

Belle II Tracking: an overview

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On behalf of the Belle II Tracking and Vertexing group

Belle II Physics week

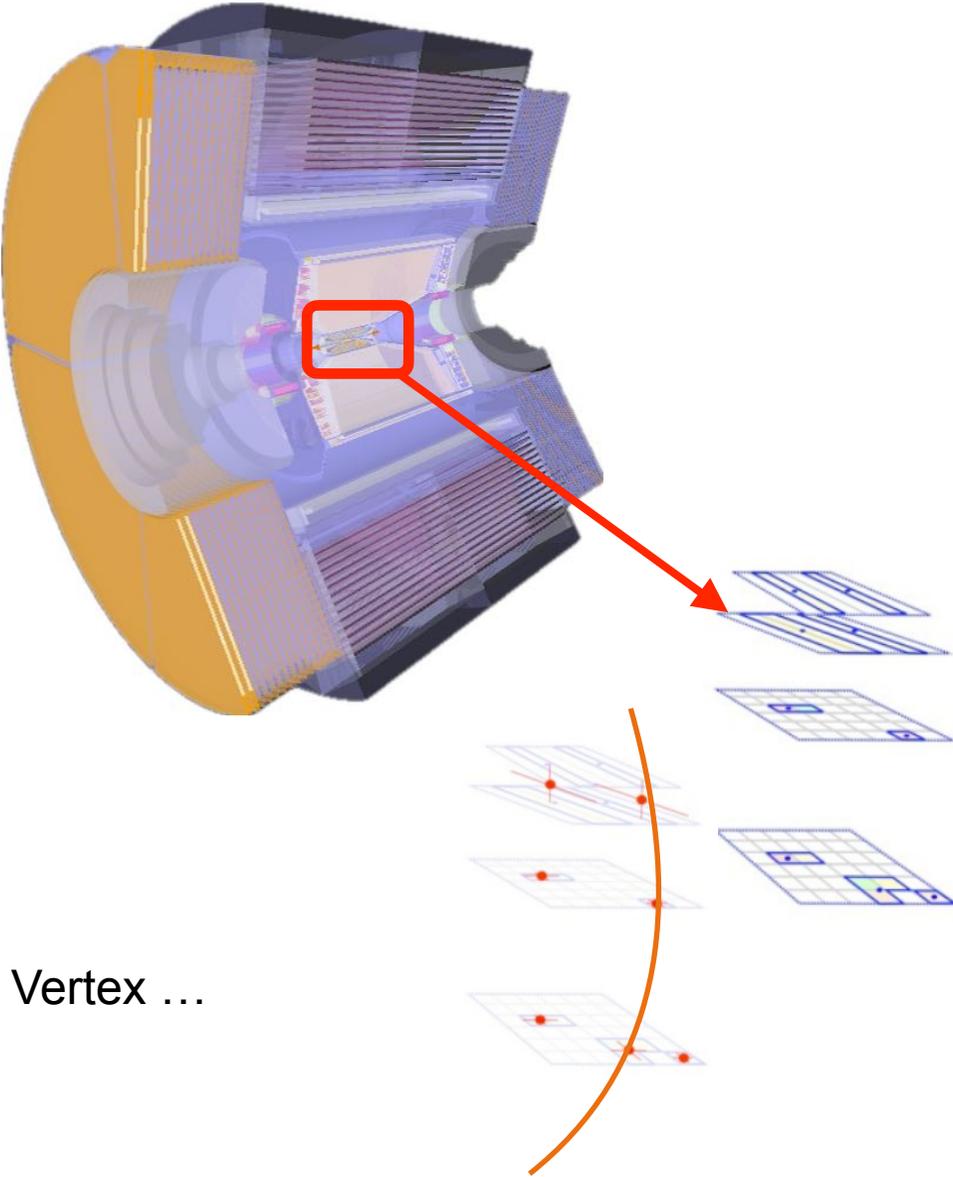
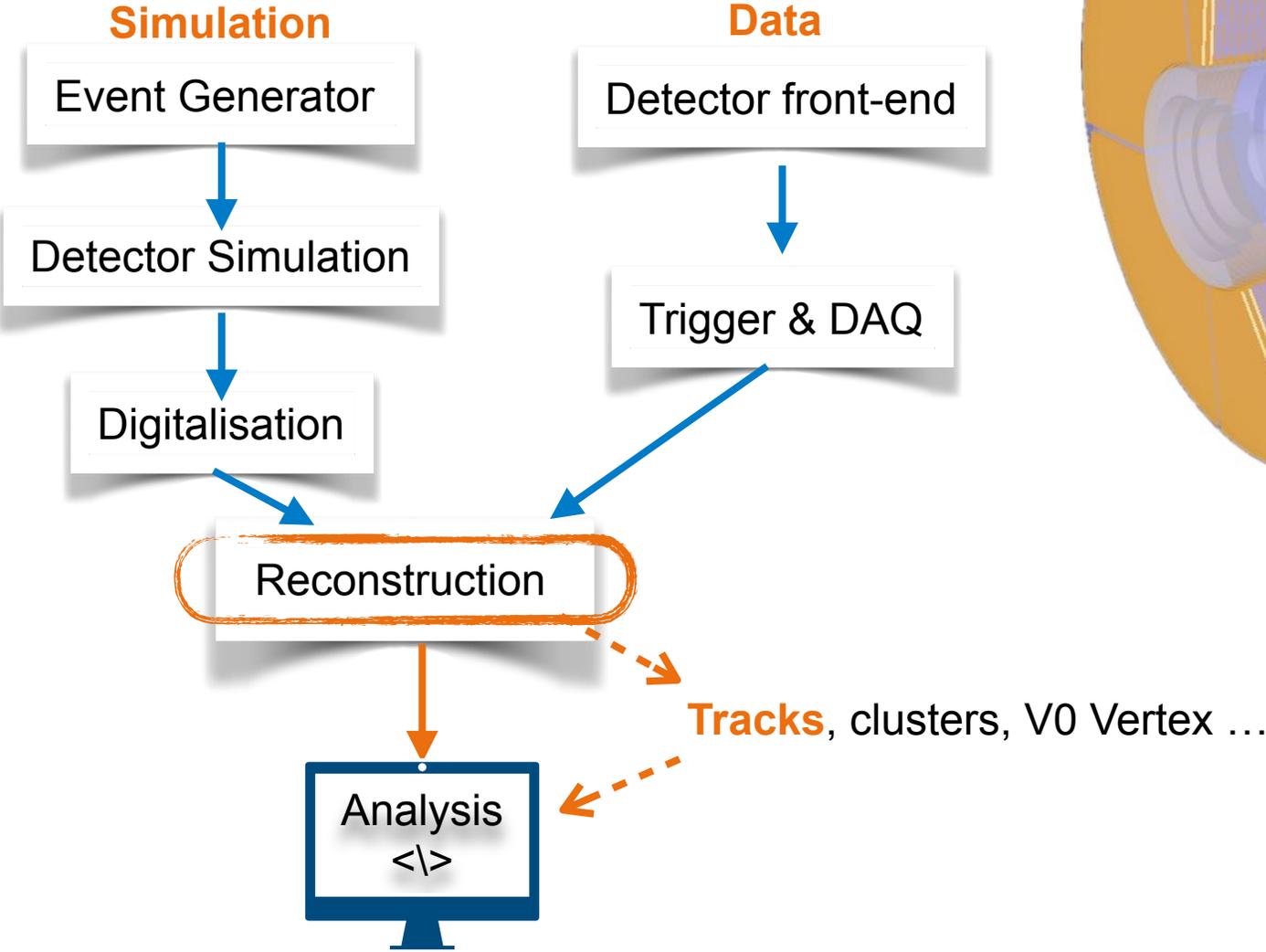
31.10.2023

Outline

- Event reconstruction at HEP
- Belle II Tracking detectors
- Belle II Tracking procedures
- Tracking performance and Inputs to the analysis
- Summary

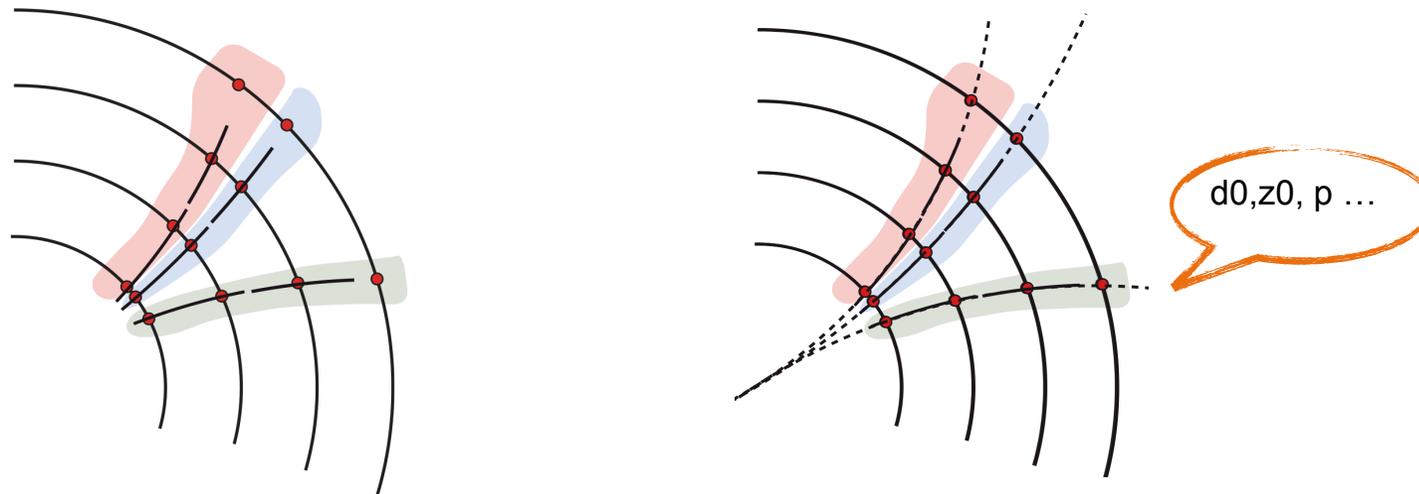
Event reconstruction at HEP

Event reconstruction at HEP



What is Tracking

- Goal: Identify **trajectories of charged particles** through the tracking detectors
 - **Track Finding**: Identifying clusters/hits of detectors that are associated with charged particles
 - **Track Fitting**: Fitting/parameterisation with the hits found by the track finding and extract the properties (eg. momentum, charge, direction, dE/dx) of the particles



- ◆ The actual design of the tracking procedure strongly depends on the **details of the experiment** and the **working point**
- ◆ Always need to **balance** the high performance and the usage of computing resources

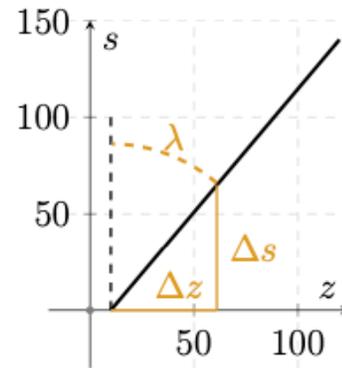
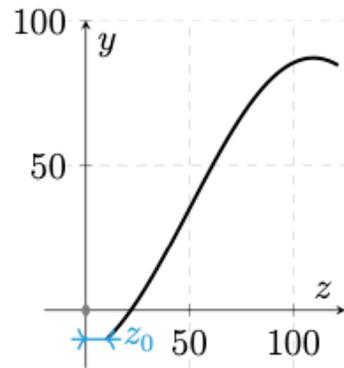
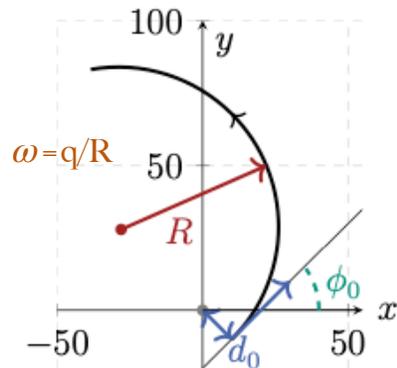
Track parameterisation

How to describe the Tracks

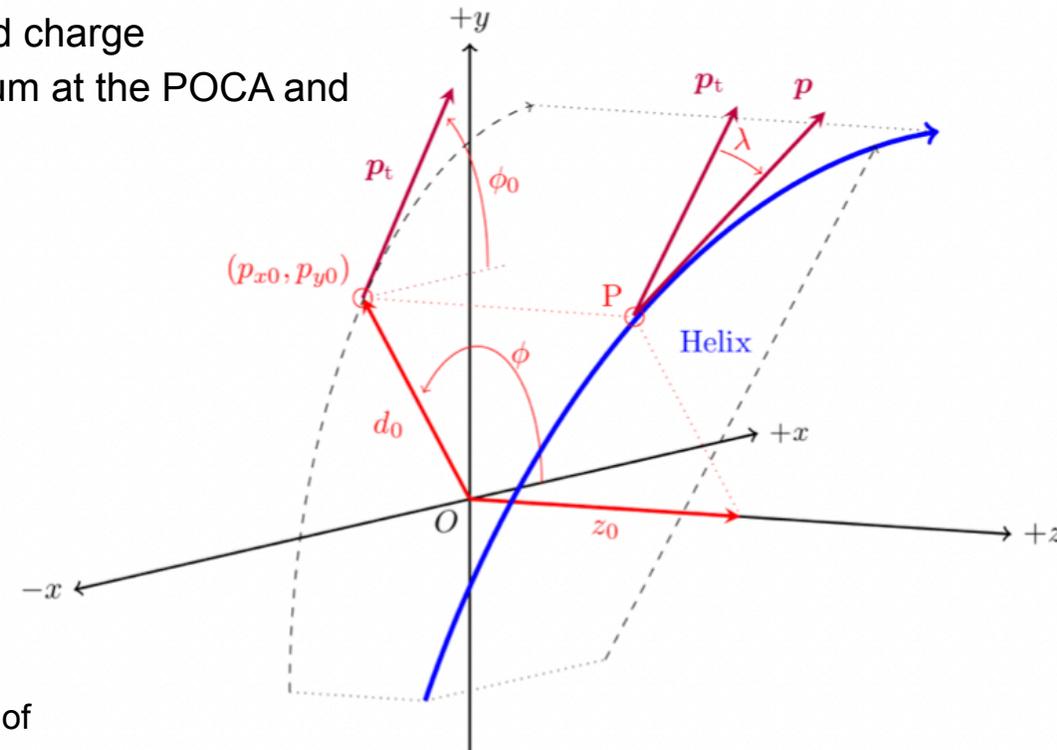
➔ The Track of a charged particle in homogeneous magnetic field can be represented by a **'Helix'**

- **5 parameters** computed at the point of the closest approach to the z axis(POCA)

- $d_0, z_0 \in [-\infty, +\infty]$: signed distance of the POCA on transverse plane and z
- $\phi_0 \in [-\pi, +\pi]$: angle defined by the transverse momentum at the POCA and the x
- $\omega \in [-\infty, +\infty]$: inverse of the curvature radius, signed with the fitted charge
- $\tan \lambda \in [-\infty, +\infty]$: the tangent of the angle defined by the momentum at the POCA and the transverse plane



(s representing the path of the helix on xy plane)

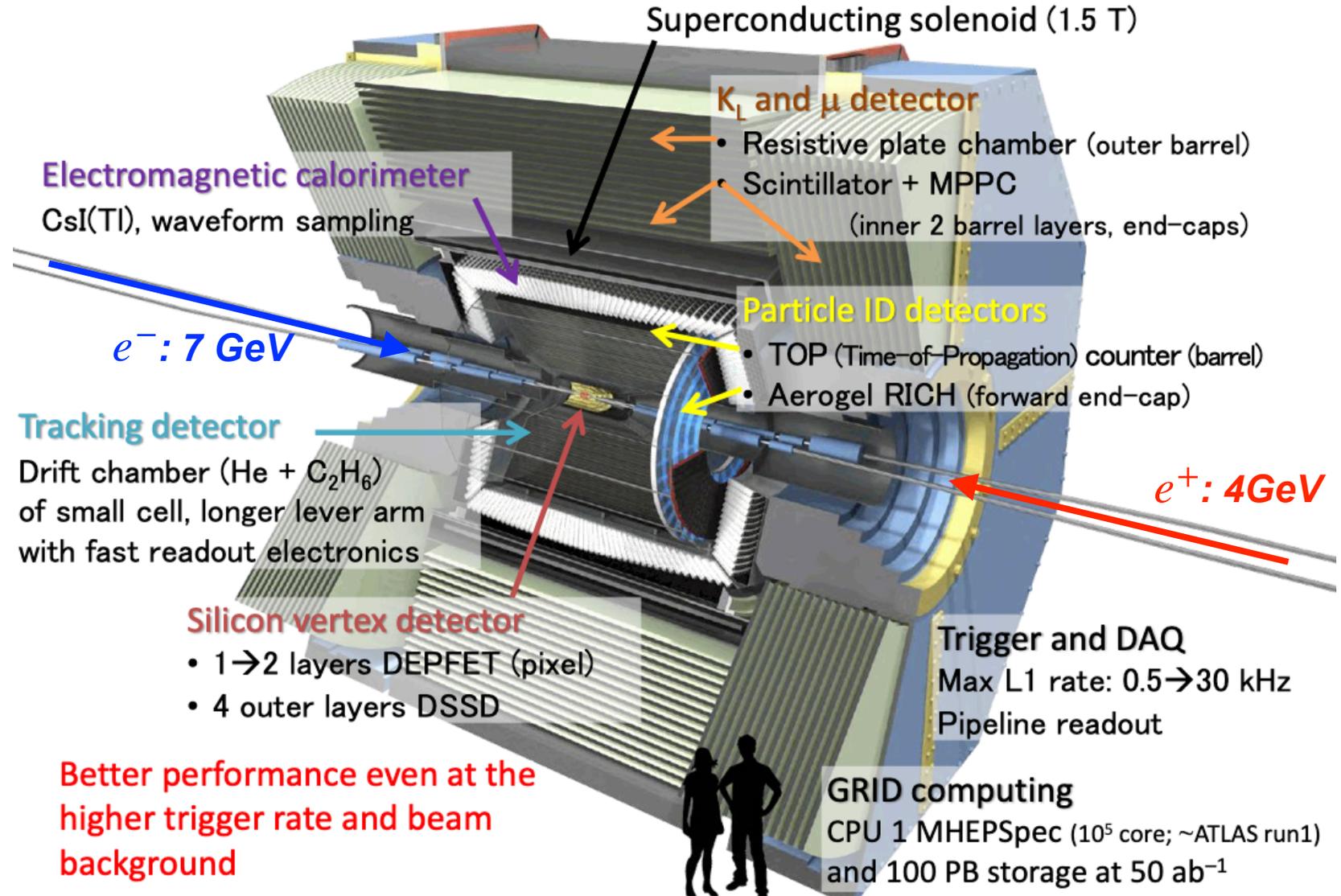


Belle II Tracking detectors

Belle II detector

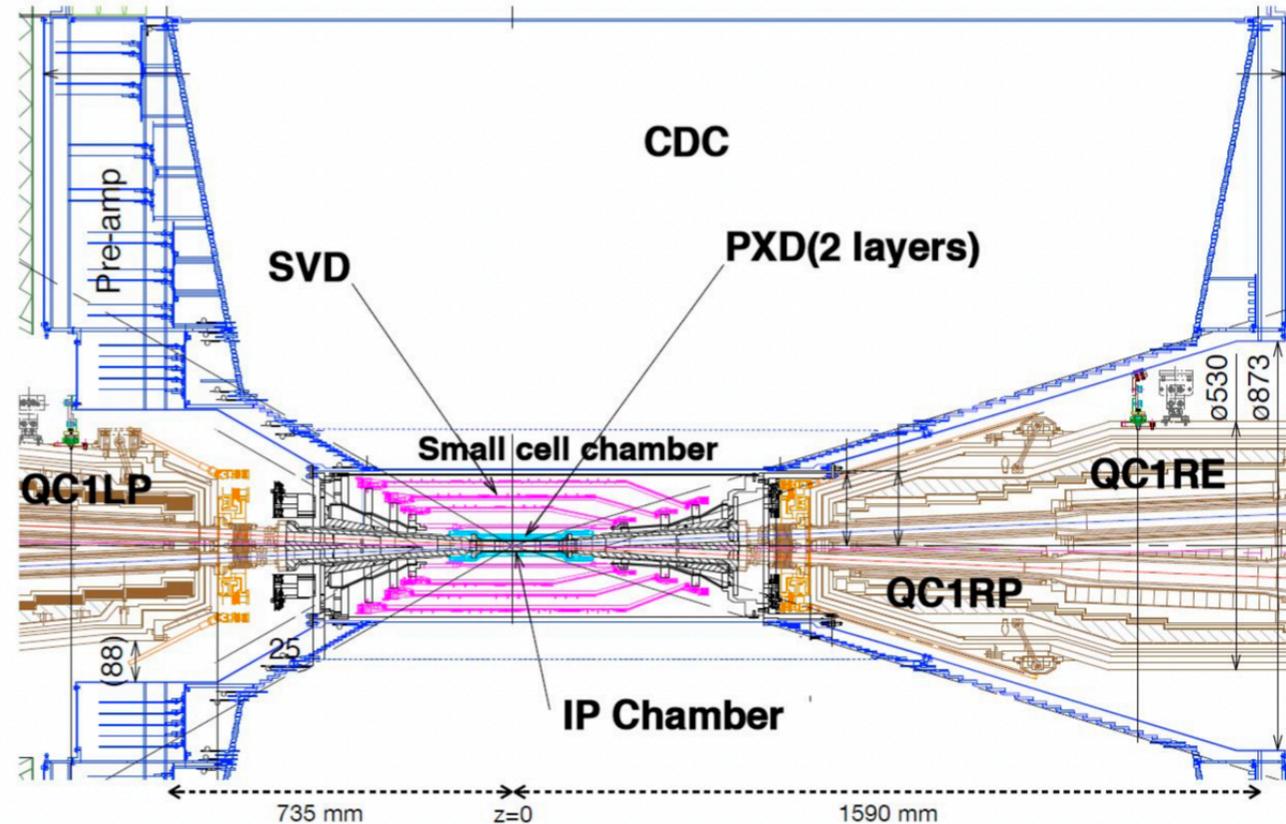
A multi-purpose system

- Pixel detector ([PXD](#))
- Silicon Vertex Detector ([SVD](#))
- Central Drift Chamber ([CDC](#))
- Calorimeter ([ECL](#))
- Aerogel Ring-Imaging Cherenkov (ARICH)
- Time-Of-Propagation (TOP) counter
- KL0 and μ detection (KLM)



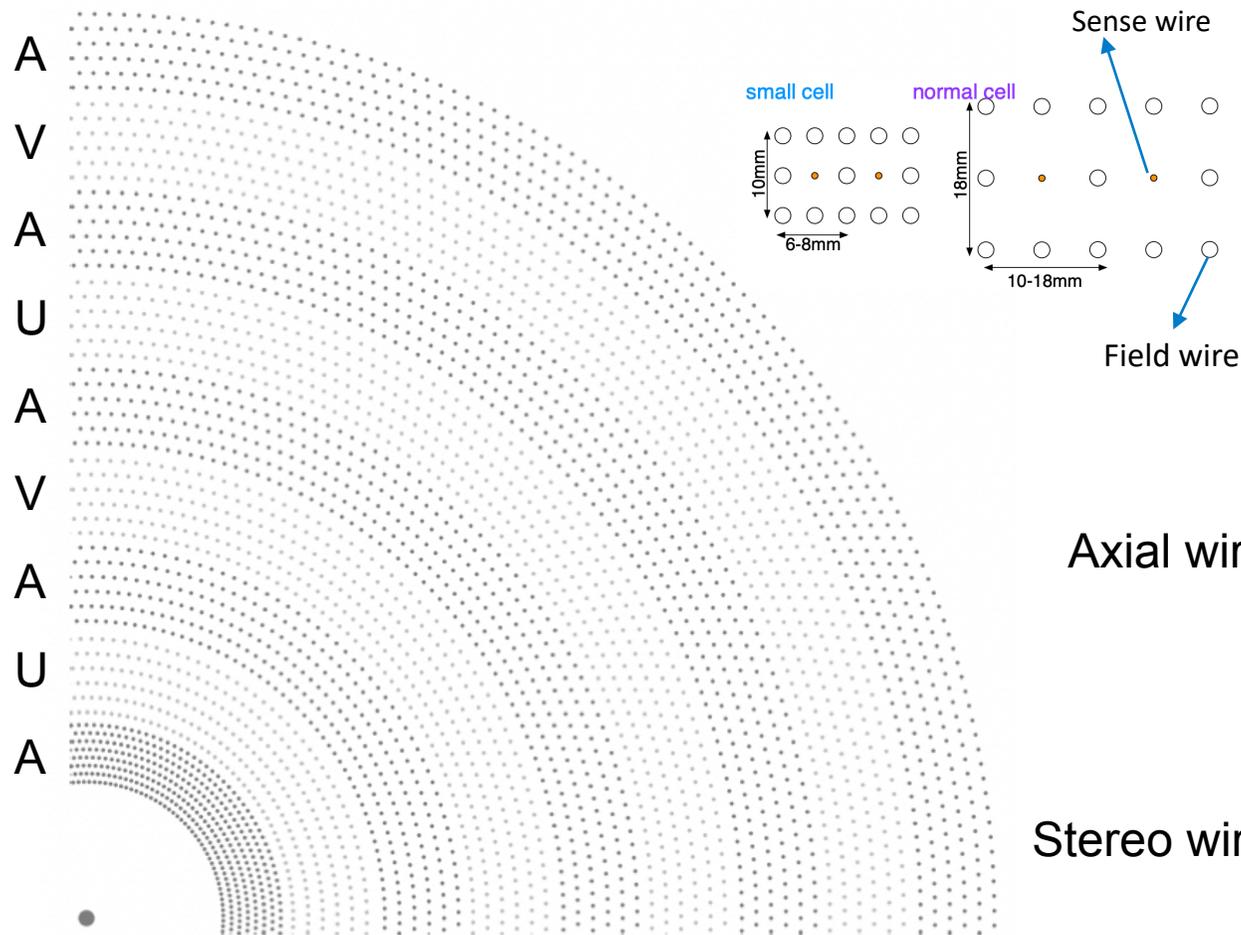
Belle II tracking detector

- Tracking sub-systems:
 - Central drift chamber **CDC**
 - Silicon vertex detector **SVD**
 - Pixel detector **PXD**
- Large volume of CDC: good transverse momentum resolution
- Silicon detector at the inner region: precise vertex resolution
- Inside **1.5 T magnetic field** provided by the superconducting solenoid
 - Homogeneous field (varies less than 1% in the entire tracking volume)



The individual tracking sub-detectors

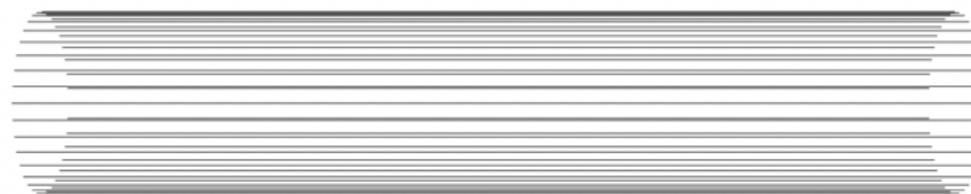
CDC: large multi-wire drift chamber
 • Tracking, Trigger, PID



