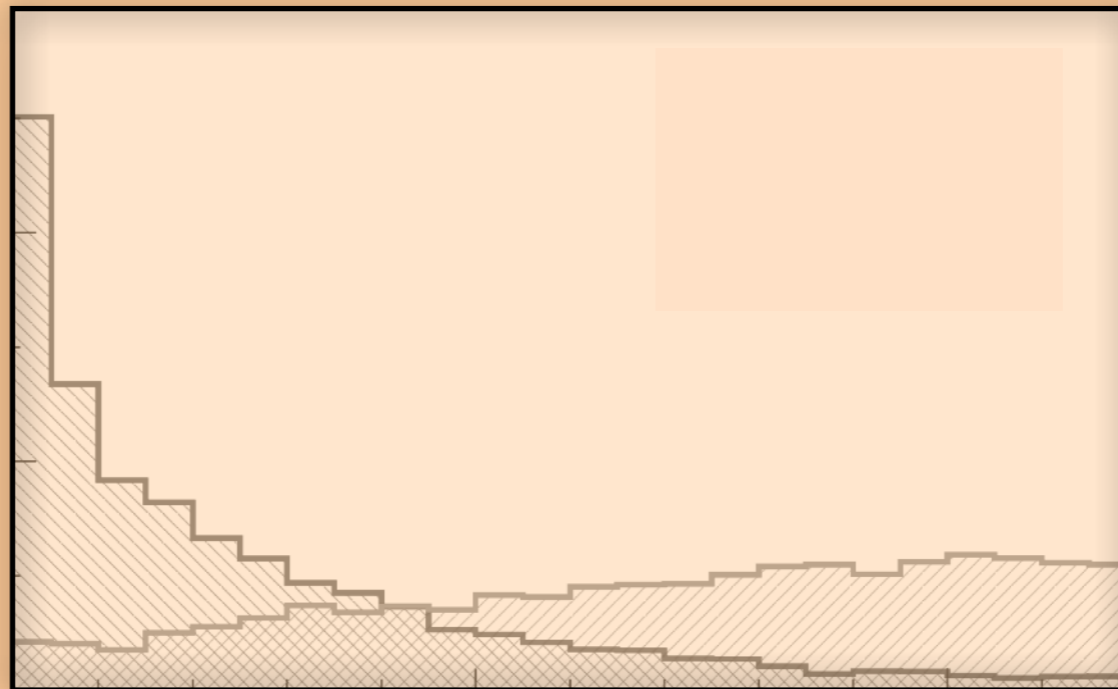


E_{ECL}: what it is and how to use it

Gaetano de Marino

2023.11.03 - Belle II Physics Week



INTRODUCTION

So far you have learned about...

ECL Chris



FEI Niharika



Belle + Belle II Xiaodong



Let's combine everything for E_{ECL} ↔ Missing energy analyses



Disclaimers

I am no expert, just a missing-energy analyst / E_{ECL} user

Inspired from Joint (S)L/EWP mini-workshop

Connection to one of the Physics-challenge

WHAT IS E_{ECL} ?

Quantity commonly referred to as E_{ECL} in Belle and E_{extra} in BaBar

E_{ECL}
BELLE

E_{extra}
BABAR

Definition

Sum of the energy deposits in the calorimeter that cannot be directly associated with the reconstructed daughters of the B_{tag} or the B_{sig}

Why is it so important?

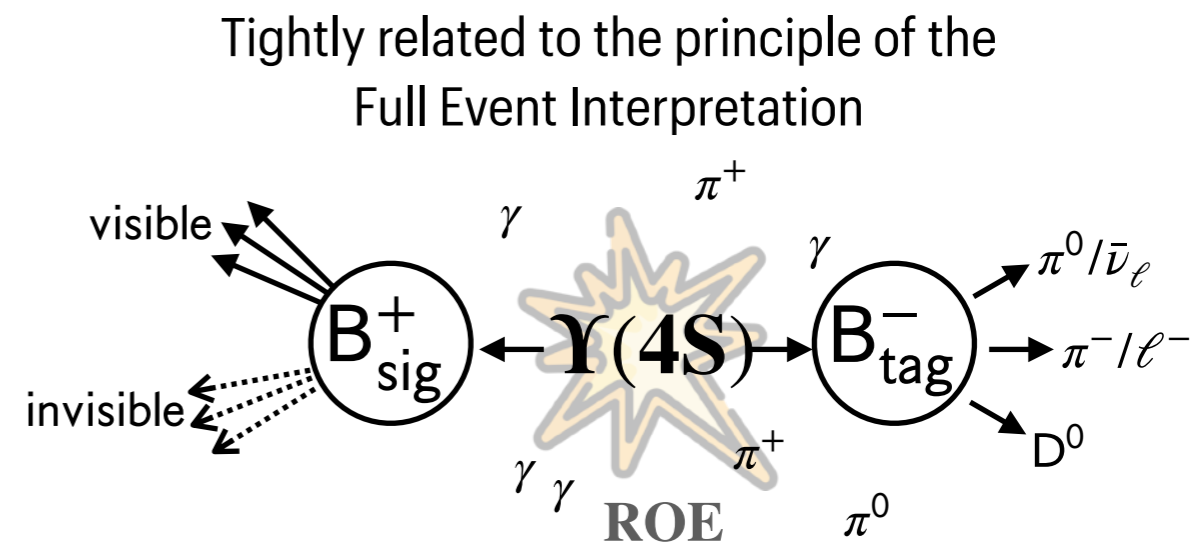
Signal events are expected to peak at or near $E_{ECL} = 0$

Background events usually contain one or more additional neutral clusters from unreconstructed particles
→ it extends to larger values with high separation between signal and background

It can be used as

Variable of signal extraction → a broad understanding of the underlying components is needed

Variable for background suppression → data/MC consistency must be checked



WHAT IS E_{ECL} ?

