Flavor Physics

Rusa Mandal
IIT Gandhinagar, India

Belle II Physics Week@KEK Tsukuba

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Outline

- Lecture I
  - Flavor of the Standard Model
  - Weak decays: Effective Theory: Operator Product Expansion
- Lecture II
  - Form factor, Penguin decays
  - Current tensions
- Lecture III
  - SMEFT, Minimal Flavor Violation
  - Flavor Model with BSM physics

- hep-ph/980647
review by A. J. Buras
- Gauge theories of weak decays
book by A. J. Buras

Aim of the lectures: to get familiar with the methods

and terms used in theory

The Standard Model

Gauge structure of the SM of Particle Physics

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

strong: color

weak: isospin

hypercharge

Fermions: three generations			$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
e_R	μ_R	τ_R	**1**	**1**	-1
$L_1 = (\nu_e, e_L)^\top$	$L_2 = \left(\nu_\mu, \mu_L\right)^\top$	$L_3 = (\nu_\tau, \tau_L)^\top$	**1**	**2**	$-\frac{1}{2}$
u_R	c_R	t_R	**3**	**1**	$\frac{2}{3}$
d_R	s_R	b_R	**3**	**1**	$-\frac{1}{3}$
$Q_1 = (u_L, d_L)^\top$	$Q_2 = (c_L, s_L)^\top$	$Q_3 = (t_L, b_L)^\top$	**3**	**2**	$\frac{1}{6}$
Gauge bosons: mediators					
	G_μ^a	$a = 1\text{–}8$	**8**	**1**	0
	W_μ^a	$a = 1, 2, 3$	**1**	**3**	0
	B_μ		**1**	**1**	0
Higgs					
$\Phi = \left(\phi^+, \phi^0\right)^\top$			**1**	**2**	$\frac{1}{2}$

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Lagrangian

$$\mathscr{L}_{\rm kin} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + (D^\mu \Phi)^\dagger (D_\mu \Phi)$$

Fermion-gauge boson interaction

Higgs-gauge boson interaction

$$\psi = \{e_R, L, u_R, d_R, Q\}$$

$$F_{\mu\nu} = \{G^a_{\mu\nu}, W^a_{\mu\nu}, B_{\mu\nu}\}$$

$$\mathscr{L}_{\rm Yuk} = -\bar{Q}\Phi Y_D d_R - \bar{Q}\Phi^c Y_U u_R - \bar{L}\Phi Y_E e_R$$

→ Higgs-fermion interaction

3 x 3 Yukawa matrices: flavour dynamics

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3 x 3 Yukawa matrices: flavour dynamics

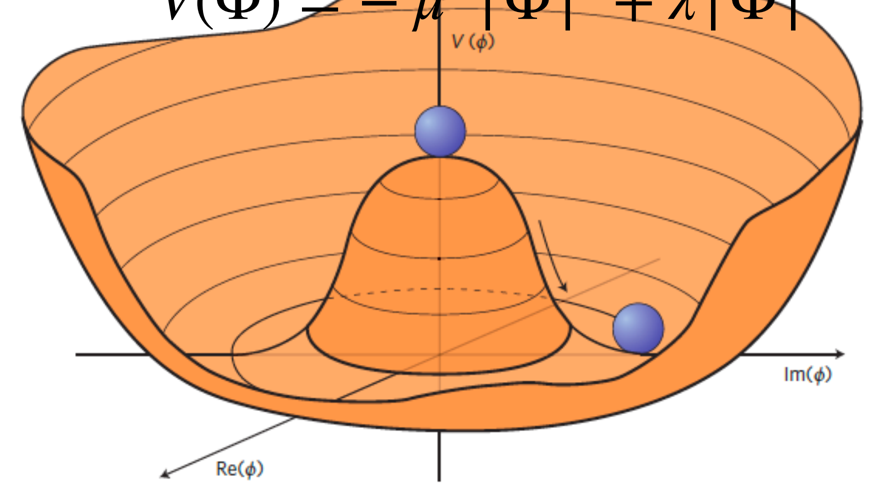
No Yukawa for neutrinos → massless in SM

$$SU(2)_L \times U(1)_Y \xrightarrow{\text{SSB}} U(1)_{\text{EM}}$$

$$\langle 0 | \Phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

→ Massive gauge bosons: $W^\pm, Z$

$$V(\Phi) = -\mu^2 |\Phi|^2 + \lambda |\Phi|^4$$



[Courtesy: CERN document server]

Flavor sector

$\mathscr{L}_{\text{kin}}$ for fermions are invariant under $[U(3)]^5$

$$Q_L \to V_L^u\, Q, \quad u_R \to V_R^u\, u_R, \quad d_R \to V_R^d\, d_R,$$
$$L \to V_L^e\, L, \quad e_R \to V_R^e\, e_R,$$

$V_{L,R}^{u,d,e}$: 3x3 unitary matrices

$\mathscr{L}_{\text{Yuk}}$ breaks $[U(3)]^5$ → $[U(1)]^4$ Baryon no. B

& 3 lepton family nos. $L_{e,\mu,\tau}$

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Can we use flavour symmetry to diagonalise all Yukawa matrices?

bi-unitary transformation $(V_L^d)^\dagger \Upsilon^D V_R^d = \hat{\Upsilon}^D, \quad (V_L^u)^\dagger \Upsilon^U V_R^u = \hat{\Upsilon}^U, \quad (V_L^e)^\dagger \Upsilon^E V_R^e = \hat{\Upsilon}^E.$

But only 3 matrices are available in quark sector: V_L^d missing

$$\mathcal{L}_{\rm Yuk} = -\bar{Q}\Phi (V_L^u)^\dagger V_L^d \hat{\Upsilon}^D d_R - \bar{Q}\Phi^c \hat{\Upsilon}^U u_R - \bar{L}\Phi \hat{\Upsilon}^E e_R$$

non-diagonal → Extra rotation for *d*-type quarks

Flavor sector

Mass basis → All Yukawa matrices are diagonal

$$d_L \to d_L' = (V_L^u)^\dagger V_L^d d_L \equiv V_{\rm CKM}\, d_L$$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Flavor sector

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Where do we see the effect of CKM rotation?

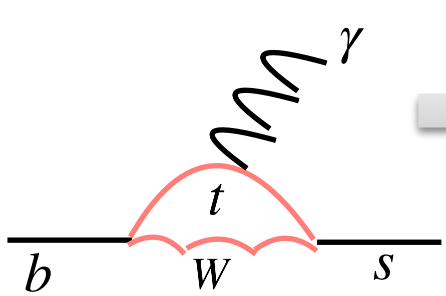
Kinetic term: $\bar{f} i \not{D} f$ → $\mathscr{L}_I^{\rm SM} \supset \frac{e}{\sin\vartheta_W \cos\vartheta_W}\left(T_3 - \sin^2\vartheta_W Q_f\right)\bar{f}\gamma^\mu Z_\mu f + eQ_f \bar{f}\gamma^\mu A_\mu f$

$$\sum_{j=1,2,3} \bar{d}_{Lj}\gamma_\mu d_{Lj} V^\mu \to \sum_{j,k=1,2,3} \bar{d}_{Lj} \underbrace{\left(V_{\rm CKM}^\dagger V_{\rm CKM}\right)_{jk}}_{=\delta_{jk}} \gamma_\mu d_{Lk} V^\mu$$

$V_\mu = \{G_\mu, Z_\mu, A_\mu\}$

neutral gauge bosons

No flavor changing neutral current@tree level



→ possible at loop level

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Flavor sector

Charged current: $\sum_{j-1,2,3} \bar{u}_j \gamma^\mu P_L d_j W_\mu^+ \longrightarrow \sum_{j,k-1,2,3} \bar{u}_j \gamma^\mu P_L V_{jk} d_k W_\mu^+ = \bar{u} \gamma^\mu V_{CKM} P_L d W_\mu^+$

- flavor violation generated in gauge interaction via Yukawa interactions in mass basis

Flavor sector

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→ flavor violation generated in gauge interaction via Yukawa interactions in mass basis

General parametrization of 3x3 unitary matrix → 3 angles + 6 phases

Not all phases physical—5 are rotated away $u_j^{L,R} \to e^{i\varphi_j^u} u_j^{L,R}, \quad d_j^{L,R} \to e^{i\varphi_j^d} d_j^{L,R}$

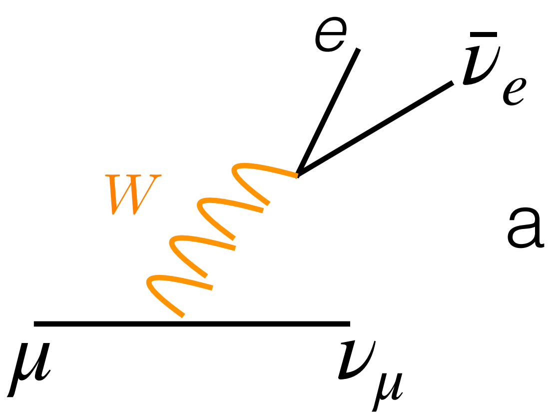
$V_{ij}^{\rm CKM} \to e^{i(\varphi_j^d - \varphi_i^u)} V_{ij}^{\rm CKM}$ → 3 angles + 1 phases

only source of CP violation

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij}$
$c_{ij} = \cos\theta_{ij}$

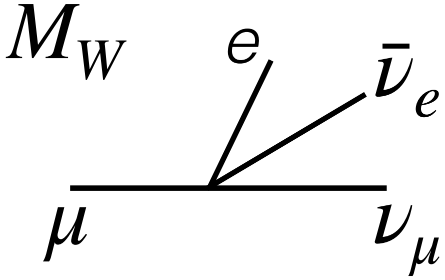
Weak decays of muons



amplitude= $-\frac{1}{8}\frac{g_2^2}{k^2 - M_W^2}[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu][\bar{e}\gamma^\mu (1-\gamma_5)\nu_e],$

$k^2 \ll M_W^2$ is good approximation as $m_\mu \ll M_W$

$$\frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu][\bar{e}\gamma^\mu (1-\gamma_5)\nu_e].$$

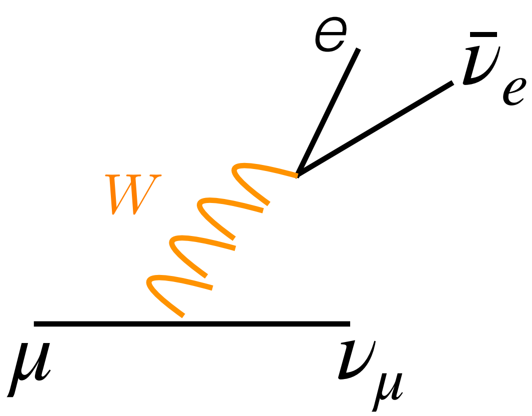


matching with Fermi theory with 4-point effective interaction $\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}$

$\mathrm{BR}(\mu \to e\nu_\mu\bar{\nu}_e) \sim 100\,\%$ → decay width of muon

used to evaluate G_F

Weak decays of muons



$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)$$

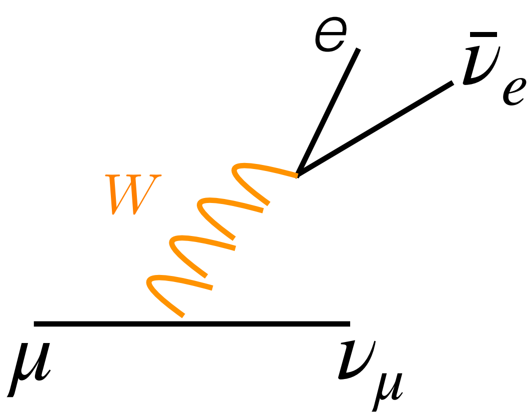
@LO with phase space factor

$$\tau_\mu^{\text{theo}} = 2.18776 \times 10^{-6}\,\text{s}$$

$$\tau_\mu^{\text{expt}} = 2.1969811(22) \times 10^{-6}\,\text{s}$$

→ very closely in agreement

Weak decays of muons



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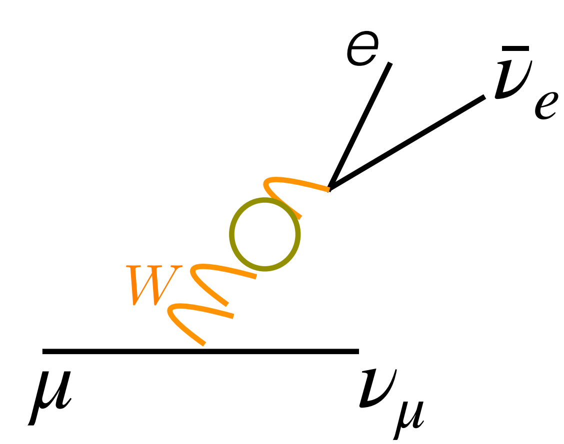
Including electro-weak corrections

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)$$



$$\tau_\mu^{\text{theo}} = 2.19699 \times 10^{-6}\,\text{s}$$

$$\tau_\mu^{\text{expt}} = 2.1969811(22) \times 10^{-6}\,\text{s}$$

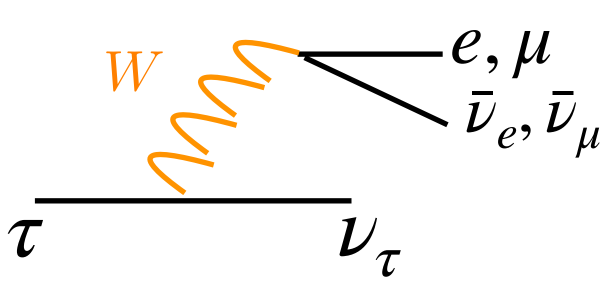
→ in perfect agreement

Weak decays of tau

Total decay width of fermion $\Gamma_f^{tot} = \frac{G_F^2 m_f^5}{192\pi^3}\left[f(m_{f'}/m_f) + \cdots\right]$

phase space + higher order in α_{EM}

BR~35%



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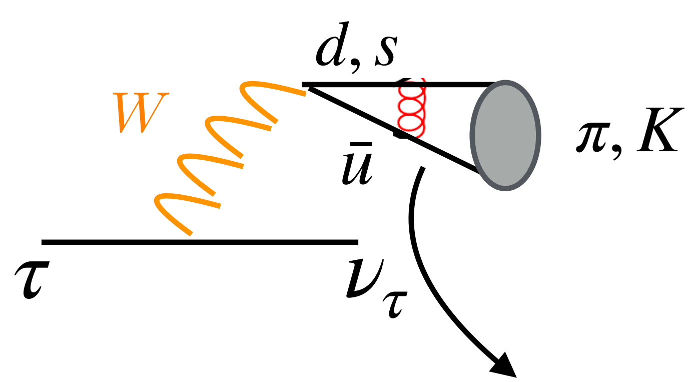


$\tau_\tau^{theo} = 3.26707 \times 10^{-13}$ s

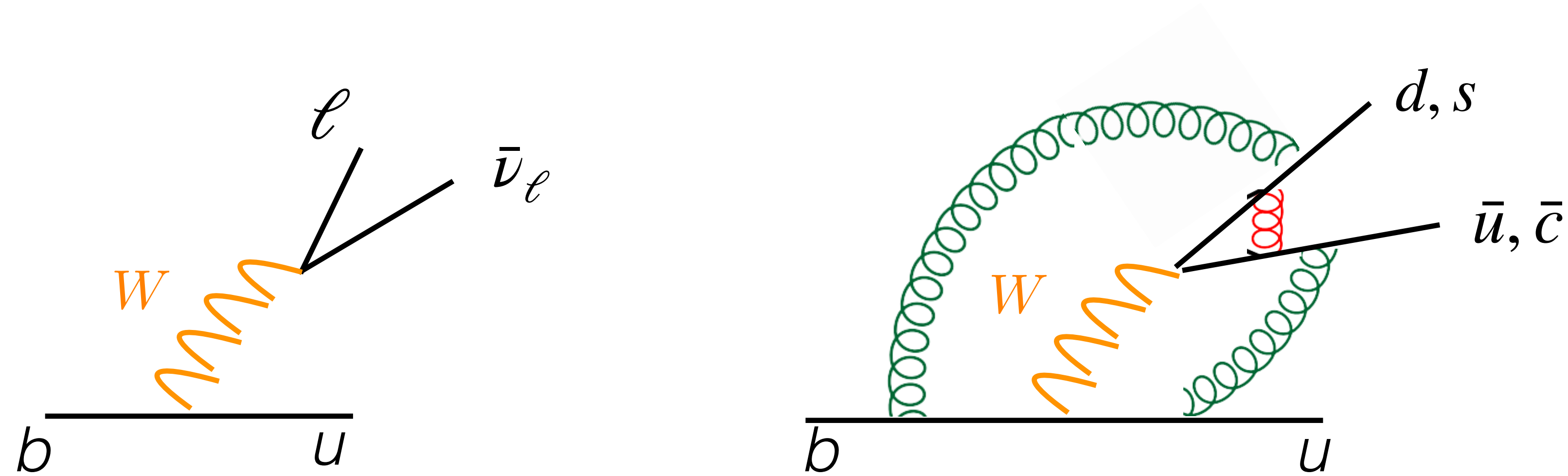
$\tau_\tau^{expt} = 2.906(1) \times 10^{-13}$ s

