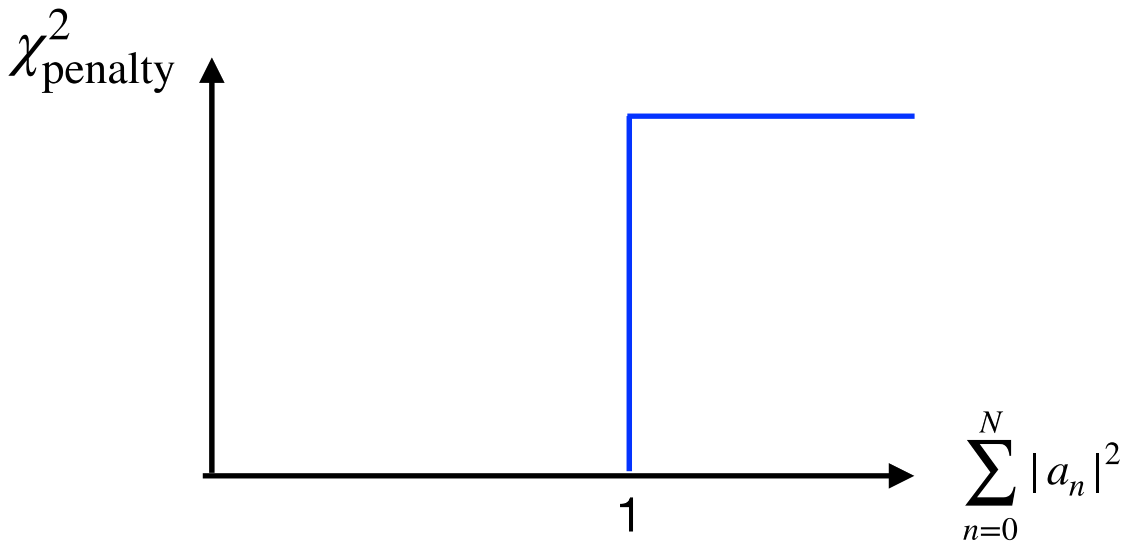
Gambino, Jung, Schacht
[arXiv:1905.08209, PLB]

Constrain contributions from higher order coefficients using **unitarity bounds**

$$\sum_{n=0}^{N} |a_n|^2 \leq 1 \qquad \sum_{n=0}^{N} \left(|b_n|^2 + |c_n|^2 \right) \leq 1$$

e.g.

$$\chi^2 \to \chi^2 + \chi^2_{\text{penalty}}$$



Nesting Procedure

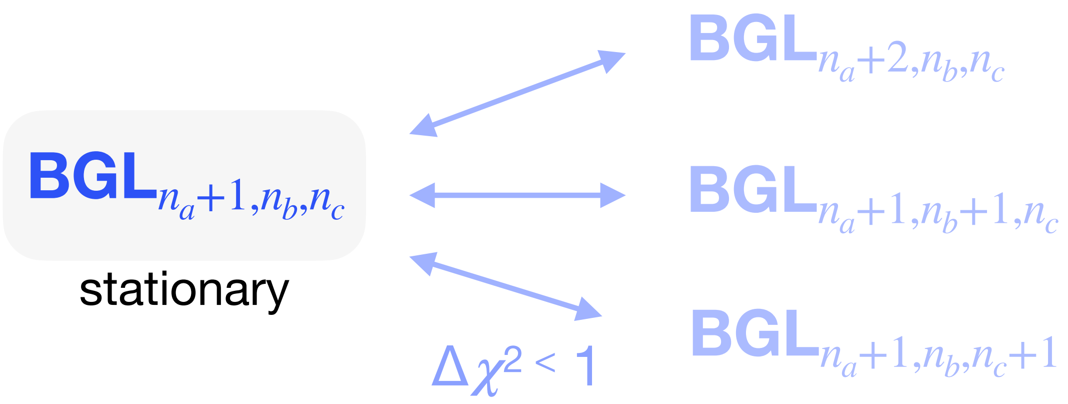
Steps:

1 Carry out nested fits with one parameter added

2 Accept descendant over parent fit, if $\Delta\chi^2 > 1$

3 Repeat 1 and 2 until you find **stationary** points

4 If multiple **stationary** points remain, choose the one with smallest N, then smallest χ^2



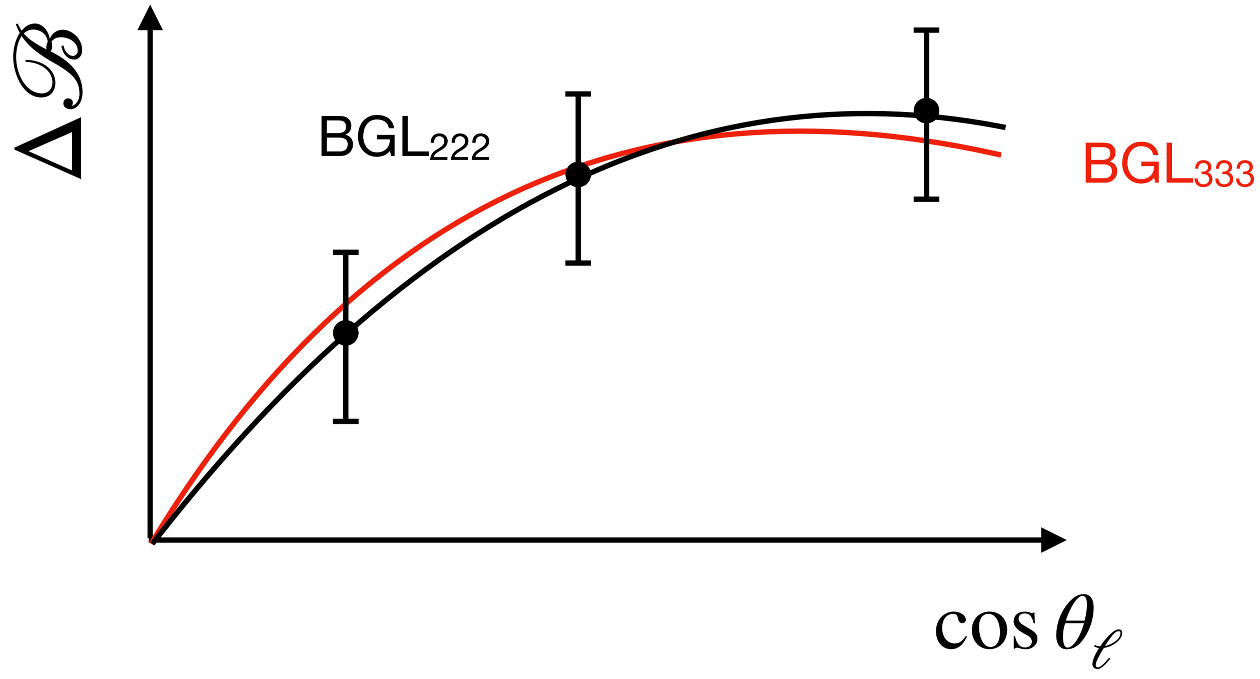
Toy study to illustrate possible bias

Use the central values of the **BGL_{222} fit** as a starting point to add **fine structure**