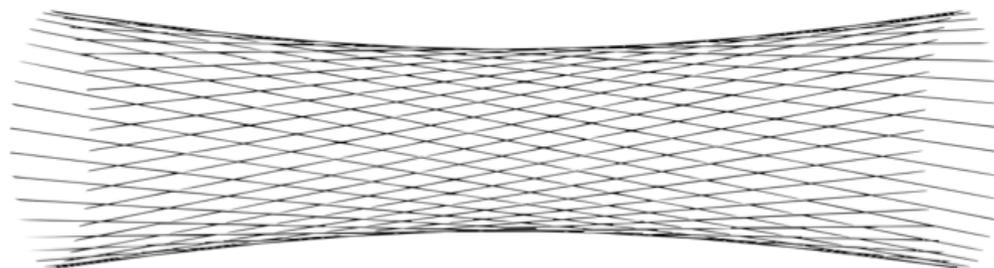
A quadrant of a slice of the CDC r-φ projection

Gas mixture	Helium, ethane
Radius [mm]	160 - 1130
Acceptance [degree]	17-150
Layers/ Super Layers	56 / 9
Stereo and axial wires	14336
Length (Z) [mm]	2325
Radiation Length	680 m
Stereo angle [mrad]	(U) 45.4-45.8
Inner -> outer	(V) -55.3 - -64.3
	(U) 63.1 - 70.0
	(V) -68.5 - -74.0

Axial wires



Stereo wires



The individual tracking sub-detectors

CDC

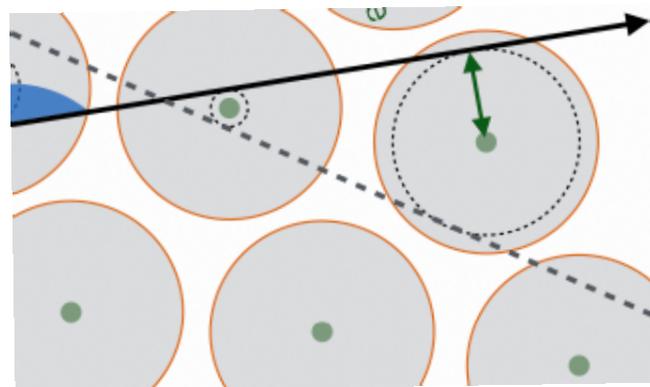
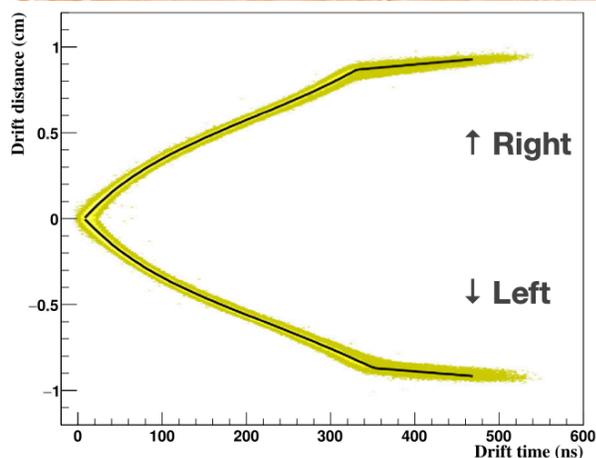
Charged particle passing through the CDC volume:

- the gas mixture is ionised
- ionised electrons drift towards the sense wires and cause avalanches within the sensitive electric field which are detected as signal

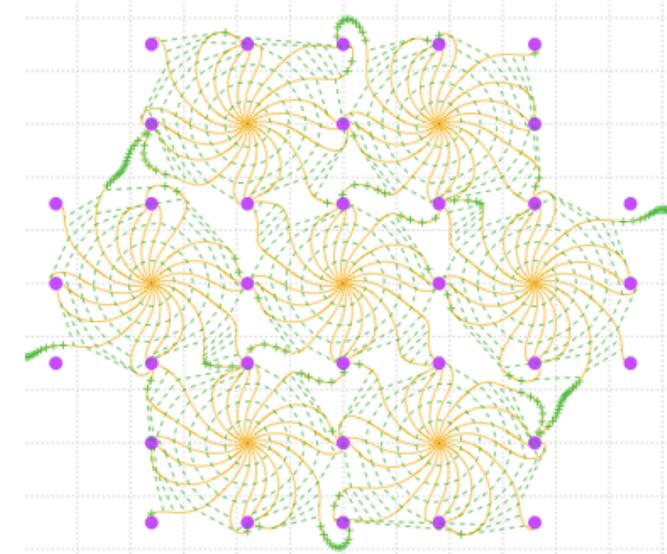
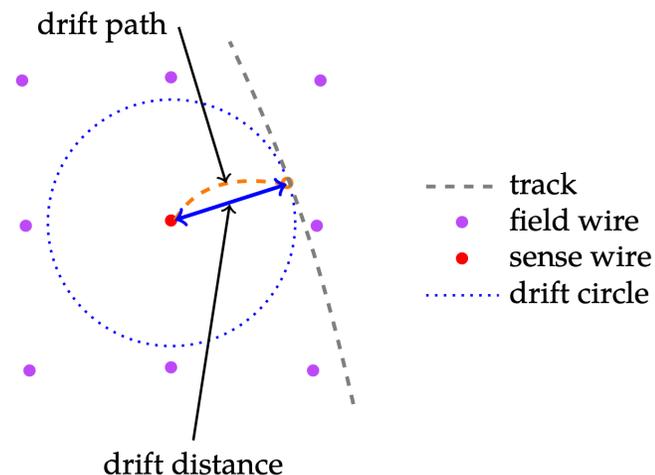
• Measurements:

- Time of arrival: time of the arrival of the signal (1ns resolution) to derive the drift distance (x-t) -> **a circle** to represent the CDC hit
 - Left/right ambiguity
- Signal amplitude (ADC): will be used for background suppression and determination of the energy loss for PID

→ Provide a measurement with spatial resolution of 120 μm on average



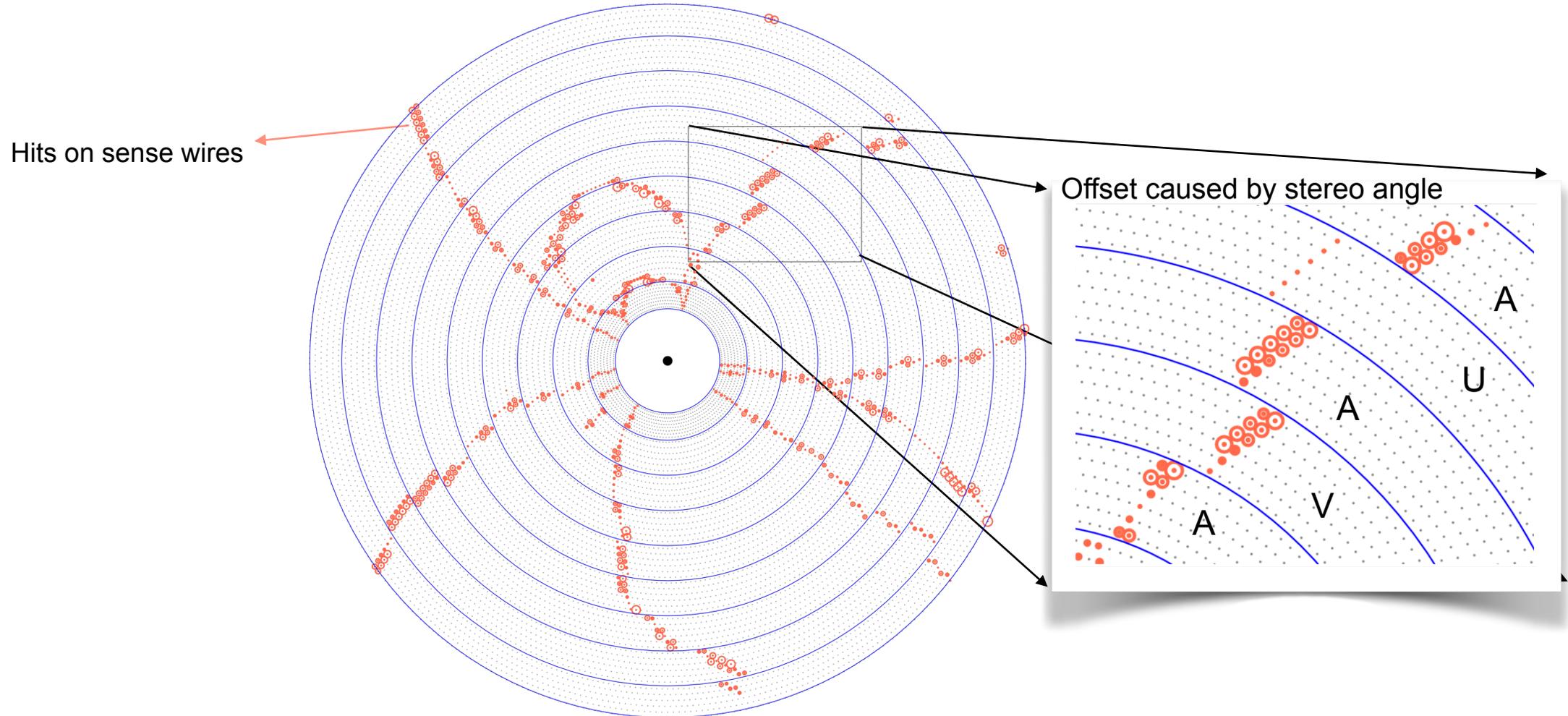
Left/right ambiguous



Field lines and isochrones

The individual tracking sub-detectors

CDC SpacePoint (xy) of a typical Y(4S) event (no beam background):

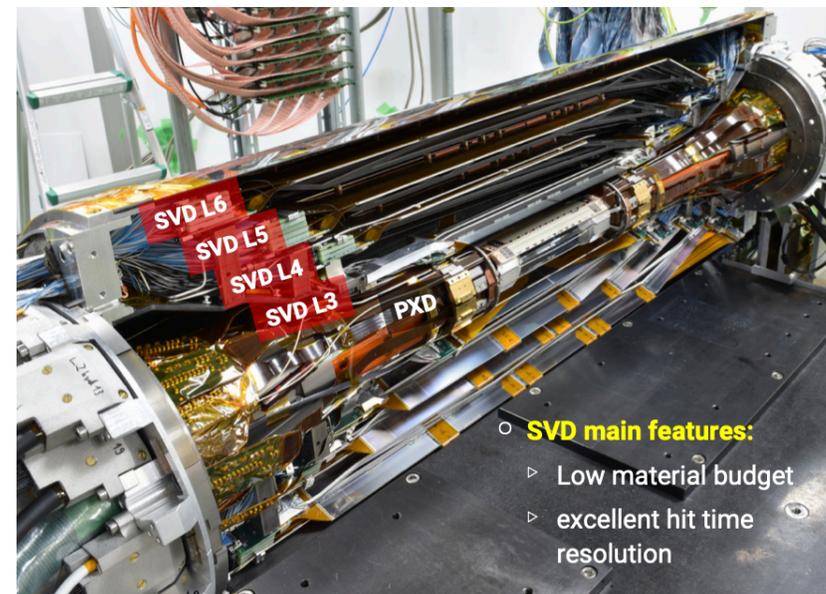


The individual tracking sub-detectors

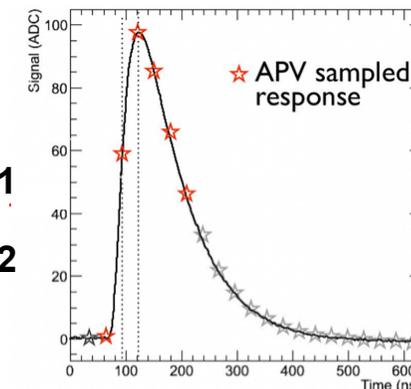
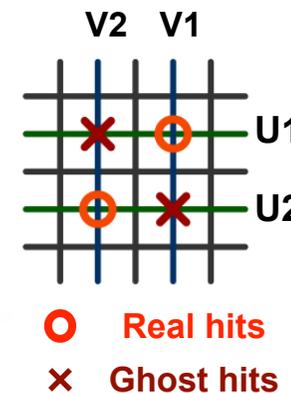
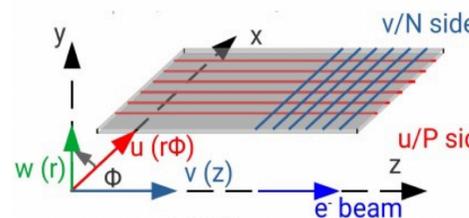
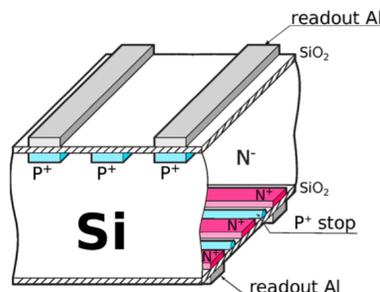
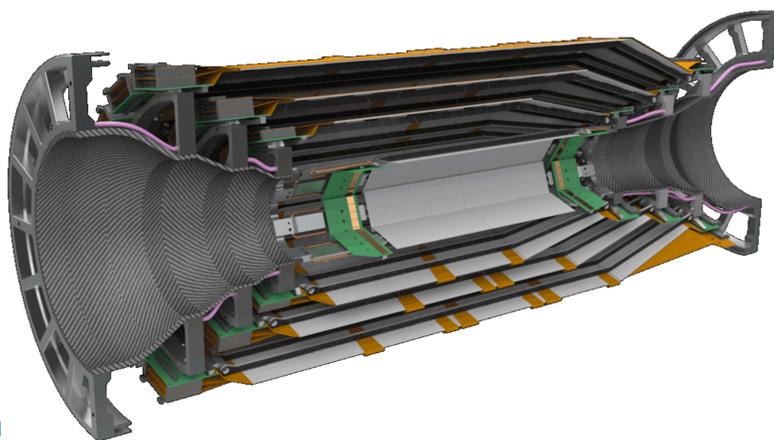
SVD

- Four layers of **double-sided silicon strip** detectors
- Rectangular sensor in barrel region and trapezoidal sensors in the forward section to **increase the coverage and minimise the material**.
 - **U**: perpendicular to the beam; **V**: parallel to the beam
 - Pitch size in U/V: 50-75um, 160-240um
- Double-sided readout: combination of measurement on U and V
 - Wrong combination: ghost hits

Position Resolution on U: 7~12 um
 V: 15~25 um
 Time Resolution: 3 – 4 ns



- **SVD main features:**
 - ▷ Low material budget
 - ▷ excellent hit time resolution



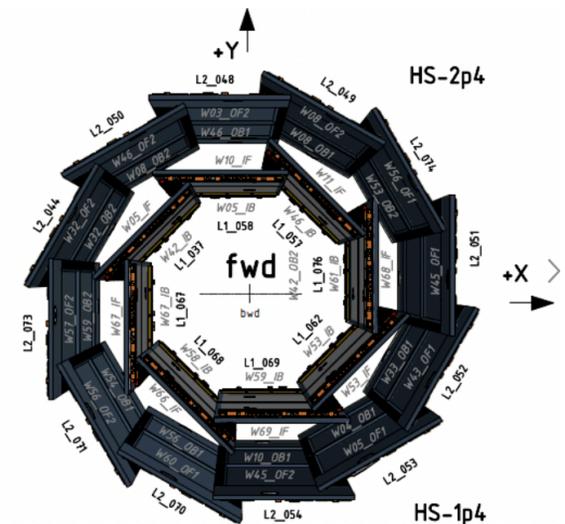
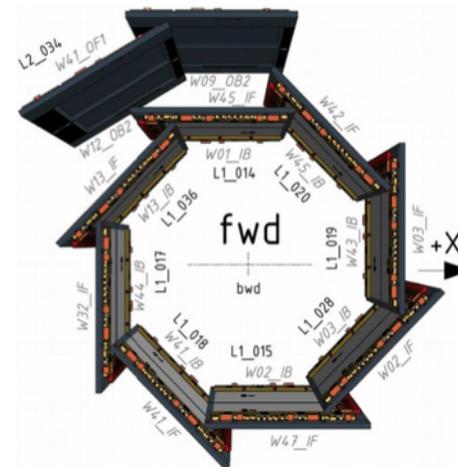
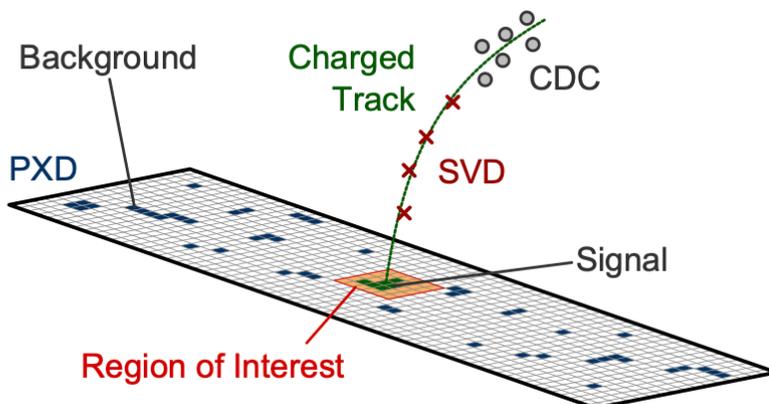
The individual tracking sub-detectors

PXD

- The **inner most 2-layer** silicon pixel detector
 - Not involved in track finding but big improvement to the track quality
- High Level Trigger ROIs (region of Interest)**: extrapolate the tracks to PXD surface and define a ROI
 - Reduce the PXD data size (as the data size is too large and dominated by beam background)

→ Accurate 3D SpacePoint with resolution: $\sim 10\mu\text{m}$ (rphi, z)

	Layer 1	Layer 2
Radius (mm)	14	22
# Ladders / modules	8 / 16	12 / 24
Sensitive thickness (μm)	75	75
Pixels per module	768 x 250	768 x 250
Pixel size (μm)	55x50 and 60x50	70x50 and 85x50
Total number of pixels	3.072×10^6	4.608×10^6



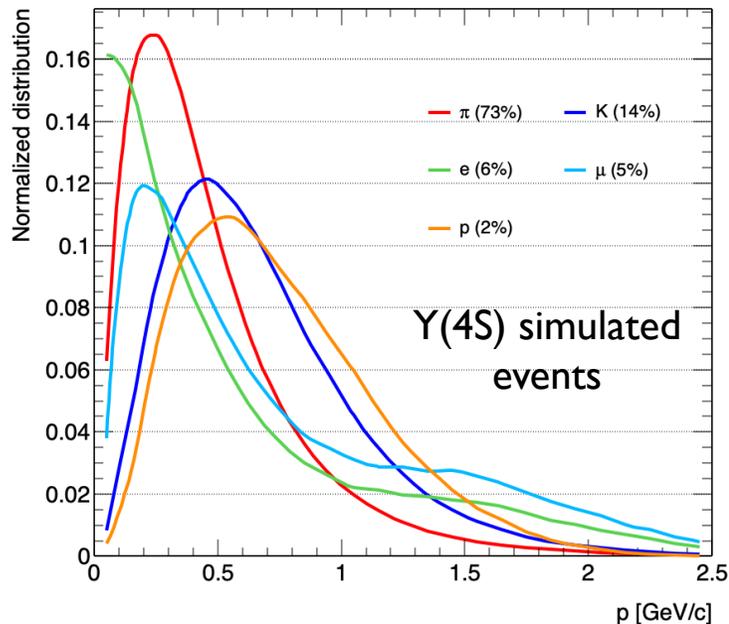
Belle II typical Y(4S) event

The objects for Tracking

→ For a typical Y(4S) event:

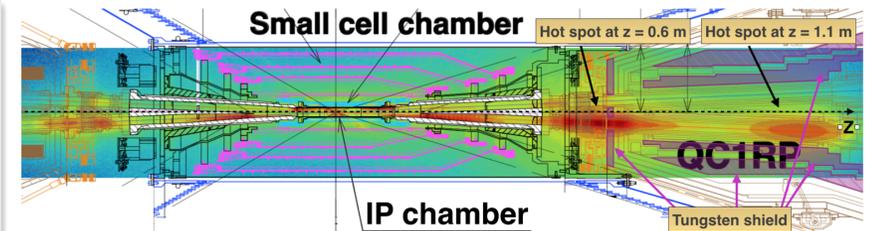
- 7GeV e⁻ on 4GeV e⁺ → $\beta\gamma = 0.28$
- Average multiplicities:
 - 11 charged tracks
 - 5 neutral pions
 - 1 neutral kaon

- Soft charged tracks momentum spectrum
 - > effect of **multiple scattering** not negligible especially for very low momentum tracks
 - $p_T < 40 \text{ MeV}/c$: not reach CDC
 - $p_T \in [40, 250] \text{ MeV}/c$: Curling in CDC



→ Significant machine Background

- **Single beam background**
 - Touscheck scattering
 - Beam-gas scattering
 - Synchrotron radiation
- **Luminosity Background**
 - Radiative Bhabha
 - two-photon process
- **Injection background**
 - L1 trigger veto when injection bunch is close to IP



hits composition@nominal lumi:

Nominal luminosity	PXD L1	SVD L3	CDC
Signal	10 hits	10 hits	500 hits
Background	39000	13000	3000

*Injection background not included

Challenges for Tracking @ Belle II

➔ The requirements to tracking:

- **High track finding efficiency:**

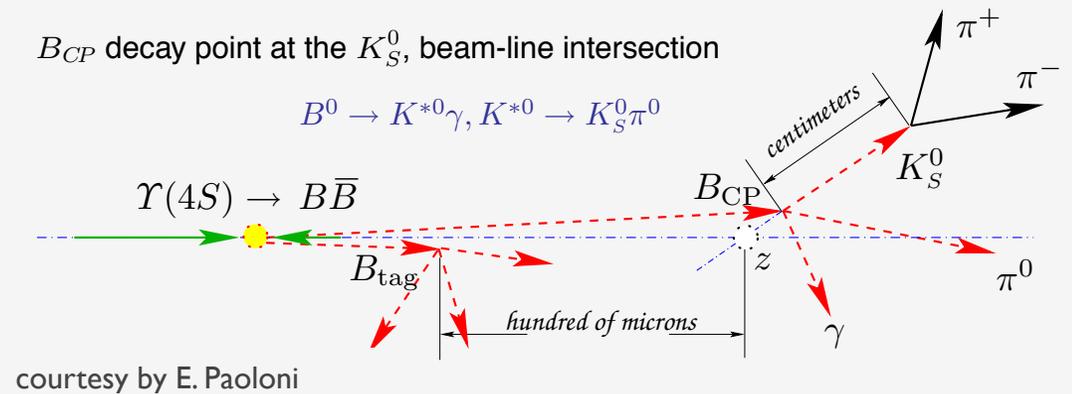
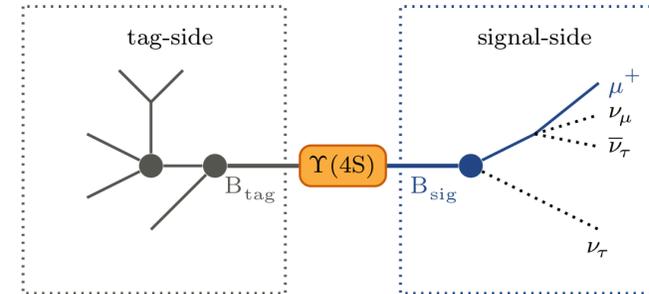
- Important for every analysis, especially for analysis with missing energy
- Analysis based on full event interpretation

- **Low fake rate:**

- Fake tracks: real but from background, real but duplicates, from random combination
- Hits from beam background dominate in the inner detector

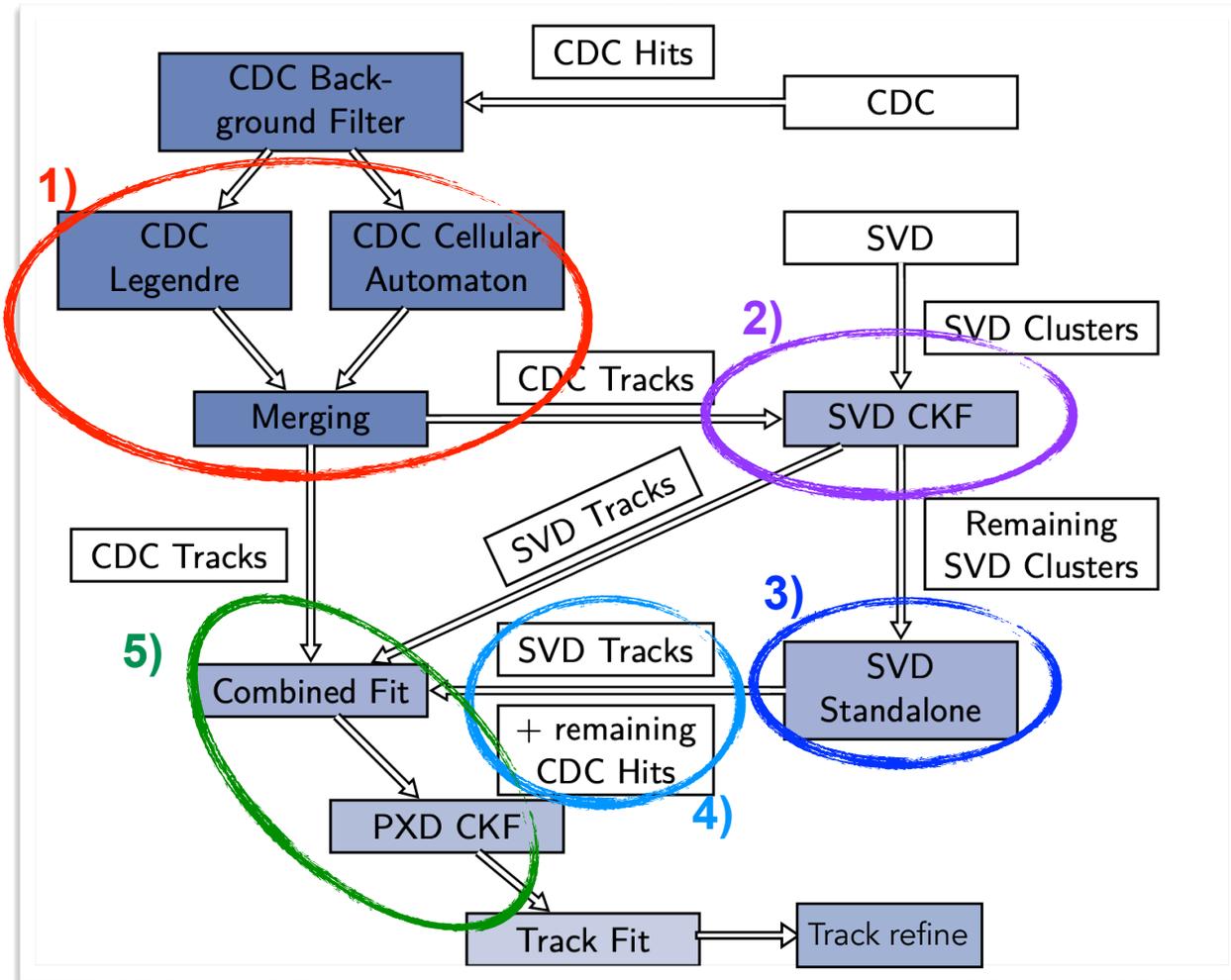
- **Good vertex position resolution:**

- Much better than 100 μm : Average secondary vertex $\sim 130\mu\text{m}$ for B decays
- Crucial for Time-dependent analysis