Gluon exchange within quarks

QED effects under control but not QCD

Tau decay is used to evaluate α_s— strong coupling constant

Weak decays of quarks



Total decay width $\Gamma_f^{\rm tot} = \dfrac{G_F^2 m_f^5}{192\pi^3}\left[f(m_{f'}/m_f) + \cdots\right]$

heavy quark masses enter
— depend on scheme

$$\begin{aligned} m_c^{\rm Pole} &= 1.471 \ \text{GeV}, & m_b^{\rm Pole} &= 4.650 \ \text{GeV} \\ \bar{m}_c(\bar{m}_c) &= 1.277 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) &= 4.248 \ \text{GeV} \\ \bar{m}_c(\bar{m}_b) &= 0.997 \ \text{GeV}, & \bar{m}_b(\bar{m}_b) &= 4.248 \ \text{GeV} \end{aligned}$$

→ Wide range of theory predictions for $\tau_b^{\rm theo}$

$\tau_b^{\rm theo} = 2.60 \times 10^{-15}\,\text{ps}$ @$\bar{m}_{c,b}(\bar{m}_b)$ — differs from $\tau_b^{\rm expt}$

Loops

Actual physics lies in loops! → Accuracy check of tree level & validity of perturbation theory

Several decays start@1-loop

Divergent pieces in loop integrals → Renormalization

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Actual physics lies in loops! → Accuracy check of tree level & validity of perturbation theory

Several decays start@1-loop

Divergent pieces in loop integrals → Renormalization → energy scale induced → Large Logarithms

$$\alpha_s(m_b)\ln(m_b^2/\mu^2)$$

$$0.2 \times 1.3$$

Resummation with RG equations

	Leading Log	Next-to leading Log	NNLL
Tree	1		
1-loop	$\alpha_s \ln$	α_s	
2-loop	$\alpha_s^2 \ln^2$	$\alpha_s^2 \ln$	α_s^2

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OPE

Weak decays of quarks involve different scales

$\mu = \mathcal{O}(M_W)$ → fundamental scale of weak interaction— small α_s

$\mathcal{O}(1\,\mathrm{GeV}) \leq \mu \leq M_W$ → α_s variation is significant
resummation of large Logs necessary

$\mu \leq \mathcal{O}(1\,\mathrm{GeV})$ → confinement effects has to be included

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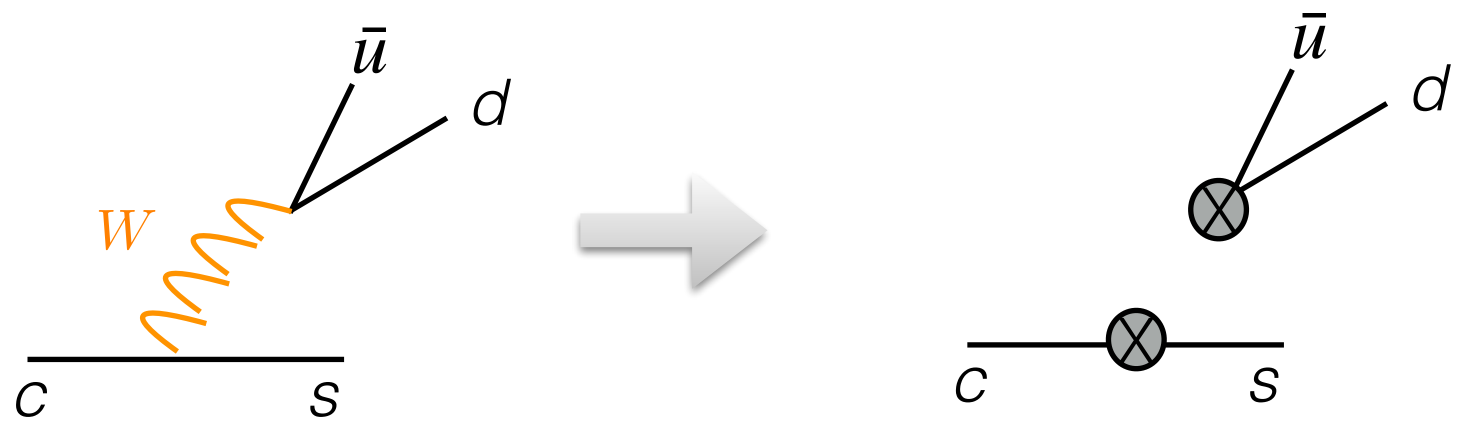
An example:

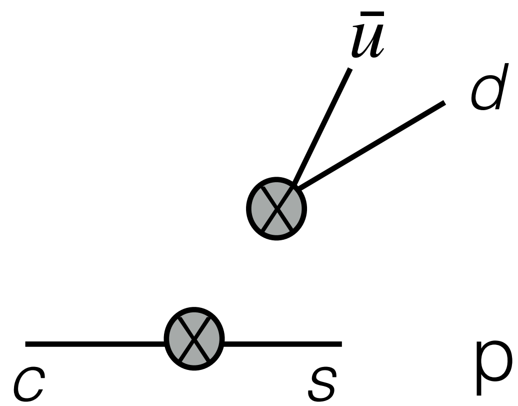


$$A = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \frac{M_W^2}{k^2 - M_W^2} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A}$$

$$= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A} + O\left(\frac{k^2}{M_W^2}\right)$$

$(\bar{f}f)_{V-A} \equiv \bar{f}\gamma_\mu(1-\gamma_5)f$

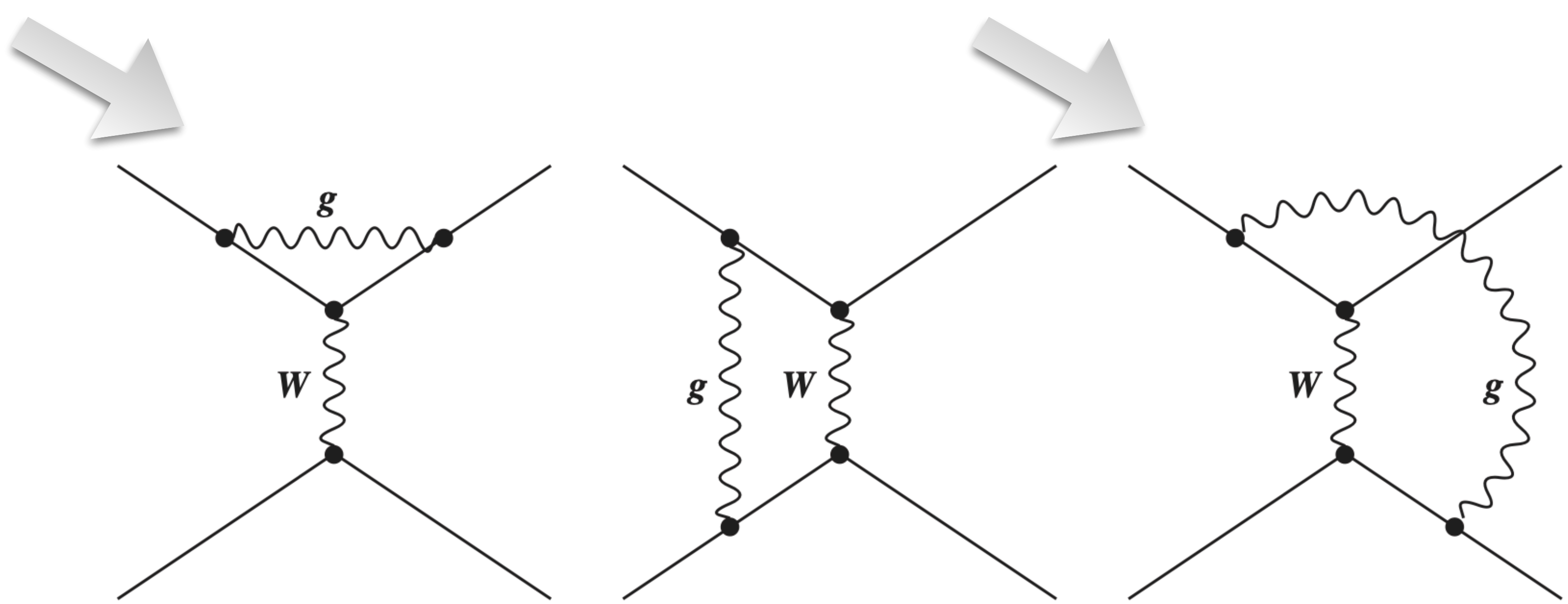
OPE



$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \, C \mathscr{Q} + \text{higher D}\,; \quad \mathscr{Q} \equiv (\bar{s}c)_{V-A}(\bar{u}d)_{V-A}$$

product of two currents expanded in series of local operators weighted by effective coupling constants— Wilson coefficients C

C=1 altered by QCD corrections + new operators induced



different colour structure

$$T^a_{\alpha\beta} T^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\gamma\beta}$$

OPE

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1(\mu) Q_1 + C_2(\mu) Q_2)$$

$$Q_1 = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A}$$

$$Q_2 = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A}$$

Amplitude of full theory should match with the amplitude produced from effective theory Hamiltonian → matching condition

$$A_{\text{full}} = A_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (C_1 \langle Q_1 \rangle + C_2 \langle Q_2 \rangle)$$

$$A_{\text{full}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \Bigg[\left(1 + 2 C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\varepsilon} + \ln \left(\frac{\mu^2}{-p^2} \right) \right) \right) S_2 + \frac{3}{N} \frac{\alpha_s}{4\pi} \ln \left(\frac{M_W^2}{-p^2} \right) S_2 - 3 \frac{\alpha_s}{4\pi} \ln \left(\frac{M_W^2}{-p^2} \right) S_1 \Bigg].$$

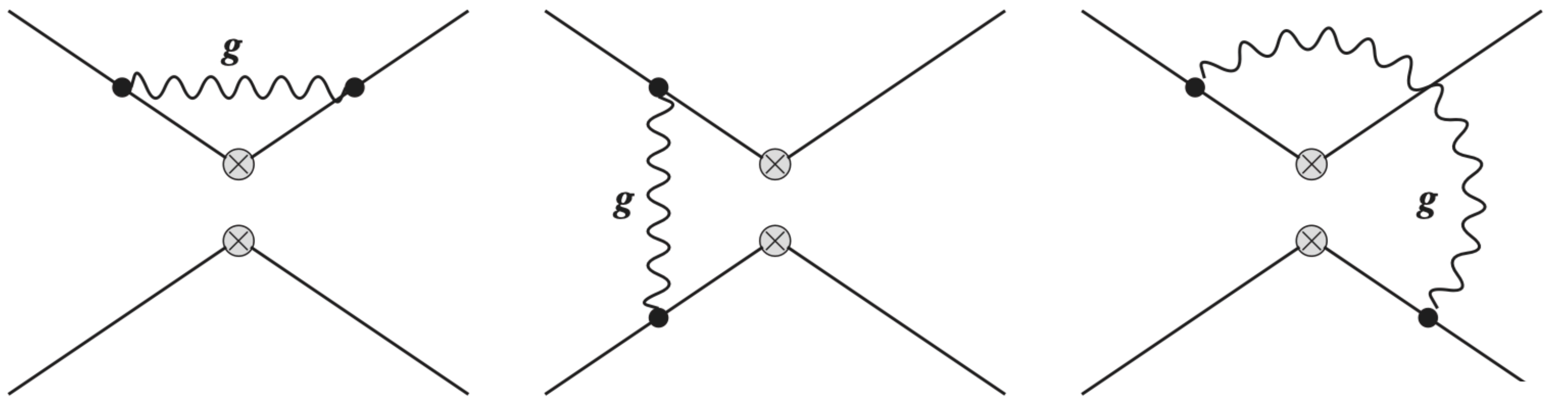
$$S_1 \equiv \langle Q_1 \rangle_{tree} = (\bar{s}_\alpha c_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A},$$

$$S_2 \equiv \langle Q_2 \rangle_{tree} = (\bar{s}_\alpha c_\alpha)_{V-A} (\bar{u}_\beta d_\beta)_{V-A},$$

Divergent pole can be absorbed in field redefinition

tree level matrix element

Matrix element



$$\langle Q_1\rangle^{(0)} = \left(1 + 2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)\right)S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)S_1 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)S_2$$

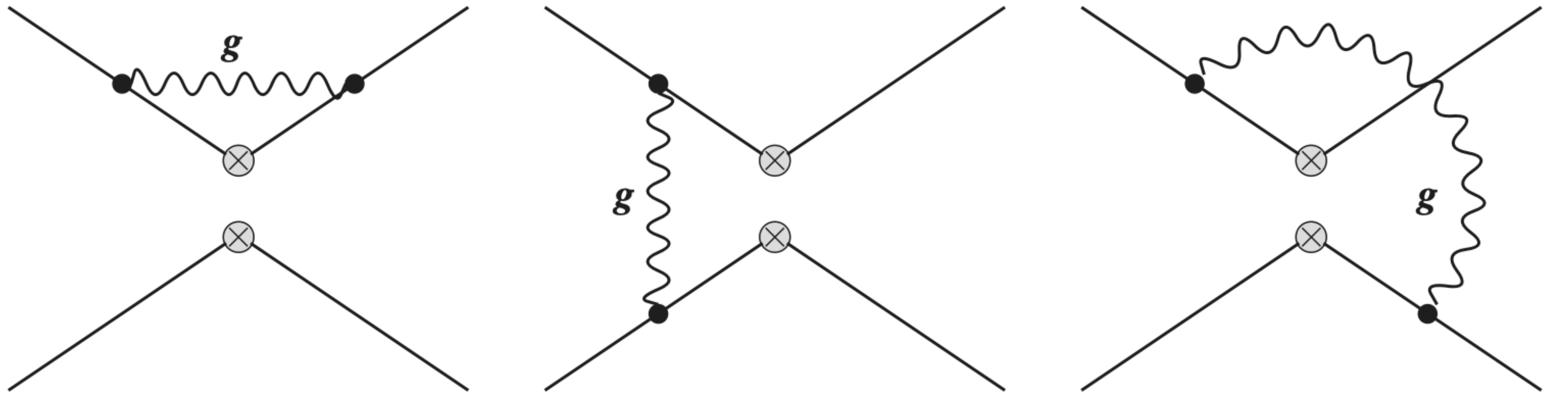
$$\langle Q_2\rangle^{(0)} = \left(1 + 2C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)\right)S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)S_2 - 3\frac{\alpha_s}{4\pi}\left(\frac{1}{\varepsilon} + \ln\left(\frac{\mu^2}{-p^2}\right)\right)S_1$$

divergences in 1st two terms absorbed in field renormalization.