	'1-times'	**'10-times'**
Parameter	Value $\times\ 10^2$	Value $\times\ 10^2$
$\tilde{a}_2$	2.6954	26.954
$\tilde{b}_2$	-0.2040	-2.040
$\tilde{c}_3$	0.5350	5.350

Create a "true" higher order Hypothesis of order **BGL_{333}**

Has fine structure element the **current data cannot resolve**



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Toy Test

Produce **ensemble** of toy measurements using **untagged covariance** & **BGL$_{333}$** central values

Each toy is fitted to build the descendant tree and carry out a **nested hypo. test** to select its preferred **BGL$n_a n_b n_c$**

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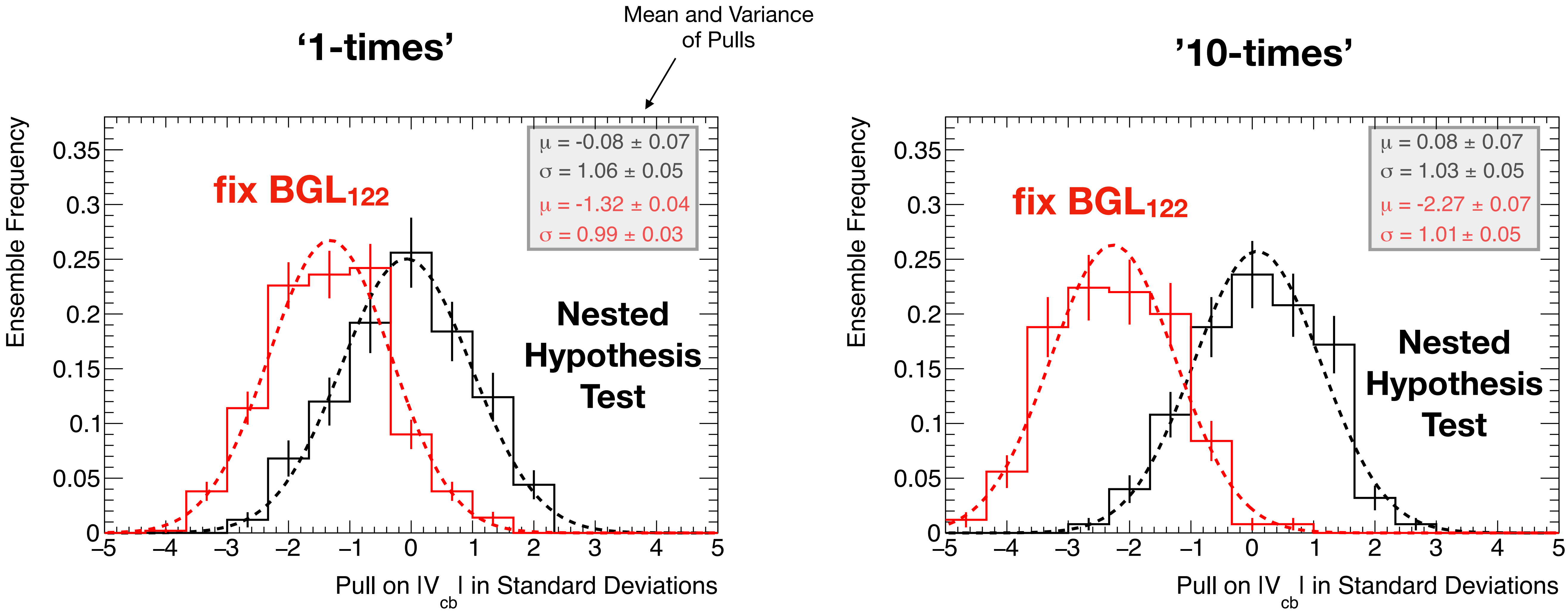
Construct Pulls

$$\textbf{Pull} = \frac{\left|V_{cb}\right|_{\textbf{true}} - \left|V_{cb}\right|_{\textbf{toy}}}{\Delta \left|V_{cb}\right|_{\textbf{toy}}}$$

As calculated from selected BGL$n_a n_b n_c$ fit of each toy

If methodology unbiased, should follow a standard normal distribution (mean 0, width 1)

Bias



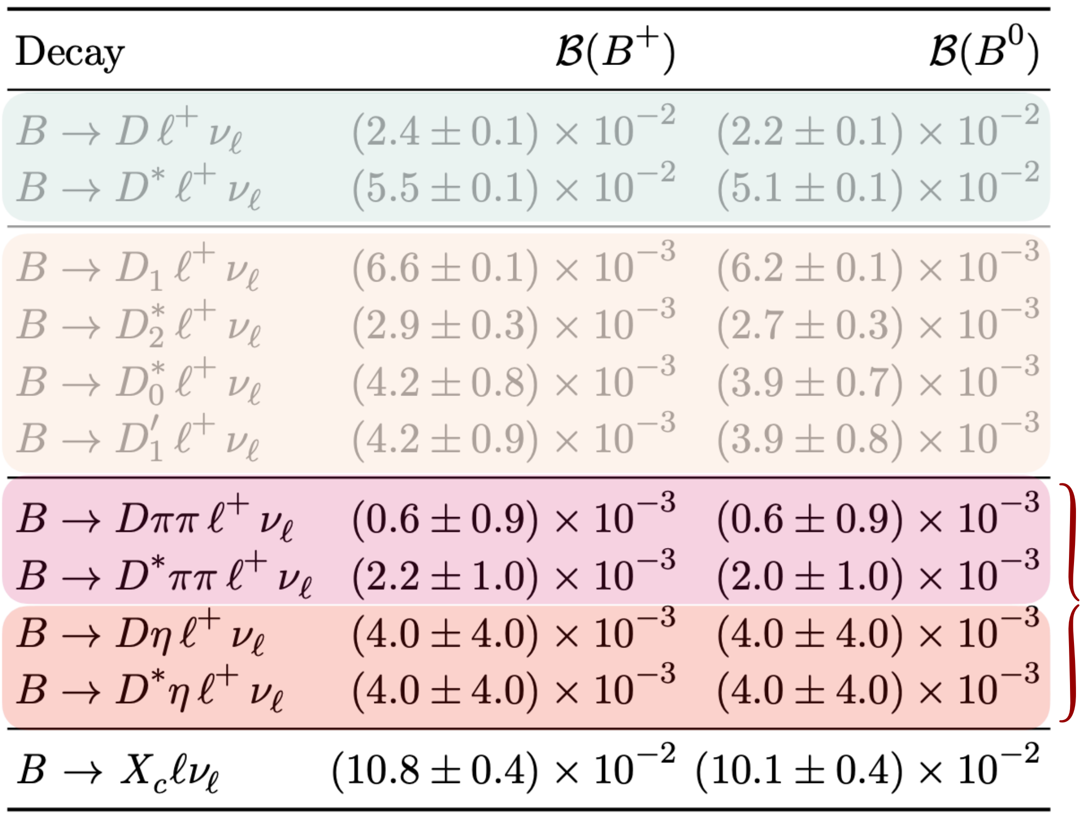
→ Procedure produces **unbiased** $|V_{cb}|$ values, just picking a hypothesis (BGL_{122}) does not

Relative Frequency of selected Hypothesis:

	BGL_{122}	BGL_{212}	BGL_{221}	BGL_{222}	BGL_{223}	BGL_{232}	BGL_{322}	BGL_{233}	BGL_{323}	BGL_{332}	BGL_{333}
1-times	6%	0%	37%	27%	6%	6%	11%	0%	2%	4%	0.4%
10-times	0%	0%	8%	38%	14%	8%	16%	3%	4%	8%	1%

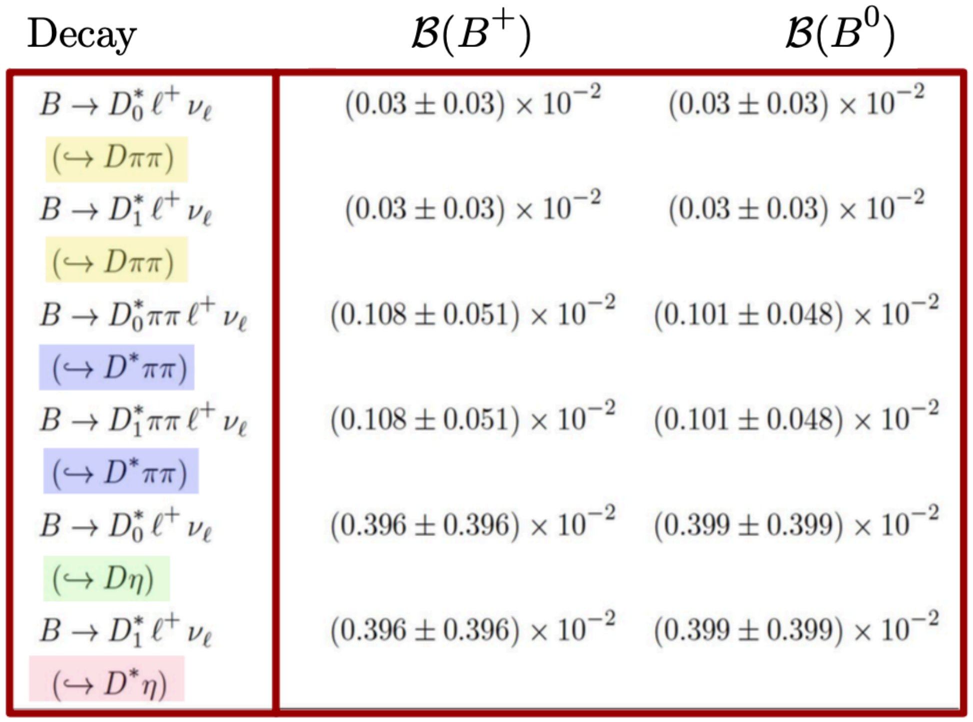
More on the Gap

Model 1:
Equidistribution of all final state particles in phase space



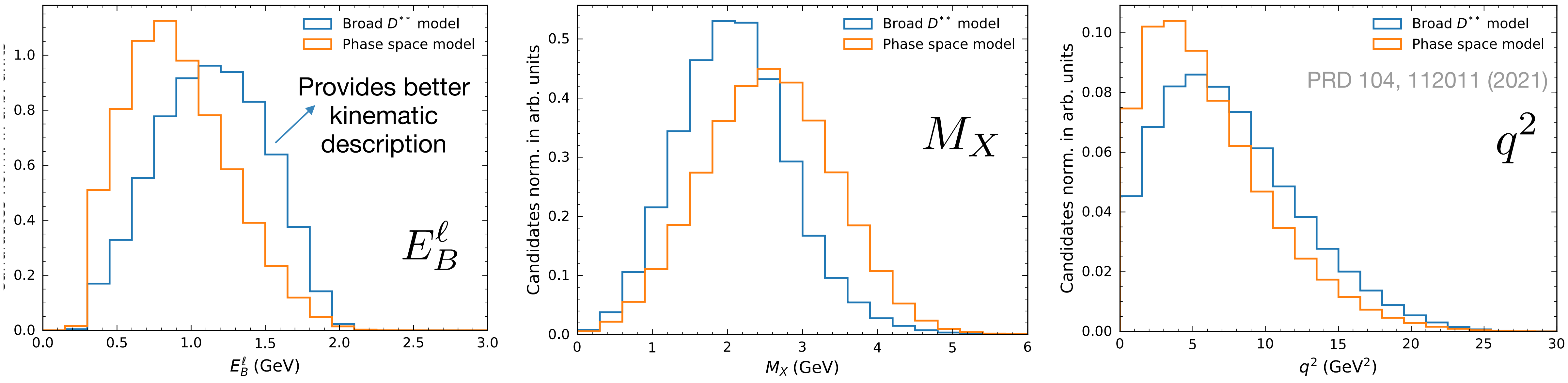
Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \to D\,\ell^+\,\nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \to D^*\,\ell^+\,\nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \to D_1\,\ell^+\,\nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \to D_2^*\,\ell^+\,\nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \to D_0^*\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \to D_1'\,\ell^+\,\nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \to D\pi\pi\,\ell^+\,\nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \to D^*\pi\pi\,\ell^+\,\nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \to D\eta\,\ell^+\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to D^*\eta\,\ell^+\,\nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \to X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Model 2:
Decay via intermediate broad D^{**} state



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \to D_0^*\,\ell^+\,\nu_\ell$ $(\hookrightarrow D\pi\pi)$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_1^*\,\ell^+\,\nu_\ell$ $(\hookrightarrow D\pi\pi)$	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \to D_0^*\pi\pi\,\ell^+\,\nu_\ell$ $(\hookrightarrow D^*\pi\pi)$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_1^*\pi\pi\,\ell^+\,\nu_\ell$ $(\hookrightarrow D^*\pi\pi)$	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \to D_0^*\,\ell^+\,\nu_\ell$ $(\hookrightarrow D\eta)$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \to D_1^*\,\ell^+\,\nu_\ell$ $(\hookrightarrow D^*\eta)$	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$