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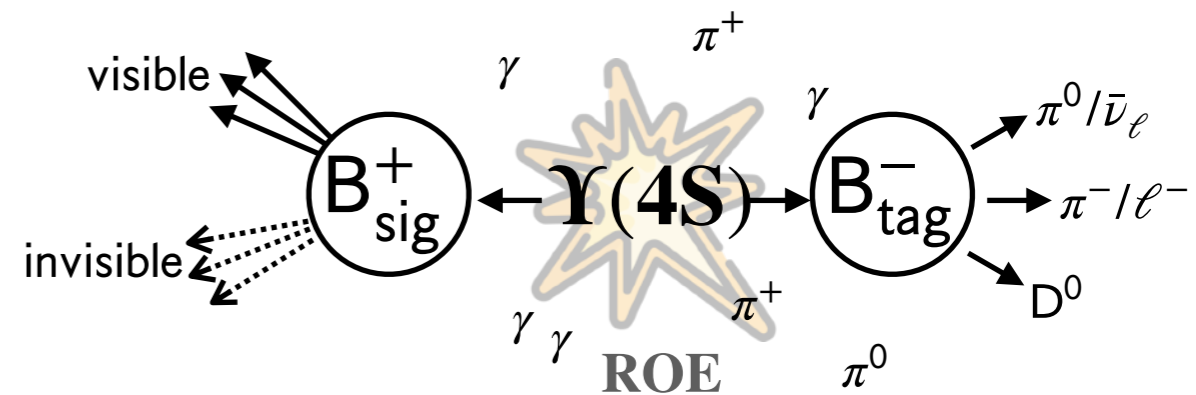
Background events usually contain one or more additional neutral clusters from unreconstructed particles
 → it extends to larger values with high separation between signal and background

A reliable reconstruction of E_{ECL} is one of the most crucial tasks in many analyses with missing energy