More divergent than full theory—effective theory is nonrenormalizable

need additional constants —operator renormalization

Matrix element



$$\langle Q_i\rangle^{(0)} = Z_q^{-2}\hat{Z}_{ij}\langle Q_j\rangle$$

Quark field renormalization

Operator renormalization

$$\hat{Z} = 1 + \frac{\alpha_s}{4\pi}\frac{1}{\varepsilon}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}$$

Renormalized operators:

$$\langle Q_1\rangle = \left(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)\right)S_1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)S_1 - 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)S_2$$

$$\langle Q_2\rangle = \left(1 + 2C_F\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)\right)S_2 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)S_2 - 3\frac{\alpha_s}{4\pi}\ln\left(\frac{\mu^2}{-p^2}\right)S_1$$

Wilson coefficients

Matching between full and EFT amplitudes gives

$$C_1(\mu) = -3\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right), \qquad C_2(\mu) = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\left(\frac{M_W^2}{\mu^2}\right)$$

remember when NO QCD: $C_1(M_W) = 0, \;\; C_2(M_W) = 1$

Operator renormalization similar to coupling constant renormalization if Wilson coefficients are thought as bare coupling constants in $\mathcal{H}_{eff}$

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Operator renormalization similar to coupling constant renormalization if Wilson coefficients are thought as bare coupling constants in $\mathscr{H}_{eff}$

Factorisation of energy scales@ $\mathcal{O}(\alpha_s)$

$$\left(1 + \alpha_s r \ln\left(\frac{M_W^2}{-p^2}\right)\right) \doteq \left(1 + \alpha_s r \ln\left(\frac{M_W^2}{\mu^2}\right)\right) \cdot \left(1 + \alpha_s r \ln\left(\frac{\mu^2}{-p^2}\right)\right)$$

full theory= WC (short distance) matrix element (long distance)

$$\int_{-p^2}^{M_W^2} \frac{dk^2}{k^2} = \int_{\mu^2}^{M_W^2} \frac{dk^2}{k^2} + \int_{-p^2}^{\mu^2} \frac{dk^2}{k^2}$$

Wilson coefficients

- WCs are independent of external states

  p^2 dropped from the expressions

  — need to be careful while regularising infrared divergences

- Operators mix under renormalization: $\hat{Z}$ is non-diagonal

→ Counter term for $\mathcal{Q}_2$ depends on the constants for both $\mathcal{Q}_2$ & $\mathcal{Q}_1$

diagonal basis: $Q_\pm = \dfrac{Q_2 \pm Q_1}{2}, \qquad C_\pm = C_2 \pm C_1$

$$C_\pm(\mu) = 1 + \left(\frac{3}{N} \mp 3\right) \frac{\alpha_s}{4\pi} \ln\left(\frac{M_W^2}{\mu^2}\right)$$

$\frac{\alpha_s}{4\pi}$ → 4%

$\ln\left(\frac{M_W^2}{\mu^2}\right)$ → Large log $\mu = 1\,\mathrm{GeV}$

Total 1st order correction amounts 60-130%

→ Naive breakdown of perturbative series

Wilson coefficients

Resum large logs via RG eqn: $$\frac{dC_\pm(\mu)}{d\ln\mu} = \gamma_\pm(g)C_\pm(\mu)$$

anomalous dimension

depends on renormalization constants

Similar to running of α_s → $$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{1-\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right)} \approx \alpha_s(M_Z)\left[1-\sum_{n=1}^{\infty}\left(\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right)\right)^n\right]$$

In RG improved perturbation theory: $$C_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\frac{\gamma_\pm^{(0)}}{2\beta_0}} C_\pm(M_W)$$ ($C_\pm(M_W)$ → 1)

@b-mass scale $C_+(\mu_b) = 0.847$ and $C_-(\mu_b) = 1.395$

→ departure from the value 1 due to QCD

Prescription

- Step-1: Matching in perturbation theory

amplitude in full theory matched to operator matrix element in effective theory → extraction of WCs $C_i(\Lambda)$

↳ mass of heavy particles integrated out

Prescription

- Step-1: Matching in perturbation theory

amplitude in full theory matched to operator matrix element in effective theory ⟶ extraction of WCs $C_i(\Lambda)$

Λ ⟶ mass of heavy particles integrated out

- Step-2: RG improved perturbation theory

using anomalous dimension of operators compute WCs at any lower scale via RG evolution $C_i(\mu) = U(\mu, \Lambda)C_i(\Lambda)$

Prescription

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- Step-2: RG improved perturbation theory

using anomalous dimension of operators compute WCs at any lower scale via RG evolution $C_i(\mu) = U(\mu, \Lambda) C_i(\Lambda)$

- Step-3: Non-perturbative calculation

hadronic matrix elements at the lower scale via methods: Lattice gauge theory, QCD sum rules

factorization between short & long distance physics

$C_i(\mu) : \mu >$ ↩ ↪ $< \mu : \langle \mathcal{Q}(\mu) \rangle$

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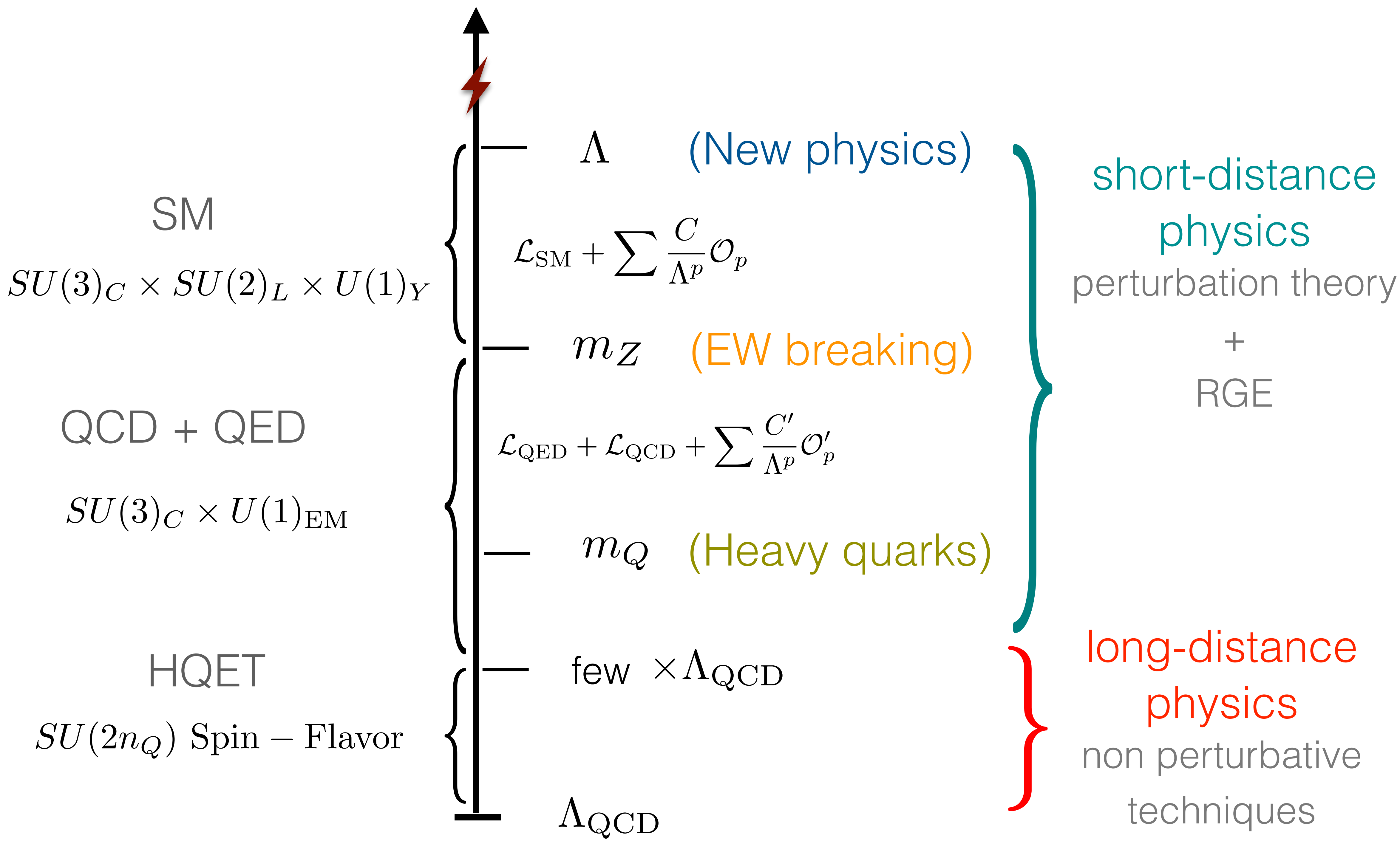
Lecture II

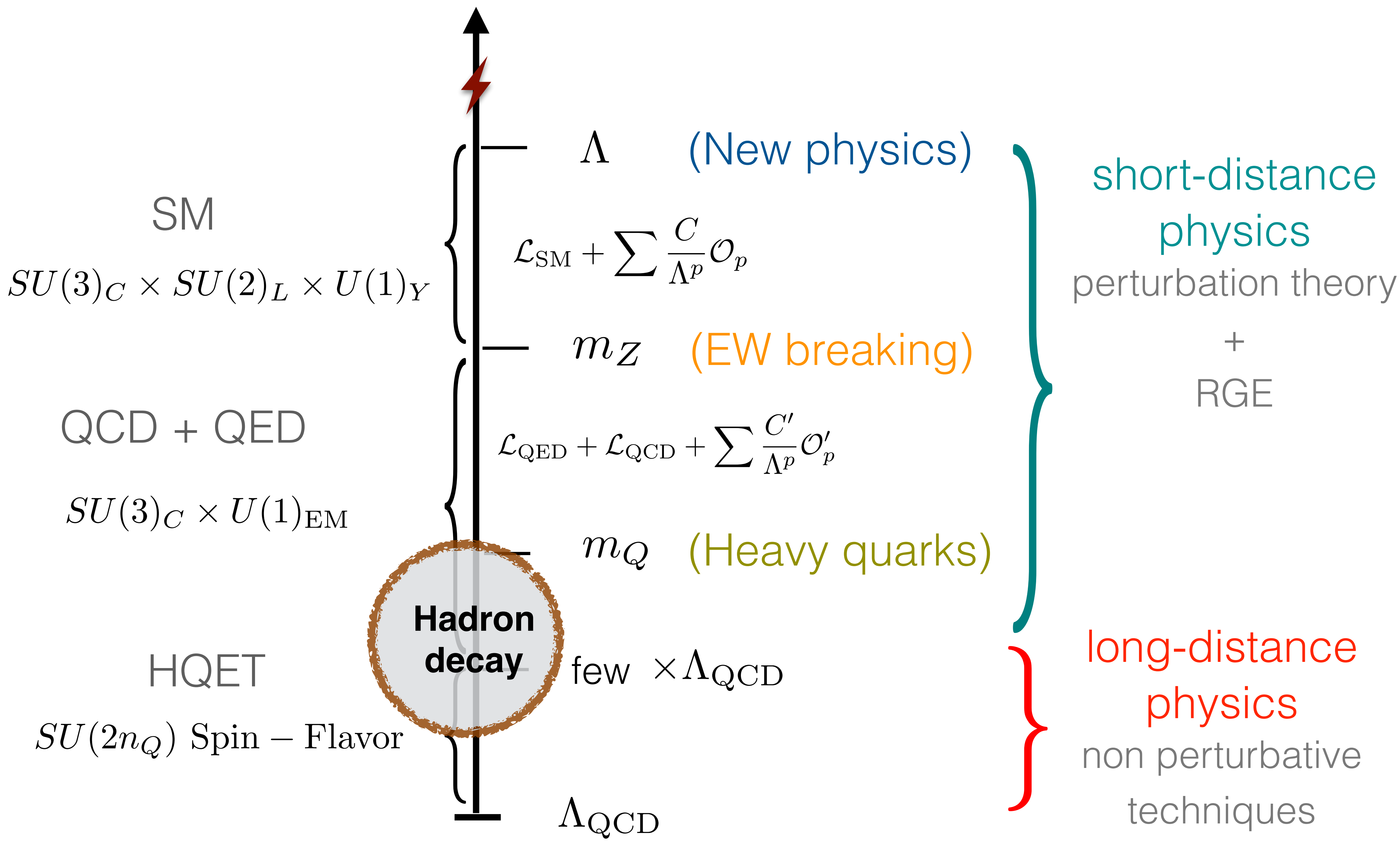
Effective theory

Effect of new particles captured in higher dimensional operators



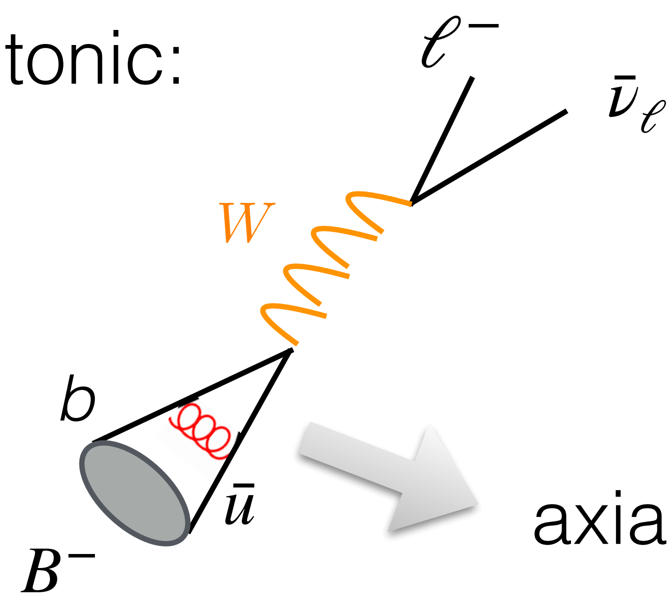
Effective theory

Effect of new particles captured in higher dimensional operators



Hadron decay

Leptonic:



decay probability: $\langle 0 | \bar{b}\Gamma u | B \rangle$

should be parity-odd

pseudoscalar

axialvector $\langle 0 | \bar{b}\gamma^\mu\gamma_5 u | B(p) \rangle = -\, i f_B p^\mu$

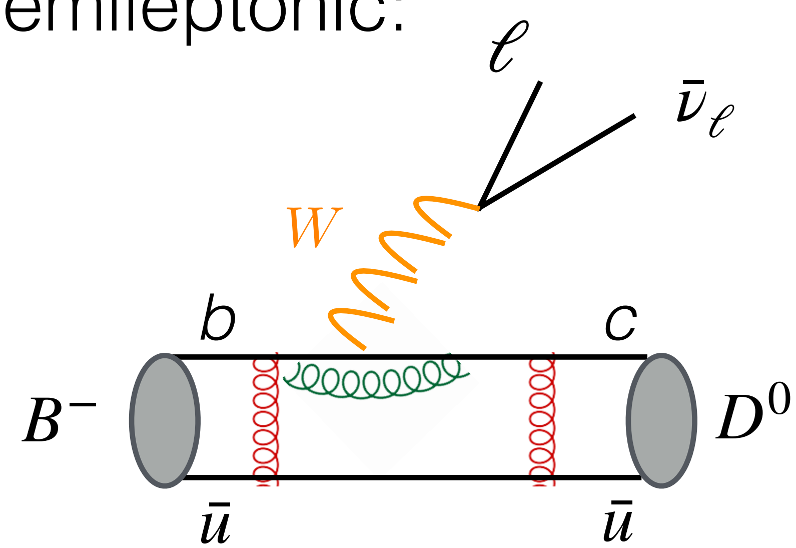
QCD effects parametrized in non-perturbative parameter
— decay constant

binding of quarks inside meson

non-perturbative techniques: Lattice QCD, QCD sum rule,
Heavy quark effective theory

Hadron decay

Semileptonic:



$$\mathscr{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} (\bar{c}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \nu)$$

parametrization of quark current when sandwiched between hadron states

binding of quarks inside meson
+
QCD interaction between initial and final state

$$\langle D(p_D) \,|\, \bar{c}\gamma^\mu b \,|\, B^-(p_B) \rangle = f_1 p_D^\mu + f_2 p_B^\mu$$

$f_{1,2}$ Form factors—depends on Lorentz scalar $(p_B - p_D)^2 = q^2$

non-perturbative techniques: Lattice QCD, QCD sum rule, Heavy quark effective theory