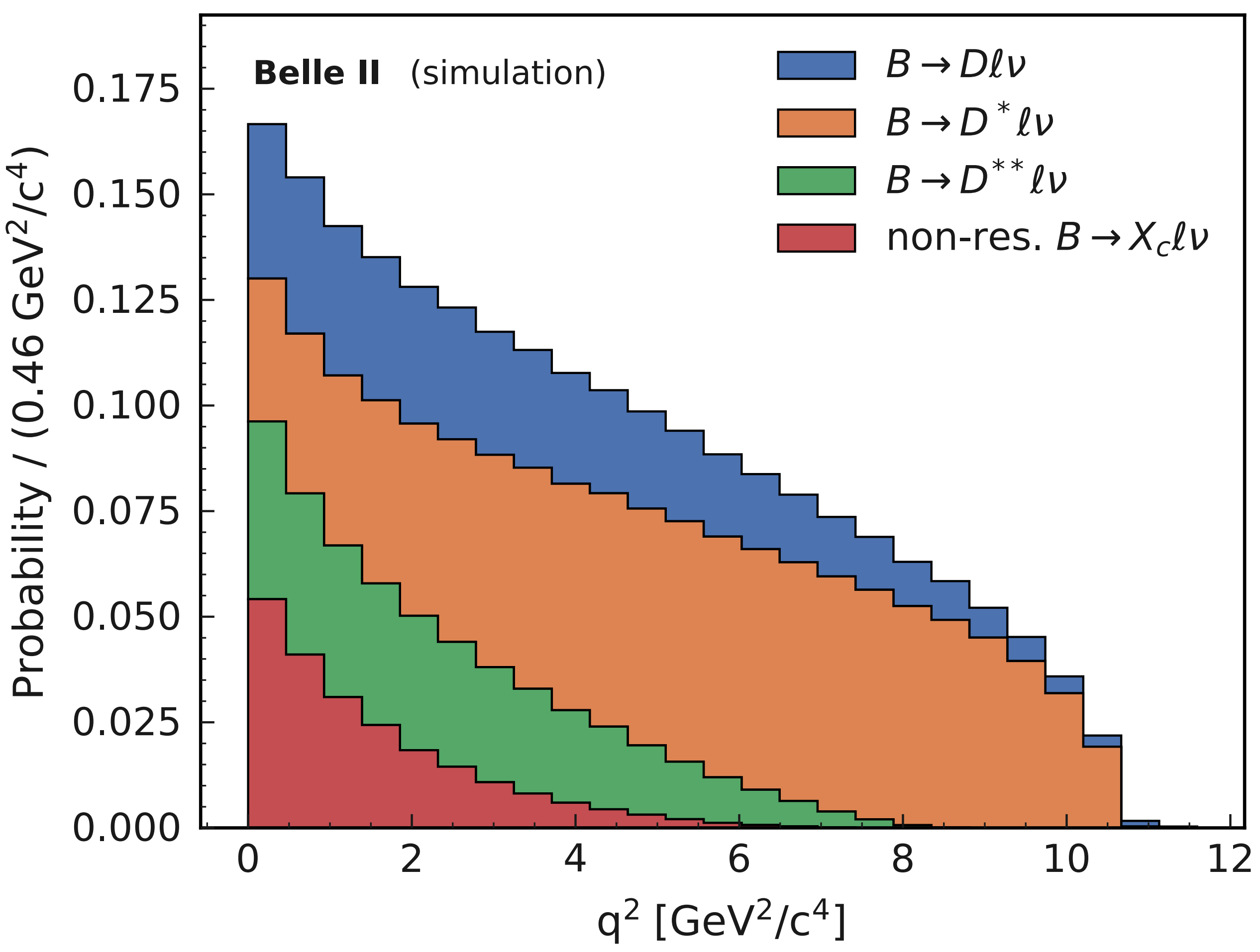
(Assign 100% BR uncertainty in systematics covariance matrix)

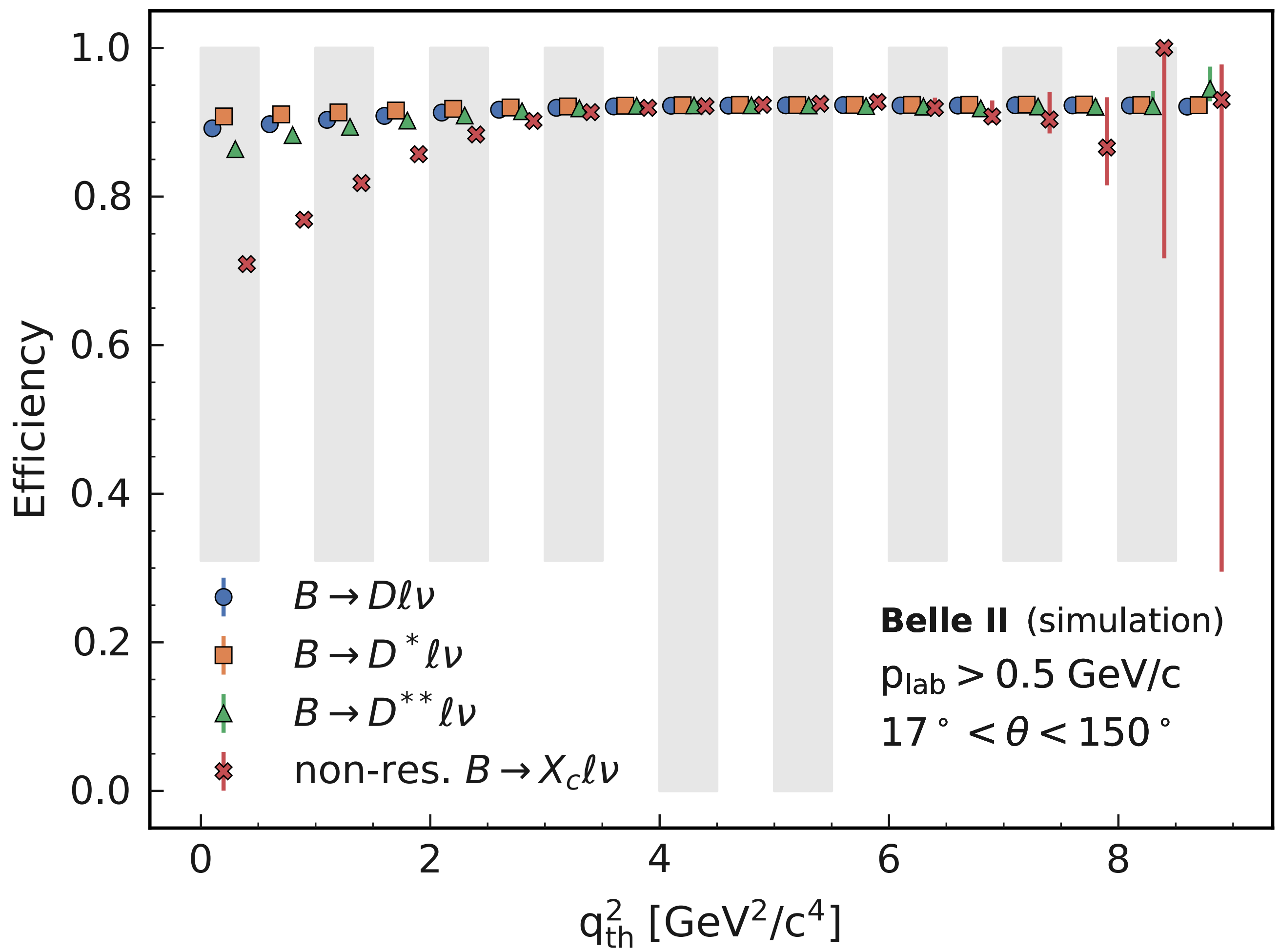


Provides better kinematic description

PRD 104, 112011 (2021)

X_c Simulation





Full Angular Information **without** going to 4D

Full angular information can be encoded into **12 coefficients** :

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_V\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\chi} = \frac{G_F^2\left|V_{cb}\right|^2 m_B^3}{2\pi^4}$$

$$\times\left\{J_{1s}\sin^2\theta_V + J_{1c}\cos^2\theta_V\right.$$

$$+\left(J_{2s}\sin^2\theta_V + J_{2c}\cos^2\theta_V\right)\cos 2\theta_\ell$$

$$+J_3\sin^2\theta_V\sin^2\theta_\ell\,\cos 2\chi$$

$$+J_4\sin 2\theta_V\sin 2\theta_\ell\,\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\,\cos\chi$$

$$+\left(J_{6s}\sin^2\theta_V + J_{6c}\cos^2\theta_V\right)\cos\theta_\ell$$

$$+J_7\sin 2\theta_V\sin\theta_\ell\,\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\,\sin\chi$$

$$\left.+J_9\sin^2\theta_V\sin^2\theta_\ell\,\sin 2\chi\right\}.$$

→ Each of these coefficients is a function of $q^2 \sim w$

↓

With some smart folding, one can "easily" determine them

Based on the ideas of:
JHEP 05 (2013) 043
JHEP 05 (2013) 137
Phys. Rev. D 90, 094003 (2014)
http://cds.cern.ch/record/1605179

8 Coefficients relevant in massless limit & SM

How can we measure these coefficients?