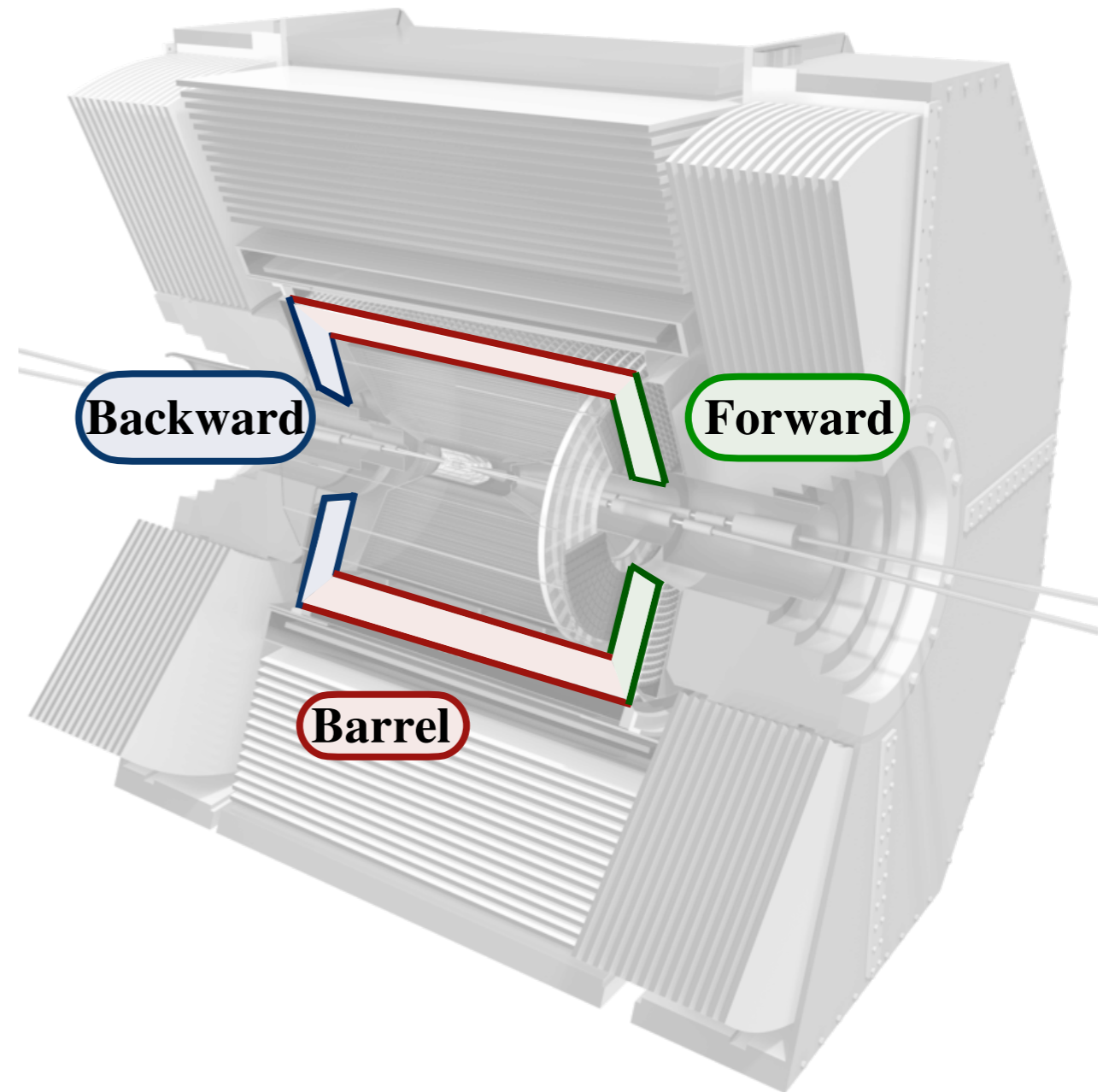
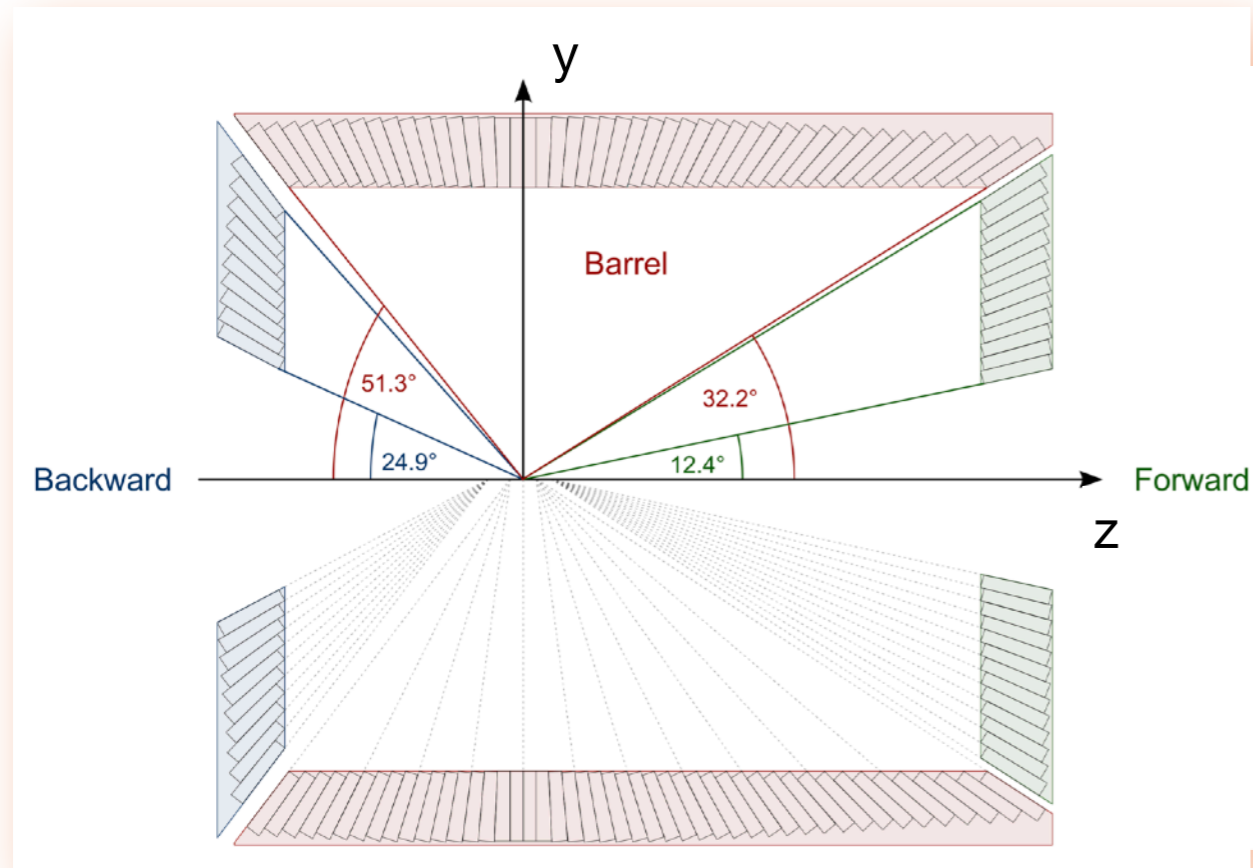
(Semi)leptonic $B \rightarrow D^{(*)}\tau\nu, \tau\nu, \dots$

EWP Penguin $B \rightarrow K\nu\nu, K\tau X, \dots$ Naturally biased toward these analyses 🧐