Belle II Tracking procedures

The Tracking procedure in Belle II



*CKF: Combinatorial Kalman Filter

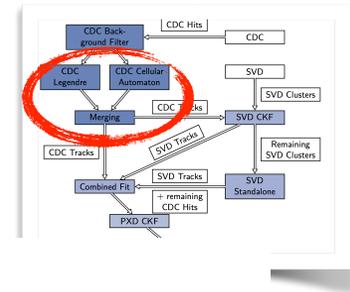
- 1) CDC tracking with
 - I. Global Legendre approach
 - II. Local approach (off by default)
 - III. Merge
- 2) Extrapolate to SVD with CKF
- 3) SVD standalone Tracking
- 4) SVD standalone extrapolate to CDC
- 5) Combine and attaching the PXD hits

**Heavily relying on filters trained on simulated events (MVAs, SectorMaps)*

[Track finding at BelleII](#)

The Tracking procedure in Belle II

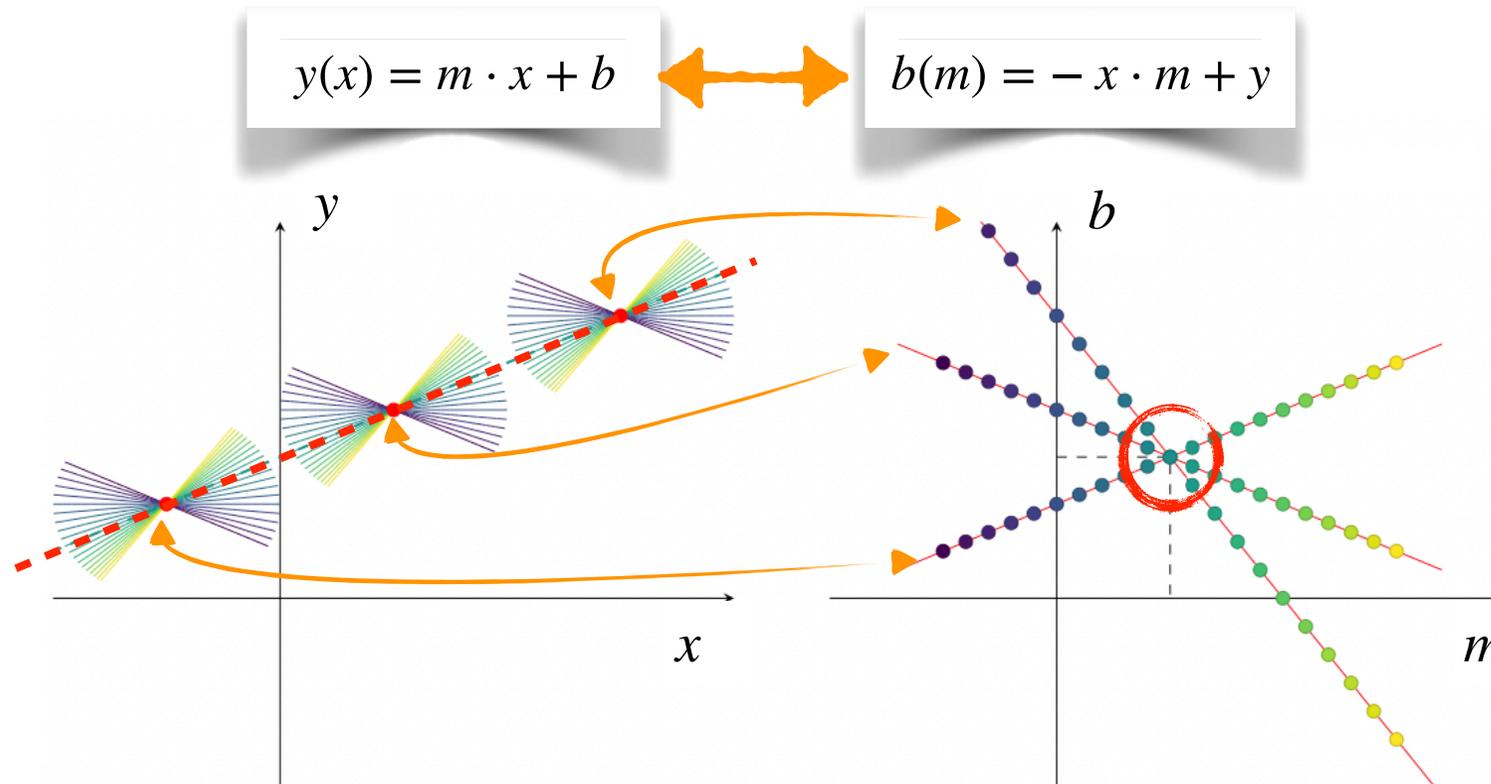
CDC Global tracking



Transformations involved:

Hough Transformation

- Transform a point in **measurement space** to a line or a curve in **Hough space** (and vice versa)
 - For multiple measurements, the line going through them corresponding to the **crossing point** of the lines in Hough space

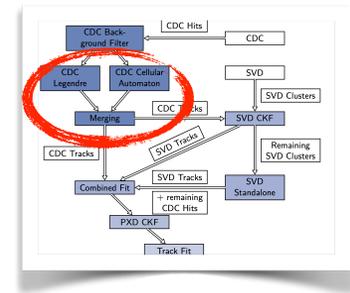


The Tracking procedure in Belle II

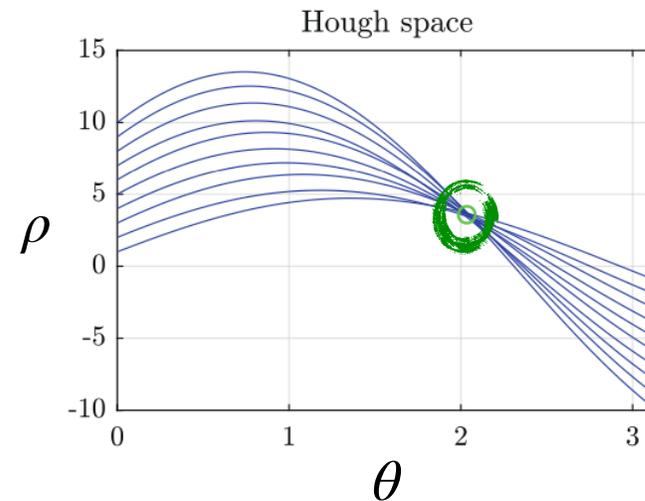
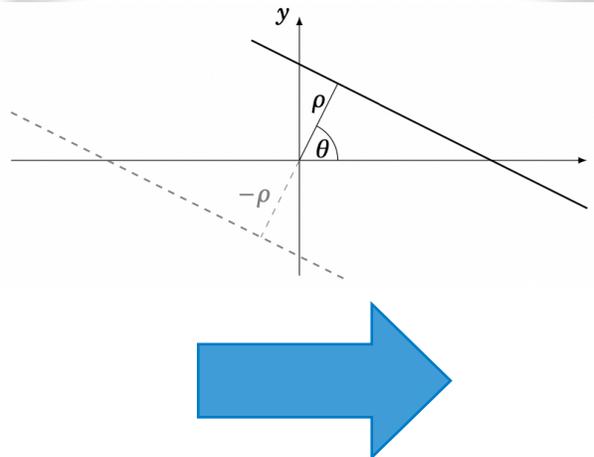
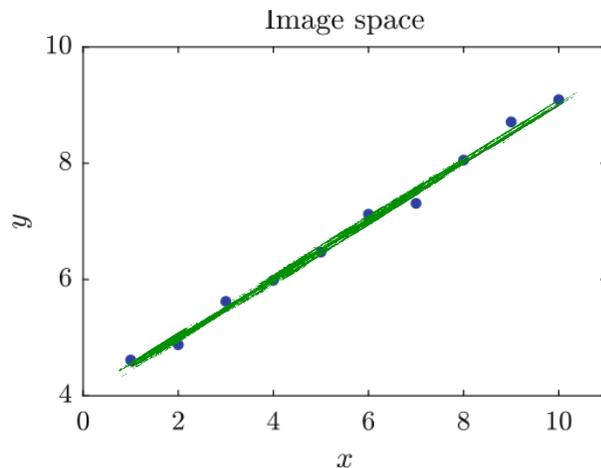
CDC Global tracking

Hough Transformation

- Transform a point in **measurement space** to a line or a curve in **Hough space** (and vice versa)
 - For vertical Tracks, both slope and intercept are infinite numbers
 - To better handle this case:
 - **The Hesse normal form of a straight line:** (ρ, θ) parameter space, where $\theta \in [0, \pi]$



$$\rho = x \cdot \cos \theta + y \cdot \sin \theta$$



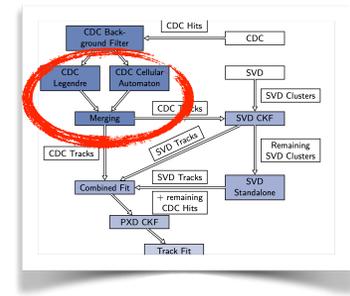
The Tracking procedure in Belle II

CDC Global tracking

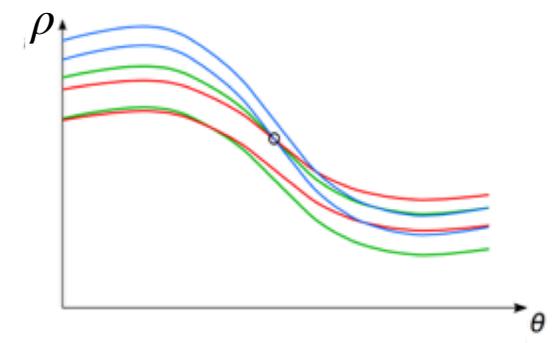
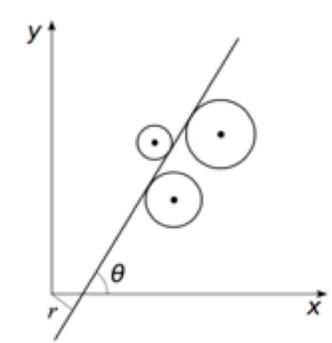
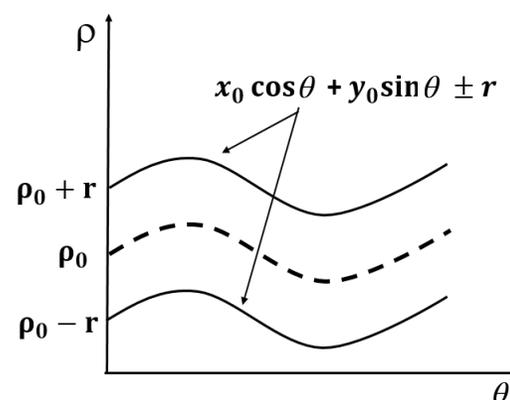
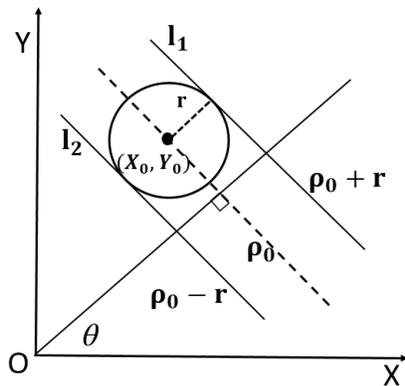
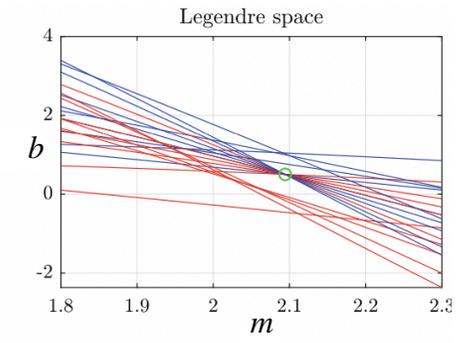
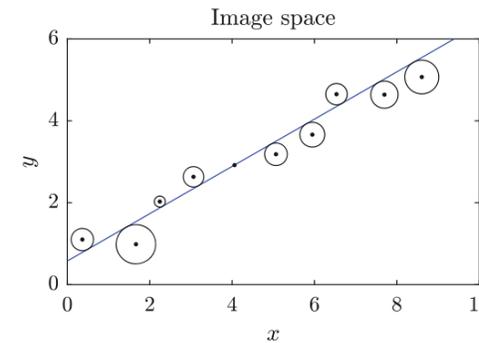
- Point -> Circle

Legendre Transformation (An extension of the Hough transform)

- Common tangent Lines to the **circles** (drift circles)

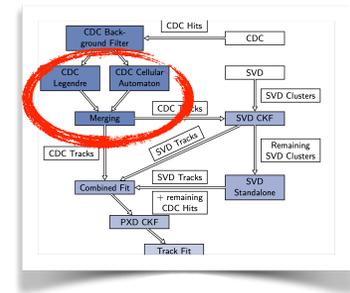


$$\rho = x \cdot \cos\theta + y \cdot \sin\theta \pm r$$



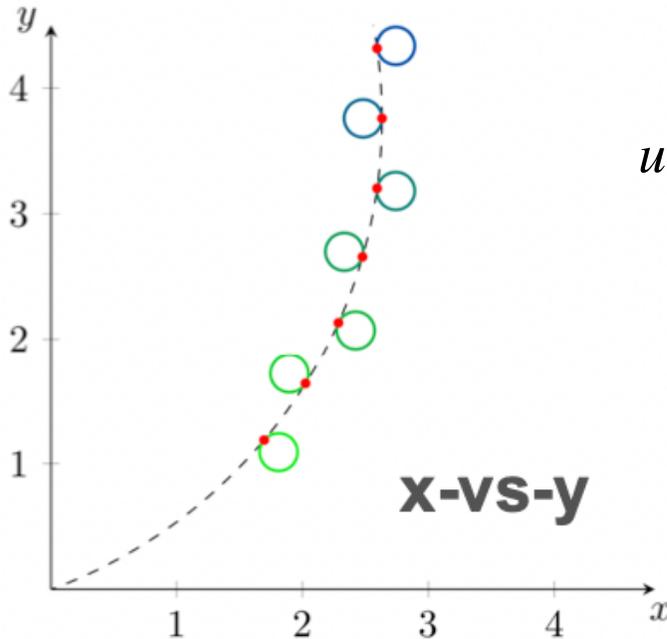
The Tracking procedure in Belle II

CDC Global tracking

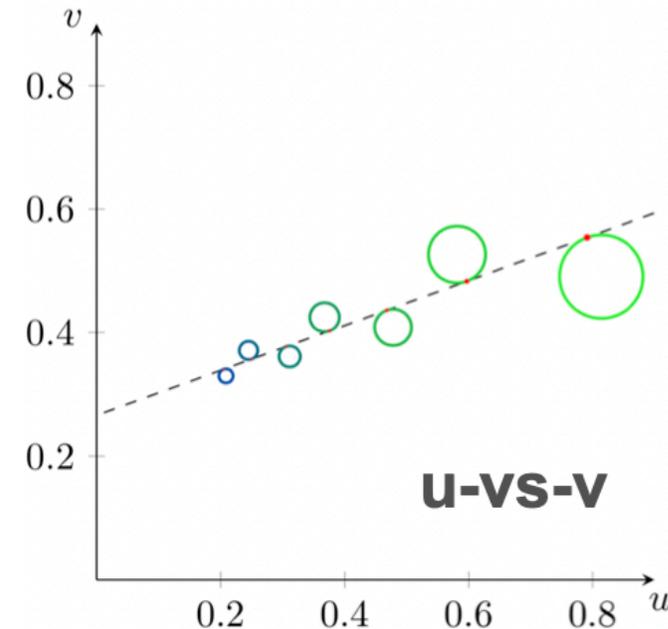
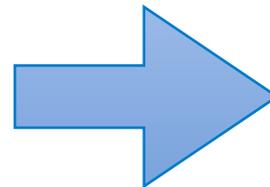


For real CDC tracks:

- Helix tracks (assumption: from the IP) instead of a straight line track
- 2-D track finding in transverse plane in this step: Only **axial layers** used in this step
- Apply firstly the **Conformal transform** (preserve angles locally after transforming)
 - Curling Track going through the IP → Straight Line
 - Circles not passing the origin transform into new circles (drift circles in Hough space)

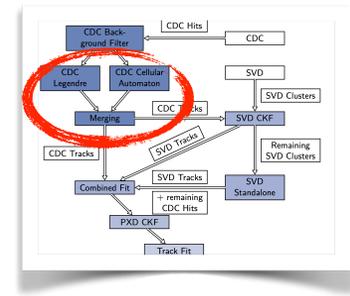


$$u = \frac{2x}{x^2 + y^2} \quad v = \frac{2y}{x^2 + y^2}$$



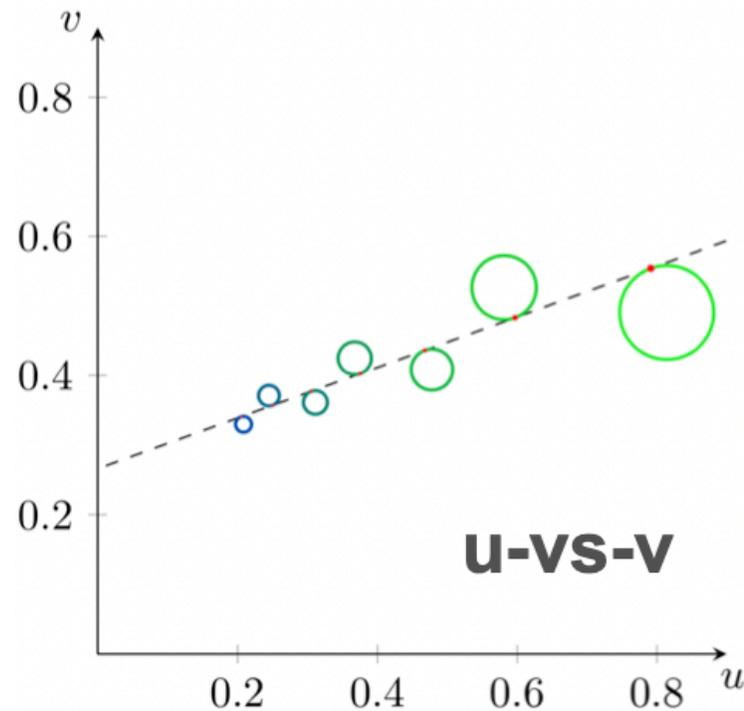
The Tracking procedure in Belle II

CDC Global tracking

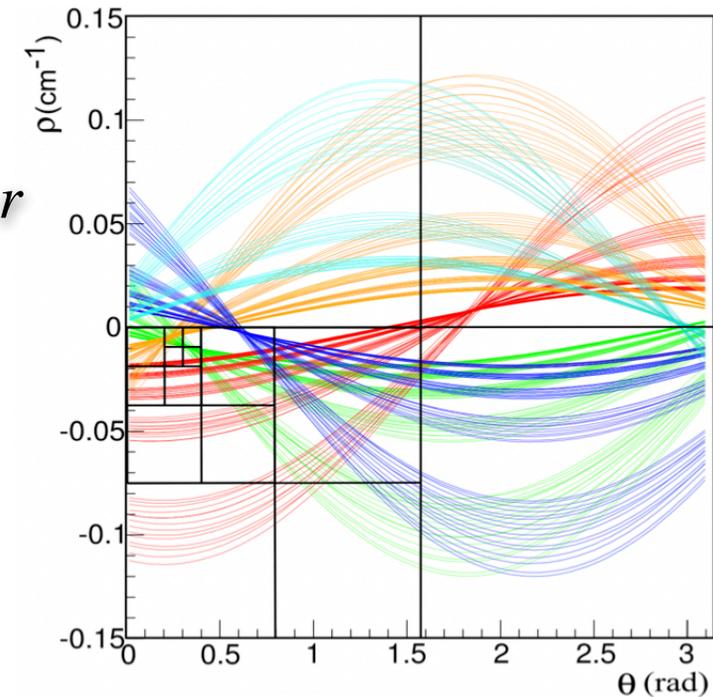
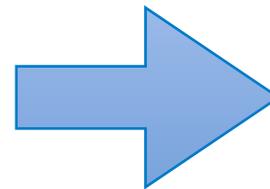


Then apply the **Legendre Transformation**

CDC Global Track finding -> find the region of maximum density in the Legendre space

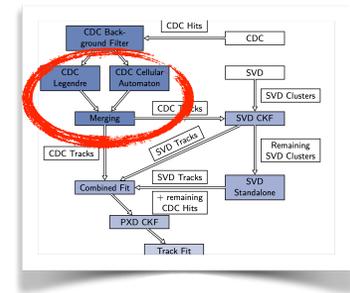


$$\rho = x \cos\theta + y \sin\theta \pm r$$



The Tracking procedure in Belle II

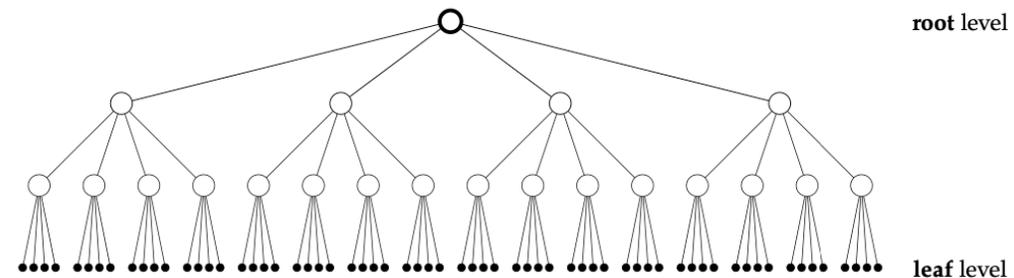
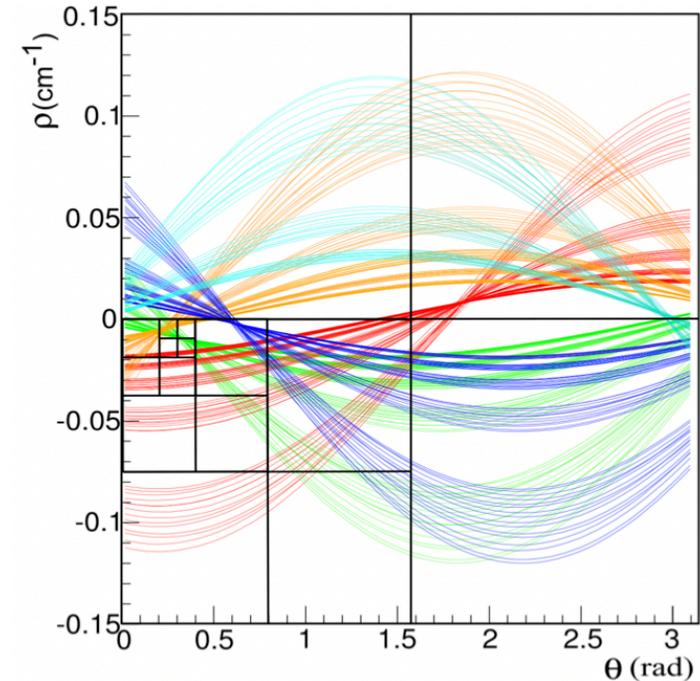
CDC Global tracking



-> find the region of maximum density in the Legendre space

- 2-dimensional binary search:
 - Iterative searching: collect the number of curves in each bin, the most populated bin is considered for the next iteration
 - QuadTree used to store the 2-D binary search
 - Until the lowermost level reached or selected bin is rejected

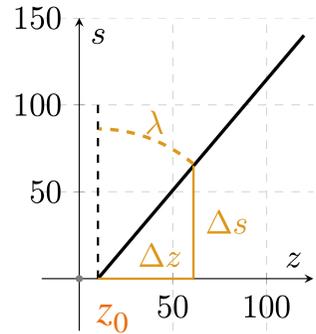
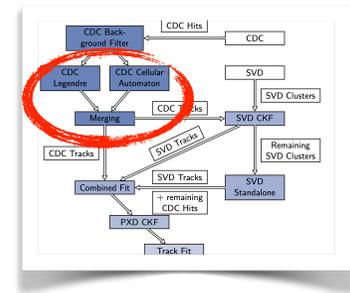
➔ Multiple tracks could be found by searching in the Legendre space iteratively



The Tracking procedure in Belle II

CDC Global tracking

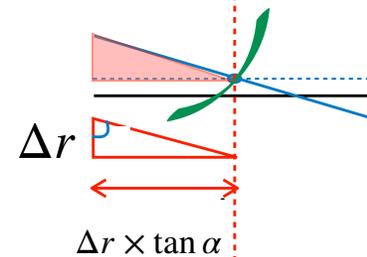
➔ Towards 3D track finding: Including the Z info



$$z_0 = z_{rec} - \tan \lambda \cdot s_{rec}$$

To be measured

z_{rec} : 2D circle fitting to $r - \phi$ trajectories to get the position information of each stereo layer
Derive the z_{rec} given the Δr and α (the stereo angle)



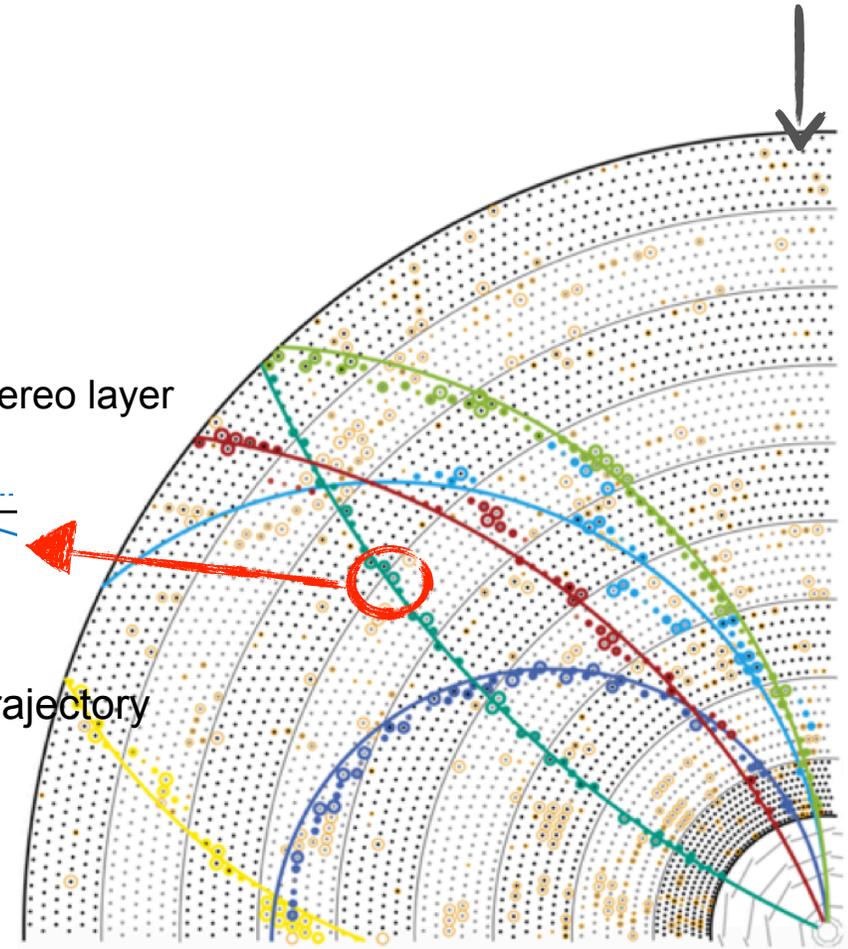
s_{rec} : the length of the path of the helix on x-y plane, also get from the $r - \phi$ trajectory

➔ Similar global search of a straight line going through (z_{rec}, s_{rec}) @page 20

- Legendre transformation to $(z_0, \tan \lambda)$ space

• Stereo-wire hits been found are added to the $r - \phi$ tracks

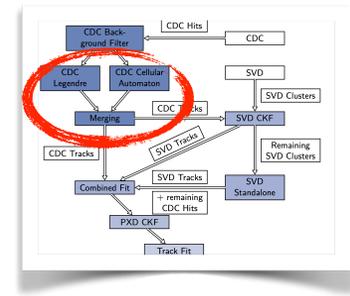
☑ Finished the CDC global track finding



The Tracking procedure in Belle II

CDC local tracking

Weighted Cellular Automaton + longest path search: focusing on short and displaced tracks



Cellular automata method (CA):

discrete **dynamical** model whose behaviour is completely specified in terms of a local relation
(A computational model with discrete cells updated synchronously)

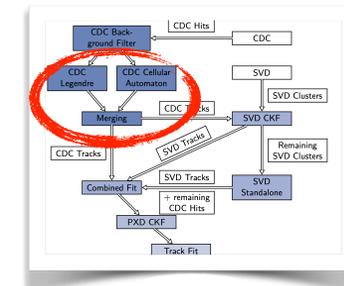
- Was created for self-replicating automata by John von Neumann (~1950)
- **Best Known CA**: 1970 Game of life (by John Horton Conway) <https://playgameoflife.com>
- ...