QCD sum rules

- QCD Sum Rule methods for non perturbative estimates

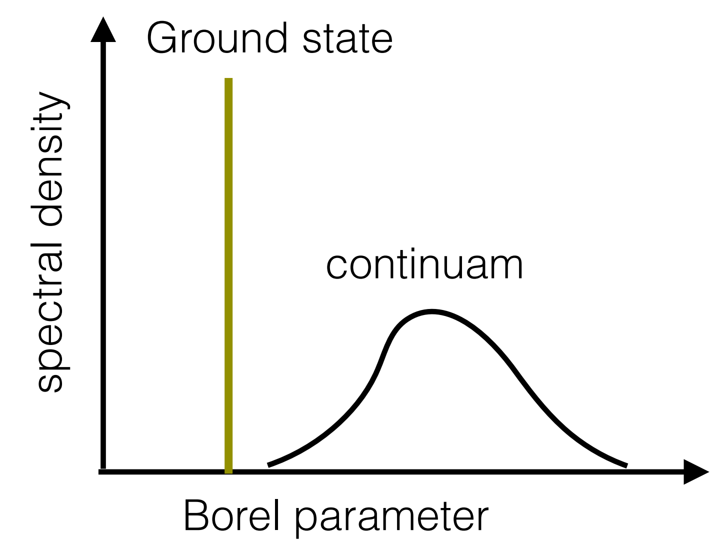
Based on Operator product expansion [Shifman *et al.* '79]

Perturbatively calculable amplitudes + Quark & gluon condensate: charactarises QCD vacuum or distribution amplitudes in LCSR [Khodjamirian *et al.* '05,'07]

Dispersion relation

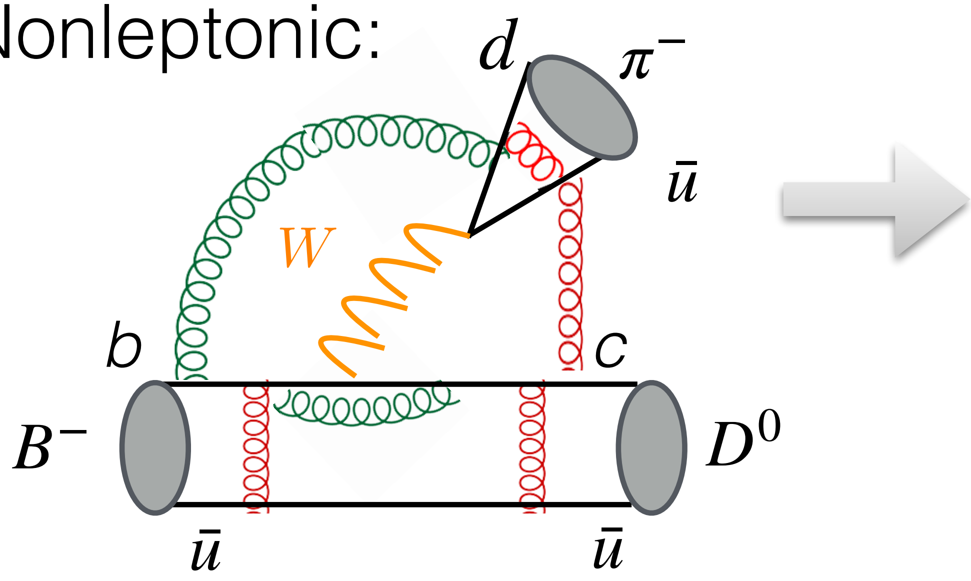
Physical hadronic parameters

- Limitations: hadronic parameter extraction depends on model ansatzs for the spectrum



Hadron decay

Nonleptonic:



Complicated QCD dynamics—
additional assumption required

$$\mathcal{H}_{\rm eff} \sim V_{cb} V_{ub}^* (\bar{c}\gamma_\mu P_L b)(\bar{u}\gamma^\mu P_L d)$$

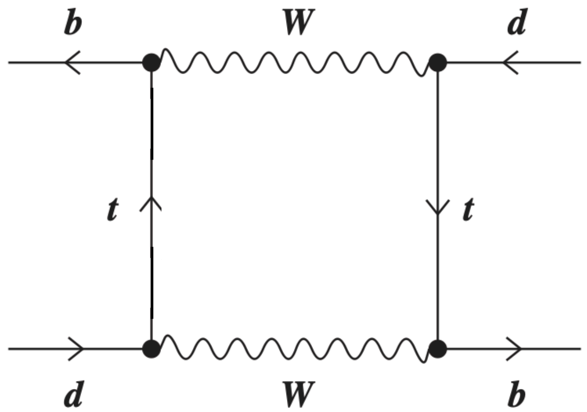
QCD factorization $\langle D^0 \pi^- | \bar{c}\gamma_\mu (1-\gamma_5) b \cdot \bar{u}\gamma^\mu (1-\gamma_5) d | B^- \rangle$

$$\approx \langle D^0 | \bar{c}\gamma_\mu (1-\gamma_5) b | B^- \rangle \cdot \langle \pi^- | \bar{u}\gamma^\mu (1-\gamma_5) d | 0 \rangle$$

$$\approx f^{B^- \to D^0}(q^2) \cdot f_\pi \, .$$

How good is the assumption?!

Box



$$\mathcal{H}_{\text{eff}}^{\Delta B=2} = \frac{G_F^2}{16\pi^2} M_W^2 \left(V_{tb}^* V_{td}\right)^2 C_Q(\mu_b)\, (\bar{b}_\alpha d_\alpha)_{V-A} (\bar{b}_\beta d_\beta)_{V-A}$$

in absence of QCD

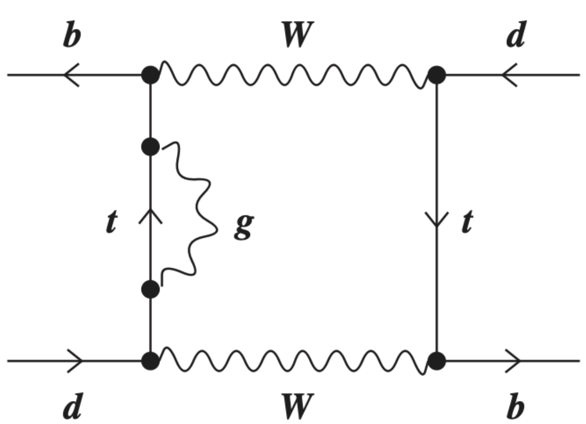
$C_Q(\mu_W) = S_0(x_t)$ loop function

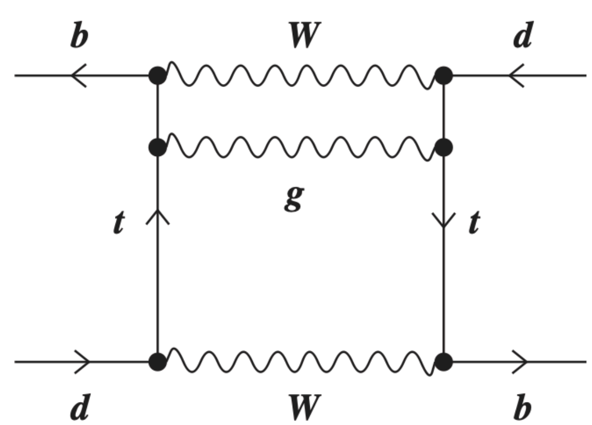
sandwiched between meson-antimeson states— calculated at meson mass scale

Use RG evolution for WCs $C_Q(\mu_b) = \left[\frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)}\right]^{6/23} S_0(x_t)$

$$\langle \bar{B}_d^0 | (\bar{b}_\alpha d_\alpha)_{V-A} (\bar{b}_\beta d_\beta)_{V-A} | B_d^0 \rangle \equiv \frac{4}{3} B_{B_d}(\mu_b) F_{B_d}^2 m_{B_d}$$

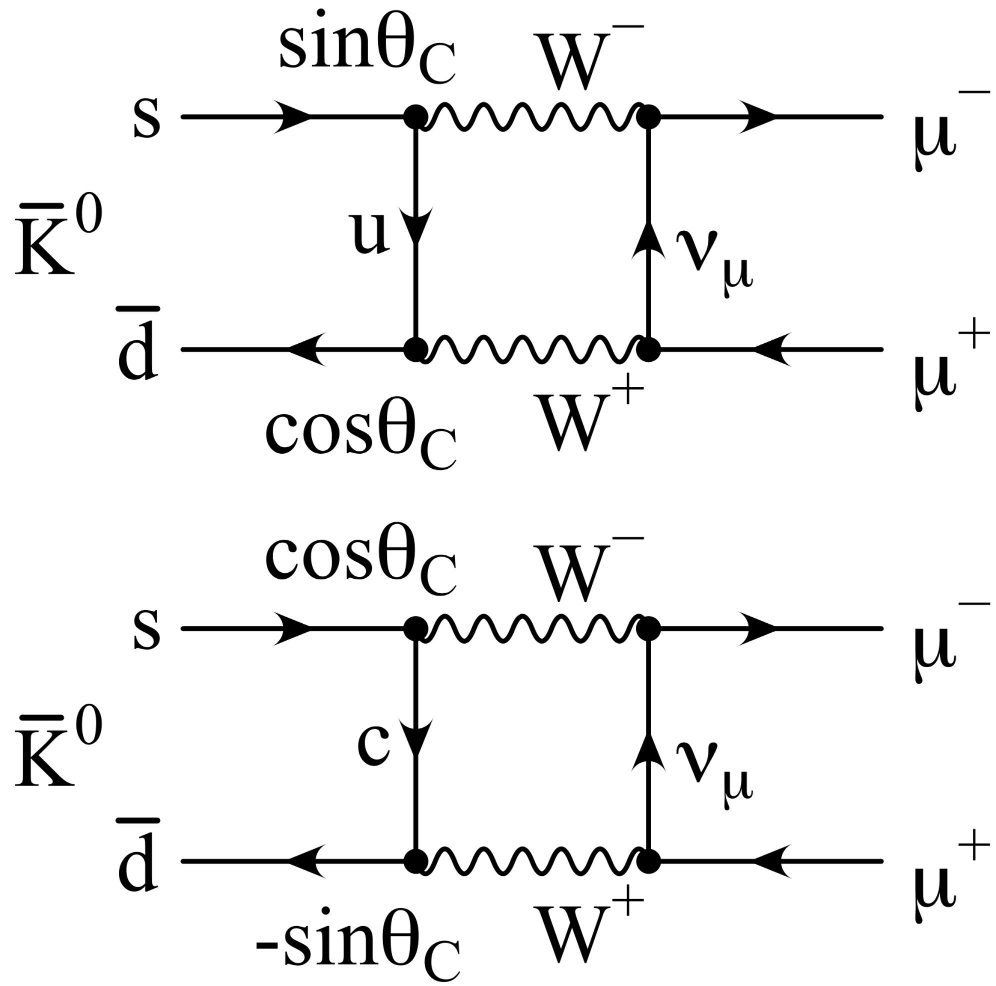
precise estimates from Lattice QCD





improve with NLO

Charm discovery



$$\frac{\mathrm{BR}(K_L \to \mu^+\mu^-)}{\mathrm{BR}(K^+ \to \mu^+\nu_\mu)} \sim 10^{-9}$$

smallness of BR predicted existence of fourth quark

Sum of two diagrams $\propto \alpha_{\mathrm{EM}}^2 (m_c^2 - m_u^2)$

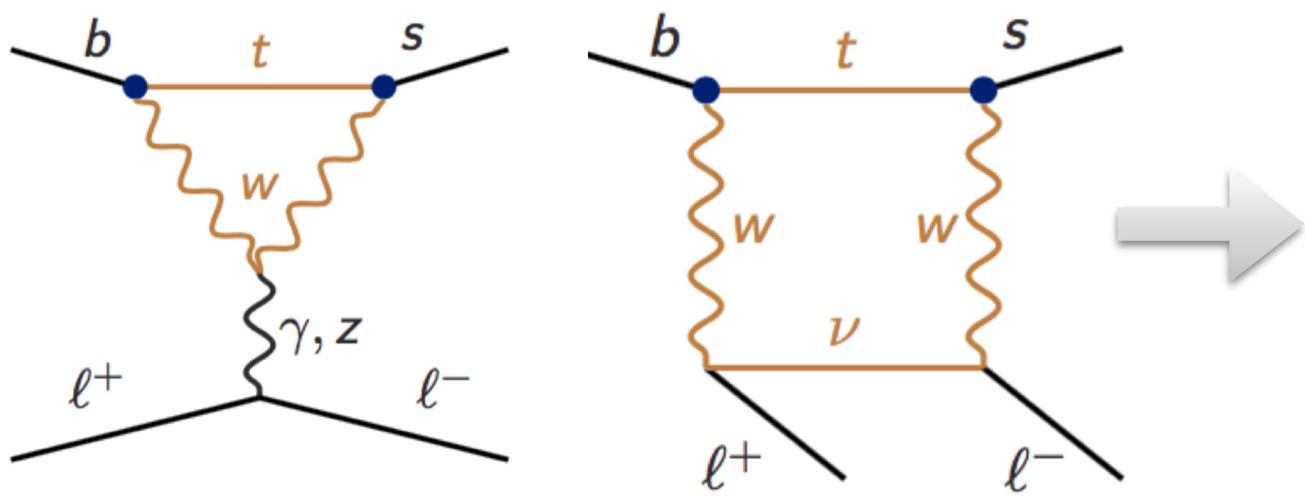
Prediction of c-quark mass

absence of FCNC at loop level due to unitarity of CKM
—broken by distinct masses of the quarks GIM Mechanism

Theory of weak decays done!

Next is to compare with data

Penguin



Top quark contribution dominates

Most general dim-6 effective Hamiltonian

$$\mathcal{H}_{\rm eff} = -\frac{4\,G_F}{\sqrt{2}}\left(\lambda_t \mathcal{H}^{(t)}_{\rm eff} + \lambda_u \mathcal{H}^{(u)}_{\rm eff}\right) \qquad \lambda_i = V_{ib}V^*_{is}$$

$$\mathcal{H}^{(t)}_{\rm eff} = C_1 Q^c_1 + C_2 Q^c_2 + \sum_{i=3}^{6} C_i Q_i + \sum_{i=7,8,9,10,P,S} (C_i Q_i + C'_i Q'_i),$$

$$\mathcal{H}^{(u)}_{\rm eff} = C_1 (Q^c_1 - Q^u_1) + C_2 (Q^c_2 - Q^u_2).$$

Penguin

dim-6 basis including new physics operators

$$Q_7 = \frac{e}{g_s^2} m_b(\mu)(\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, \qquad Q_7' = \frac{e}{g_s^2} m_b(\mu)(\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu},$$

$$Q_8 = \frac{1}{g_s} m_b(\mu)(\bar{s}\sigma_{\mu\nu} T^a P_R b) G^{\mu\nu\, a}, \qquad Q_8' = \frac{1}{g_s} m_b(\mu)(\bar{s}\sigma_{\mu\nu} T^a P_L b) G^{\mu\nu\, a},$$

$$Q_9 = \frac{e^2}{g_s^2} (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu), \qquad Q_9' = \frac{e^2}{g_s^2} (\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \mu),$$

$$Q_{10} = \frac{e^2}{g_s^2} (\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \gamma_5 \mu), \qquad Q_{10}' = \frac{e^2}{g_s^2} (\bar{s}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \gamma_5 \mu),$$

$$Q_S = \frac{e^2}{16\pi^2} m_b(\mu)(\bar{s} P_R b)(\bar{\mu}\mu), \qquad Q_S' = \frac{e^2}{16\pi^2} m_b(\mu)(\bar{s} P_L b)(\bar{\mu}\mu),$$

$$Q_P = \frac{e^2}{16\pi^2} m_b(\mu)(\bar{s} P_R b)(\bar{\mu}\gamma_5 \mu), \qquad Q_P' = \frac{e^2}{16\pi^2} m_b(\mu)(\bar{s} P_L b)(\bar{\mu}\gamma_5 \mu).$$