Tightly related to the principle of the Full Event Interpretation



ECL, REMINDER



thetaInECLAcceptance **ptInBECLAcceptance**

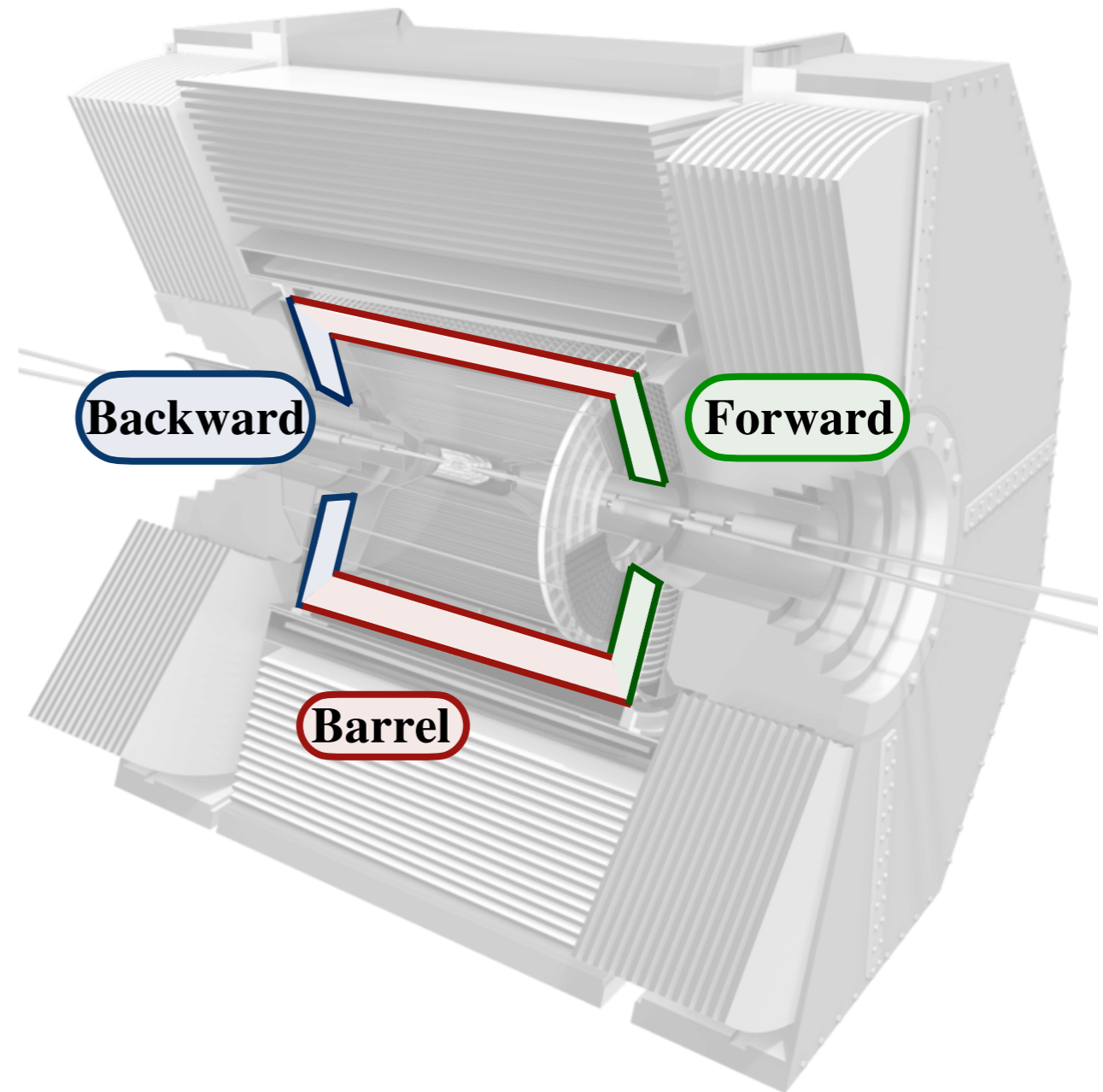
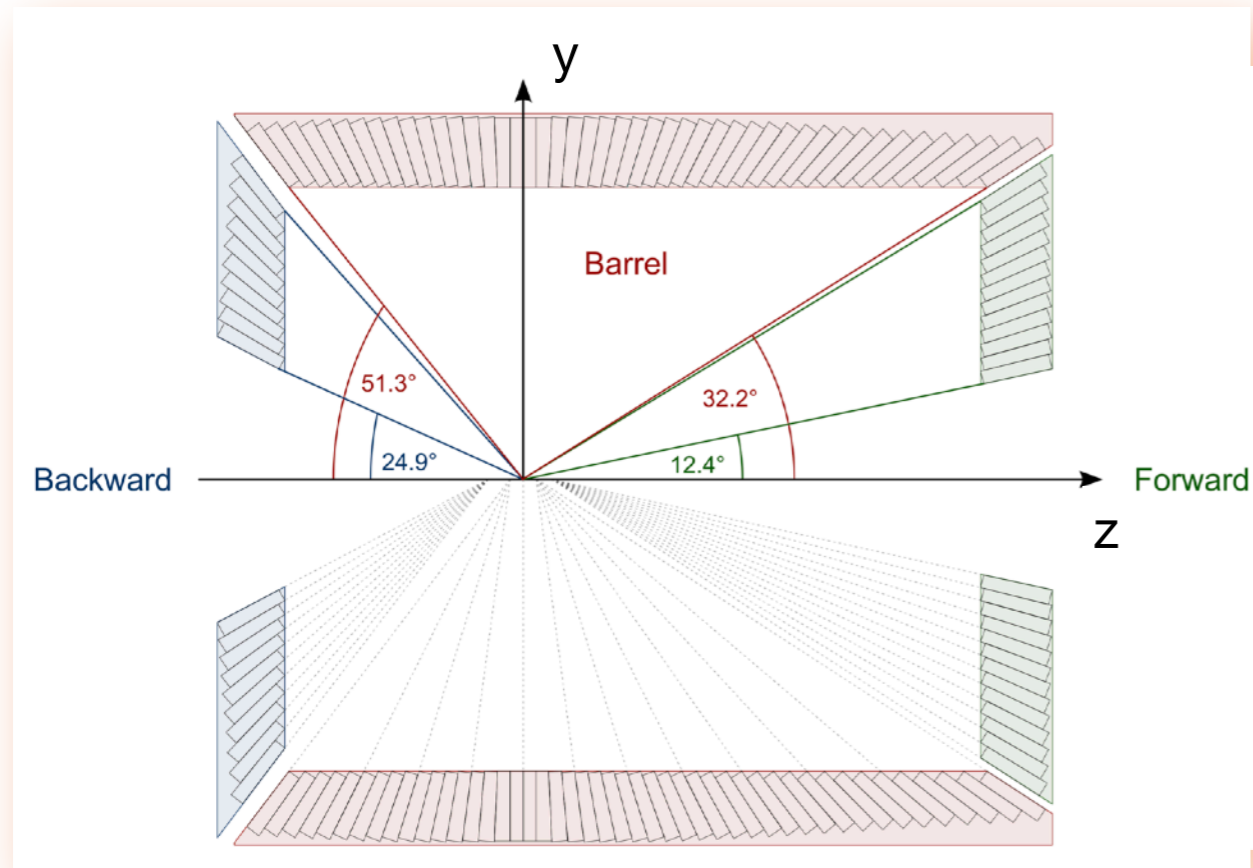
- Backward $\theta \in [130.7^\circ, 155.1^\circ]$ - $p_t > 0.28 \text{ GeV}/c$
- Barrel $\theta \in [32.2^\circ, 128.7^\circ]$
- Forward $\theta \in [12.4^\circ, 31.4^\circ]$