Characteristics:

- **Cell**: basic unit of CA
- Each cell could take several different **discrete states**
- **Rules** of evolution are Local: characterised by local connectivity of the cell
- Cells' states **updated simultaneously** at each step

The Tracking procedure in Belle II

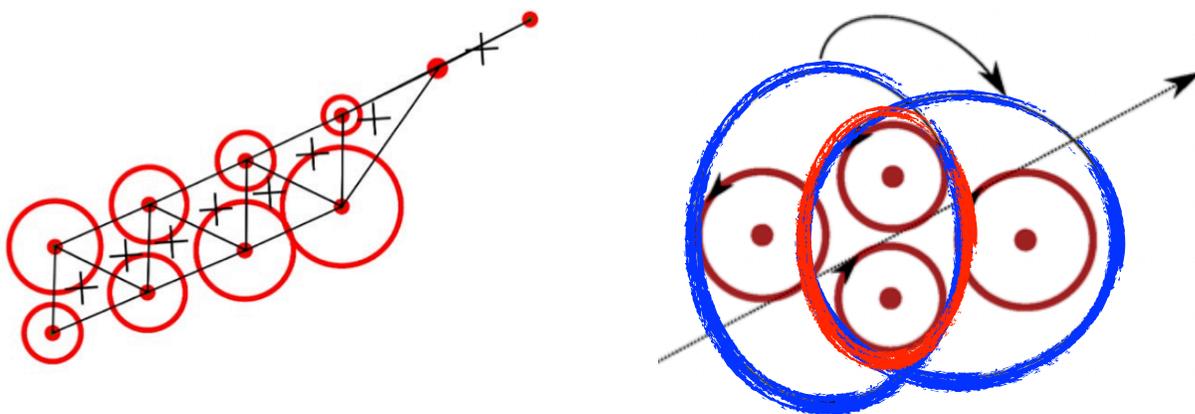
CDC local tracking



Weighted Cellular Automaton + longest path search: focusing on short and displaced tracks

Two stages: 1. Segments building (within Super Layers) 2. Track building

1. Build **Triplets** of wire hits
2. Construct the graph **edges**: searching neighbours of each Triplet + cellular Automaton to remove the short path
3. Build **Segment**: Iteratively retrieve the **paths with highest E**
4. Make both possible orientation of each segment available for the next step



E_i is the E (total weight) of a path to the vertex i :

$$E_i = \sum \omega_{ij} + \sum \Theta_j$$

Θ_j : the weight for triplet j

ω_{ij} : the weight for the edge of ij

Linear fit to the three/four hits and weight assigned based on the χ^2 of the fit

6. Combine segments with global found tracks using a mva-based method

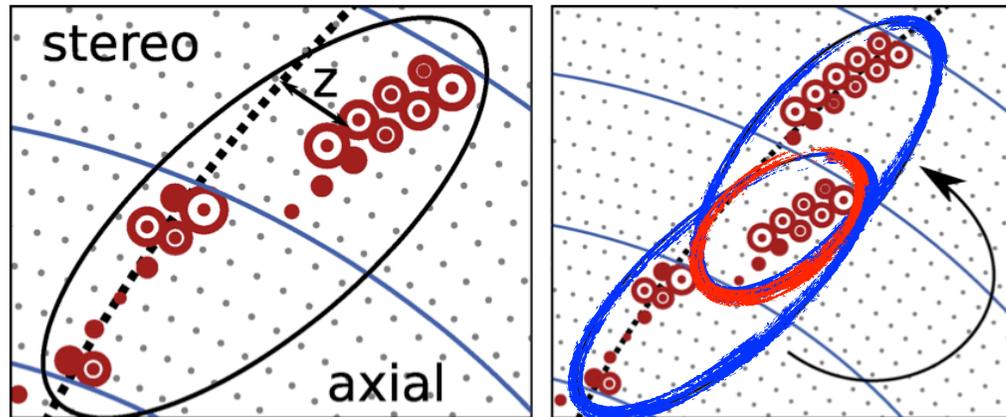
The Tracking procedure in Belle II

CDC local tracking

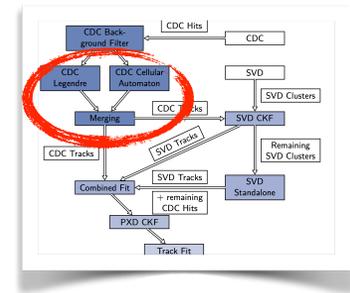
weighted Cellular automaton + path search: focusing on short and displaced tracks

Two stages: 1. Segments building 2. Track building

1. Build **doublets** of segments (between axial- and stereo-superlayers)
2. Construct the graph **edges** by searching neighbours of each segment doublets
3. Retrieve the longest paths
4. Decide whether the tracks should be interpreted as reversed

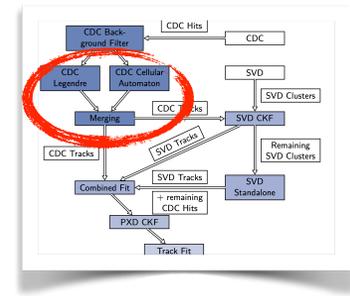


*Currently not used due to non-negligible fake rate



The Tracking procedure in Belle II

Combination of local and global Tracks



CDC Global track finding: capable to reconstruct tracks with several missing layers

CDC Local track finding: short and displaced tracks (off at the moment)

Then, reach out to SVD using **CKF** (Combinatorial Kalman Filter)

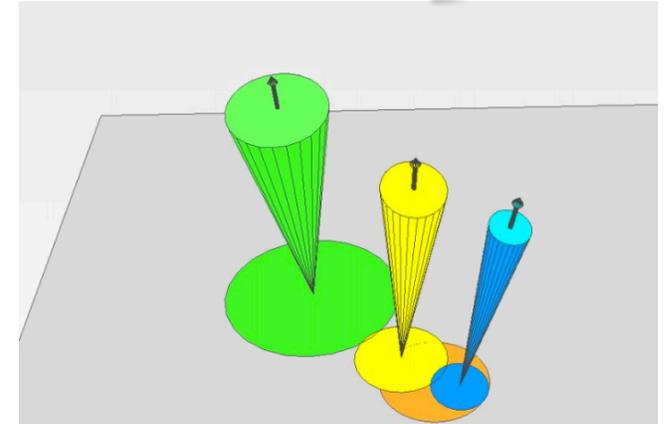
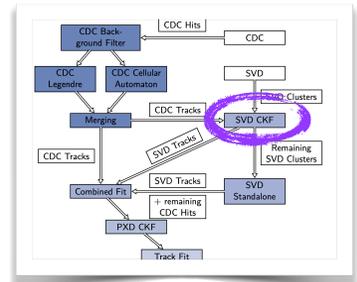
- Introduction to Kalman Filter

Kalman Filter

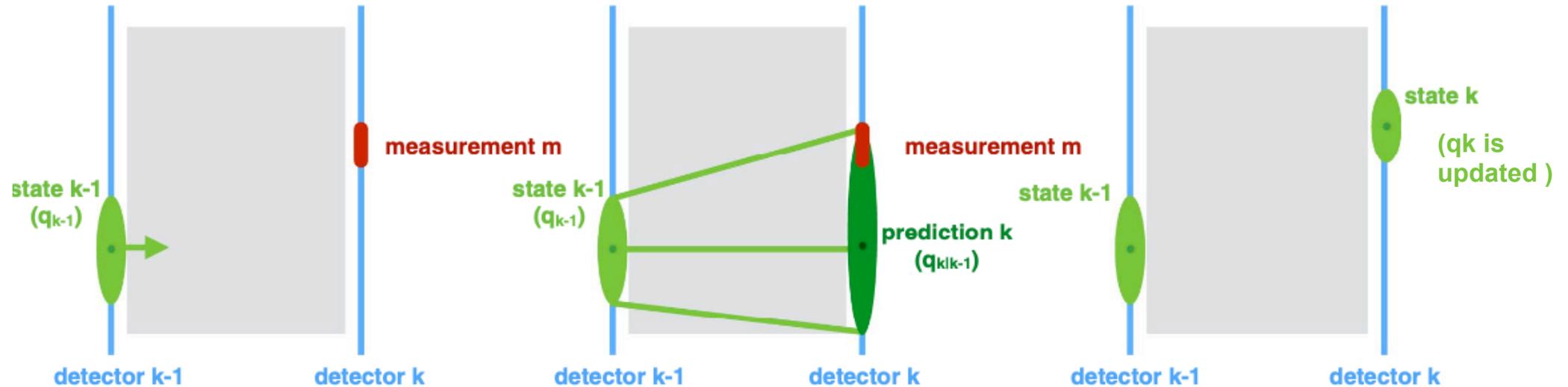
General idea

A famous method for track fitting: progressively perform a least square fit

- Extrapolate from $k-1$ to k : **prediction** + **filtering**
 - Prediction: extrapolate the state to next detector plane
 - Filtering: update the predicted state with the measurement
- Backward **smoothing** to update all the states when forward filtering is done
- Propagate in inhomogeneous magnetic field (Runge-Kutta-Nystrom method)
- Material effects included



Typical procedures:



The Tracking procedure in Belle II

Extrapolate to SVD and attach the SVD hits

CKF (Combinatorial Kalman Filter)

Perform a full combinatorial exploration when there is multiple next-hit candidates

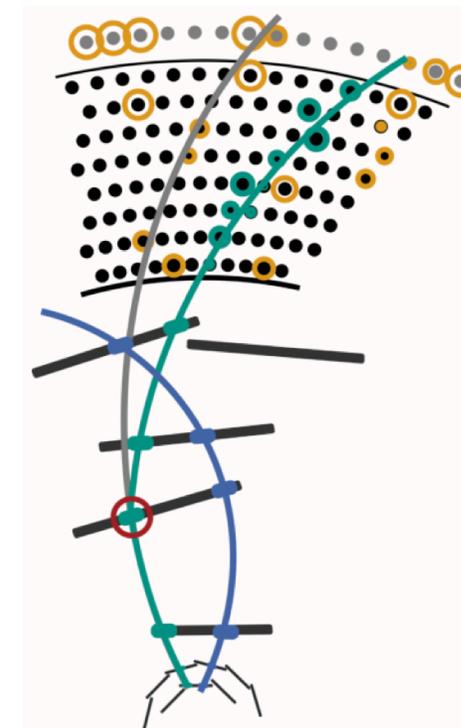
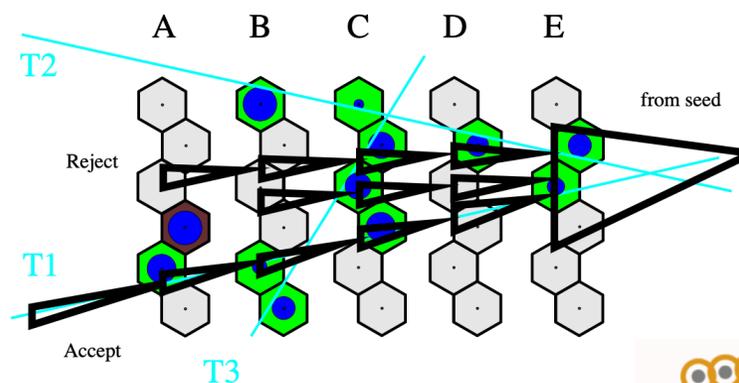
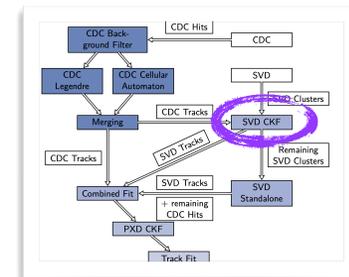
- duplicate the track candidate and treat as different tracks
- select best candidate

Reach out to SVD hits using CKF:

1. CKF:

- Extrapolate to SVD in both direction: low momentum tracks curl and can pass SVD multiple times
- MVA-based filter to attach the signal SVD cluster

2. **Combined CDC-SVD track refitted** with full material effects included



The Tracking procedure in Belle II

SVD standalone Track finding

VXDTF2: SVD standalone pattern recognition algorithm

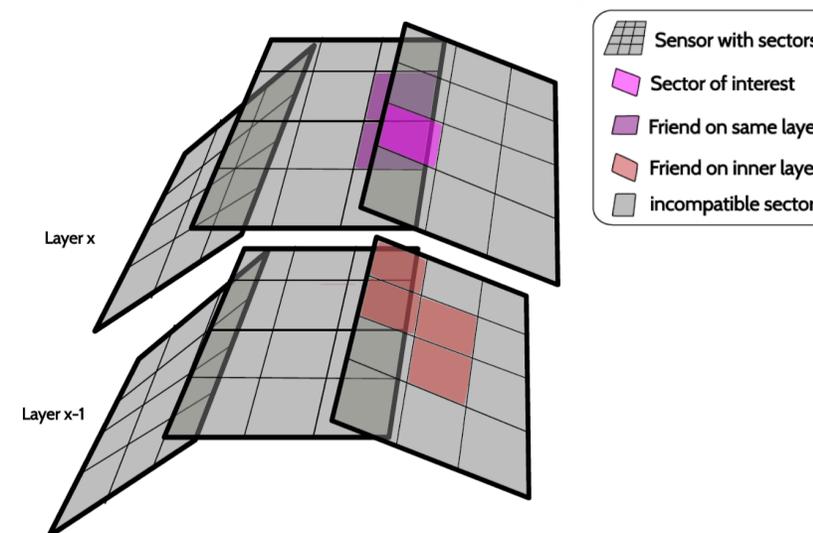
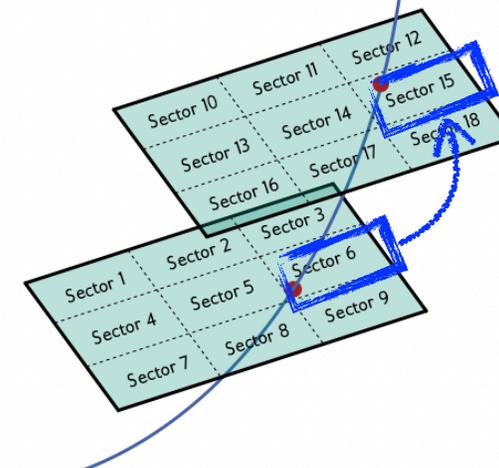
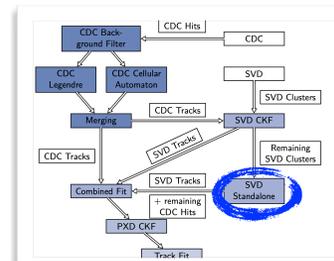
1. Sector map filters: reduce the combinations
2. Cellular automaton: identify track candidates
3. Best candidate selection

1. Sector map

Data structure holds information about the **relations and filters** of the space points in different region(sector)

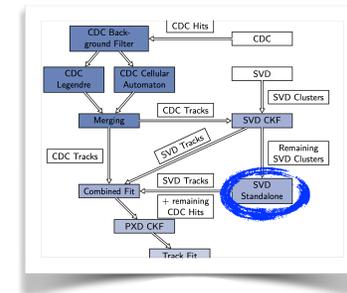
How to build the sector map:

- Sub-divide sensors into virtual **Sectors (3x3)**
- ‘**friends sectors**’: two sectors connected by one track
- **Training:** Use MC events to learn which sectors are friend
 - Training samples: Y(4S) events, Bhabha
- Store the possible ‘friendships’ into a so called “**Sector Map**”



The Tracking procedure in Belle II

SVD standalone Track finding



VXDTF2: SVD standalone pattern recognition algorithm

1. Sector map filters: reduce the combinations
2. Cellular automaton: identify track candidates
3. Best candidate selection

1. Sector map

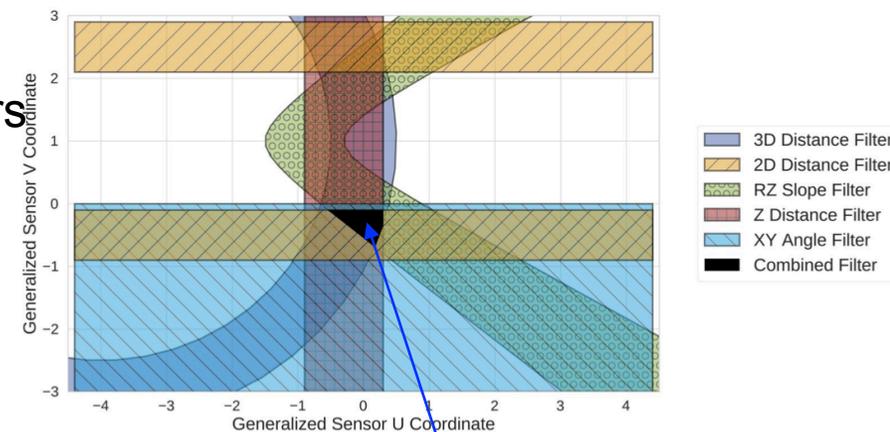
Also holds selection criteria (filters) on the space points on friend sectors to reduce the number of combinations of hits

- **Filters:** defined individually for each sector combination (2- or 3- hits filters)

- Geometrical quantities (distance, ϕ , θ -direction)
- SVD timing information
- Trained with same MC samples

e.g. Combination of filters' effects with a given space point on another sensor

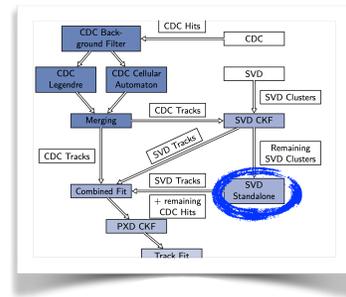
❖ Capable of adapting to **different detector conditions** (defects, misalignments...)



consider SP only
in this allowed
region

The Tracking procedure in Belle II

SVD standalone Track finding



VXDTF2: SVD standalone pattern recognition algorithm

1. Sector map filters: reduce the combinations
2. Cellular automaton: identify track candidates
3. Best candidate selection

2. **cellular automaton** (helps to gather the longest path):

Beginning with the nodes on the outermost layers

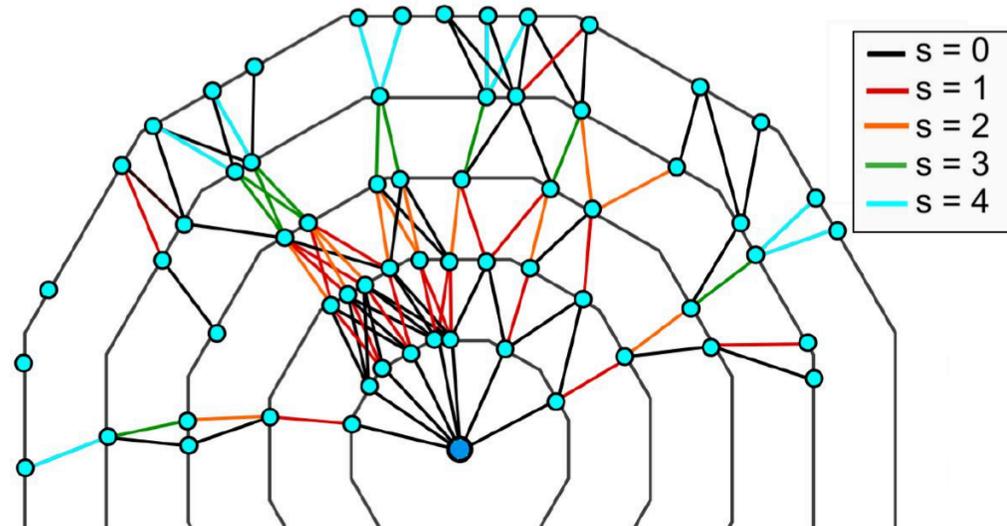
Cell: segments (pairs of hits)

Rules:

Check step: for each cell if there is at least one inner neighbour with the same state.

(If yes increase the state by 1: $s(t+1)=s(t)+1$)

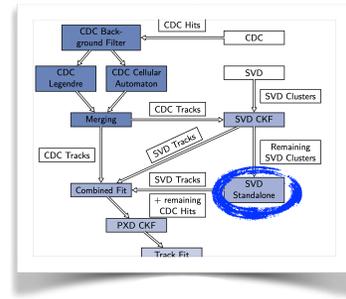
Repeated until all states of all the cells became stable



Final situation: outer cell with higher states and inner Segment with lower states

The Tracking procedure in Belle II

SVD standalone Track finding

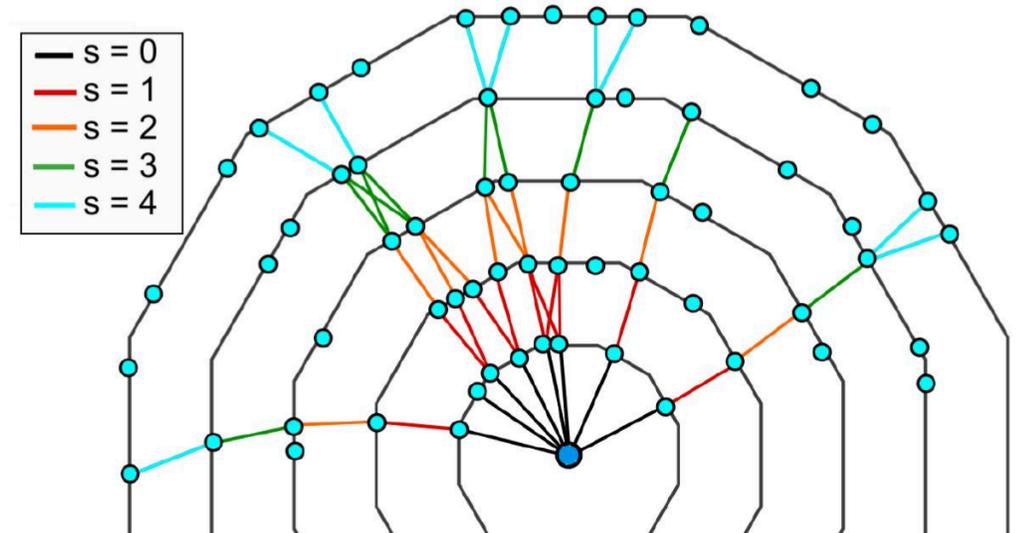


VXDTF2: SVD standalone Track finding

1. Sector map: geometrical relation and selection criteria for each relation
2. Cellular automaton
3. best path selection

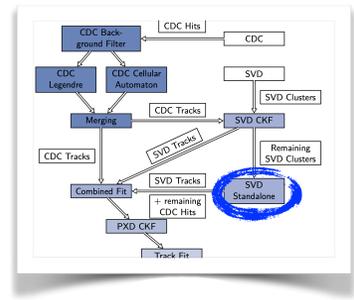
2. cellular automaton:

- Select the outer-most cells that have a state larger than a threshold (=3 in this case) and collect the next inner neighbour if the state is s-1 iteratively



The Tracking procedure in Belle II

SVD standalone Track finding



VXDTF2: SVD standalone pattern recognition algorithm

1. Sector map filters: reduce the combinations
2. Cellular automaton: identify track candidates
3. Best candidate selection

3. Best path selection:

- Quality for each track estimated by a fast fit
- Tracks are sorted and picked according to their qualities
- Tracks selected after are required not to have any SVD cluster shared with the tracks picked before

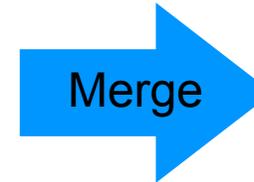
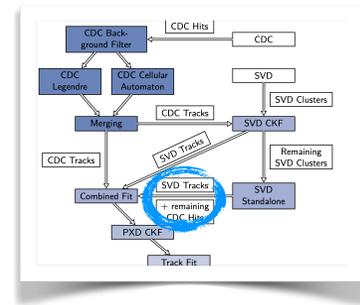
The Tracking procedure in Belle II

SVD standalone extrapolate to CDC

Use CKF to extrapolate SVD(standalone) tracks into CDC

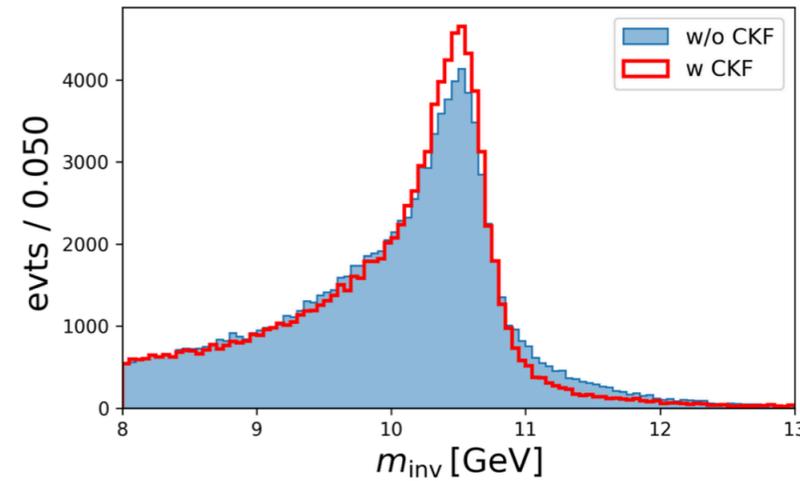
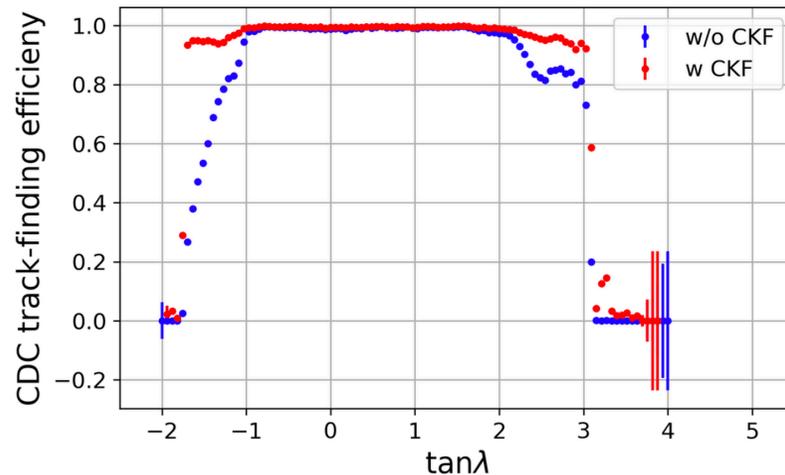
→ attach CDC hits(remaining ones)