In SM (neglecting small q^2 dependence)

$$C_7^{\text{eff}} = -0.2957, \quad C_8^{\text{eff}} = -0.1630, \quad C_9 = 4.114, \quad C_{10} = -4.193.$$

$$C'_{7-10}, C_S^{(\prime)}, C_P^{(\prime)} = 0$$

Penguin

- The amplitude $\mathcal{A}\left(B(p) \to K^*(k)\ell^+\ell^-\right)$

$$= \frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \Bigg[\Bigg\{ C_9 \langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle - \frac{2C_7}{q^2}\langle K^*|\bar{s}i\sigma^{\mu\nu}q_\nu(m_b P_R + m_s P_L)b|\bar{B}\rangle$$

$$- \frac{16\pi^2}{q^2} \sum_{i=\{1-6,8\}} C_i \mathcal{H}_i^\mu \Bigg\} \bar{\ell}\gamma_\mu \ell + C_{10}\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle \bar{\ell}\gamma_\mu\gamma_5\ell \Bigg]$$

WCs combines

C_{1-6}

Penguin

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$$-\frac{16\pi^2}{q^2}\sum_{i=\{1-6,8\}} C_i\mathcal{H}_i^\mu\Bigg\}\bar{\ell}\gamma_\mu\ell + C_{10}\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle\,\bar{\ell}\gamma_\mu\gamma_5\ell\Bigg]$$

WCs combines C_{1-6}

parametrization with form-factors

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Penguin

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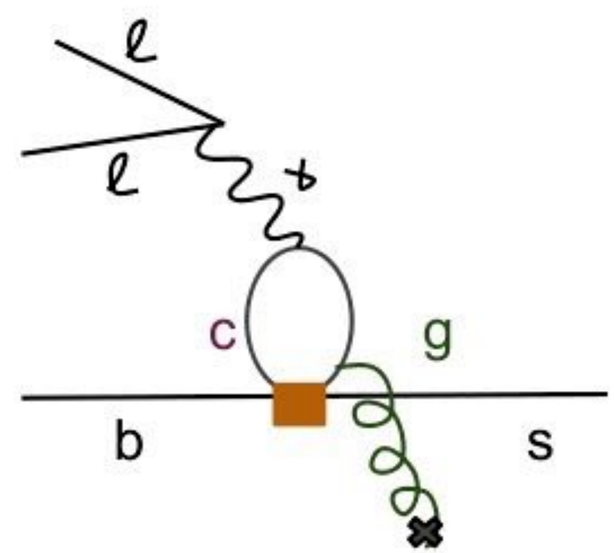
WCs combines C_{1-6}

parametrization with form-factors

non-local operator
for non factorization contributions

$$\mathcal{H}_i^\mu \sim \left\langle K^*|i\int d^4x\, e^{iq\cdot x} T\{j^\mu_{em}(x), \mathcal{O}_i(0)\}|\bar{B}\right\rangle$$



[Khodjamirian *et. al* '10]

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Penguin

Rich in terms of information $\mathcal{M}_{(m,n)}(\bar{B} \to \bar{K}^* V^*) = \epsilon^{*\mu}_{\bar{K}^*}(m)\, M_{\mu\nu}\, \epsilon^{*\nu}_{V^*}(n)$

polarization of virtual gauge boson

Project out components with different polarization vectors

helicity amplitudes $H_m = \mathcal{M}_{(m,m)}(B \to K^* V^*) \quad m = 0, +, -.$

transversity amplitudes $A_{\perp,\parallel} = (H_{+1} \mp H_{-1})/\sqrt{2}, \ \ A_0 \equiv H_0$

Penguin

Rich in terms of information $\mathcal{M}_{(m,n)}(\bar{B} \to \bar{K}^* V^*) = \epsilon^{*\mu}_{\bar{K}^*}(m)\, M_{\mu\nu}\, \epsilon^{*\nu}_{V^*}(n)$

polarization of virtual gauge boson

Project out components with different polarization vectors

→ helicity amplitudes $H_m = \mathcal{M}_{(m,m)}(B \to K^* V^*) \quad m = 0, +, -.$

transversity amplitudes $A_{\perp,\parallel} = (H_{+1} \mp H_{-1})/\sqrt{2}, \;\; A_0 \equiv H_0$

$$\mathcal{M}(\bar{B} \to \bar{K}^* V^*(\to \mu^+\mu^-))(m) \propto \epsilon^{*\mu}_{K^*}(m)\, M_{\mu\nu} \sum_{n,n'} \epsilon^{*\nu}_{V^*}(n)\epsilon^{\rho}_{V^*}(n')\, g_{nn'}\, (\bar{\mu}\gamma_\rho P_{L,R}\mu)$$

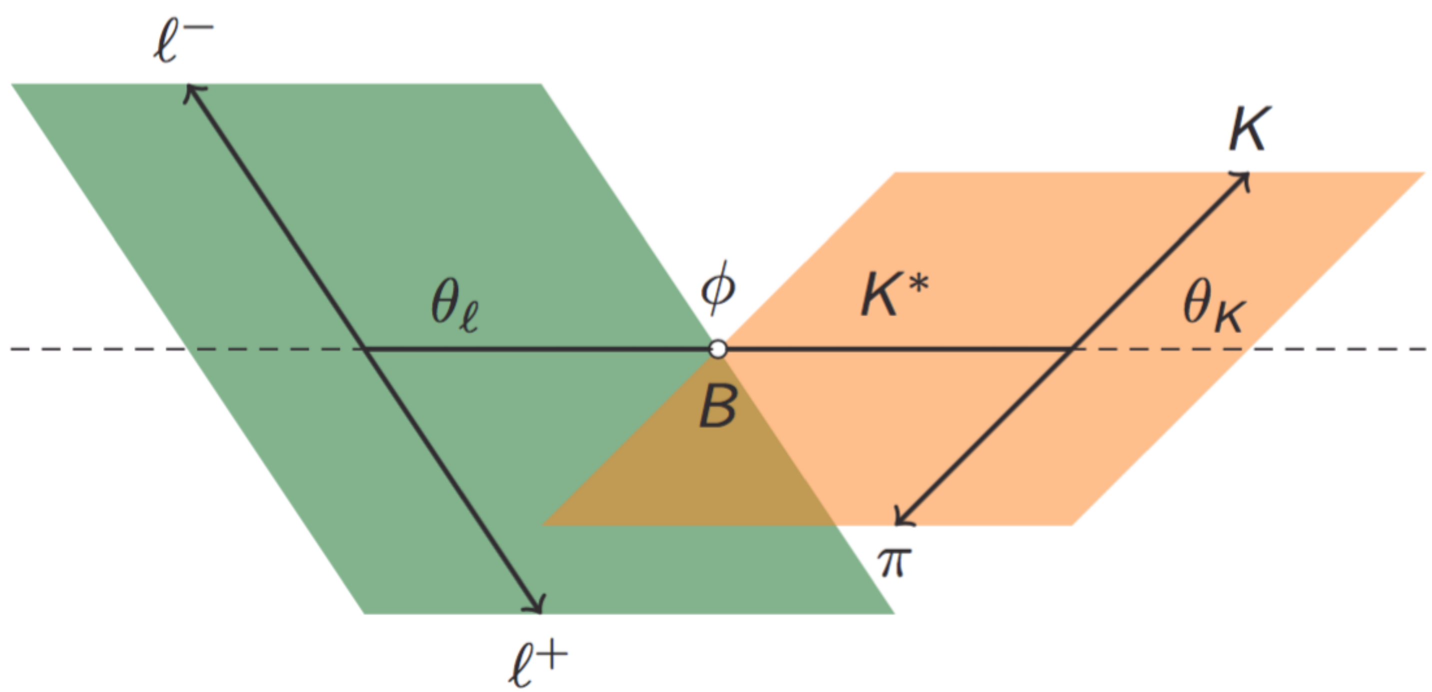
→ total 7 TAs in SM

lepton chirality adds further

angular observables constructed with different helicity combinations

Angular analysis

Angular analysis in well known helicity frame



The differential distribution $\dfrac{d^4\Gamma(B \to K^* \ell^+ \ell^-)}{dq^2\, d\cos\theta_l\, d\cos\theta_k\, d\phi}$

$$= \frac{9}{32\pi}\Big[I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_l + I_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi$$
$$+ I_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + I_5 \sin 2\theta_K \sin\theta_l \cos\phi + I_6^s \sin^2\theta_K \cos\theta_l$$
$$+ I_7 \sin 2\theta_K \sin\theta_l \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi\Big]$$

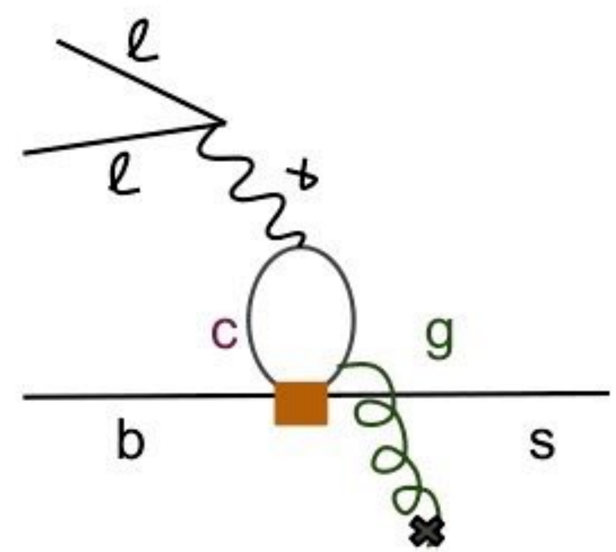
Angular analysis

- $I_i =$ short distance + long distance

Wilson coefficients:
perturbatively calculable

Form-factors:
non-perturbative estimates
from LCSR, HQET, Lattice …
tremendous effort since past

Non-factorizable
contributions:



no quantitative computation

- Need to construct observables with less dependency on non-perturbative estimates

Form factors

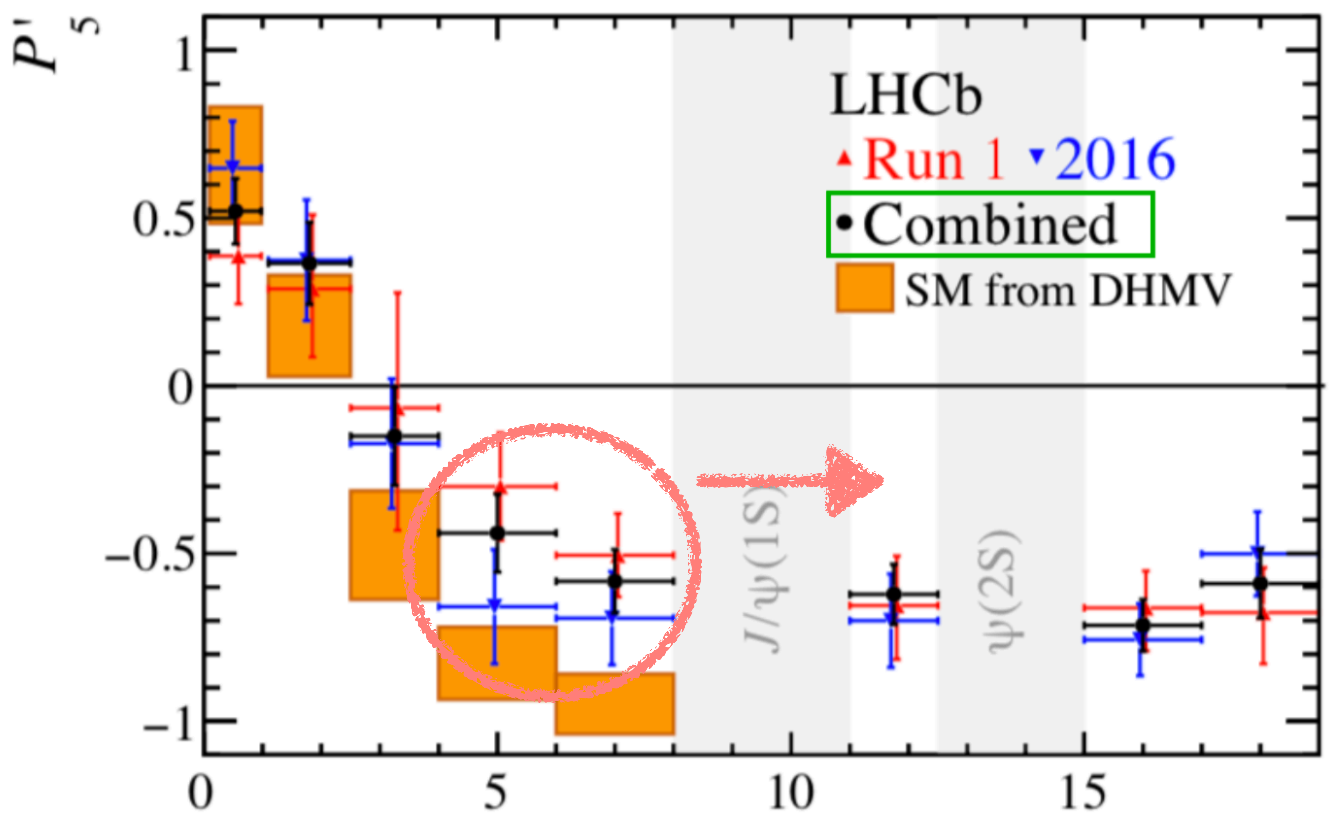
$$\langle \bar{K}^*(k)|\bar{s}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle = -i\epsilon^*_\mu(m_B+m_{K^*})A_1(q^2)$$
$$+\, i(2p-q)_\mu(\epsilon^*\cdot q)\,\frac{A_2(q^2)}{m_B+m_{K^*}}$$
$$+\, iq_\mu(\epsilon^*\cdot q)\,\frac{2m_{K^*}}{q^2}\left[A_3(q^2)-A_0(q^2)\right]$$
$$+\, \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,\frac{2V(q^2)}{m_B+m_{K^*}},$$

combination of $A_{1,2}$

$$\langle \bar{K}^*(k)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|\bar{B}(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,2T_1(q^2)$$
$$+\, T_2(q^2)\left[\epsilon^*_\mu(m_B^2-m_{K^*}^2)-(\epsilon^*\cdot q)\,(2p-q)_\mu\right]$$
$$+\, T_3(q^2)(\epsilon^*\cdot q)\left[q_\mu-\frac{q^2}{m_B^2-m_{K^*}^2}\,(2p-q)_\mu\right],$$