Crystal configuration is unchanged from the Belle times, covering about 90% of the solid angle in the CM frame

New reconstruction software for higher efficiency at low energies

New readout electronics to mitigate the pile-up background due to higher background levels at Belle II

ECL, REMINDER



thetaInECLAcceptance **ptInBECLAcceptance**

- Backward $\theta \in [130.7^\circ, 155.1^\circ]$ - $p_t > 0.28 \text{ GeV}/c$
- Barrel $\theta \in [32.2^\circ, 128.7^\circ]$
- Forward $\theta \in [12.4^\circ, 31.4^\circ]$

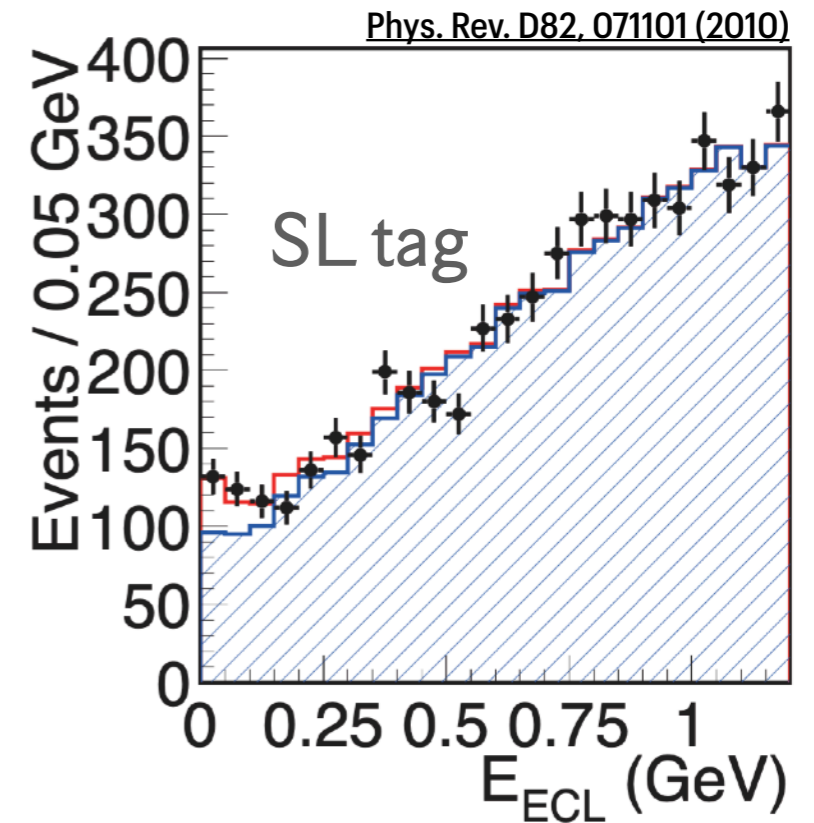
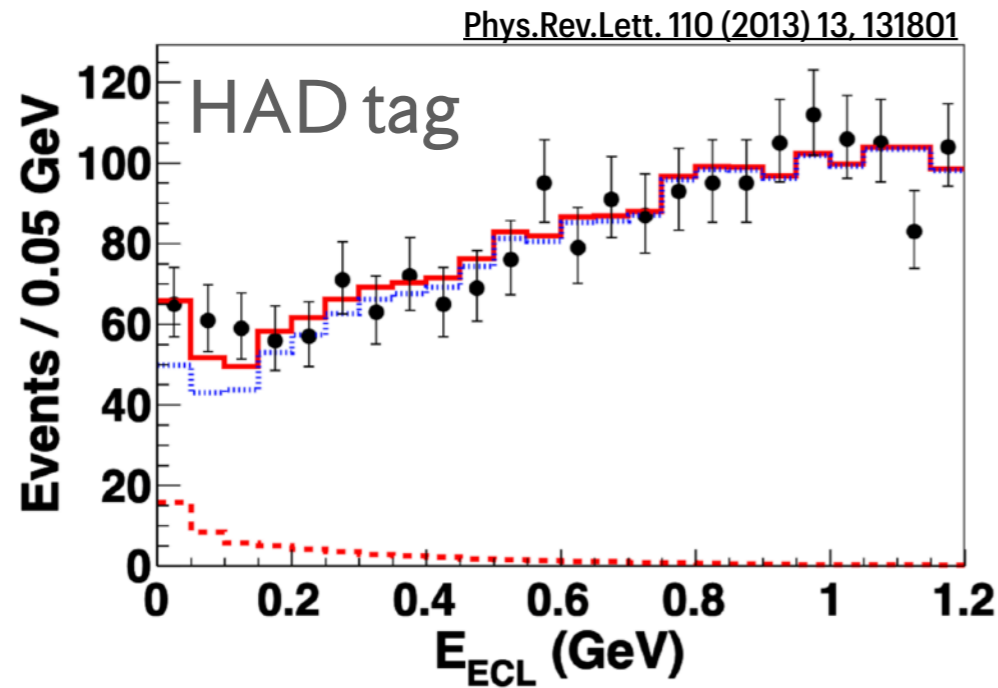
Photon ID