Step 1: bin up phase-space in $q^2 \sim w$ in however many bins you can afford

Step 2: Determine the # of signal events in specific phase-space regions

The coefficients are related to a weighted sun of events in a given q^2 bin

$$J_i = \frac{1}{N_i} \sum_{j=1}^{8} \sum_{k,l=1}^{4} \eta_{ij}^{\chi}\, \eta_{ik}^{\theta_\ell}\, \eta_{il}^{\theta_V} \left[\chi^i \otimes \theta_\ell^j \otimes \theta_V^k \right]$$

Normalization Factor

Weights

Phase space region

J_i	η_i^{χ}	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+,a,a,+\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+,a,a,+\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-,b,b,-\}$	$\{-,c,c,-\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-,b,b,-\}$	$\{+,d,d,+\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-,c,c,-\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+,d,d,+\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$$

FB, Z. Ligeti, S. Turczyk, Phys. Rev. D 90, 094003 (2014)

E.g. for J_3: **Split** χ into **2 Regions**

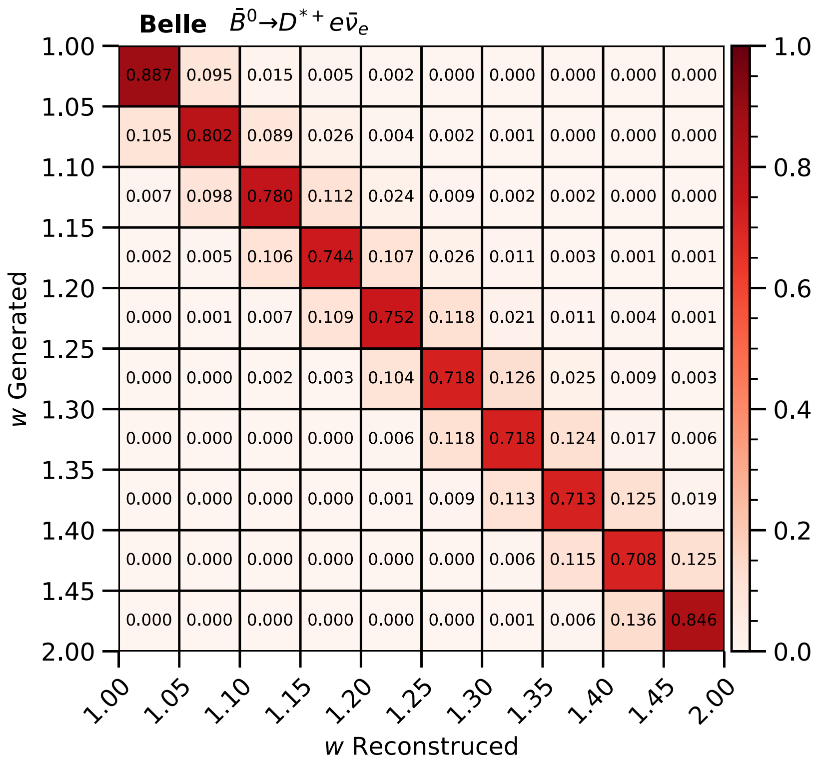
$'+' : \chi \in [0,\pi/4], [3/4\pi,5/4\pi], [7/4\pi,2\pi]$ $\tilde{N}_+$

$'-' : \chi \in [\pi/4,3/4\pi], [5/4\pi,7/4\pi]$ $\tilde{N}_-$

Step 3: Reverse Migration and Acceptance Effects

Resolution effects: events with a **given "true" value** of $\{q^2, \cos\theta_\ell, \cos\theta_V, \chi\}$ can **fall into different reconstructed** bins

E.g. w migration matrix



0.887	0.095	0.015	0.005	0.002	0.000	0.000	0.000	0.000	0.000
0.105	0.802	0.089	0.026	0.004	0.002	0.001	0.000	0.000	0.000
0.007	0.098	0.780	0.112	0.024	0.009	0.002	0.002	0.000	0.000
0.002	0.005	0.106	0.744	0.107	0.026	0.011	0.003	0.001	0.001
0.000	0.001	0.007	0.109	0.752	0.118	0.021	0.011	0.004	0.001
0.000	0.000	0.002	0.003	0.104	0.718	0.126	0.025	0.009	0.003
0.000	0.000	0.000	0.000	0.006	0.118	0.718	0.124	0.017	0.006
0.000	0.000	0.000	0.000	0.001	0.009	0.113	0.713	0.125	0.019
0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.115	0.708	0.125
0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.006	0.136	0.846

arXiv:2301.07529 **[hep-ex]**

$$\begin{pmatrix}\tilde{N}_+^{q_1^2}\\ \tilde{N}_-^{q_1^2}\\ \tilde{N}_+^{q_2^2}\\ \tilde{N}_-^{q_2^2}\\ \tilde{N}_+^{q_3^2}\\ \tilde{N}_-^{q_3^2}\\ \tilde{N}_+^{q_4^2}\\ \tilde{N}_-^{q_4^2}\\ \tilde{N}_+^{q_5^2}\\ \tilde{N}_-^{q_5^2}\end{pmatrix} = \mathscr{M} \begin{pmatrix}N_+^{q_1^2}\\ N_-^{q_1^2}\\ N_+^{q_2^2}\\ N_-^{q_2^2}\\ N_+^{q_3^2}\\ N_-^{q_3^2}\\ N_+^{q_4^2}\\ N_-^{q_4^2}\\ N_+^{q_5^2}\\ N_-^{q_5^2}\end{pmatrix}$$

← Unfolded yields

Migration matrix

Bkg subtracted yields

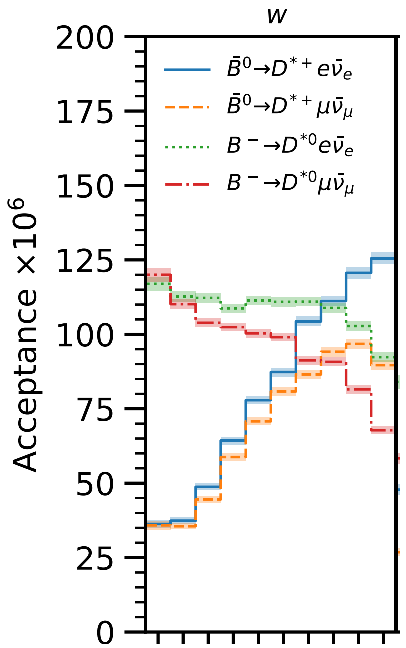
Acceptance x Efficiency Corrections:

$$N_+^{q_i^2} \cdot e^{-1}_{\text{eff},+,q_i^2} = n_+^{q_i^2}$$

$$N_-^{q_i^2} \cdot e^{-1}_{\text{eff},-,q_i^2} = n_-^{q_i^2}$$

← Acceptance / Eff. corrected yields

Unfolded yields



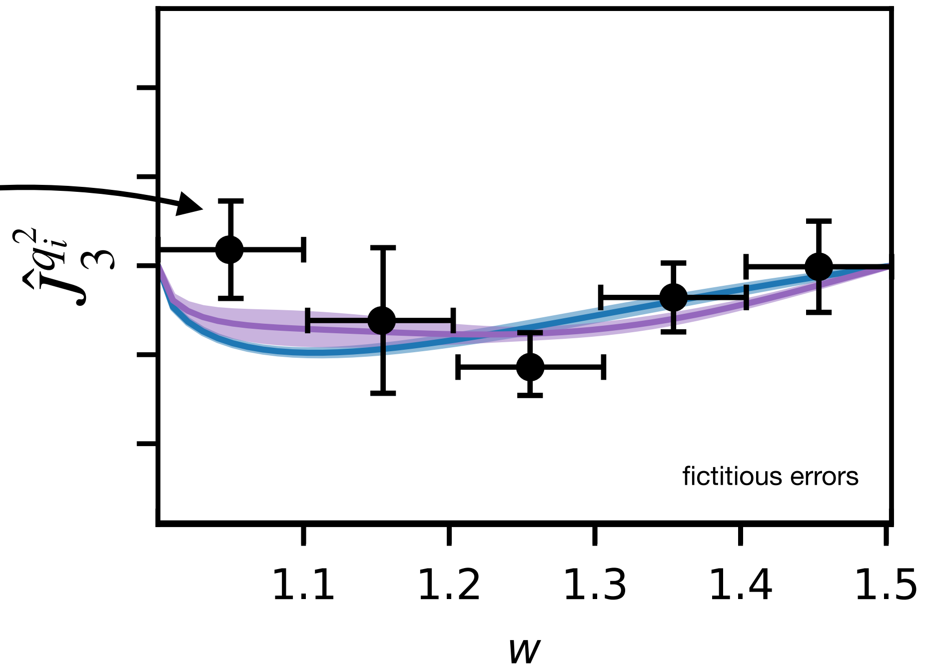
arXiv:2301.07529 **[hep-ex]**

Step 4: Calculate J_i for a given w/q^2 bin

$$\begin{matrix} n_+^{q_i^2} \\ n_-^{q_i^2} \end{matrix} \rightarrow \hat{J}_3^{q_i^2} = \boxed{\frac{1}{\Gamma}} \times \frac{n_+^{q_i^2} - n_-^{q_i^2}}{\boxed{4(4/3)^2}}$$

Normalization

$$\Gamma = \frac{8}{9}\pi\left(3\sum_i J_{1c}^{q_i^2} + 6\sum_i J_{1s}^{q_i^2} - \sum_i J_{2c}^{q_i^2} - 2\sum_i J_{2s}^{q_i^2}\right)$$



More **involved** for the **other** coefficients: need full experimental covariance between all measured w/q^2 bins and coefficients (statistical overlap, systematics)

SM:

$$\{J_{1s}^{q_i^2}, J_{1c}^{q_i^2}, J_{2s}^{q_i^2}, J_{2c}^{q_i^2}, J_{3}^{q_i^2}, J_{4}^{q_i^2}, J_{5}^{q_i^2}, J_{6s}^{q_i^2}\}$$

e.g. **5 x 8** = **40 coefficients**

or full thing (SM + NP)
with **5 x 12** = **60 coefficients**

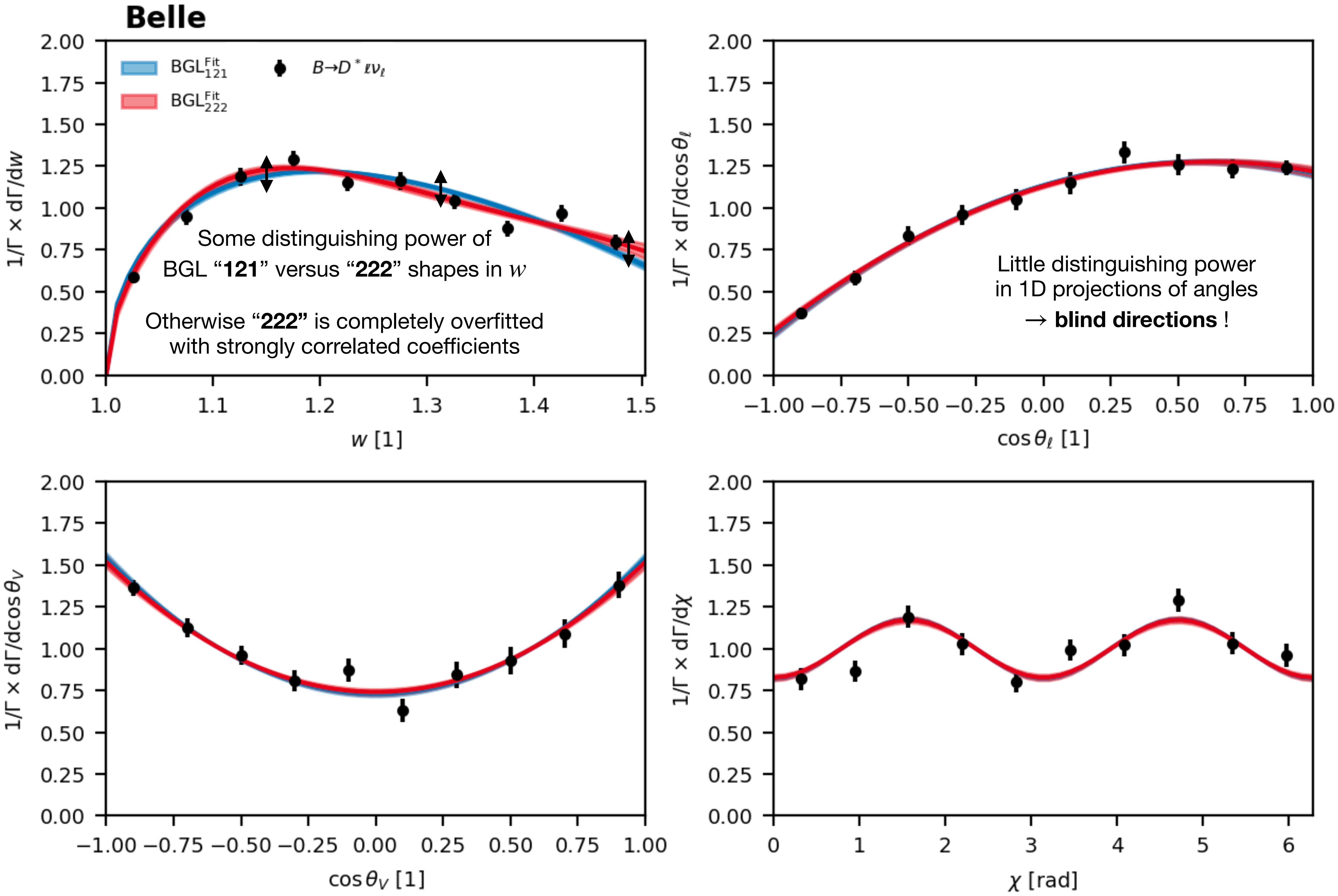
J_i	η_i^χ	$\eta_i^{\theta_\ell}$	$\eta_i^{\theta_V}$	normalization N_i
J_{1s}	$\{+\}$	$\{+, a, a, +\}$	$\{-, c, c, -\}$	$2\pi(1)2$
J_{1c}	$\{+\}$	$\{+, a, a, +\}$	$\{+, d, d, +\}$	$2\pi(1)(2/5)$
J_{2s}	$\{+\}$	$\{-, b, b, -\}$	$\{-, c, c, -\}$	$2\pi(-2/3)2$
J_{2c}	$\{+\}$	$\{-, b, b, -\}$	$\{+, d, d, +\}$	$2\pi(-2/3)(2/5)$
J_3	$\{+,-,-,+,+,-,-,+\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$
J_4	$\{+,+,-,-,-,-,+,+\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_5	$\{+,+,-,-,-,-,+,+\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_{6s}	$\{+\}$	$\{+,+,-,-\}$	$\{-, c, c, -\}$	$2\pi(1)2$
J_{6c}	$\{+\}$	$\{+,+,-,-\}$	$\{+, d, d, +\}$	$2\pi(1)(2/5)$
J_7	$\{+,+,+,+,-,-,-,-\}$	$\{+\}$	$\{+,+,-,-\}$	$4(\pi/2)(4/3)$
J_8	$\{+,+,+,+,-,-,-,-\}$	$\{+,+,-,-\}$	$\{+,+,-,-\}$	$4(4/3)^2$
J_9	$\{+,+,-,-,+,+,-,-\}$	$\{+\}$	$\{+\}$	$4(4/3)^2$