- Improves the finding efficiency of CDC hits
- Significant improvement at Large $|\tan \lambda|$
- Improves the momentum resolution of the full track



CDC-SVD tracks

2-track events (data), $\lambda \equiv \frac{\pi}{2} - \theta$

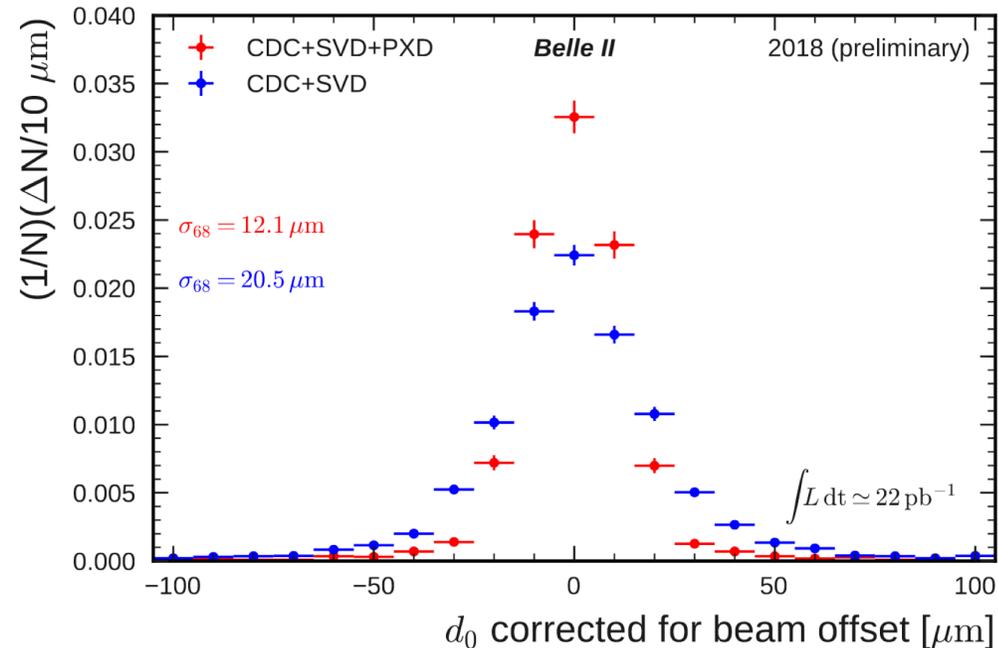
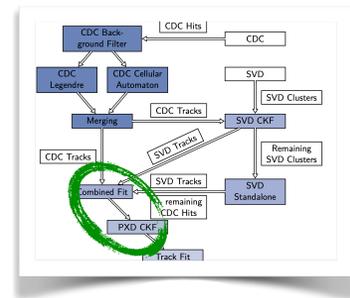


The Tracking procedure in Belle II

CDC-SVD tracks -> PXD

Use CKF to add PXD hits

- Improve position resolution significantly
e.g. the impact parameter: d_0

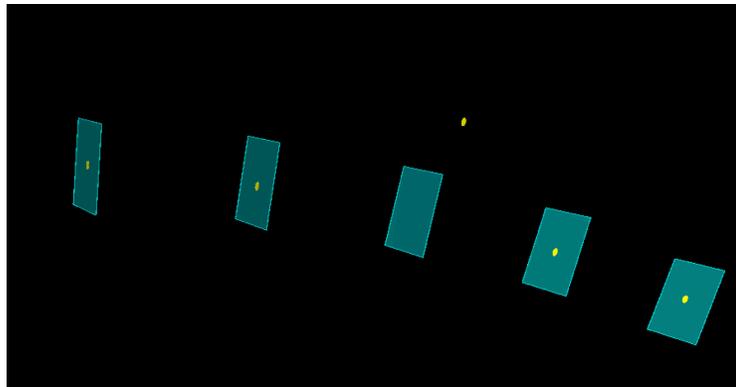
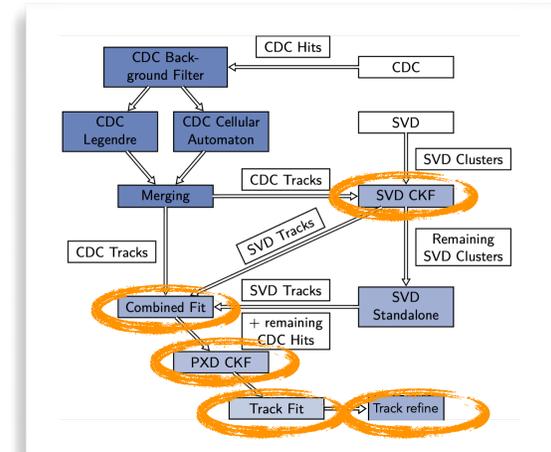


☑ Track finding complete!

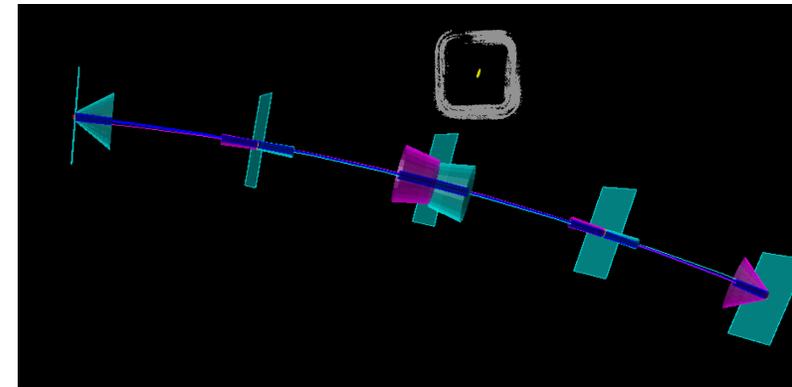
Track Fitting

DAF fitter from GENFIT

- **GENFIT** is an experiment-independent framework for track reconstruction
Widely used in different experiments
- **DAF** (Deterministic Annealing Filter)
 - Iterative Kalman Filter with reweighted observations
 - Designed for track fitting in presence of outlier and background hits
 - Capable of **outlier rejection, L/R ambiguity resolution**
 - Outliers: wrongly assigned hits (background hit found during track finding step)
 - Hits are weighted according to their residual to the smoothed track
 - **At most** 5 iterations per track



- Fit with three different mass hypotheses (π, k, p)

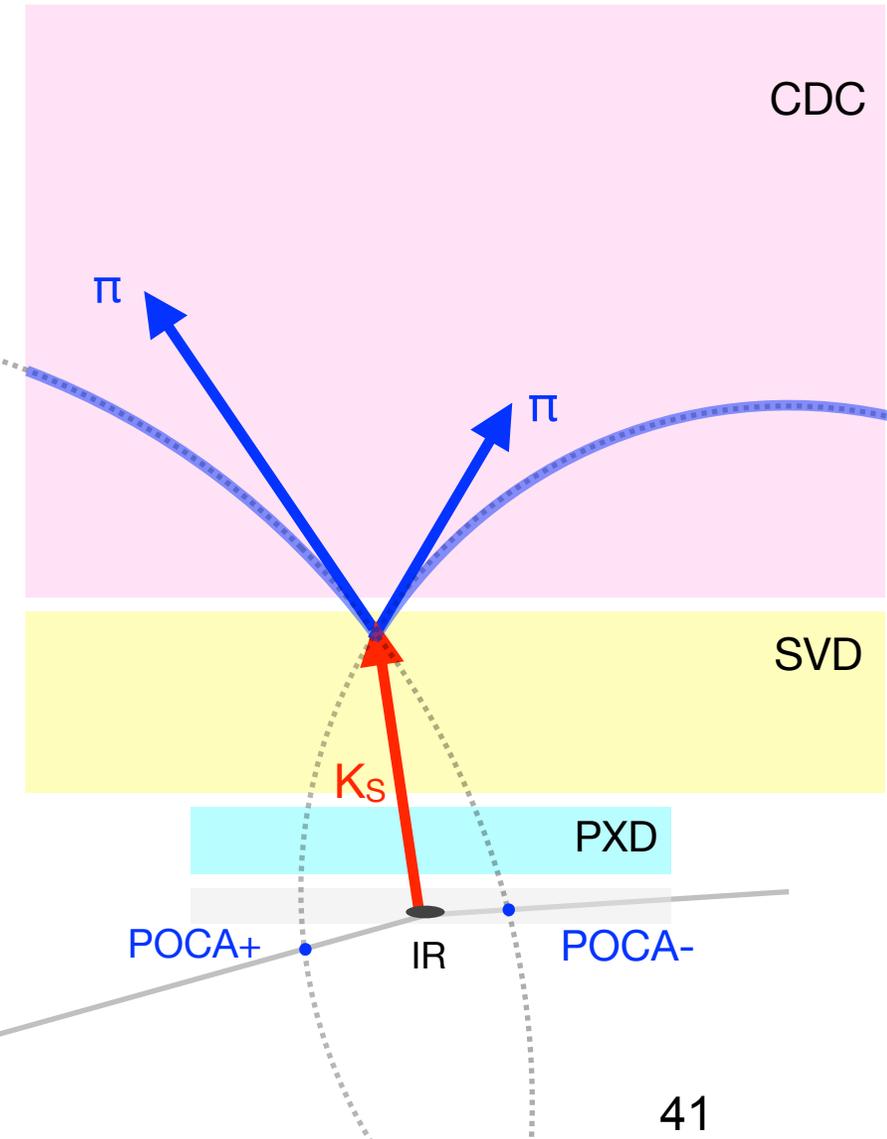


The Tracking procedure in Belle II

V0 Finder

- (Refining step before V0 Finder to correct the wrong charge assignment for low momentum track in transverse plane. @Backup Page50)
 - **V0s** are K_s^0 , Λ^0 and converted Photons ... have **displaced vertexes** located **outside the beam pipe**
 - By default, the track parameters extrapolated to POCA are stored in mDST
 - Includes the correction for material effects and energy loss
 - Not True for the daughters of V0s since they are not produced at the POCA
- > special Treatment needed: **V0Finder**

a K_s decay seen in the transverse plane:

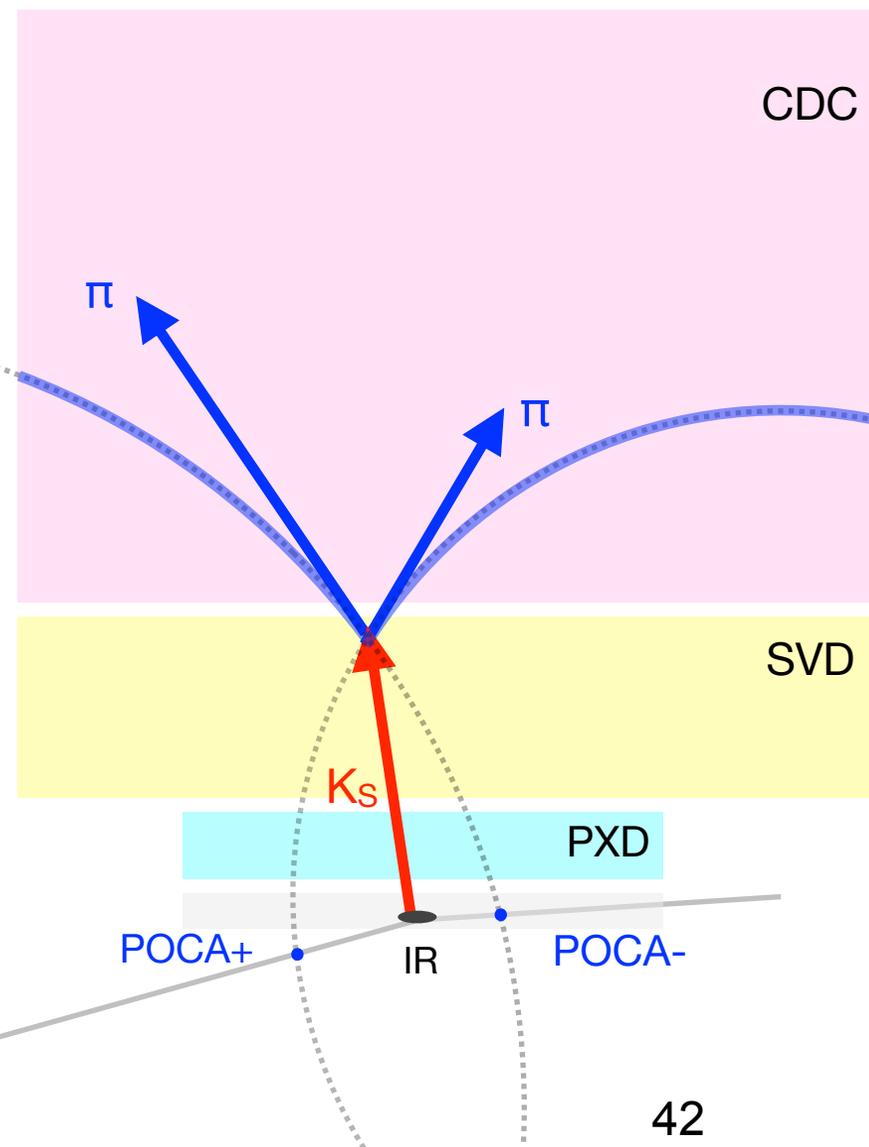


The Tracking procedure in Belle II

V0 Finder

- **V0Finder**: Pairs up all positive and negative tracks and tries to find vertexes between them during tracking (mDST production):
 - Extra information needed:
 - Geometry material, magnetic field map, hits attached to tracks
 - Not available at analysis level
 - ➔ Return V0s outside of the beam pipe
 - Contain the references **to two tracks** and fitResults with the parameters of the helix **at the decay vertex position**
 - `stdV0s.stdKshorts`, `stdV0s.stdLambdas` for analysis
- V0s could be also build in analysis (include the V0s inside the beampipe)
 - Merged list available for analysis

a K_s decay seen in the transverse plane:

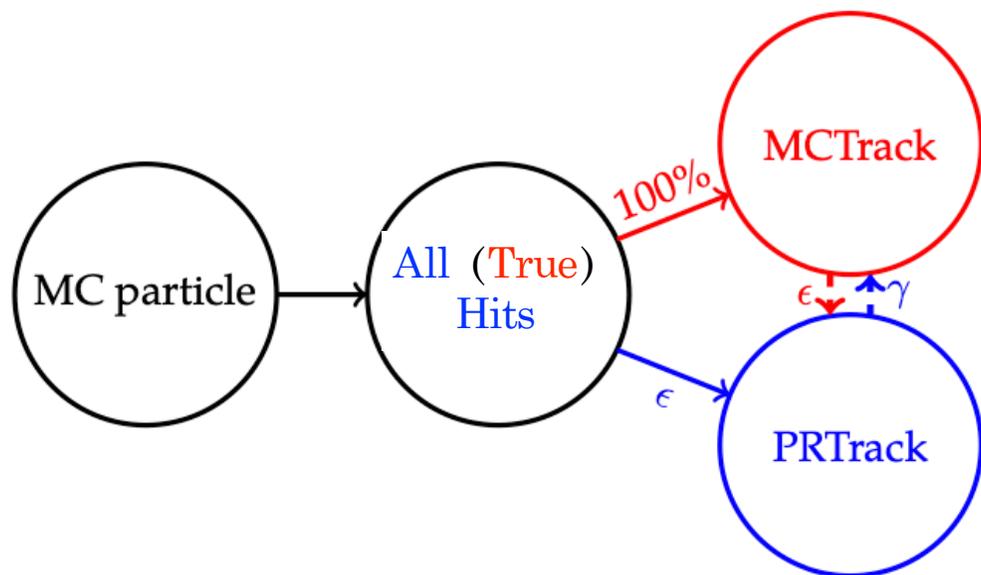


Tracking Performance and Inputs to Analysis

Tracking performance in MC

Tracking Finding efficiency definition:

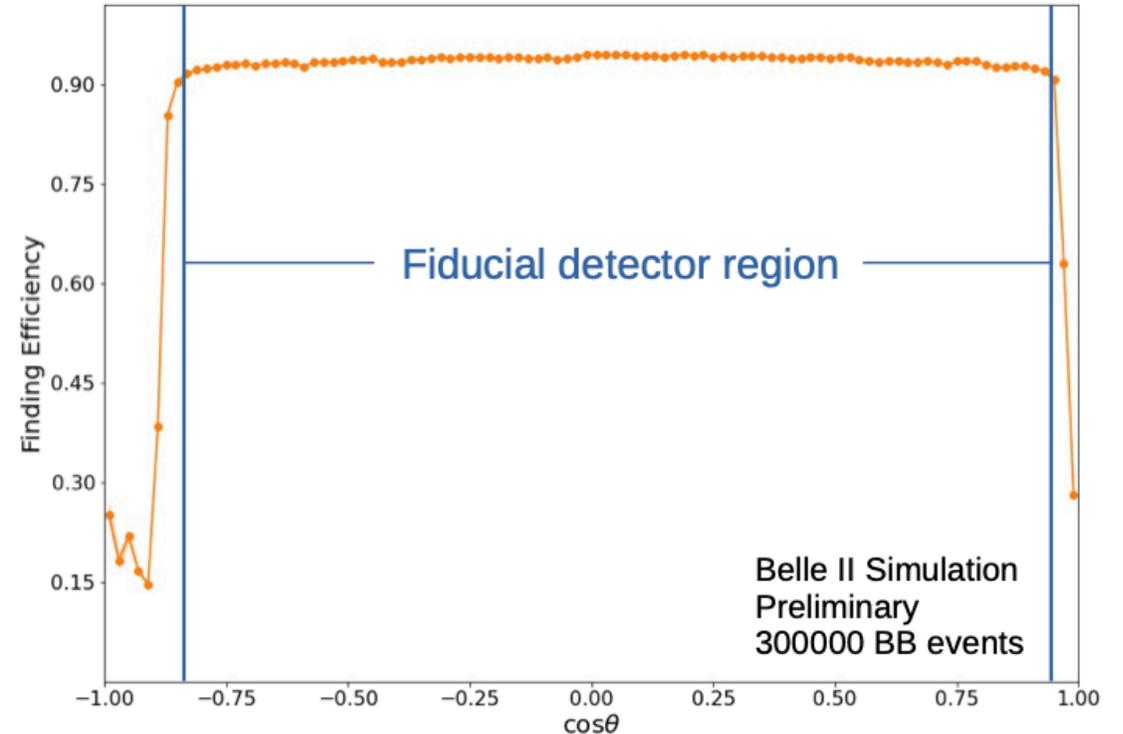
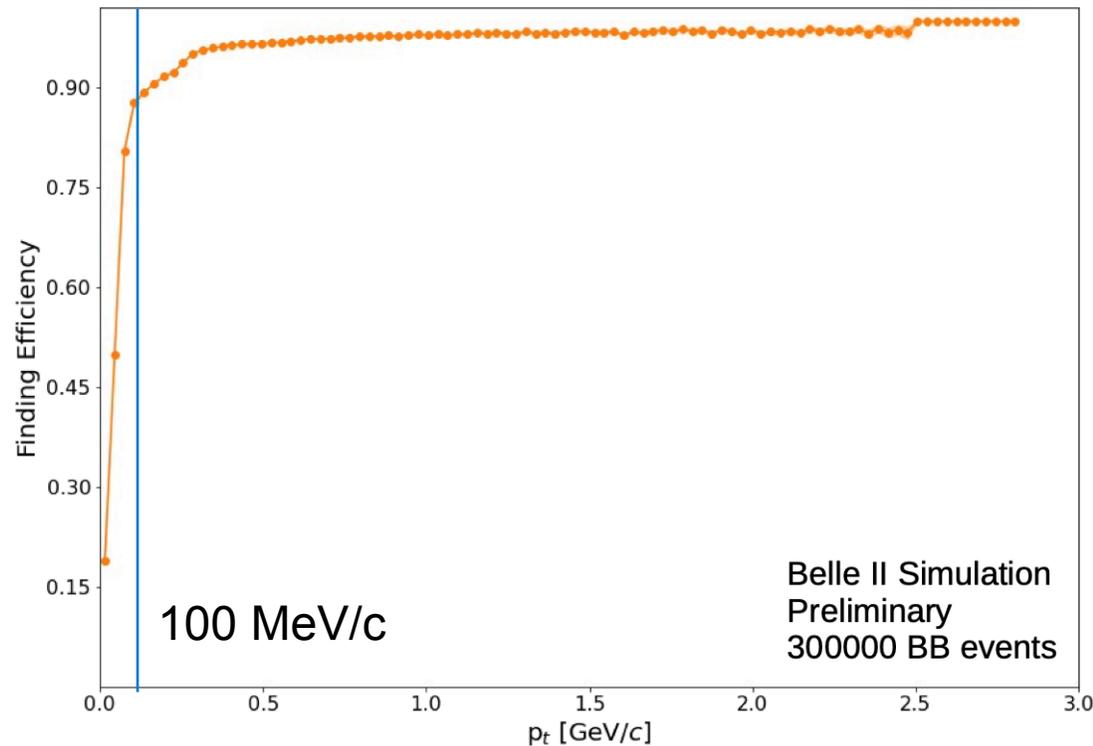
- by comparing **MCTracks with PRTracks**
 - From truth: Generator → Geant4 → True hits → Monte Carlo Track finder → **MCTracks**
(The ideal Tracks only limited by the Detector acceptance, efficiency and resolution)
 - From PR: ... → Track Finding → (pattern recognition) **PRTracks**



- **Hit efficiency (ϵ)**: fraction of the hits in **MCTrack** that are contained in **PRTrack**
- **Hit Purity (γ)**: Fraction of hits in a **PRTrack** contained in the corresponding **MCTrack**.
- **Matched**: hit purity > 66% and hit efficiency exceeds 5%
- **Finding efficiency**: fraction of matched MC Tracks over all MC-tracks
- **Fake rate**: faction of unmatched PR Tracks over all PR Tracks
 - (Also include beam background as the beam background doesn't have MC information)

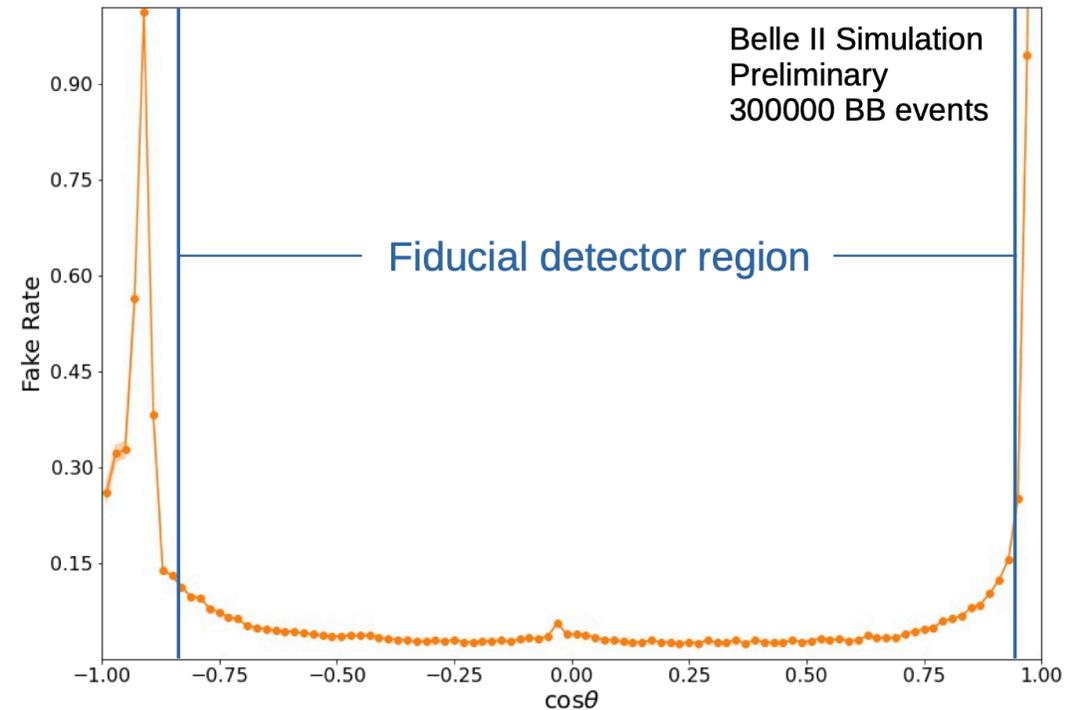
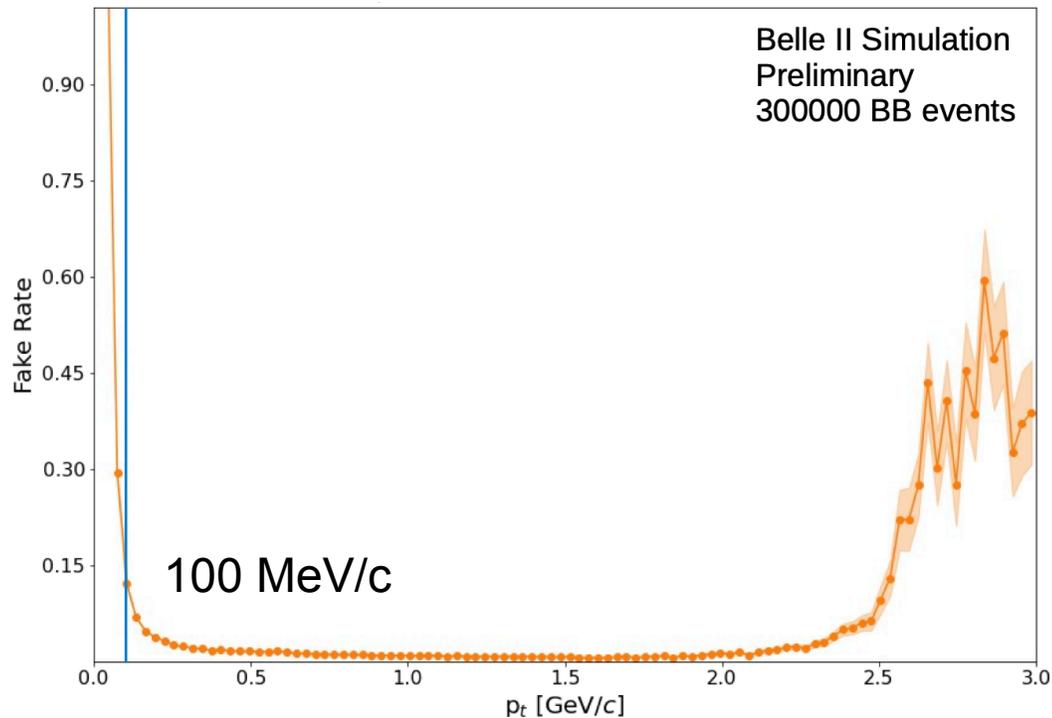
Tracking performance in MC

- **Track finding efficiency** vs p_T and $\cos\theta$ (300 k BB events)
- Above 90% for most of the phase space covered by Belle II detector
 - **93.6% on average** (to be updated with new beam background files)
- Track finding efficiency drops for low p_T ($< 100\text{MeV}/c$) tracks:
 - Due to small number of hits, multiple scattering and higher level of background



Tracking performance in MC

- **Fake rate** vs p_T and $\cos \theta$ (300 k BB events)
 - Fake rate peaking at forward and backward direction -> probably real fakes from luminosity background
 - Low p_T -> probably combinatorial effects + two photon process
 - High p_T -> random combinations of hits and low number of tracks from Bhabha events



The information passing to the analysis

Direct inputs to analysis (objects in mdst):

- Reconstructed data saved in well-defined format (mini data-summary-table / **mdst**)
- **mdst -> analysis**

MDST_OBJECTS = (

```
'ECLClusters',
'ECLClustersToTracksNamedBremsstrahlung',
'EventLevelClusteringInfo',
'EventLevelTrackingInfo',
'EventLevelTriggerTimeInfo',
'KLMClusters',
'KlIds',
'PIDLikelihoods',
'SoftwareTriggerResult',
'TrackFitResults',
'Tracks',
'TRGSummary',
'V0s' )
```

Relations between objects

Belle2::Particle
Belle2::ParticleList

Track + PIDLikelihood

-> **Charged Particles** (k, π, e, μ, p)

ECL/KLM cluster

-> **Neutral Particles**

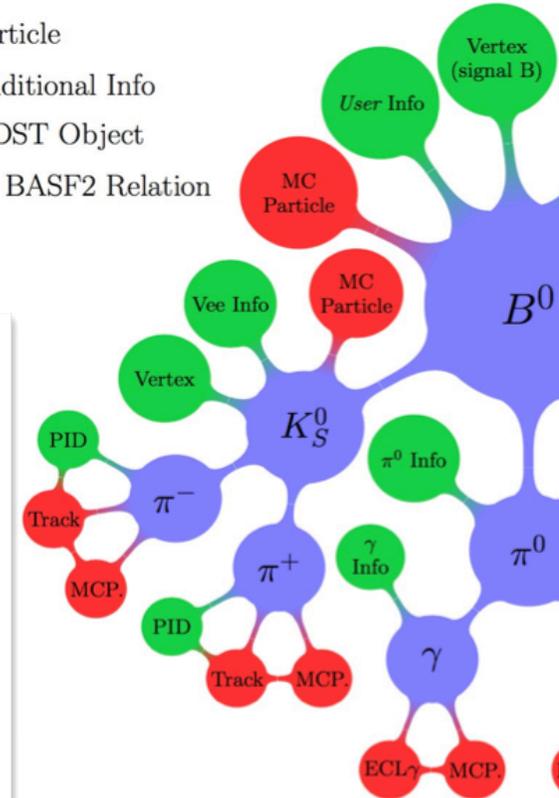
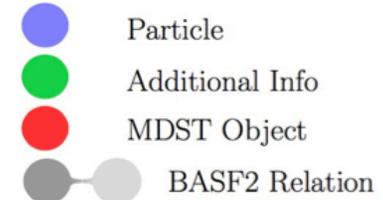
V0s

-> K_S^0, Λ^0 or converted photons ...