- is based on parameters describing the shower shape of ECL clusters not matched to a reconstructed track
- relies on the fact that EM showers from an incident photon is cylindrically symmetric in the lateral direction and the energy deposition decreases exponentially with the distance from the incident axis

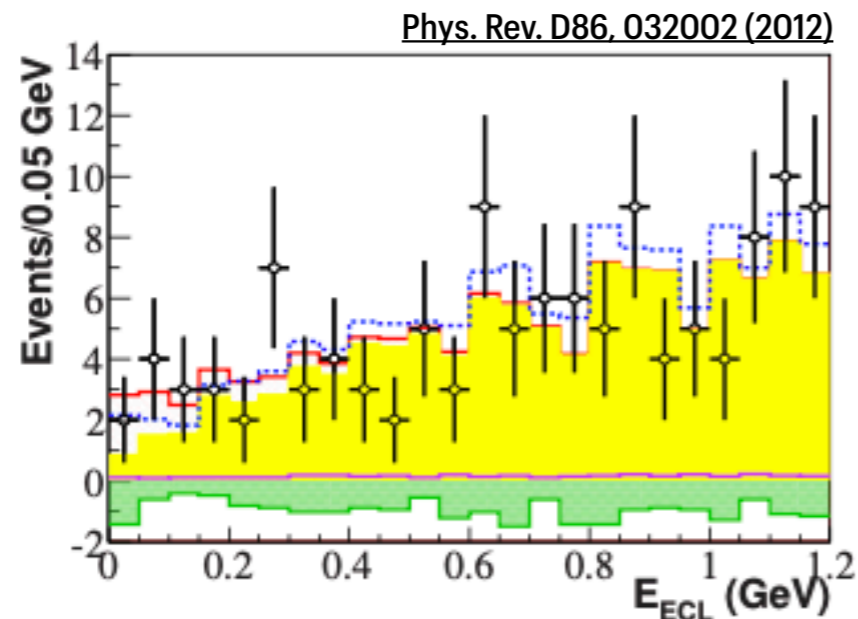
E_{ECL} FOR MISSING ENERGY

A plethora of analyses have used E_{ECL} as extraction variable!