$$a = 1 - 1/\sqrt{2}, b = a\sqrt{2}, c = 2\sqrt{2} - 1, d = 1 - 4\sqrt{2}/5$$

FB, Z. Ligeti, S. Turczyk, Phys. Rev. D 90, 094003 (2014)

1D versus Full Angular Sensitivities

Errors and central values from 1D projection fits of arXiv:2301.07529 (Table XVI)



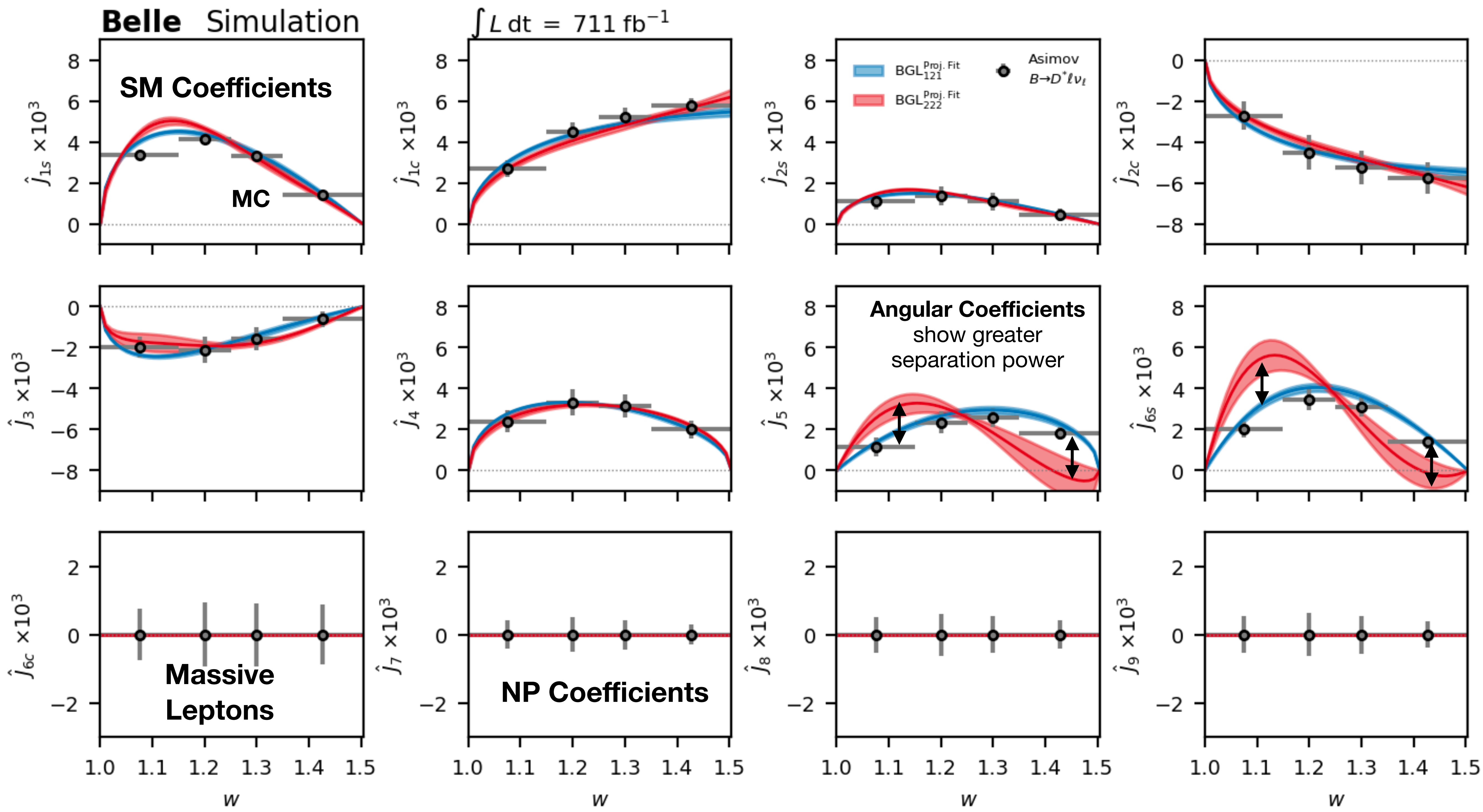
Some distinguishing power of BGL "**121**" versus "**222**" shapes in w

Otherwise "**222**" is completely overfitted with strongly correlated coefficients

Little distinguishing power in 1D projections of angles → **blind directions** !

1D versus Full Angular Sensitivities

Errors and central values from
1D projection fit of
arXiv:2301.07529 (Table XVI)
Data points: **Asimov Fit using MC (!)**