Total 7 form factors parametrize SM+ new physics matrix elements

Tensions



$$P'_5 = S_5/\sqrt{F_L(1-F_L)}$$

Tension in angular observable $\sim 3\sigma$

$$\mathcal{H}_{\rm eff} = \sum_i C_i \mathcal{O}_i^{\rm SM} + \sum_j C_j^{\rm NP} \mathcal{O}_j^{\rm NP}$$

new Lorentz structure → new WCs

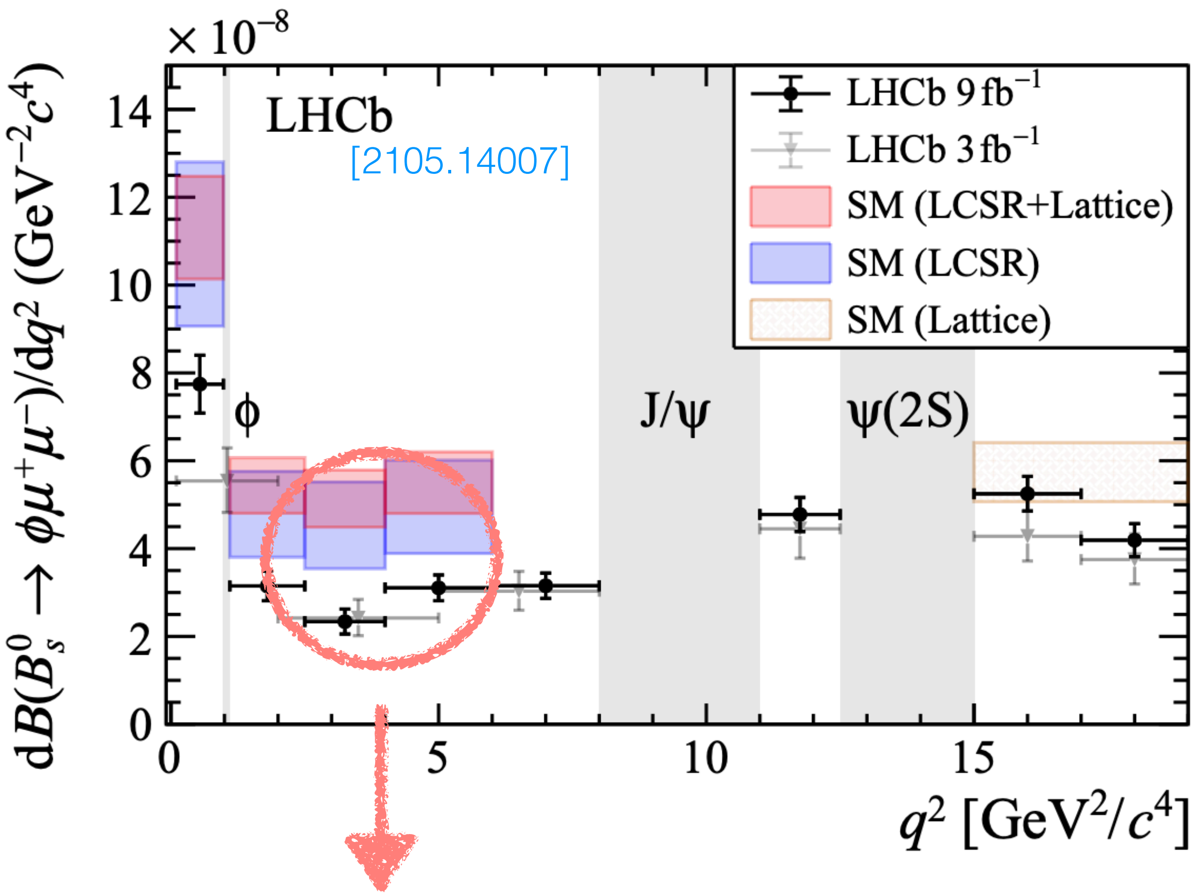
SM Lorentz structure → modification to old WCs $C_i = C_i^{\rm SM} + C_i^{\rm NP}$

Tensions

Same WCs appear in other channels with same partonic level transition

$B_s \to \phi\mu^+\mu^- : \ C_{7,9,10}$

$B_s \to \mu^+\mu^- : C_{10}$



Tension in differential distribution $\sim 2\sigma$

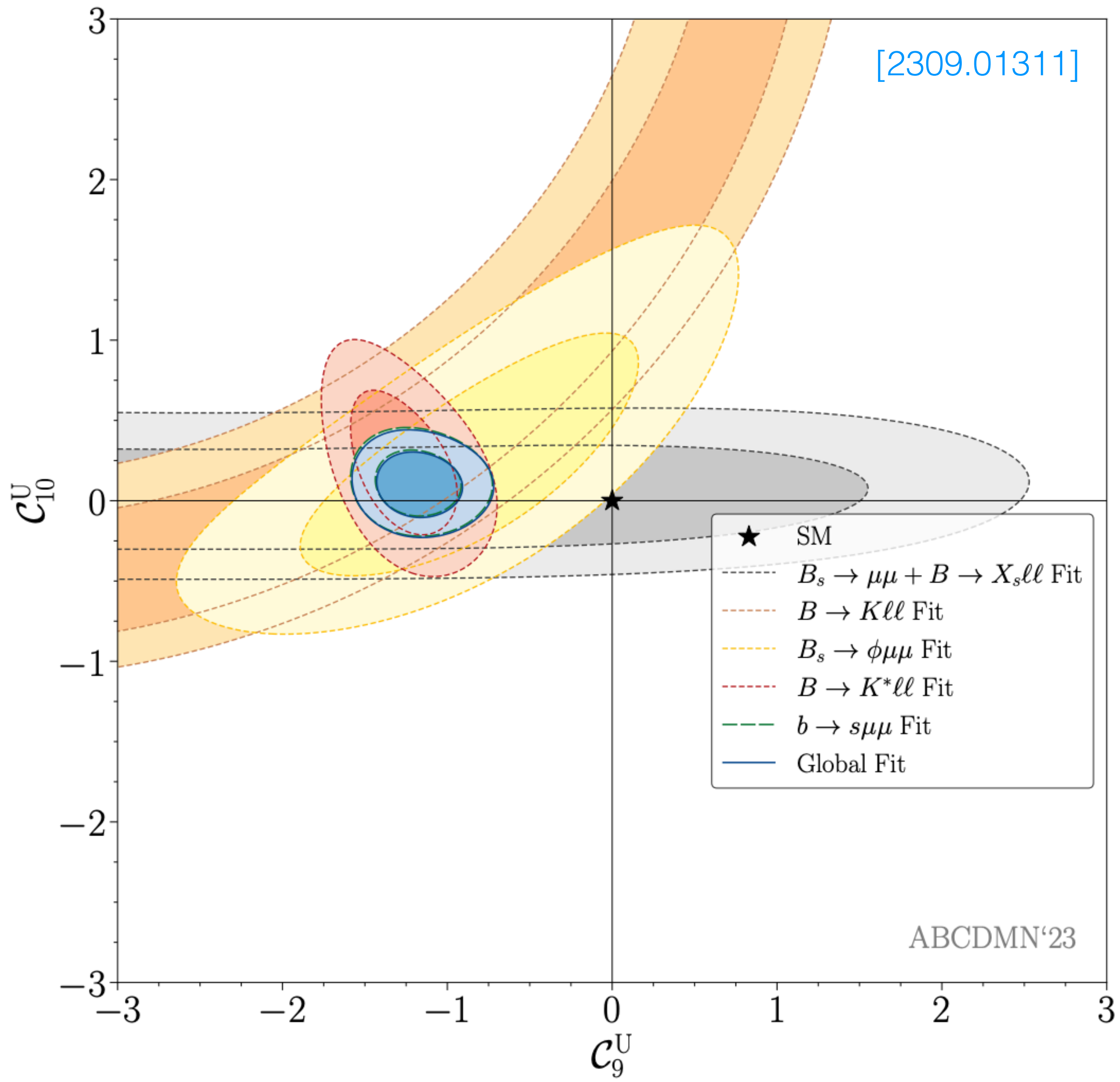
$$\mathcal{B}^{\text{Av.}}_{B_s\to\mu^+\mu^-} = (3.52^{+0.32}_{-0.30}) \times 10^{-9}$$

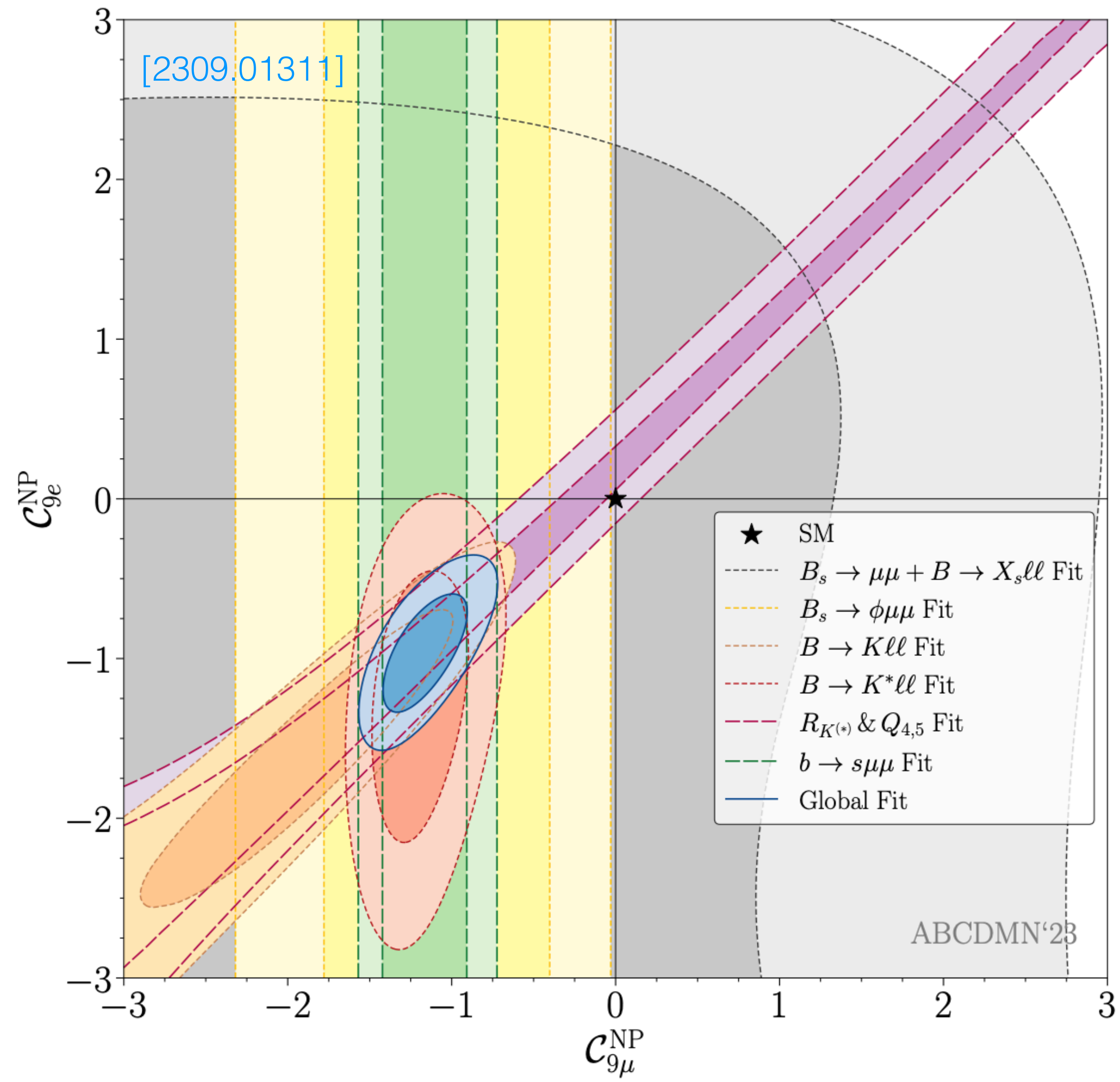
$$\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (3.66 \pm 0.14) \times 10^{-9}$$