$B^+ \rightarrow \tau^+ \nu_\tau$ with hadronic and semileptonic B-tagging

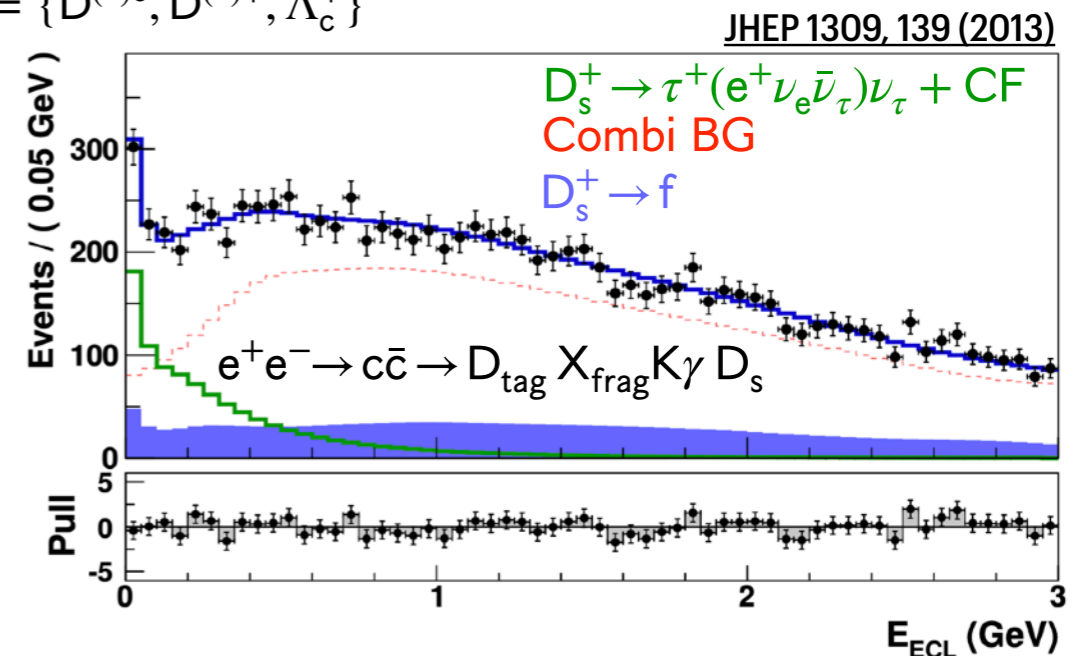


$B^+ \rightarrow$ invisible with hadronic B-tagging



$D_s^+ \rightarrow \tau^+(e^+ \nu_e \bar{\nu}_\tau) \nu_\tau$ with full reco of charmed hadron

$$D_{tag} = \{D^{(*)0}, D^{(*)+}, \Lambda_c^+\}$$



ROE IN BELLE / BELLE II

```
if inputParticlelists is None:
    inputParticlelists = []
fillParticleList('pi+:all', '', path=path)
if fillWithMostLikely:
    from stdCharged import stdMostLikely
    stdMostLikely(chargedPIDPriors, '_roe', path=path)
    inputParticlelists = [f'{ptype}:mostlikely_roe' for ptype in ['K+', 'p+', 'e+', 'mu+']]
import b2bii
if not b2bii.isB2BII():
    fillParticleList('gamma:all', '', path=path)
    fillParticleList('K_L0:roe_default', 'isFromKLM > 0', path=path)
    inputParticlelists += ['pi+:all', 'gamma:all', 'K_L0:roe_default']
else:
    inputParticlelists += ['pi+:all', 'gamma:mdst']
```

Belle II***BELLE***

HOW TO COMPUTE E_{ECL} IN BELLE II?

roeExtra(*yourMask*)

Total extra energy from ECLClusters belonging to ROE

```
auto roephotons = roe->getPhotons(maskName);
ROOT::Math::PxPyPzEVector total4vector;
for (auto* photon : roephotons)
{
    total4vector += photon->get4Vector();
}
const auto& frame = ReferenceFrame::GetCurrent();
auto frameRoe4Vector = frame.getMomentum(total4vector);
return frameRoe4Vector.energy();
};
```

roeNeextra(*yourMask*)

Total extra energy from neutral ECLClusters belonging to ROE

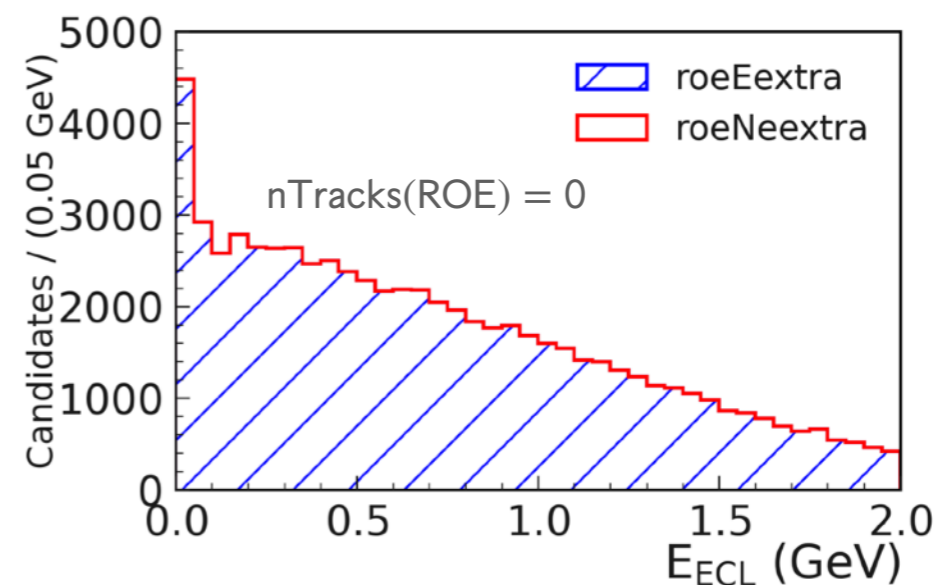
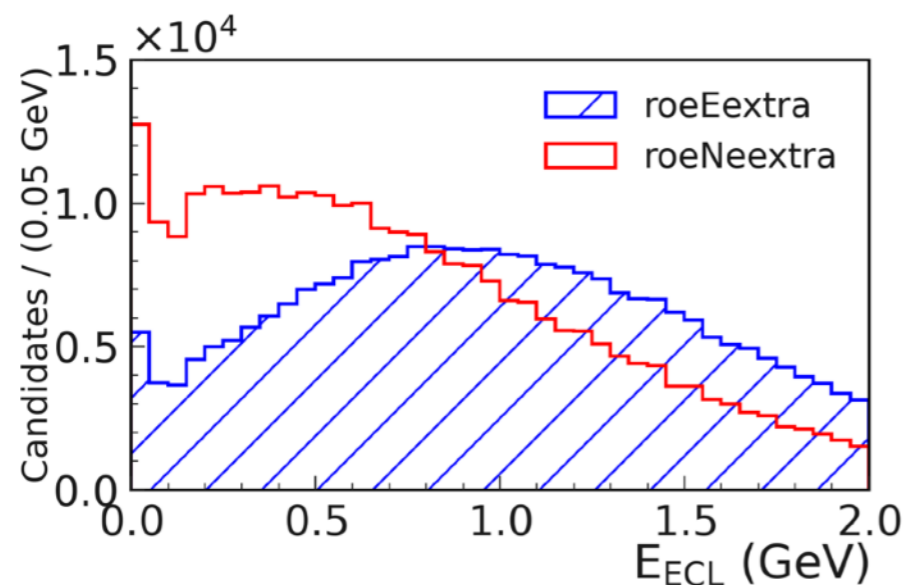
```
auto roeClusters = roe->getPhotons(maskName);

for (auto& roeCluster : roeClusters)
    if (roeCluster->getECLClusterEHypothesisBit() ==
        ECLCluster::EHypothesisBit::c_nPhotons)
        extraE += roeCluster->getECLClusterEnergy();

auto roeChargedParticles = roe->getChargedParticles(maskName);

for (auto& roeChargedParticle : roeChargedParticles)
{
    if (roeChargedParticle->getECLCluster())
        extraE += roeChargedParticle->getECLClusterEnergy();
}
```