SM Coefficients

MC

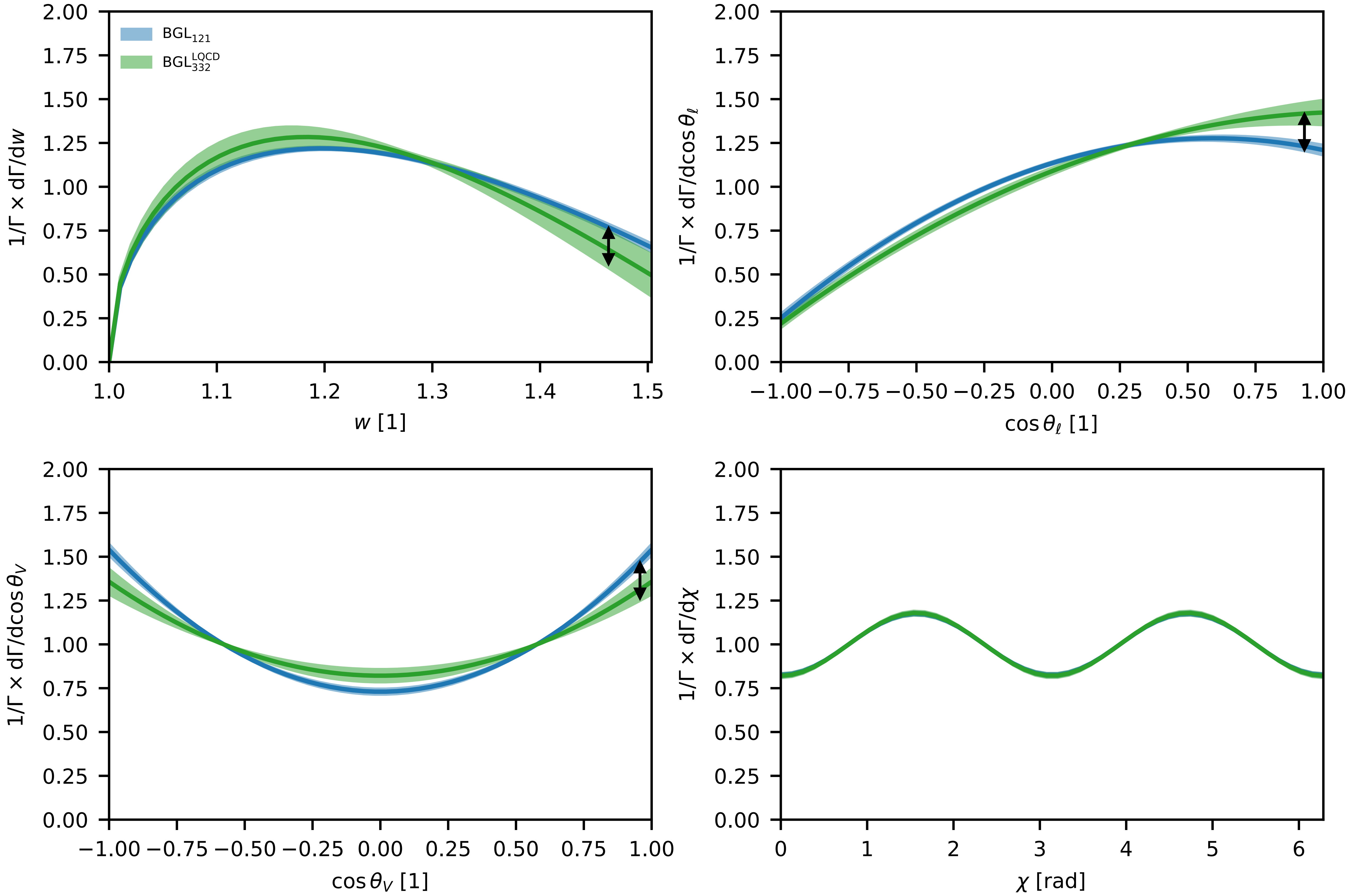
Angular Coefficients
show greater
separation power

Massive Leptons

NP Coefficients

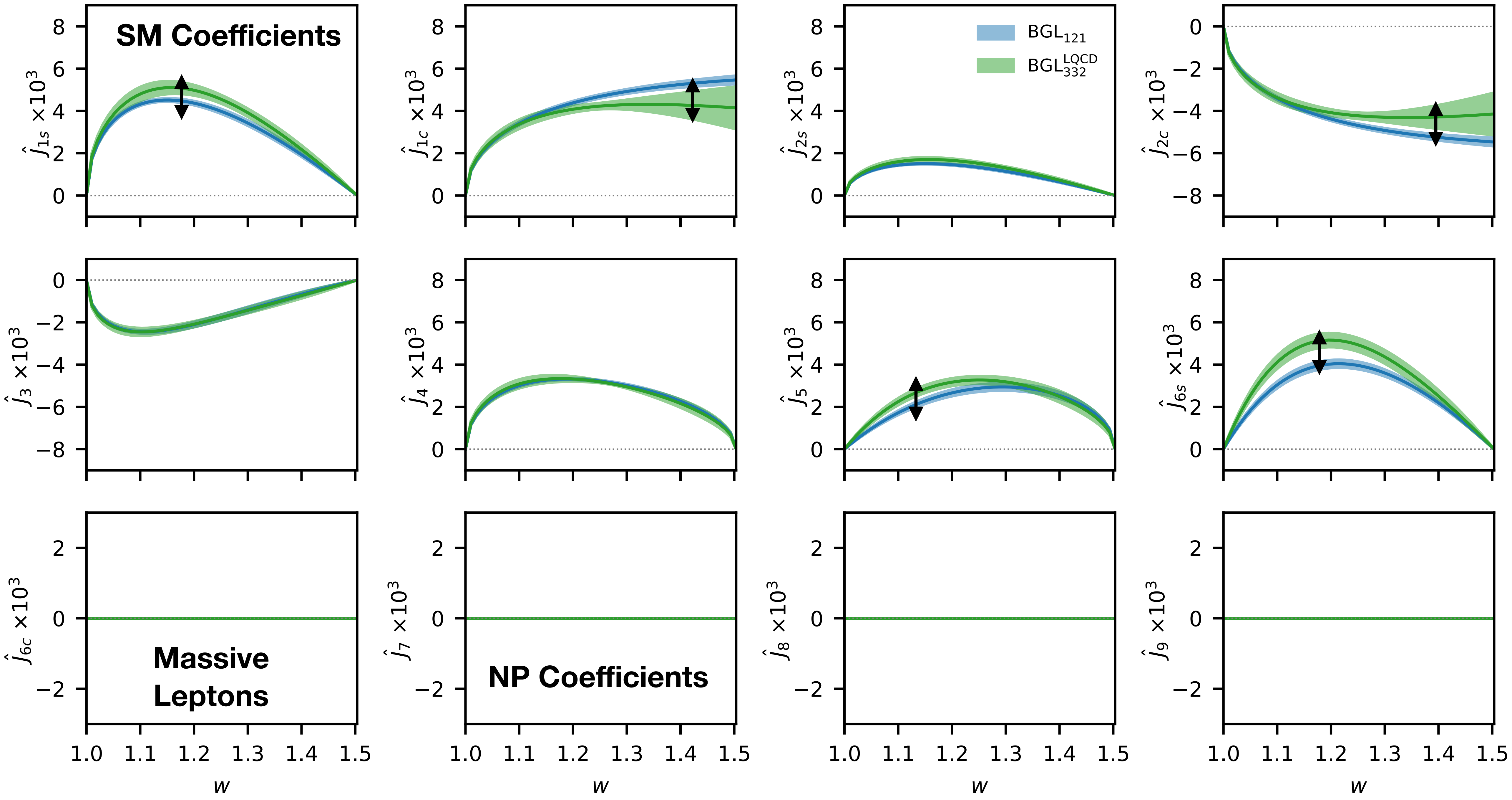
1D versus Full Angular Sensitivities

BGL121 1D projection fit of arXiv:2301.07529 (Table XVI) or
FNAL/MILC prediction
[arXiv:2105.14019]



1D versus Full Angular Sensitivities

BGL121 1D projection fit of arXiv:2301.07529 (Table XVI) or **FNAL/MILC prediction** [arXiv:2105.14019]



Angular Coefficients also will allow us to better **investigate** what is going on with **lattice** versus **data tensions**..