Tension in BR $\sim 2\sigma$

Any modification to WCs has to be consistent with other channels

Tensions



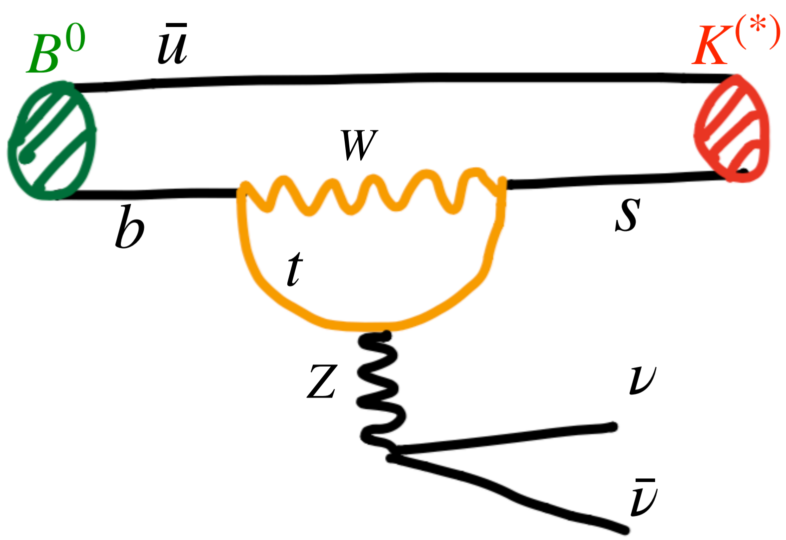


lepton flavor universal is preferred

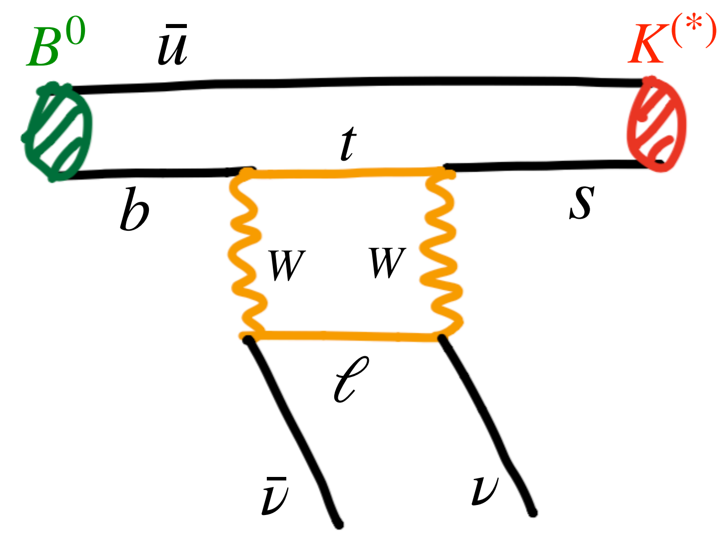
lepton flavor non-universal

Further Penguins

Mode with same quarks but charged leptons → Neutrinos



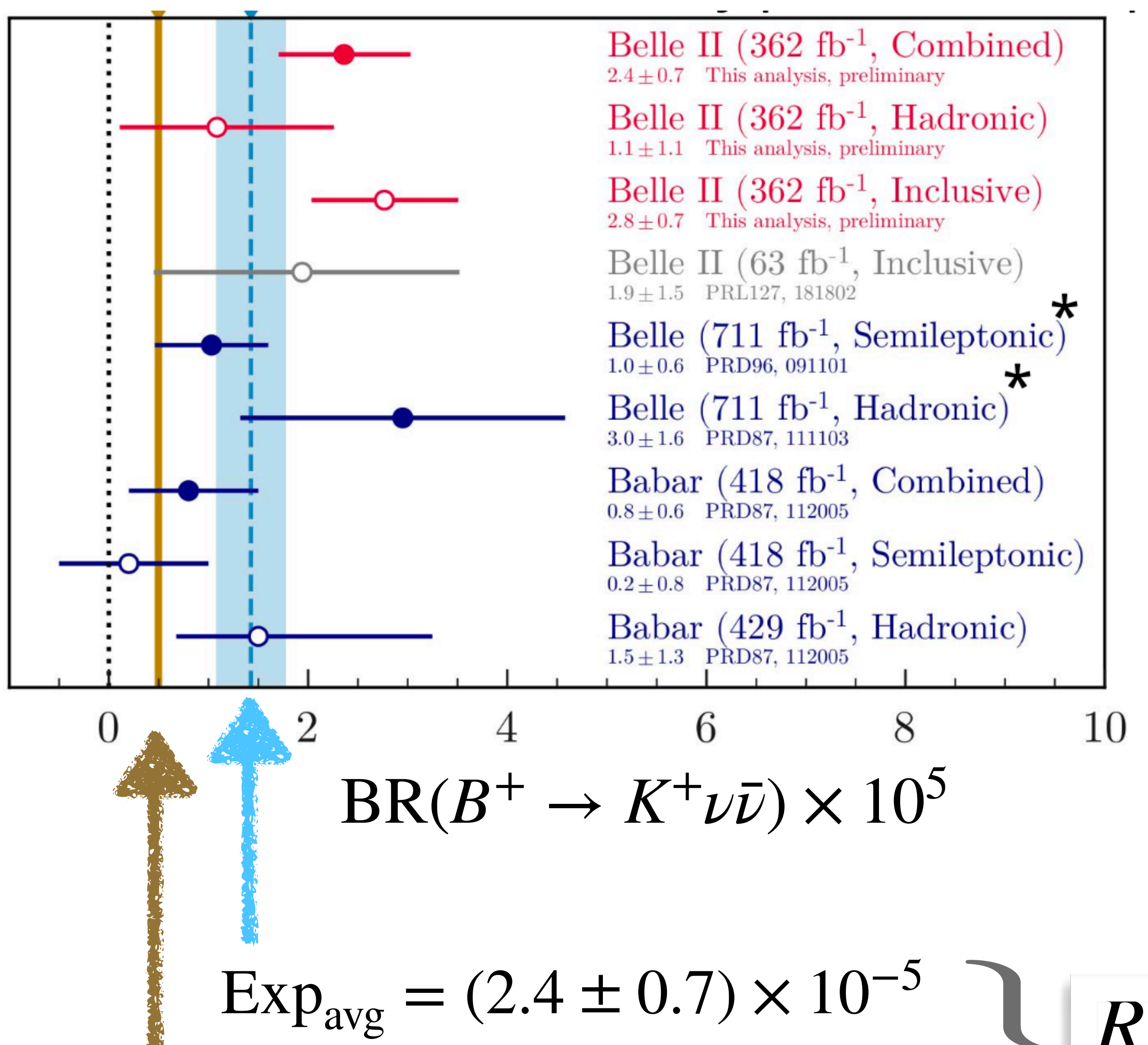
$\approx$

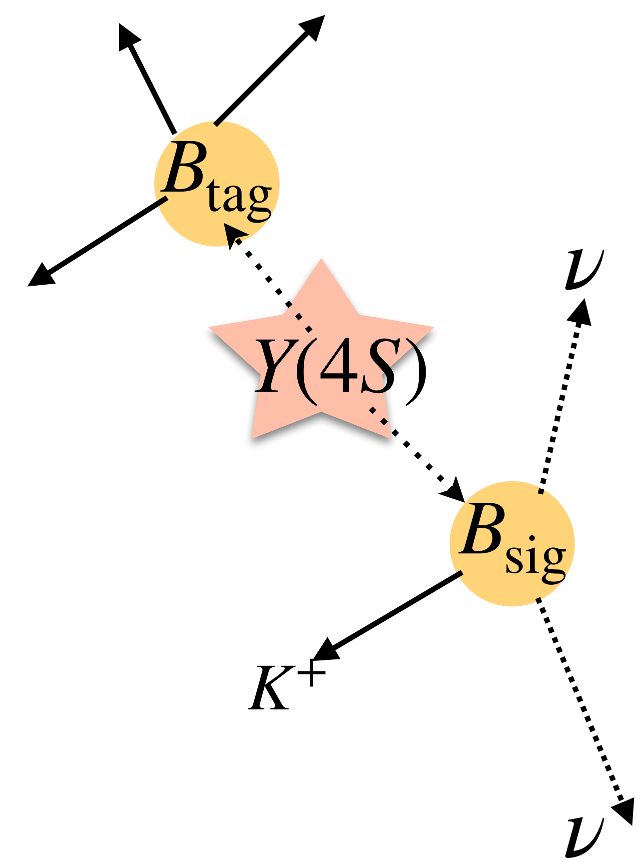


- ☐ No photon pole contribution— Z-penguin & box contribute equally
- ☑ Theoretically much cleaner than $B \to K^* \ell^- \ell^+$
- ☐ Experimentally quite challenging due to two missing neutrinos— — No signal has been observed so far

Further Penguins

- Inclusive tagging technique from Belle II has higher efficiency~4%





$$\text{Exp}_{\text{avg}} = (2.4 \pm 0.7) \times 10^{-5}$$

$$\text{SM} = (4.6 \pm 0.5) \times 10^{-6}$$

$$R_K^{\nu} = 5.2 \pm 1.6$$

Further Penguins

- Effective Hamiltonian with all possible dim-6 operators for $b \to s\nu\bar{\nu}$

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}}\frac{\alpha_{\rm EM}}{4\pi}V_{tb}V_{ts}^*\left(C_{LL}^{\rm SM}\delta_{\alpha\beta}[\mathcal{O}_{LL}^V]^{\alpha\beta} + \sum_{\substack{X=S,V,T\\ A,B=L,R}}[C_{AB}^X]^{\alpha\beta}[\mathcal{O}_{AB}^X]^{\alpha\beta}\right)$$

SM FCNC contribution

$C_{LL}^{\rm SM} = -2X_t/s_w^2$ = -12.7

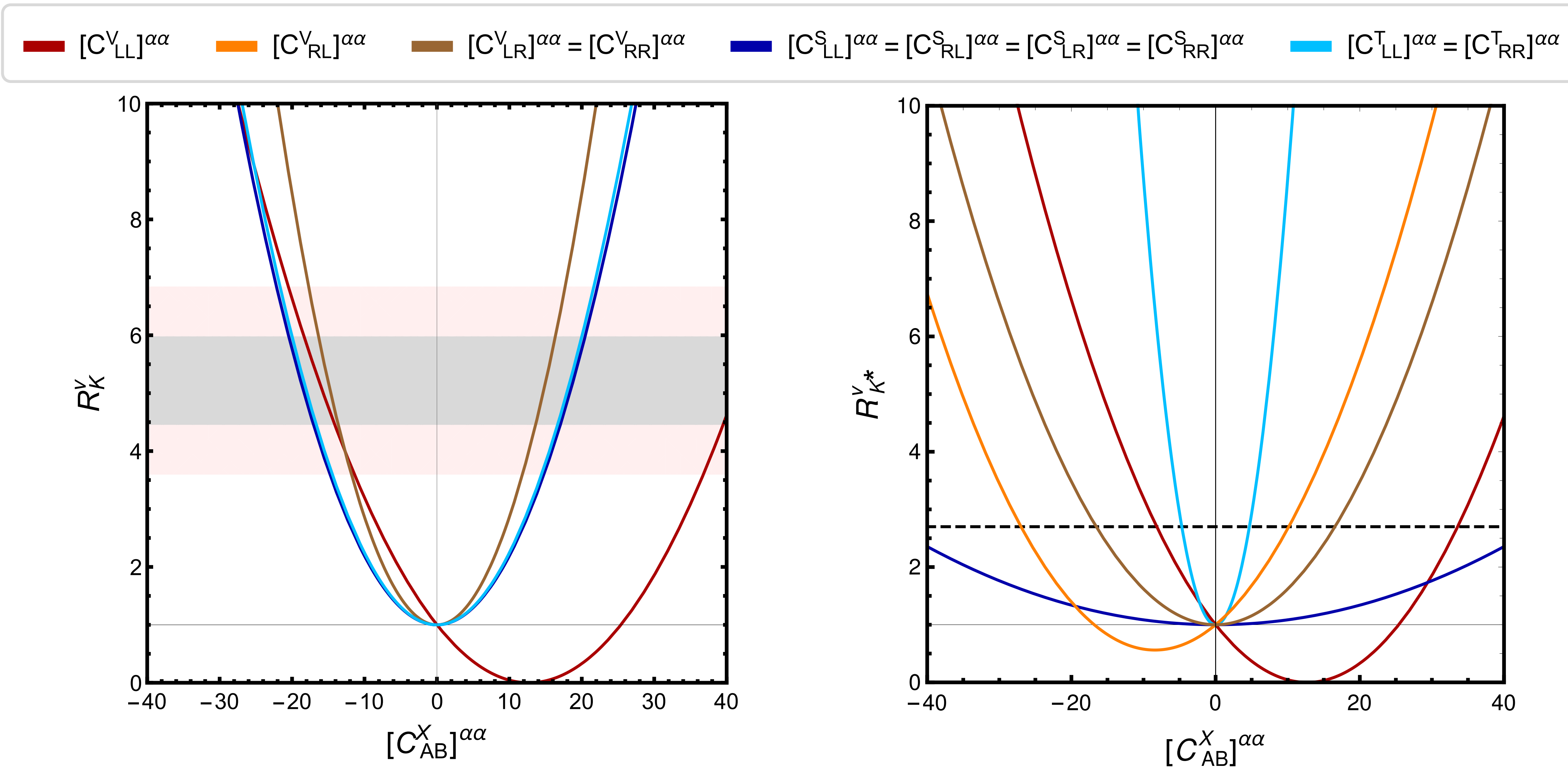
Includes light right-handed neutrinos

$$[\mathcal{O}_{AB}^V]^{\alpha\beta} \equiv (\bar{s}\,\gamma^\mu P_A b)\left(\bar{\nu}^\alpha\gamma_\mu P_B\nu^\beta\right)\,,$$
$$[\mathcal{O}_{AB}^S]^{\alpha\beta} \equiv (\bar{s}\,P_A b)\left(\bar{\nu}^\alpha P_B\nu^\beta\right)\,,$$
$$[\mathcal{O}_{AB}^T]^{\alpha\beta} \equiv \delta_{AB}\;(\bar{s}\,\sigma^{\mu\nu}P_A b)\left(\bar{\nu}^\alpha\sigma_{\mu\nu}P_B\nu^\beta\right)$$

- Observables: Branching ratio, differential distribution in q^2
  Longitudinal polarization fraction in $B \to K^*\nu\bar{\nu}$

Rusa Mandal, IIT Gandhinagar

Further Penguins



Variation with individual Wilson coefficients

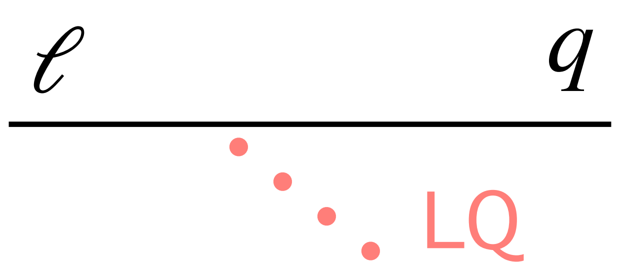
➡ All operators can achieve the expected range

Consider some motivated BSM particle contributing to the mode

Simplified Model— Only one particle at a time

Leptoquarks

[2107.01080]



Idea from '70s: R-parity violating SUSY, GUTs

Mediators	Spin	Interaction terms	Operators
$S_3(\bar{3}, 3, 1/3)$	0	$+\overline{Q^c}\, Y_{S_3}\, i\tau_2\, \boldsymbol{\tau}\cdot\mathbf{S_3}\, L$	$\mathcal{O}^V_{LL}$
$\tilde{R}_2(3, 2, 1/6)$	0	$-\bar{d}_R\, Y_{\tilde{R}_2}\, \tilde{R}_2^T\, i\tau_2 L + \overline{Q}\, Z_{\tilde{R}_2}\, \tilde{R}_2\, \nu_R$	$\mathcal{O}^V_{RL}, \mathcal{O}^V_{LR}, \mathcal{O}^{S,T}_{LL}, \mathcal{O}^{S,T}_{RR}$
$S_1(\bar{3}, 1, 1/3)$	0	$+\overline{Q^c}\, i\tau_2\, Y_{S_1} L\, S_1 + \overline{u^c_R}\, \tilde{Y}_{S_1}\, S_1\, e_R + \overline{d^c_R}\, Z_{S_1}\, S_1\, \nu_R$	$\mathcal{O}^{S,V,T}_{RR}, \mathcal{O}^{S,V,T}_{LL}$
$U_3^\mu(3, 3, 2/3)$	1	$+\overline{Q}\, \gamma^\mu \tau^a\, Y_{U_1} L\, U^a_{1\mu}$	$\mathcal{O}^V_{LL}$
$V_2^\mu(\bar{3}, 2, 5/6)$	1	$+\overline{d^c_R}\, \gamma^\mu Y_{V_2} V^T_{2\mu} i\tau_2\, L + \overline{Q^c_L}\, \gamma^\mu \tilde{Y}_{V_2} i\tau_2\, V_{2\mu} e_R$	$\mathcal{O}^S_{RL}$
$\bar{U}_1^\mu(3, 1, -1/3)$	1	$+\bar{d}_R\, Z_{\bar{U}_1}\, \gamma^\mu \bar{U}_{1\mu}\, \nu_R$	$\mathcal{O}^V_{RR}$

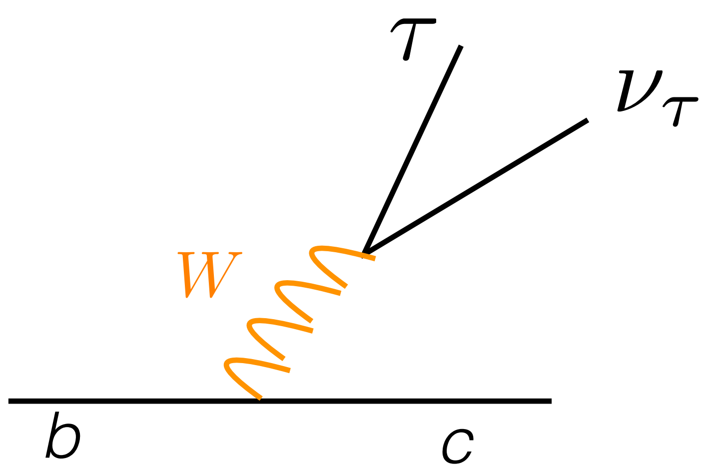
Rusa Mandal, IIT Gandhinagar

Leptoquarks

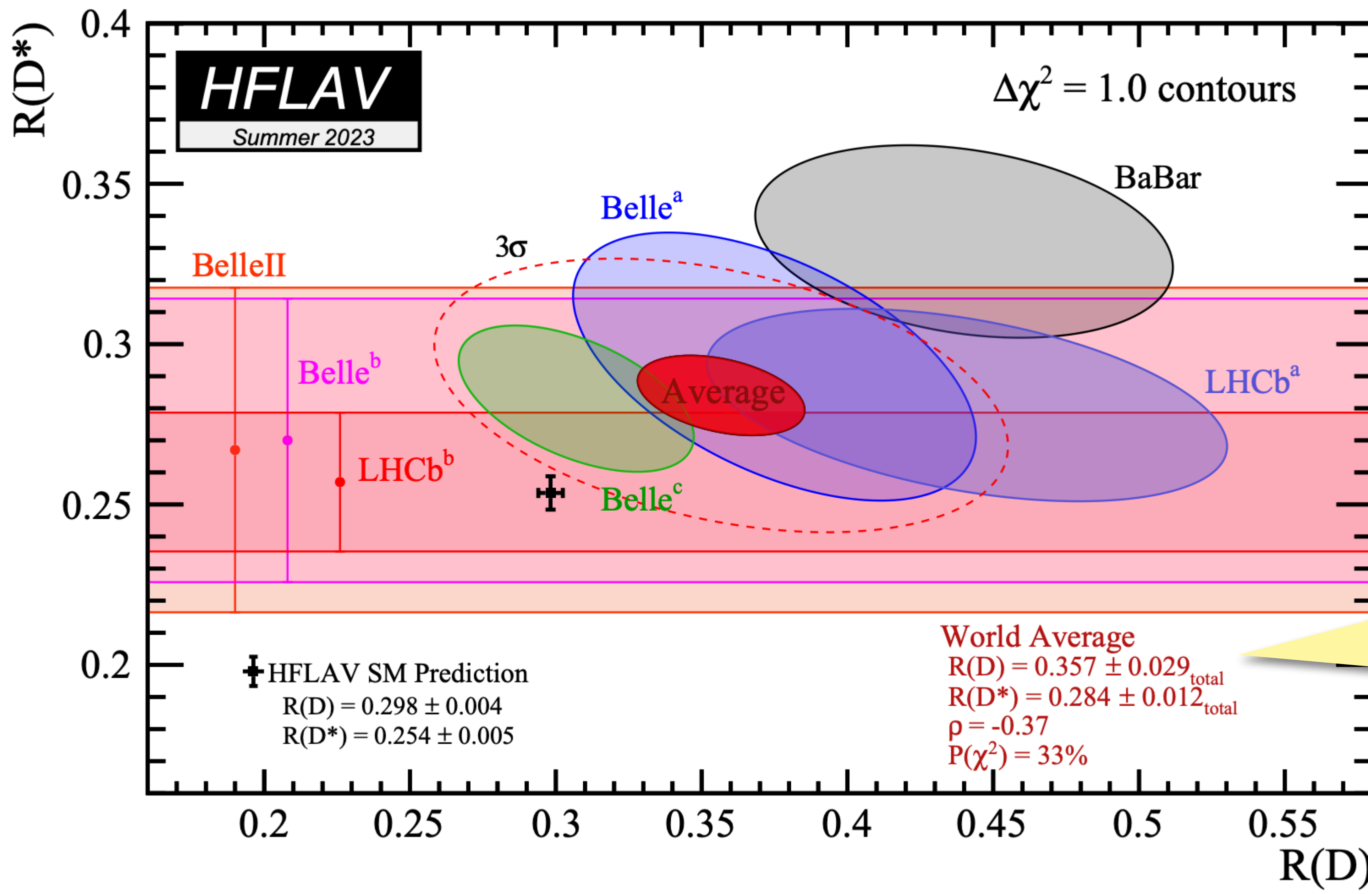
Mediators	Spin	Angular obs	$R(D)$	$R(D^*)$	R_K^ν
$S_3(\bar{3}, 3, 1/3)$	0	✓	✗	✗	✓
$\tilde{R}_2(3, 2, 1/6)$	0	✗	— no effect —		no effect
+ RHN			✗	✗ ←	✓
$S_1(\bar{3}, 1, 1/3)$	0	no effect	✓	✓	✓
+ RHN			✗	✗ ←	✓
$U_3^\mu(3, 3, 2/3)$	1	✓	✗	✗ ←	✓
$V_2^\mu(\bar{3}, 2, 5/6)$	1	✗	✓	✗	✓
$\bar{U}_1^\mu(3, 1, -1/3)$	1	no effect	— no effect —		✓