(stdV0s.stdKshorts, stdV0s.stdLambdas)

...

addExtraInfo() for collecting additional information



Later on: Particle combinations, MC Truth matching, particle vertexing...

General recommendations

Cuts recommendation and tools/suggestions for systematic uncertainty

▶ Track cleanup:

For B/D/Tau/... decays (short lived mesons)

- $|dz| < 3 \text{ cm}$, $dr < 0.5 \text{ cm}$ w.r.t. beamspot: select the tracks from IP (cut off those from beam-material interaction)

For displaced vertex, Ks/Lambda/... don't use it

▶ Reliable PID

$$N_{CDCHits} > 20$$

(Based on the PID performance study,
will change for rel-08)

- Always check your samples before applying the cuts

• Tools for Systematic uncertainty determination

TrackingEfficiency: randomly remove tracks from the provided particle list to estimate the uncertainty due to tracking efficiency

TrackingMomentum: scale/smear momenta of the particles according to a scaling factor

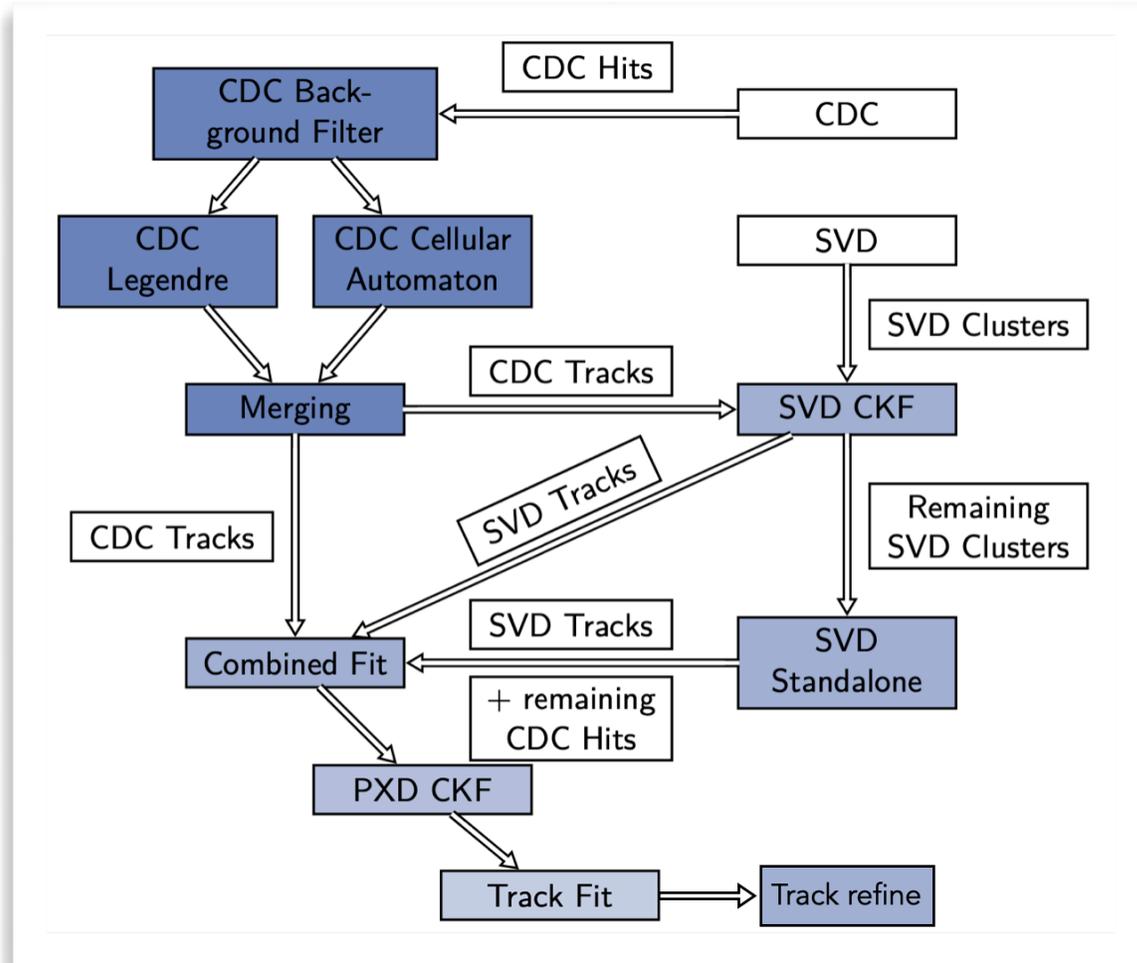
✦ Corrections & Systematic recommendation:

proc13+prompt VS MC15rd

Topic	Status	Deliverable
data-MC Slow pion tracking efficiency	finished, under review	Value + Uncertainty
data-MC Mid to high momentum tracking efficiency	ongoing	Uncertainty
data-MC Ks efficiency	ongoing	Value + Uncertainty
data global momentum scale	ready	the correction must be
data cos-theta dependent momentum scale & other effects including energy loss	ongoing	

[Tracking performance page](#)

Summary



Thanks!

More details in [Track finding at Belle II](#)

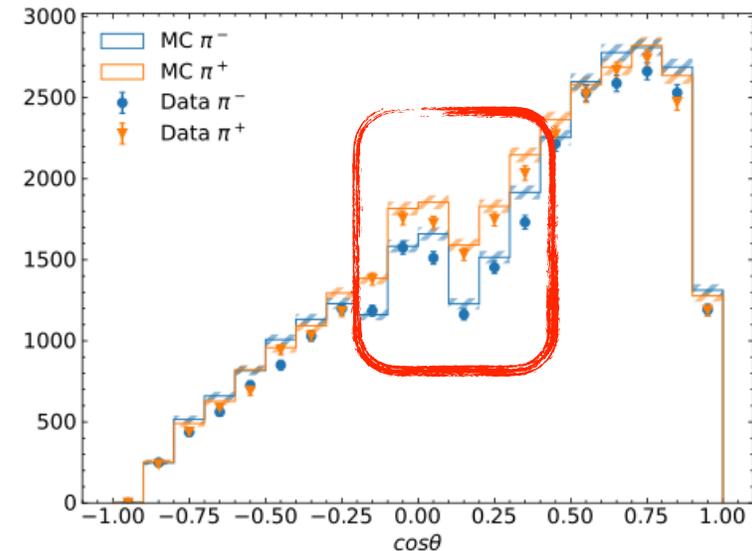
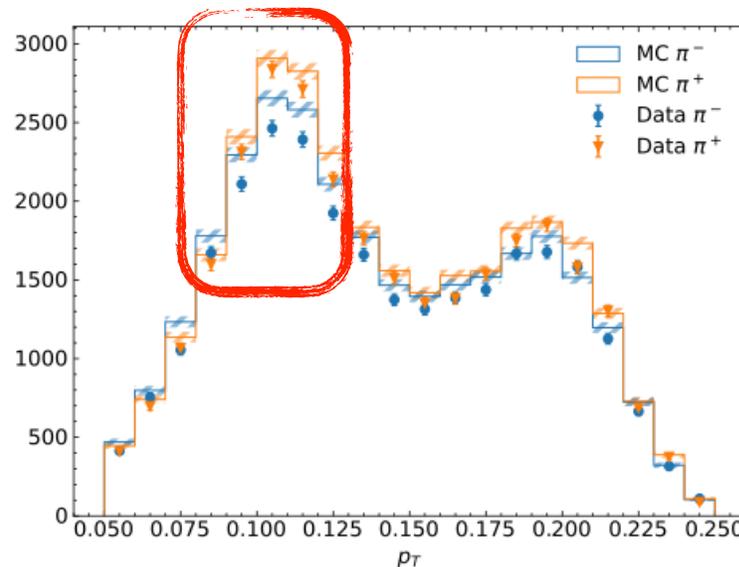
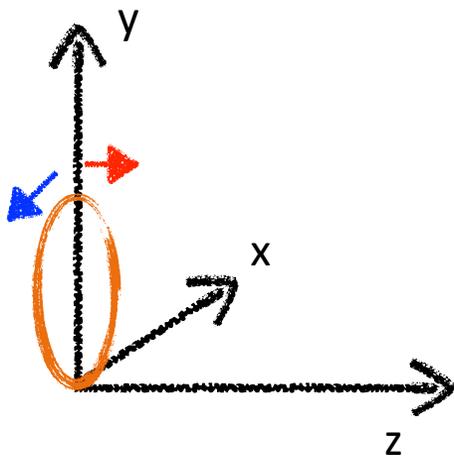
Backup

The Tracking procedure in Belle II

Refining: flip & refit

☑ Tracking is completed at this point

- ▶ Significant **Charge asymmetry** observed for **low momentum tracks in the transverse plane**
 - Which was further found to be related with the **mis-assignment** of direction for low Pt tracks in transverse plane



Charge asymmetry for π_S in slow pion efficiency study

The Tracking procedure in Belle II

Refining: flip & refit

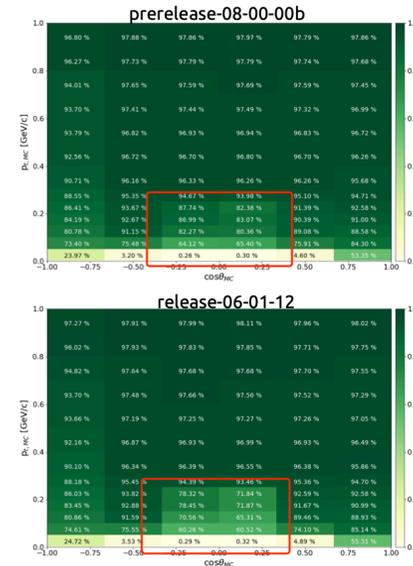
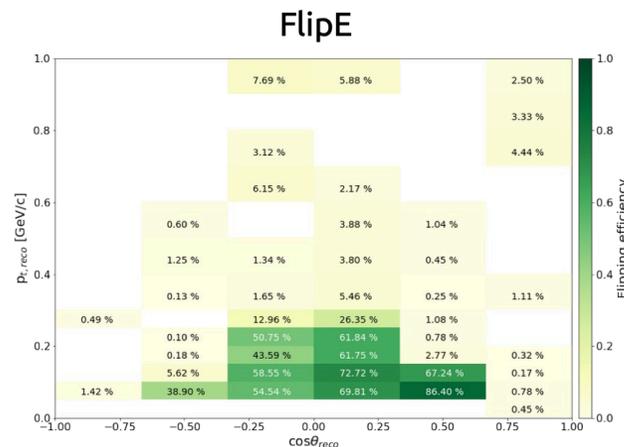
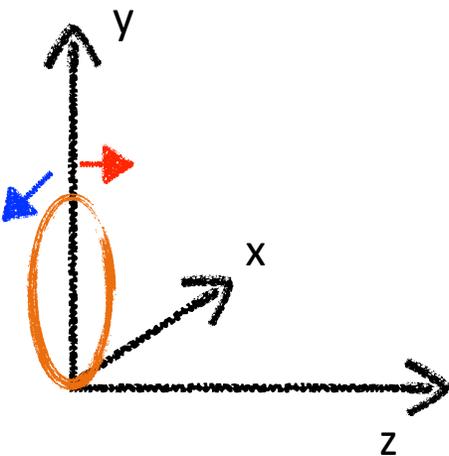
☑ Tracking is completed at this point

► Significant **Charge asymmetry** observed for **low momentum tracks in the transverse plane**

- Which was further found to be related with the **mis-assignment** of direction for low Pt tracks in transverse plane

☑ A refining step was added to fix the mis-assignment as much as possible:

- Two MVA involved
- Low level information + fit Results of same track with different direction
- ➔ Correct **~50% of the charge mis-assignment with high efficiency(99%) and less than 1%** (of the total tracks) refitted
- ➔ Increase the charge finding efficiency and partially cure the Charge asymmetry

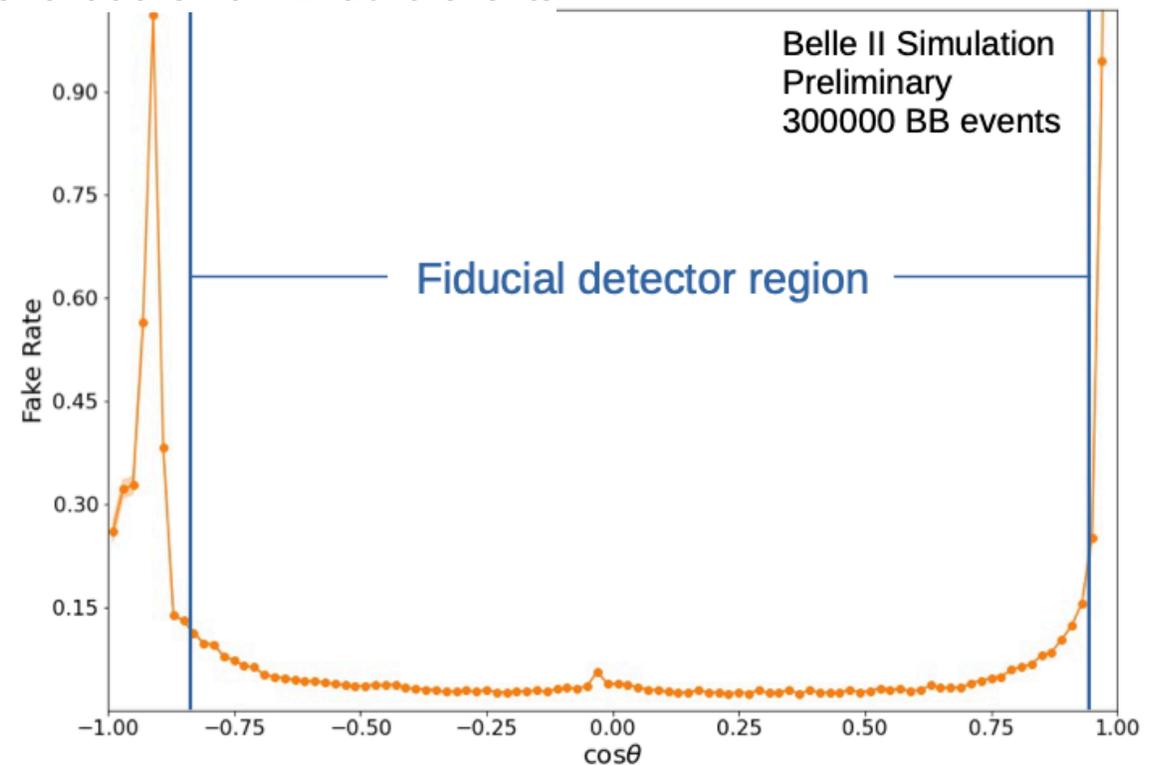
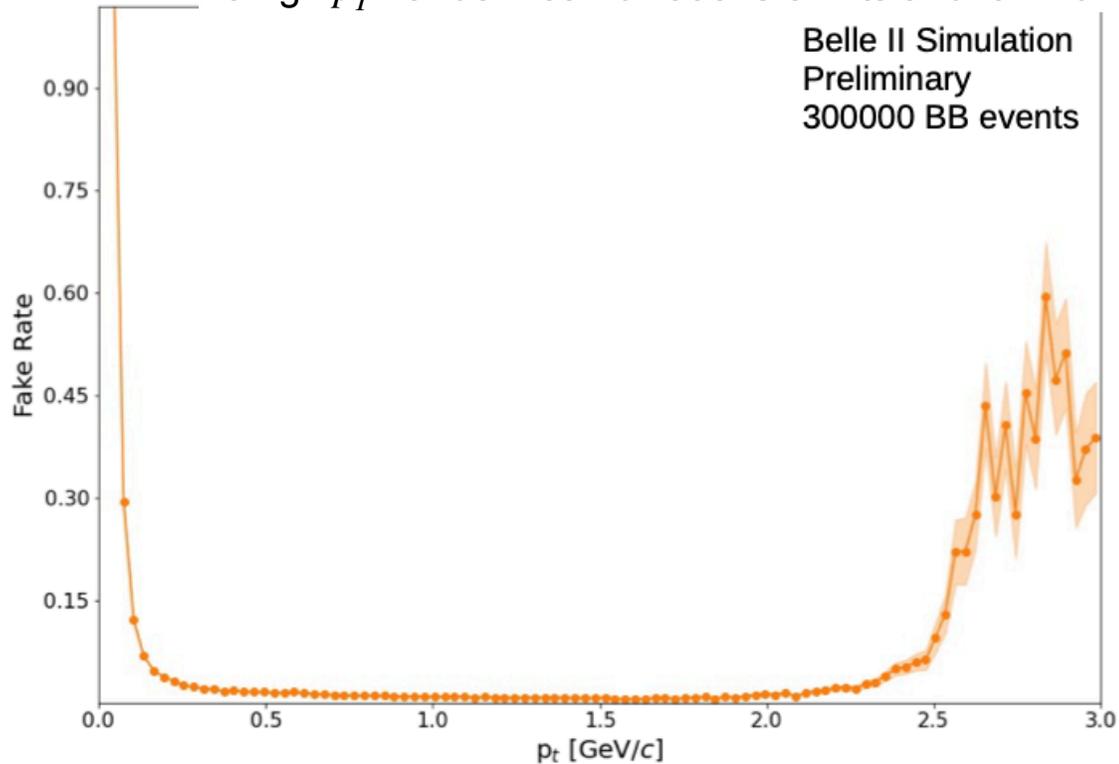


MC tracking validation:

	CA [%]
mixed	-0.142 ± 0.037
	0.324 ± 0.037
charged	-0.027 ± 0.038
	0.343 ± 0.04
uubar	-0.026 ± 0.048
	0.225 ± 0.048
ddbar	-0.057 ± 0.048
	0.176 ± 0.048
ssbar	-0.056 ± 0.051
	0.187 ± 0.051
ccbar	-0.041 ± 0.044
	0.244 ± 0.044

Tracking performance

- Backup
- Fake rate vs p_T and $\cos \theta$
 - Increased fake rate for low and high p_T , $\cos \theta$ regions
 - More small angle $e^+e^- \rightarrow e^+e^-(\gamma)$ scattering and $e^+e^- \rightarrow e^+e^-(f\bar{f})$ (two photon process) with low p_T and in very forward and backward directions -> actual tracks from physic backgrounds
 - At high p_T : random combinations of hits and low number of tracks from Bhabha events



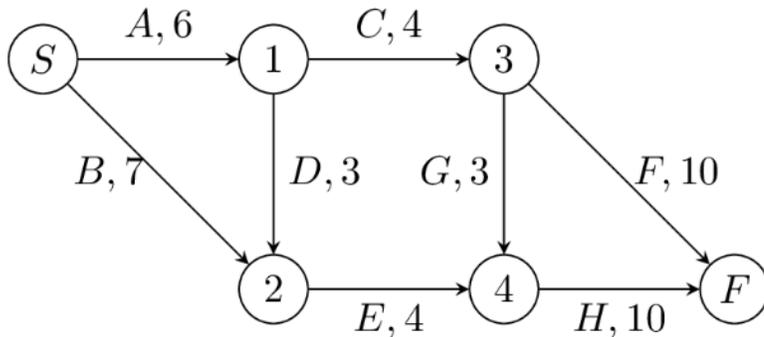
The Tracking procedure in Belle II

CDC local tracking

*Usage In our case:

- ▶ Helps to solve the **longest path problem** on directed acyclic graphs
 - acyclic graphs: no closed loop formed
- ▶ longest path problem: A typical problem in graph theory
 - Complexity: at most $O(v!)$, $O(v+e)$ with topological sorting

Path: a sequences of edges and they are distinct with each other
 v : the number of vertex
 e : number of edge



Longest path: A-C-G-H or A-D-E-H

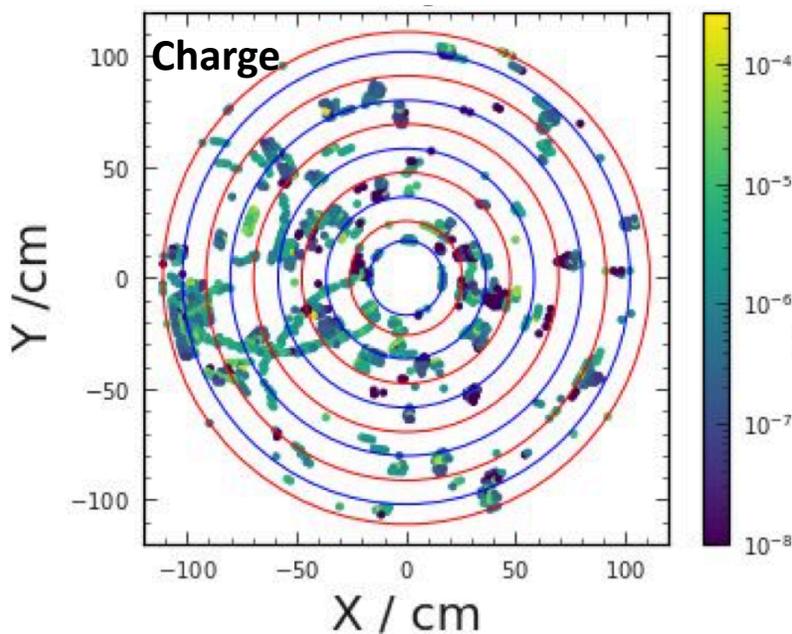
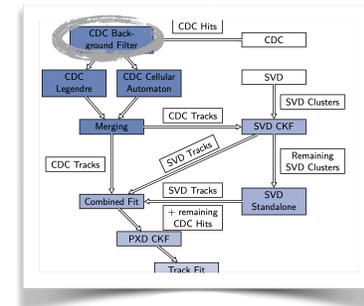
[Graph-Theory-Notes/s_graphalgorithms_longest-paths](#)

- ▶ Better performance than other methods ($O(v)$)

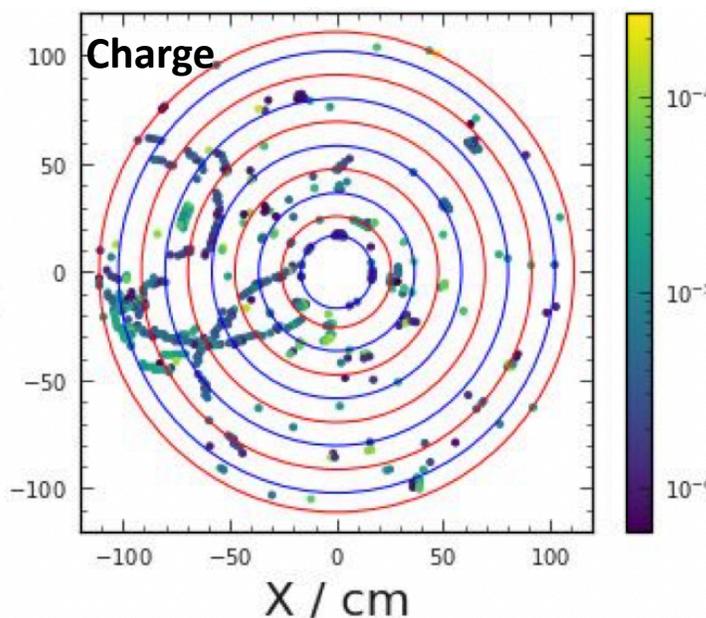
The Tracking procedure in Belle II

CDC background filter

- CDC Hits component: 500 vs 3000 (signal hits vs background hits @full lumi)
- Main background hits: **beam background + cross talk**
- Filters: **ADC counts + ASIC cross talk filters**
 - ASIC cross talk: ASICs sharing same connector with the one which has signal hits fired with low ADC
 - Can be suppressed by the Time&ADC information/relations
- ➔ Background hits and hits from cross-talk are reduced