B anomalies

- Exciting discrepancies observed in charged current B decays also



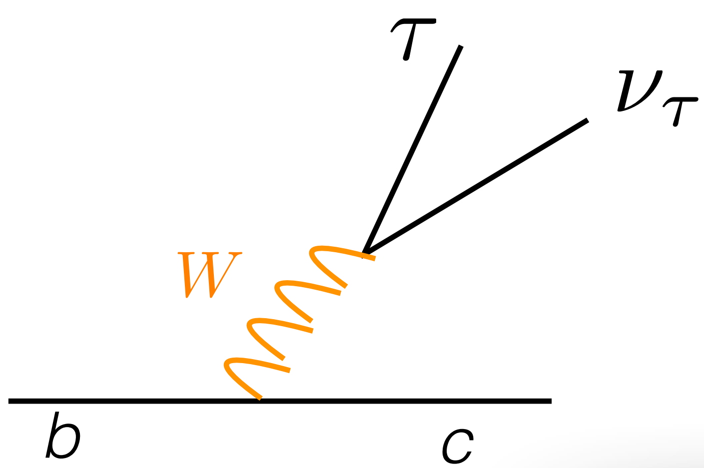
$$R(D^{(*)}) \equiv \frac{\text{BR}(B \to D^{(*)}\tau\nu)}{\text{BR}(B \to D^{(*)}\ell\nu)}, \quad \ell \in \{e, \mu\}$$



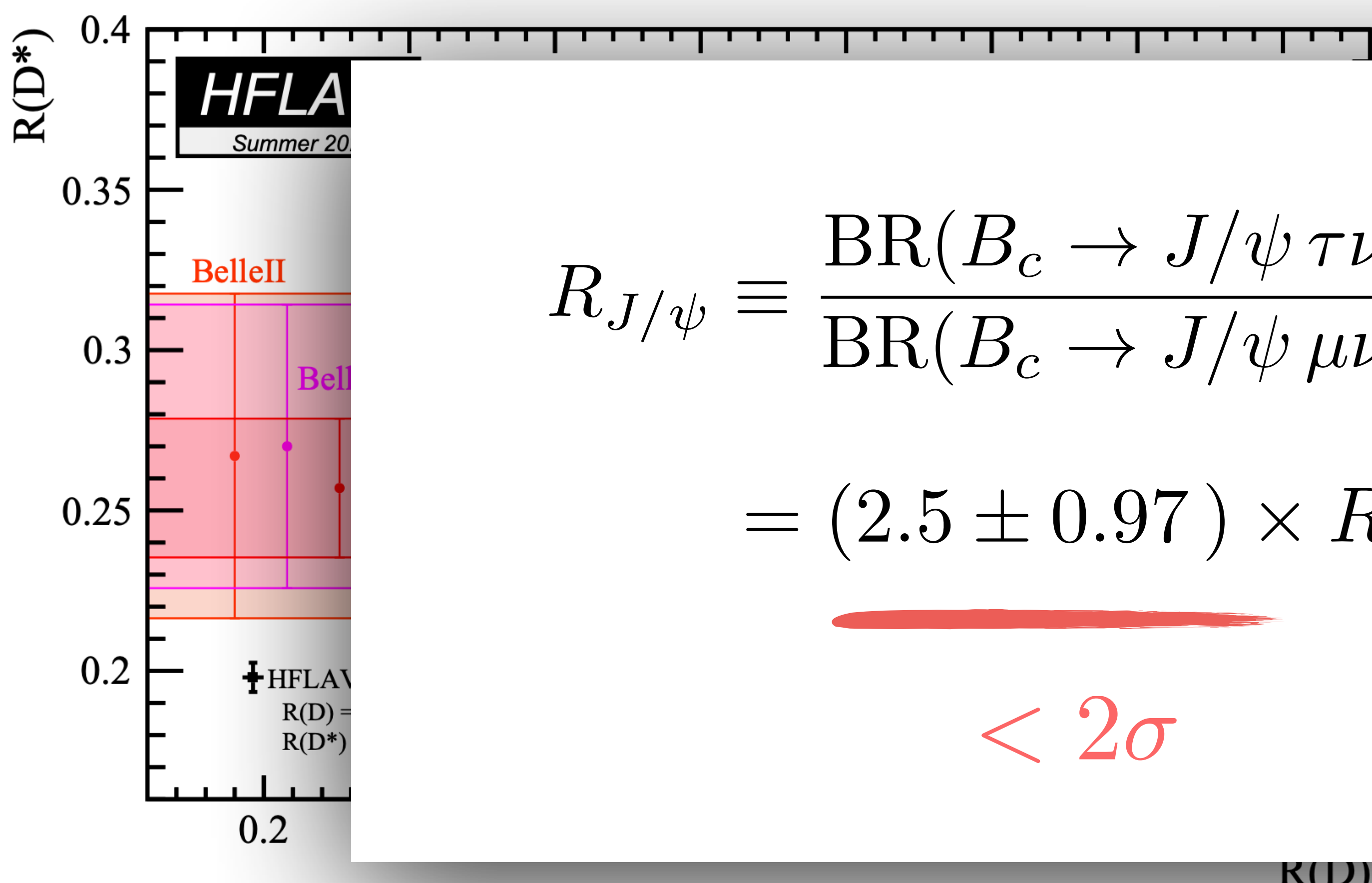
combined deviation
$\sim 3\sigma$

B anomalies

- Exciting discrepancies observed in charged current B decays also



$$R(D^{(*)}) \equiv \frac{\text{BR}(B \to D^{(*)}\tau\nu)}{\text{BR}(B \to D^{(*)}\ell\nu)}, \quad \ell \in \{e, \mu\}$$



[LHCb,1711.05623]

$$R_{J/\psi} \equiv \frac{\text{BR}(B_c \to J/\psi\,\tau\nu)}{\text{BR}(B_c \to J/\psi\,\mu\nu)}$$

$$= (2.5 \pm 0.97\,) \times R_{J/\psi}^{\text{SM}}$$

$< 2\sigma$

deviation

σ

Hamiltonian

- Most general dim-6 BSM Hamiltonian for $b \to c\ell\bar{\nu}$

$$\mathcal{H}_{\rm eff} = \frac{4G_F V_{cb}}{\sqrt{2}} \left\{ \mathcal{O}_{LL}^{V} + \sum_{\substack{X=S,V,T \\ M,N=L,R}} C_{MN}^{X} \mathcal{O}_{MN}^{X} \right\}$$

Wilson coefficients:
perturbatively calculable

All $C_{MN}^{X} = 0$ in the SM

→ Simple dynamics

$$\mathcal{O}_{MN}^{S} \equiv (\bar{c} P_M b)\left(\bar{\ell} P_N \nu\right),$$
$$\mathcal{O}_{MN}^{V} \equiv (\bar{c}\gamma^{\mu} P_M b)\left(\bar{\ell}\gamma_{\mu} P_N \nu\right),$$
$$\mathcal{O}_{MN}^{T} \equiv (\bar{c}\sigma^{\mu\nu} P_M b)\left(\bar{\ell}\sigma_{\mu\nu} P_N \nu\right).$$

→ Sandwiched between mesons
form factors: non-perturbative

BSM physics induce new Wilson coefficients

Fits

Obs	Deviation	
R_D	2.0σ	3.3σ (combined)
R_{D^*}	2.2σ	
$P_\tau^{D^*}$	–	
$F_L^{D^*}$	1.7σ	
$d\Gamma^{D^{(*)}}/dq^2$	–	

In terms of Wilson coefficients

[2210.10751]

	Pull	Best Fit Point
C_{LL}^V	4.4σ	$+0.08(2)$
C_{RL}^V	1.9σ	$-0.05(3)$
C_{LL}^S	3.0σ	$+0.17(5)$
C_{RL}^S	3.8σ	$+0.20(5)$
C_{LL}^T	3.4σ	$-0.03(1)$

1-D fit

New Physics

- Fit to all measured observables in $B \to D^{(*)}\ell\bar{\nu}$ including differential BR in q^2 in EFT approach motivated by UV mediators

[2004.06726]

Mediators	Operators	Pull*	R_D	R_{D^*}	$F_L^{D^*}$	$P_\tau^{D^*}$
	$\mathcal{O}_{LL}^{V}, \mathcal{O}_{LR}^{S,V,T}, \mathcal{O}_{RR}^{S,V,T}$	2.4	✓	✓	✓	✓
	$\mathcal{O}_{LR}^{S,V,T}, \mathcal{O}_{RR}^{S,V,T}$	2.5	✓	✓	✗	✓
$S_1(\bar{3}, 1, 1/3)$	$\mathcal{O}_{RR}^{S,V,T}, \mathcal{O}_{LL}^{S,V,T}$	3.3	✓	✓	✗	✓
$\tilde{R}_2(3, 2, 1/6)$	$\mathcal{O}_{RR}^{S,T}$	2.9	✓	✓	✗	✓
$U_1^{\mu}(3, 1, 2/3)$	$\mathcal{O}_{RR}^{V}, \mathcal{O}_{LR}^{S}, \mathcal{O}_{LL}^{V}, \mathcal{O}_{RL}^{S}$	2.6	✓	✓	✗	✓
$\tilde{V}_2^{\mu}(3, 2, -1/6)$	$\mathcal{O}_{LR}^{S}$	1.9	✓	✗	✗	✓
$V_\mu(1, 1, -1)$	$\mathcal{O}_{RR}^{V}$	3.7	✓	✓	✗	✓
$\phi(1, 2, 1/2)$	$\mathcal{O}_{XY}^{S}$	2.5	✓	✓	✓	✓

*needs to be updated

Observables

- *CP* averaged asymmetries for vector boson final state

$$A_{FB}^{D^{(*)}} = \frac{1}{\Gamma_f^{D^{(*)}}} \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_l \, \frac{d^2(\Gamma^{D^{(*)}} - \bar{\Gamma}^{D^{(*)}})}{dq^2 d\cos\theta_l}$$

$$A_4 = \frac{1}{\Gamma_f^{D^*}} \left[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2} \right] d\phi \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_D \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} + \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$$A_5 = \frac{1}{\Gamma_f^{D^*}} \left[\int_{-\pi/2}^{\pi/2} - \int_{\pi/2}^{3\pi/2} \right] d\phi \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_D \int_{-1}^1 d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} - \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

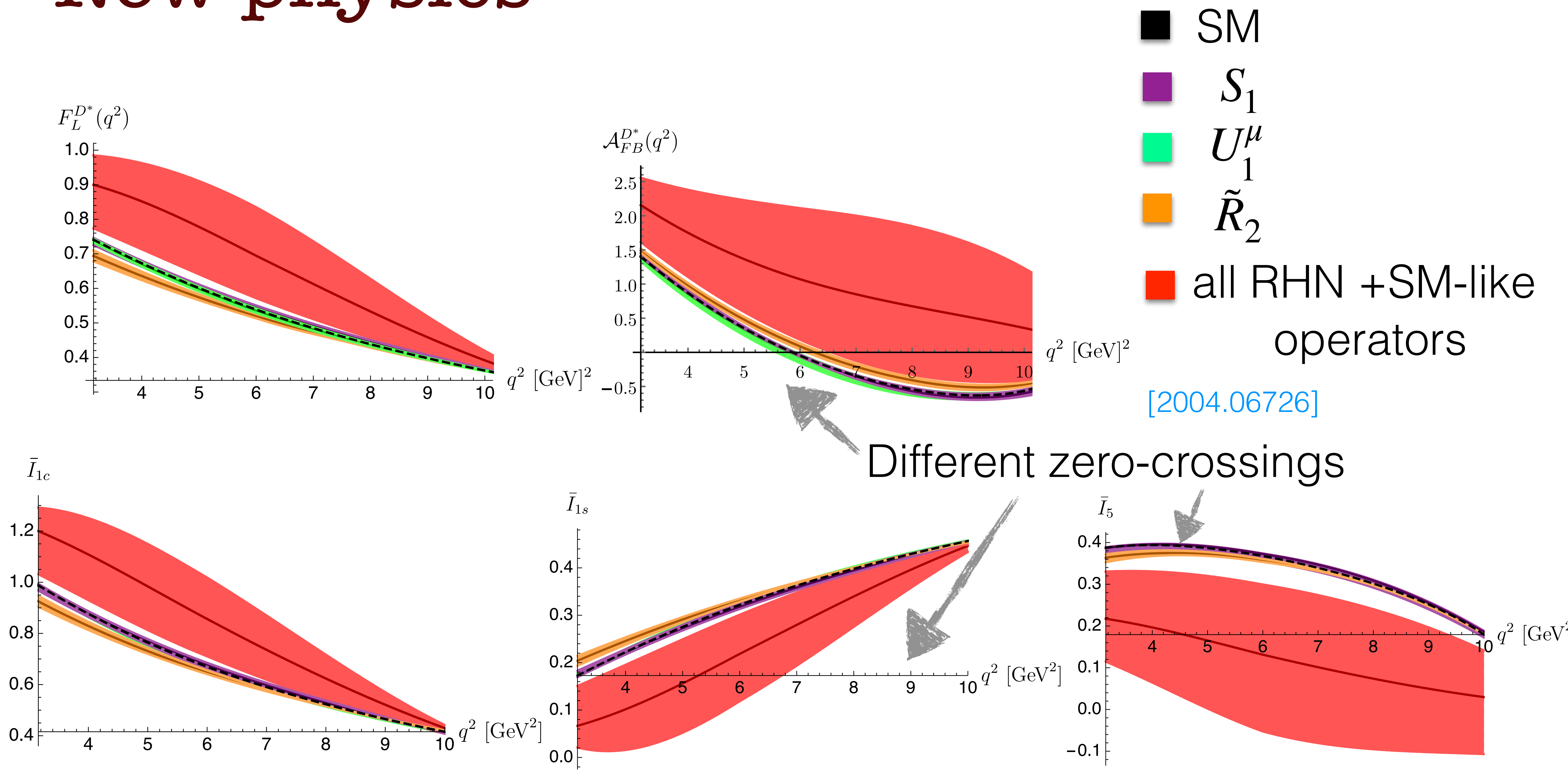
$$A_7 = \frac{1}{\Gamma_f^{D^*}} \left[\int_0^{\pi} - \int_{\pi}^{2\pi} \right] d\phi \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_D \int_{-1}^1 d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} + \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$$A_8 = \frac{1}{\Gamma_f^{D^*}} \left[\int_0^{\pi} - \int_{\pi}^{2\pi} \right] d\phi \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_D \left[\int_0^1 - \int_{-1}^0 \right] d\cos\theta_l \, \frac{d^4(\Gamma^{D^*} - \bar{\Gamma}^{D^*})}{dq^2 d\cos\theta_l d\cos\theta_D d\phi}$$

$A_{3,4,5}, A_{FB} \propto$ Real part of the amplitude

$A_{7,8,9} \propto$ Imaginary part → **Null tests of SM**

New physics



Easily distinguishable in various q^2 region

Crucial to identify NP mediators

Rusa Mandal, Siegen U.

Each tensions might point towards different
New Physics scenarios

Are they correlated?
Is there a global picture