All Hits



Hits after **ADC+ASIC cuts**

	ADC > 18	ADC > 18 and ASIC filter
Signal	95.4%	93.2%
Background	54.1%	31.1%

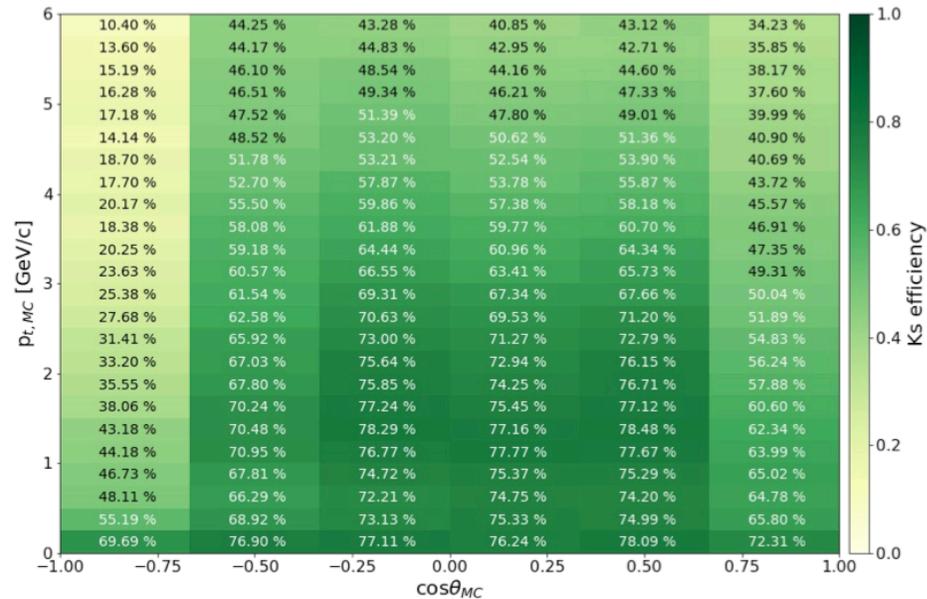
V0s

Dedicated MC samples to evaluate the V0 finding

- K_S : Generic MCri, $D \rightarrow K_S \pi \pi$, $K^0 K^0 Y_{ISR}$
- Λ_0 : Generic MCri, $\Lambda_c \rightarrow \Lambda^0 \pi$

Eg. Rel8 validation

K_S efficiency in $D \rightarrow K_S \pi \pi$

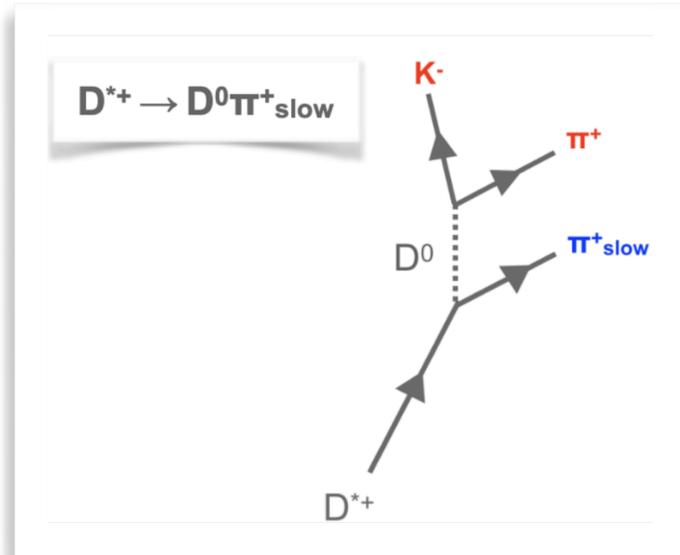


K_S fake rate in $D \rightarrow K_S \pi \pi$

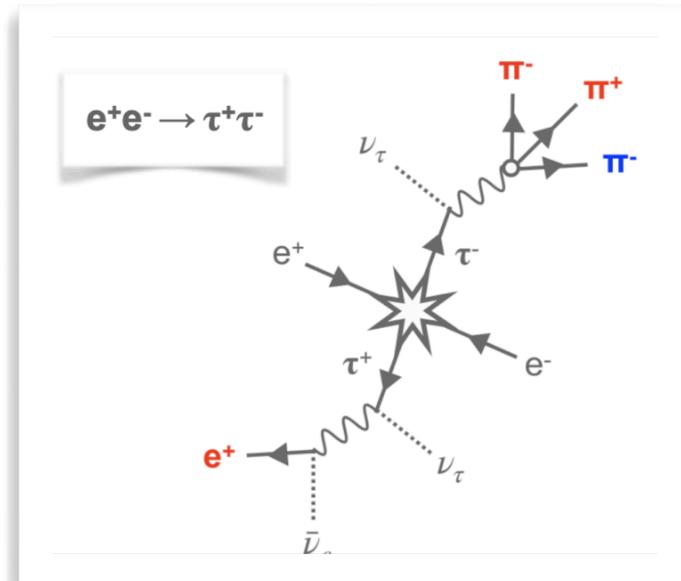


Validation with data

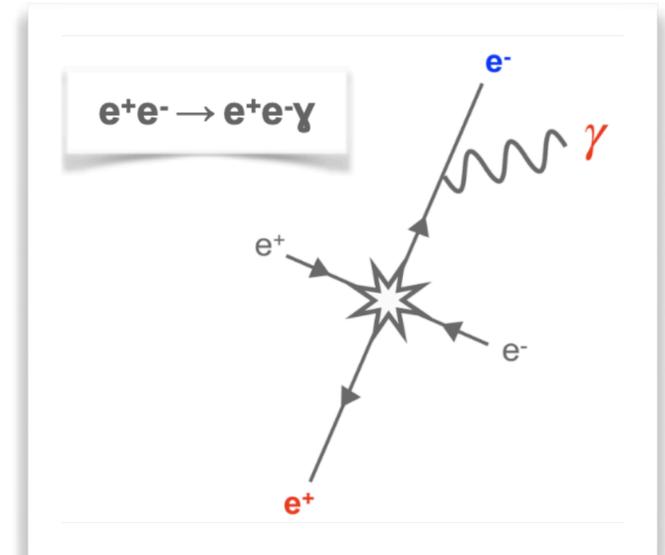
Tag-and-probe studies focusing on a variety of final states to cover a larger track momentum spectrum:



$p_T \lesssim 200 \text{ MeV}$



$200 \text{ MeV} \lesssim p_T \lesssim 2 \text{ GeV}$



$p_T \gtrsim 2 \text{ GeV}$

Validation example backup

Tag-and-probe studies focusing on a variety of final states to cover a larger track momentum spectrum:

$e^+e^- \rightarrow \tau^+\tau^-$ with one τ decays to a single charged particle while the other decays into 3 charged particles

Tagging:

3 good quality Tracks

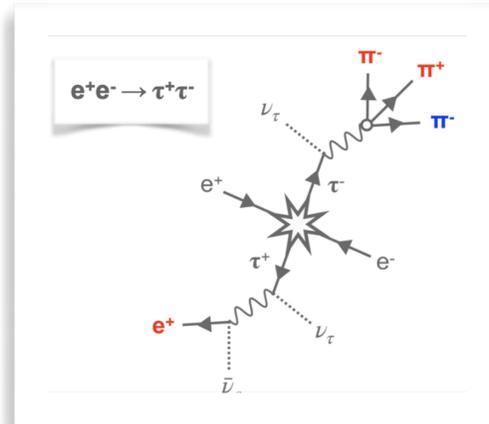
Total charge of ± 1

Probing:

The fourth Track

Tracking finding efficiency ϵ

$$\epsilon \cdot A = \frac{N_4}{N_3 + N_4}$$



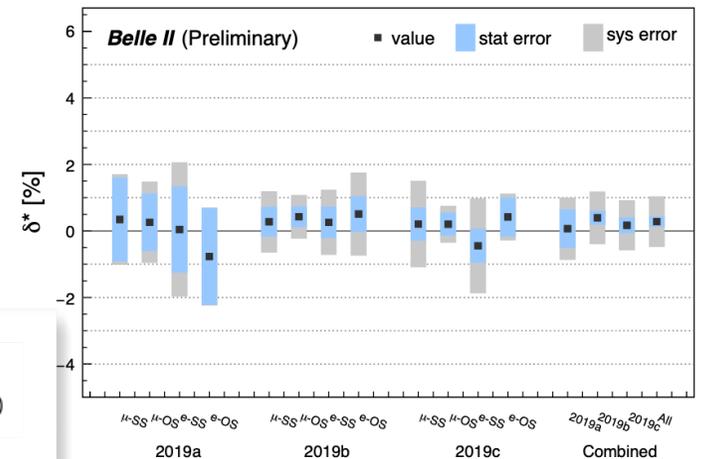
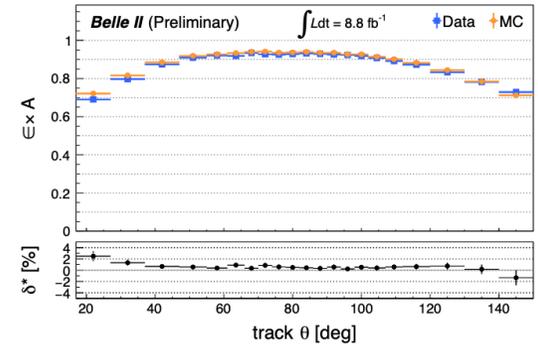
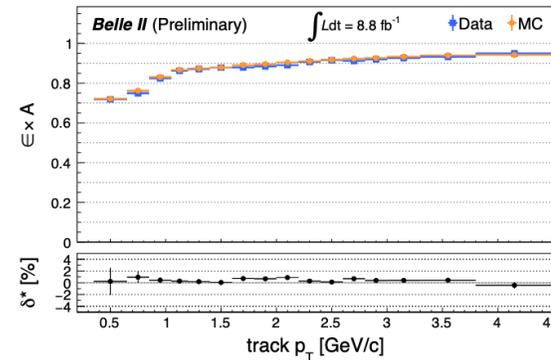
$200 \text{ MeV} \lesssim p_T \lesssim 2 \text{ GeV}$

A: detector acceptance

N_4 or N_3 : Number of events where 4 or 3 tracks were found

Data-MC discrepancy :
(2019 data to MCri)

$$\delta_{\text{efficiency}}^* = 0.28 \pm 0.15 \text{ (stat)} \pm 0.73 \text{ (sys)} \%$$



Ks systematic

Provide K_S^0 efficiency correction factor and uncertainty

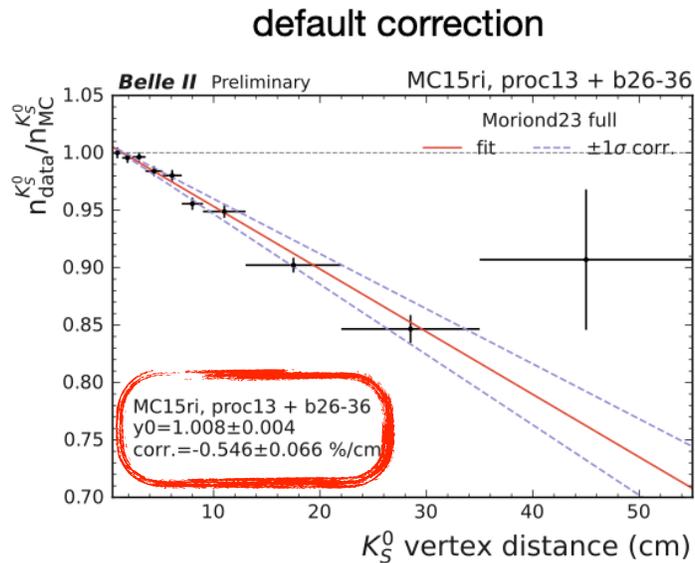
- K-short studied in

- $D^{*+} \rightarrow [\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-] \pi^+$ (S. Dreyer)
- $D^* \rightarrow [D(K_S^0 \pi^0) \pi^+]$ (J. Skorupa)
- $\tau^- \rightarrow \pi^- K_S^0 \nu (>= 0 \pi^0)$ (P. Leo)

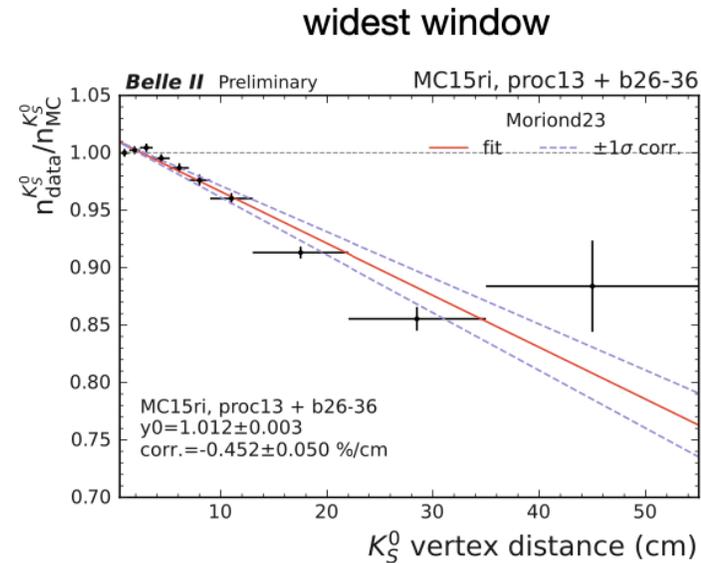
- Eg: $D^{*+} \rightarrow [\bar{D}^0 \rightarrow K_S^0 \pi^+ \pi^-] \pi^+$

<https://docs.belle2.org/record/2174?ln=en>

- Extract Ks candidates and perform a linear fit to the data/mc ratio to extract correction factor as *slope x <distance>*



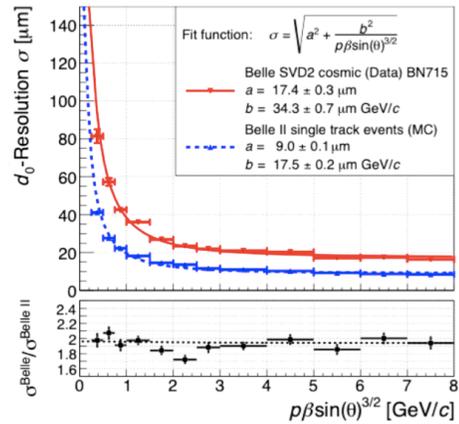
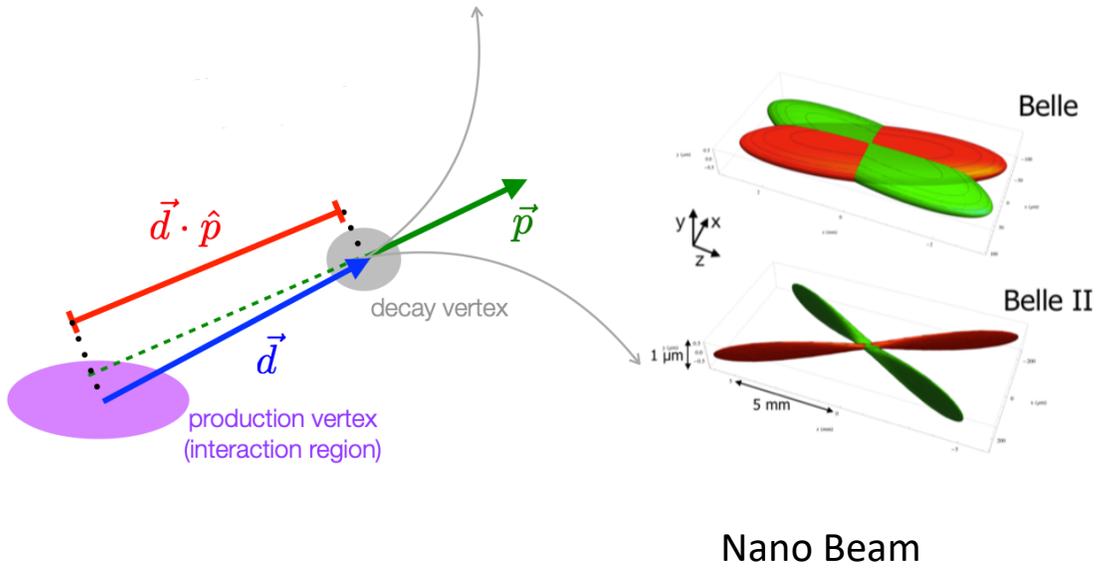
(0.48, 0.516) MeV/c²



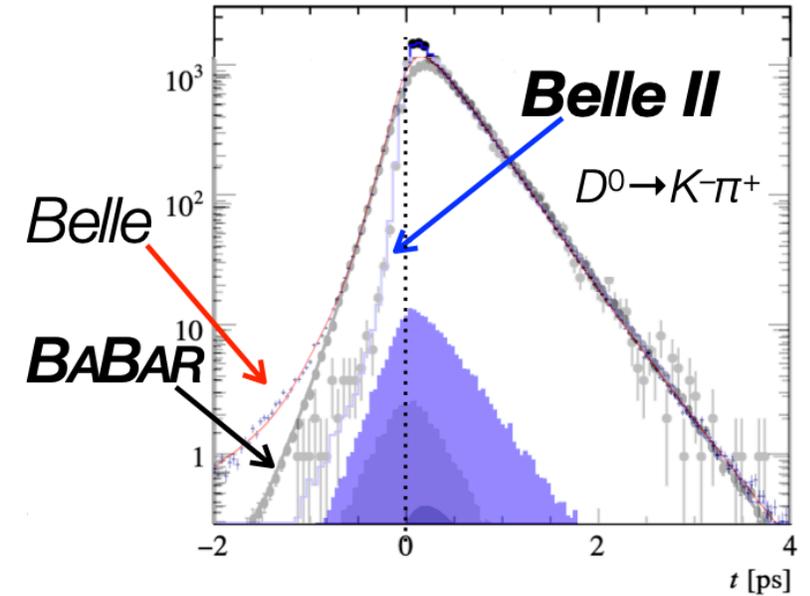
(0.45, 0.55) MeV/c²

Charm lifetimes

- D^0 lifetime: Fit to proper time distribution of production (D^{*+} -tagged) and decay vertex (D^0 candidates from $D^{*+} \rightarrow D^0 (\rightarrow K^+ \pi^-) \pi_s^+$)
- **Belle II vs Belle:** Benefit from 'Nano beam' + better vertexing (with PXD)
- **World-best D^0 , D^+ , D_s^+ and Λ_c^+ lifetimes**



d_0 resolution (Belle vs Belle II)



Talk from eps

Conformal transformation

$$\begin{aligned}r^2 &= (x - r \cdot \cos \varphi)^2 + (y - r \cdot \sin \varphi)^2 \\0 &= x^2 + y^2 - 2r \cdot (x \cdot \cos \varphi + y \cdot \sin \varphi) \\ \frac{1}{r} &= \frac{2x}{x^2 + y^2} \cdot \cos \varphi + \frac{2y}{x^2 + y^2} \cdot \sin \varphi.\end{aligned}$$

$$x' = \frac{2x}{x^2 + y^2}, \quad y' = \frac{2y}{x^2 + y^2}.$$

$$\rho = x \cdot \cos \theta + y \cdot \sin \theta$$

In conformal transformation, as shown in Fig. 2, a hit position (x, y) in xy plane ($r\phi$ plane) is transformed into a position (X, Y) in the conformal plane as,

$$X = \frac{2x}{x^2 + y^2}, \quad Y = \frac{2y}{x^2 + y^2}. \quad (1)$$

Through the conformal transformation, a circle which passes through the origin $(0,0)$ is transformed into a line,

$$x_c X + y_c Y = 1, \quad (2)$$

where, x_c, y_c are the center of the circle. The inverse of the distance between the line and the origin in the conformal plane corresponds to the radius of the circles. Meanwhile, a circle which does not pass through the origin in the xy plane is transformed into a new circle. The new circle, still does not pass through the origin in the conformal plane,

$$\begin{aligned}\left(X - \frac{2x_c}{x_c^2 + y_c^2 - R^2}\right)^2 + \left(Y - \frac{2y_c}{x_c^2 + y_c^2 - R^2}\right)^2 = \\ \left(\frac{2R}{x_c^2 + y_c^2 - R^2}\right)^2,\end{aligned} \quad (3)$$

where, x_c, y_c and R are the center and the radius of the circle in the natural plane, respectively.

Conformal transformation

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$$\left(X - \frac{2x_c}{x_c^2 + y_c^2 - R^2} \right)^2 + \left(Y - \frac{2y_c}{x_c^2 + y_c^2 - R^2} \right)^2 = \left(\frac{2R}{x_c^2 + y_c^2 - R^2} \right)^2, \quad (3)$$

where, x_c, y_c and R are the center and the radius of the circle in the natural plane, respectively.

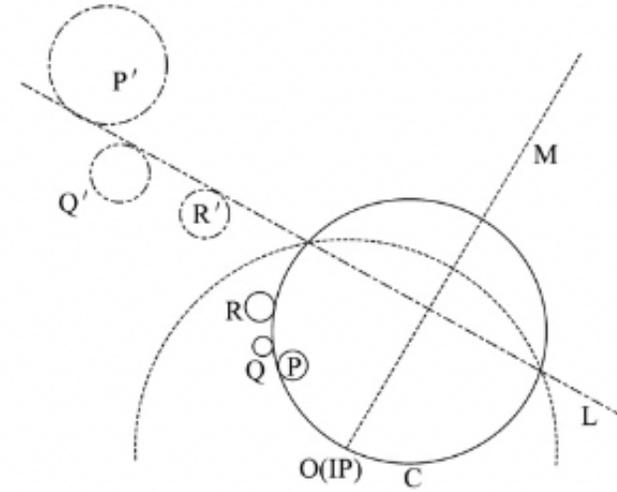
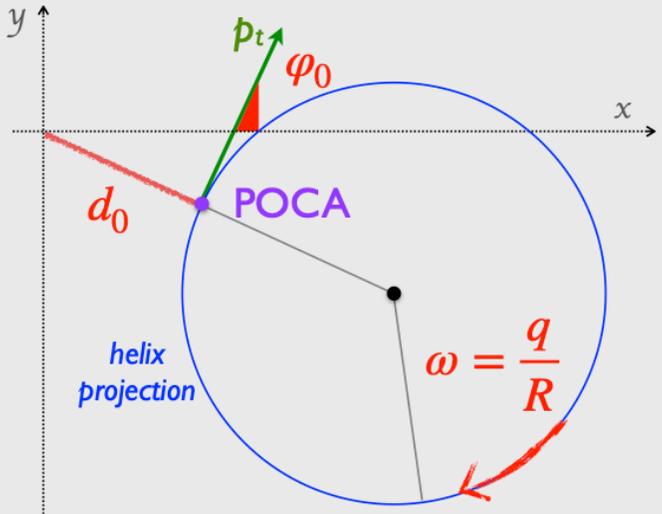


Fig. 2. After conformal transformation, the circle C which passes O(IP) is transferred into line L; the circles P, Q and R which do not pass O are transferred into new circles P', Q' and R'.

Track Parameterisation

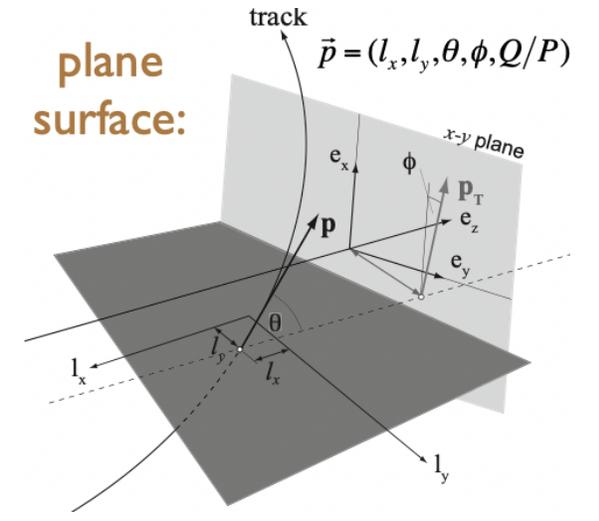
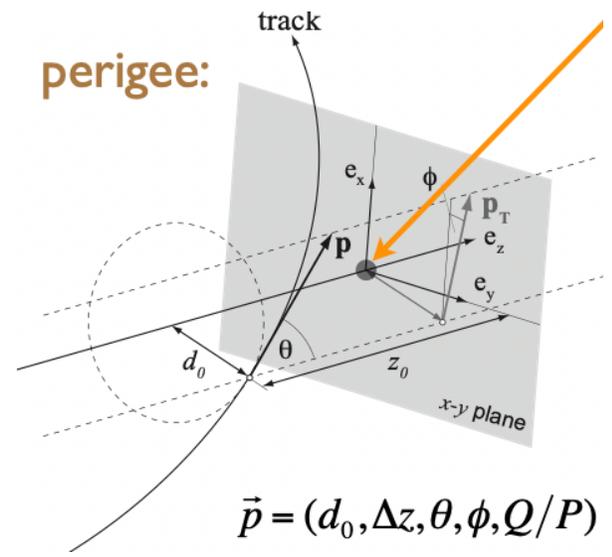
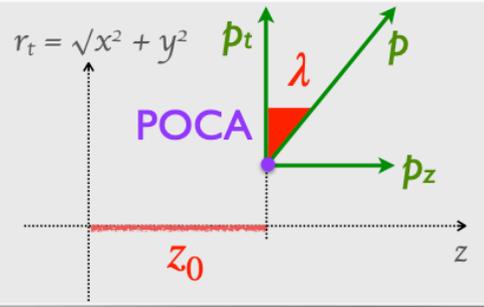
TRANSVERSE PLANE



- POCA = Point Of Closest Approach to the origin
- d_0 is the 2d signed distance of the POCA from the z axis, the sign depends on the angular momentum of the track (>0 in the fig.)
- φ_0 is the angle between p_t and the x axis at the POCA, $\varphi_0 \in [-\pi, \pi]$
- the sign of ω , the curvature, is the same as the charge of the track (>0 in the fig.)

LONGITUDINAL VIEW

- $\tan\lambda$ is the ratio of p_z and p_t , $\lambda \in [-\pi, \pi]$
- z_0 is the signed distance of the POCA from the transverse plane



Kalman filter-based Track fit

- A kalman filter is a progressive way of performing a least square fit
 - How it works:
 - Estimate starting parameter $P_{0|0}$
 - Iterate over all hits $1 \dots K$
 1. Take $P(k-1|k-1)$ at point $k-1$
 2. Propagate to point k to get predicted parameter $p(k|k-1)$
 3. Update predicted parameters with measurement m_k to get $p(k|k)$
 4. Repeat until reach the last hite
- Material effects are included as the error

1. propagate p_{k-1} and its covariance C_{k-1} :

$$\mathbf{q}_{k|k-1} = \mathbf{f}_{k|k-1}(\mathbf{q}_{k-1|k-1})$$

$$\mathbf{C}_{k|k-1} = \mathbf{F}_{k|k-1} \mathbf{C}_{k-1|k-1} \mathbf{F}_{k|k-1}^T + \mathbf{Q}_k$$

with $\mathbf{Q}_k \sim$ noise term (M.S.)

2. update prediction to get $\mathbf{q}_{k|k}$ and $\mathbf{C}_{k|k}$:

$$\mathbf{q}_{k|k} = \mathbf{q}_{k|k-1} + \mathbf{K}_k [\mathbf{m}_k - \mathbf{h}_k(\mathbf{q}_{k|k-1})]$$

$$\mathbf{C}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{C}_{k|k-1}$$

with $\mathbf{K}_k \sim$ gain matrix :

$$\mathbf{K}_k = \mathbf{C}_{k|k-1} \mathbf{H}_k^T (\mathbf{G}_k + \mathbf{H}_k \mathbf{C}_{k|k-1} \mathbf{H}_k^T)^{-1}$$

Smoother:

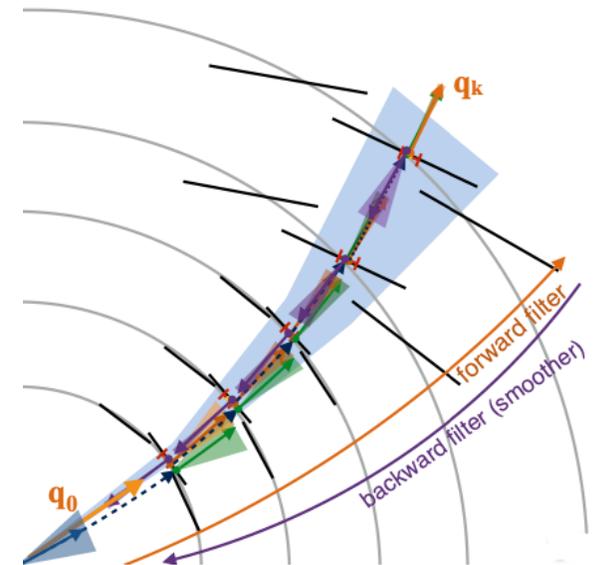
proceeds from layer $k+1$ to layer k :

$$\mathbf{q}_{k|n} = \mathbf{q}_{k|k} + \mathbf{A}_k (\mathbf{q}_{k+1|n} - \mathbf{q}_{k+1|k})$$

$$\mathbf{C}_{k|n} = \mathbf{C}_{k|k} - \mathbf{A}_k (\mathbf{C}_{k+1|k} - \mathbf{C}_{k+1|n}) \mathbf{A}_k^T$$

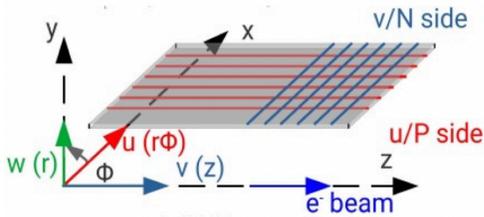
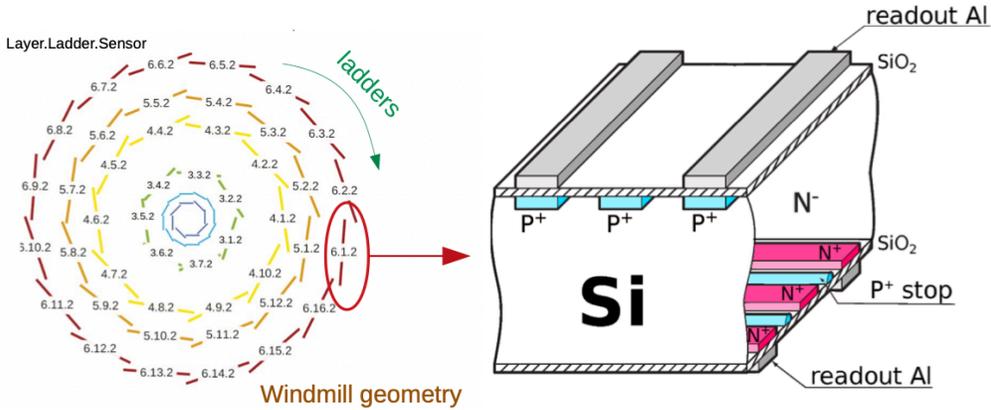
with $\mathbf{A}_k \sim$ smoother gain matrix :

$$\mathbf{A}_k = \mathbf{C}_{k|k} \mathbf{F}_{k+1|k}^T (\mathbf{C}_{k+1|k})^{-1}$$



The individual tracking sub-detectors

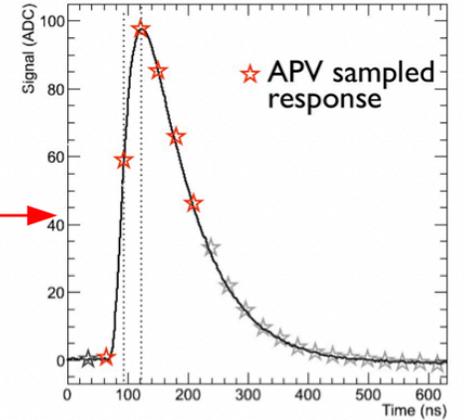
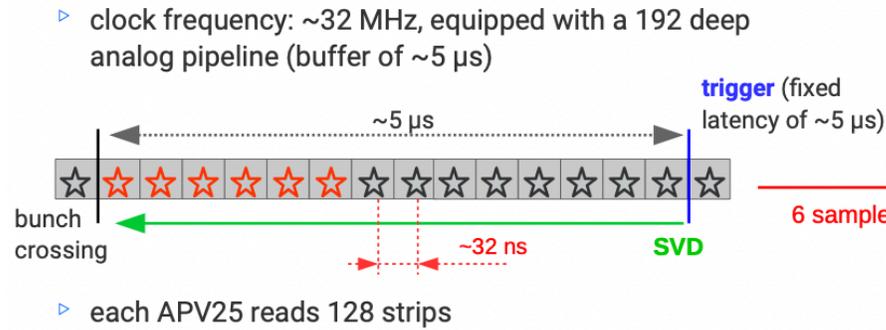
SVD: Double-sided Strip Detector (DSSD)



3D spacePoint + time info:

Resolution on U: 7~12 μm
 V: 15~25 μm
 Time resolution : 3 – 4 ns

Physics events are triggered at 30kHz
 the frequency of bunch crossing is 256 MHz



$$t_{hit} = \frac{\sum_n t_n \cdot A_n}{\sum_n A_n}$$

n : sample
 A_n : amplitude of the sample
 t_n : time of the sample

Tracking performance: definitions

Hit efficiency: how efficient the pattern recognition is in identifying all the hits belonging to a single particle

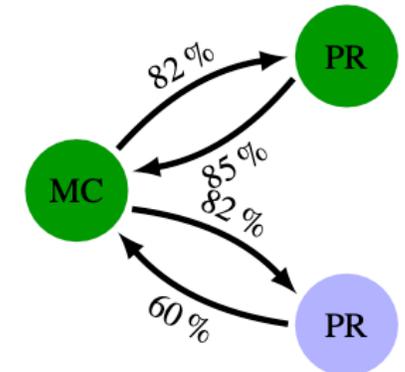
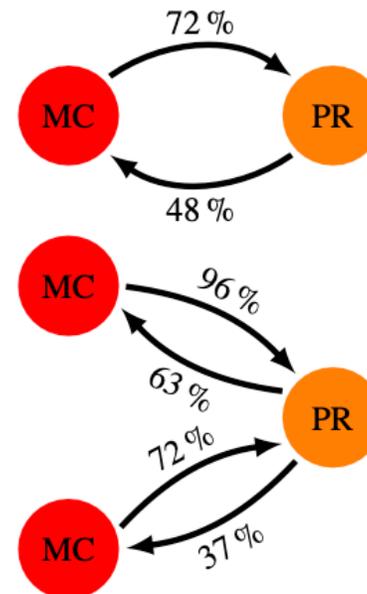
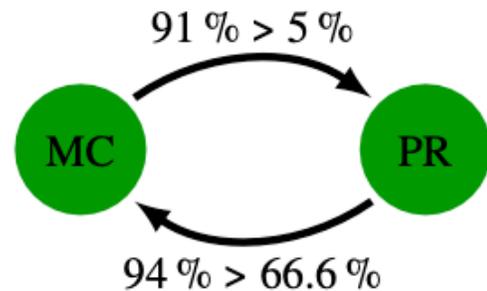
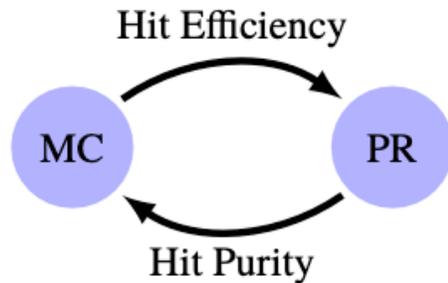
Check whether the hits in **MCTrack** are contained in **PRTrack**

Hit Purity: how precise the Pattern recognition is in identifying the hits belonging to one MCTracks

Fraction of hits in a PRtracks contained in the corresponding MCtrack.

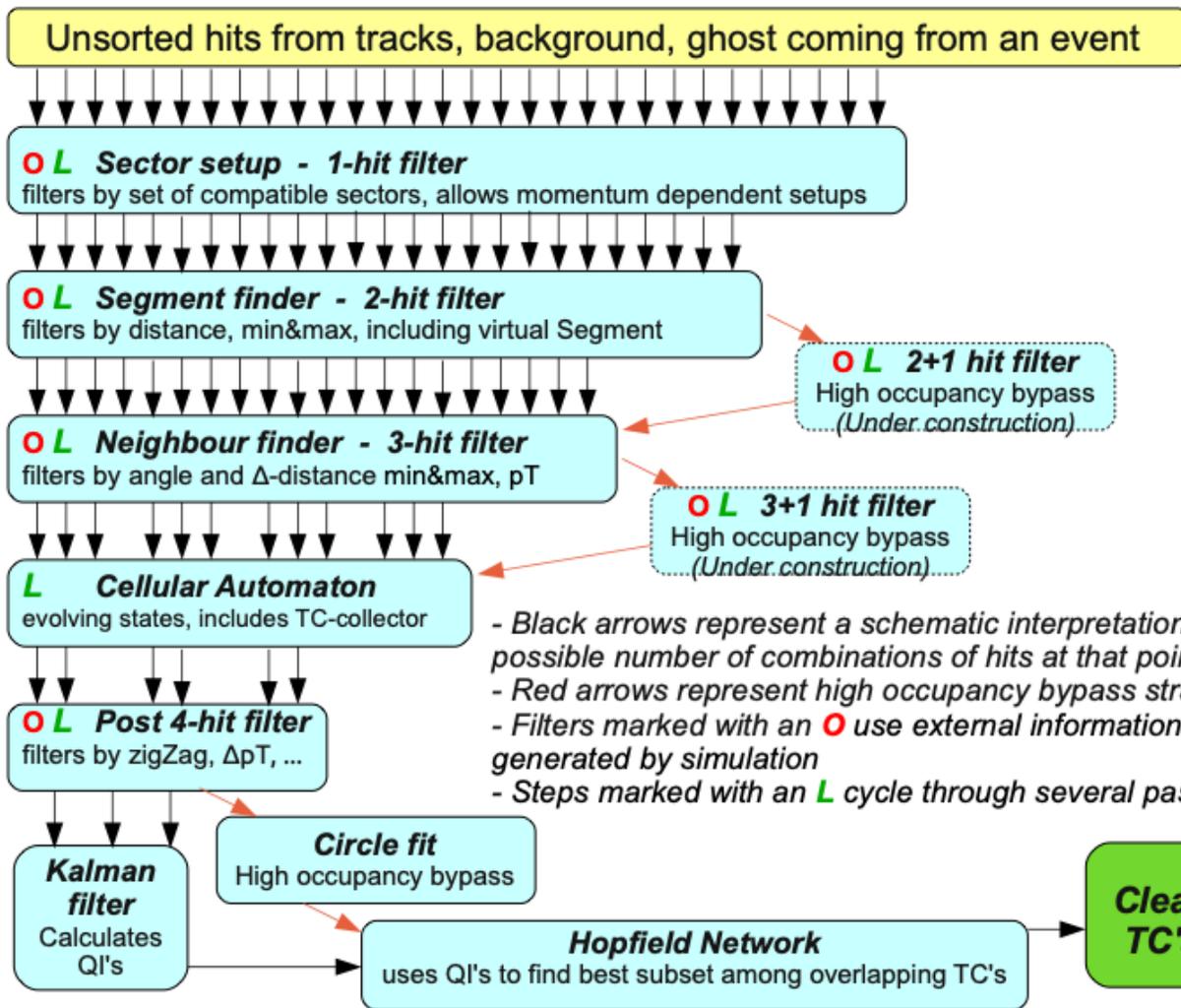
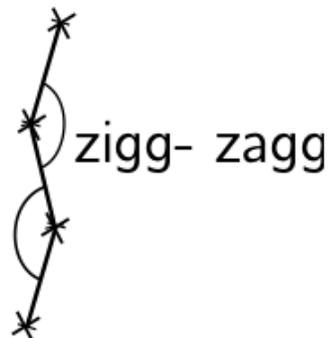
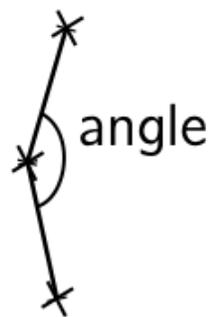
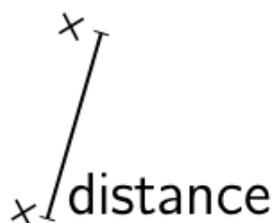
Matched: hit purity > 66% and hit efficiency exceeds 5% (to cover low momentum tracks curling in the detectors)

Finding efficiency: fraction of matched MC Tracks over all MC-tracks



Overview of VXD track finder

Schematic view of the low momentum track finder in Belle II



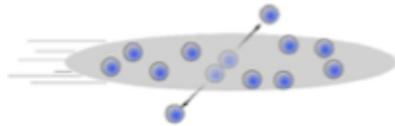
Belle II beam backgrounds

Beam background

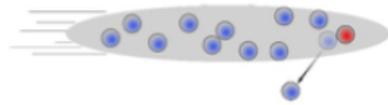
- particles that deviate from the nominal orbit are lost by hitting the beam pipe or other machine apparatus
- The generated shower particles might reach the detector and increase the hit rate
- Real Tracks but not belong to the triggered collision
- Deteriorate the detector's physics performance and damage the sensors

Beam background type:

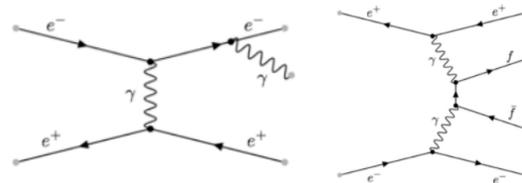
Touschek scattering:



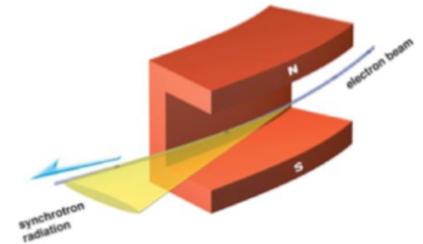
Beam-gas scattering:



Luminosity background:



Synchrotron radiation:



Injection background

Large beam loss accidents

Other none-physic factor: noise hits, cross talk ...

<https://browse.arxiv.org/pdf/2302.01566.pdf>

Beam spot parameter measurement

Resolution of the d_0 depends on:

$$\sigma_{68}(d_0) = \sqrt{\sigma_i^2 + (\sigma_x \sin \phi_0)^2 + (\sigma_y \cos \phi_0)^2}$$

$\sigma_{68}(\cdot)$: half width of the median containing 68% of the distribution

d_0 : Transverse impact parameter at POCA

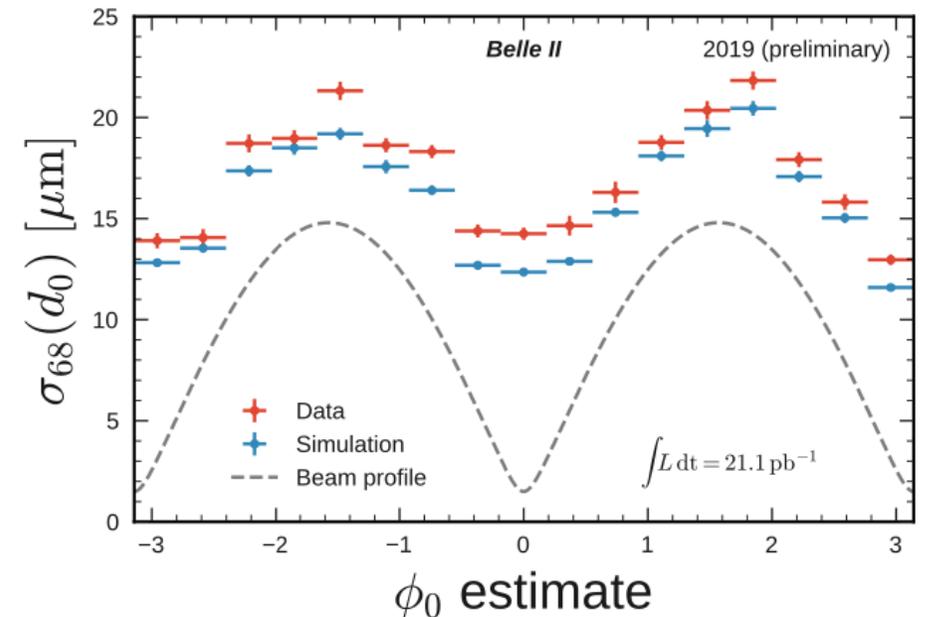
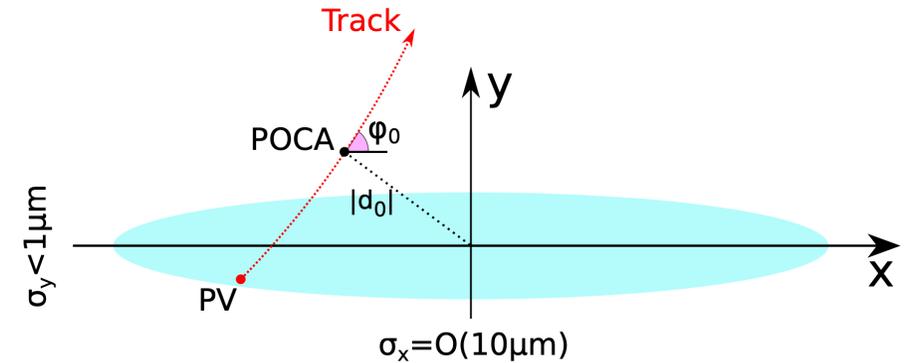
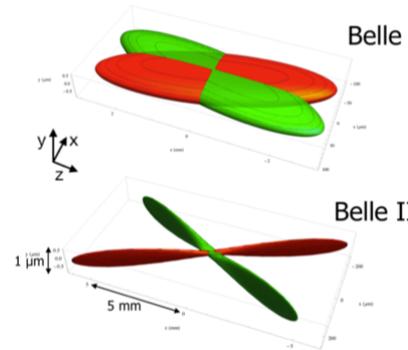
ϕ_0 : azimuthal angle at the point of closet approach

σ_x, σ_y : the beam spot size on x and y

σ_i : intrinsic resolution of tracking

Select tracks in dimuon events requiring:

- $P > 1 \text{ GeV}/c$
- $dr < 0.5 \text{ cm}$ and $|dz| < 2 \text{ cm}$
- # PXD hits ≥ 1 and # SVD hits ≥ 8 # CDC hits ≥ 20
- $9.5 < M < 11.5 \text{ GeV}/c^2$
- Exclude events with >1 candidates

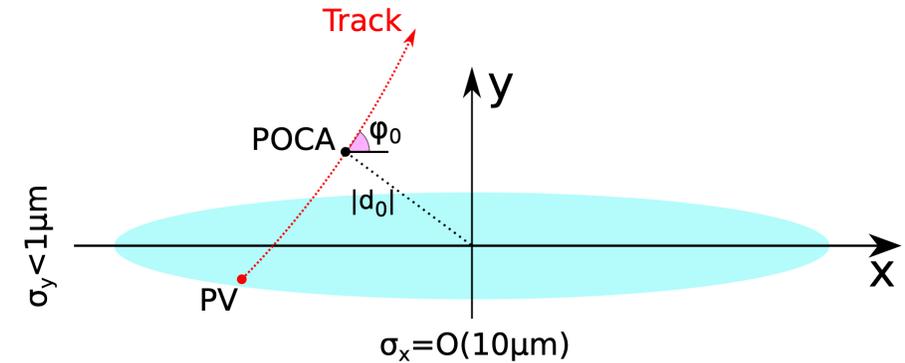


Beam spot parameter measurement

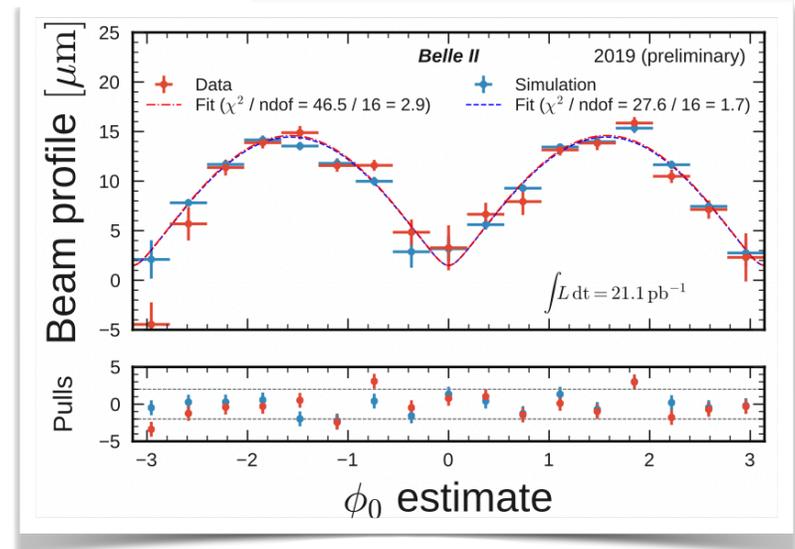
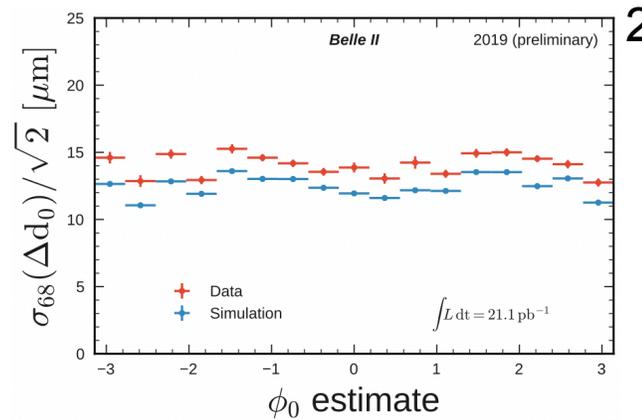
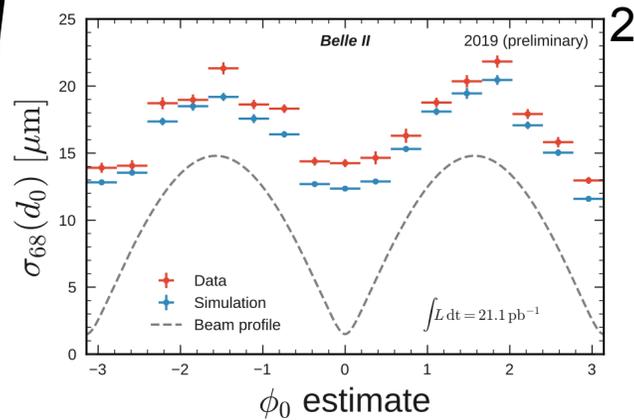
Resolution of the d_0 depends on:

$$\sigma_{68}(d_0) = \sqrt{\sigma_i^2 + (\sigma_x \sin \phi_0)^2 + (\sigma_y \cos \phi_0)^2}$$

σ_i : Δd_0 from 2-track events ($\sigma_i = \sigma_{68}(\Delta d_0)/\sqrt{2}$)



- Extract the beam spot parameters from the distribution of the primary vertex position



- Cross check & feed back to experiment parameters

Track-based alignment

Alignment is not perfect: installation, earthquake, temperature changes ...

The problem = Least squares minimisation problem

Millepede II: solve linear least Squares Fits with a Large Number of Parameters

Minimisation of the following function: (Only global parameters are determined)

$$F(p, q) = \sum_j F_j(p, q_j) \quad \text{with } F_j(p, q_j) = \frac{1}{2} \sum_i \frac{z_i^2}{\sigma_i^2}$$

p : global (calibration)

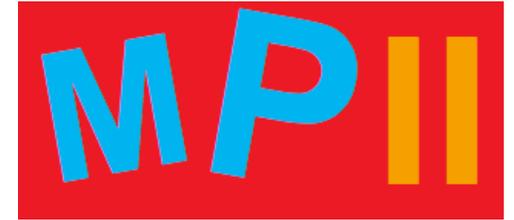
q_j : local (track, data set) parameters

z_i : the residual between the measured values and the parametrisation

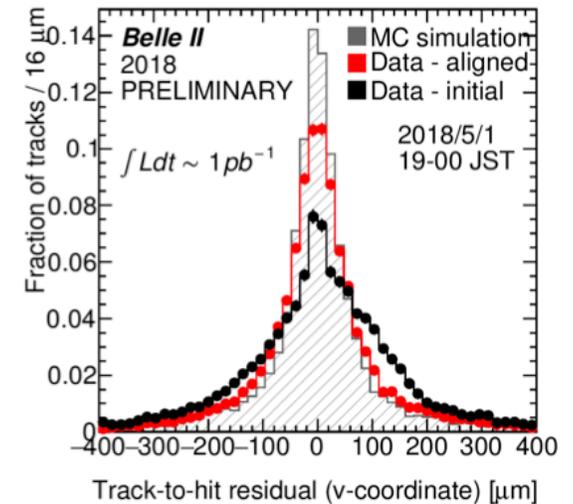
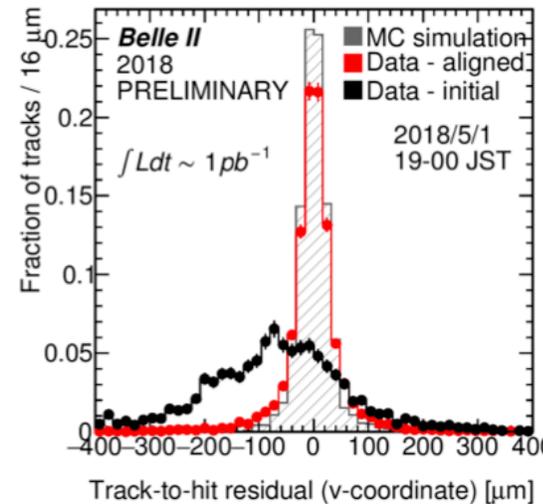
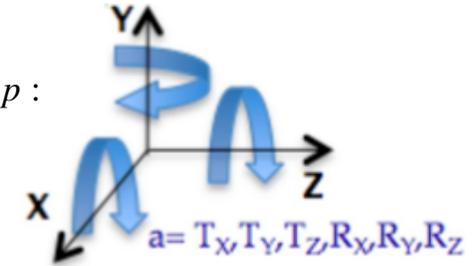
σ_i : inverse variance of the measured value

Eg: alignment after first collision

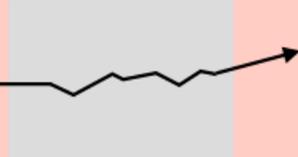
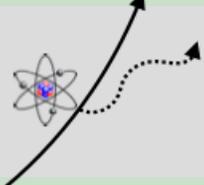
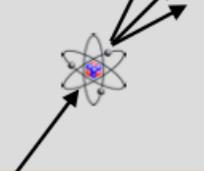
track-to-hit residuals from all PXD (left) and SVD (right) sensors after standard reconstruction and track fitting is performed



typical p :



Interactions

Interactions most relevant to Tracking				
Type	particles	parameter	characteristics	effect
Ionisation loss 	all charged particle	effective density $A/Z * \rho$	small effect in tracker, small dependence on p	increases momentum uncertainty
Multiple Scattering 	all charged particle	radiation length X_0	almost gaussian average effect 0, depends $\sim 1/p$	deflects particles, increases measurement uncertainty
Bremsstrahlung 	all charged particle, dominant for e	radiation length X_0	energy loss proportional $\sim E$, highly non-gaussian, depends $\sim 1/m^2$	introduces measurement bias and inefficiency
Hadronic Int. 	all hadronic particles	nuclear interaction length Λ_0	incoming particle lost, rather constant effect in p	main source of track reconstruction inefficiency