The two definitions coincide if the number of charged tracks (*yourMask*) is set to zero



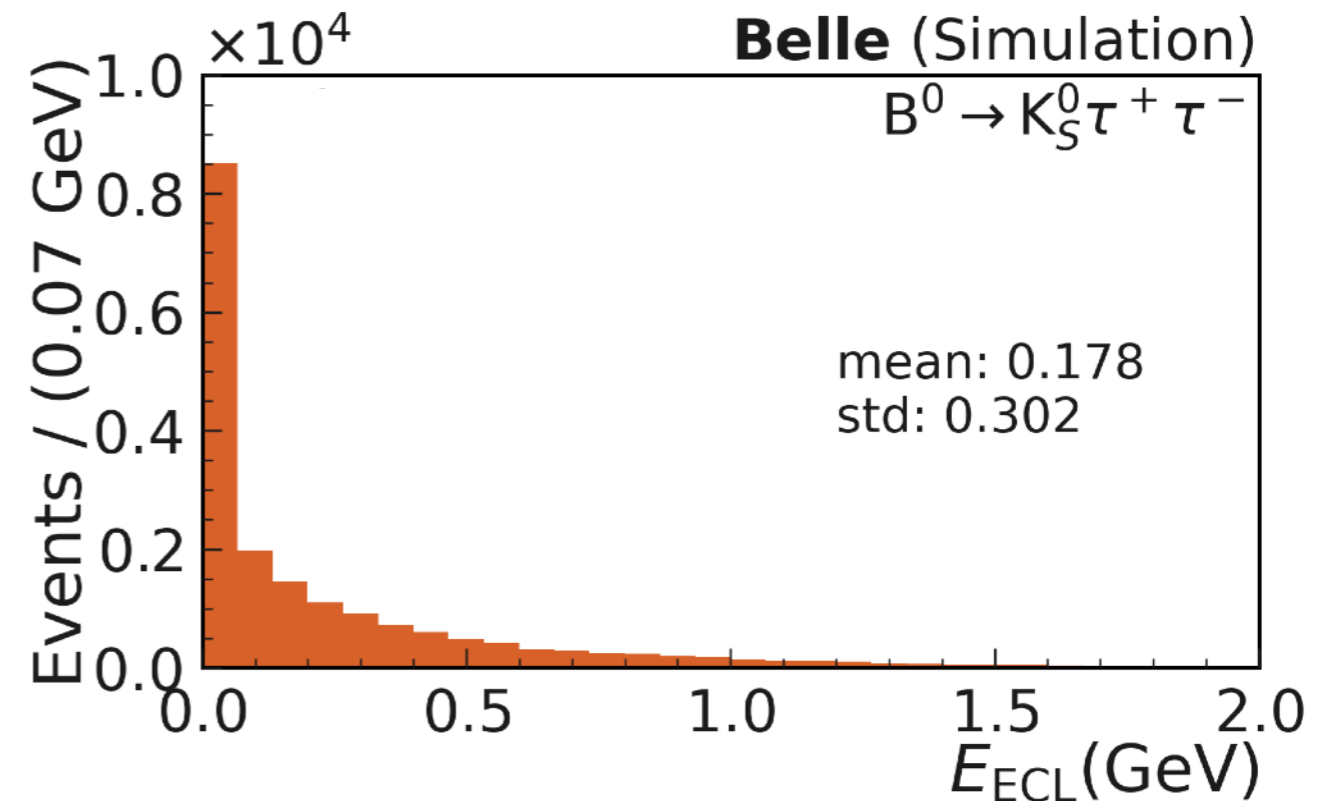
Will be focusing on the neutral E_{ECL}

HOW DOES E_{ECL} LOOK LIKE?

Once signal and tag B's are reconstructed, the process producing the cluster in the calorimeter is

- Physics

Unassigned photons from the mis-reconstructed B_{sig} and B_{tag} caused by a particle of the direct decay chain of the $Y(4S)$ +secondaries



HOW DOES E_{ECL} LOOK LIKE?

Once signal and tag B's are reconstructed, the process producing the cluster in the calorimeter is

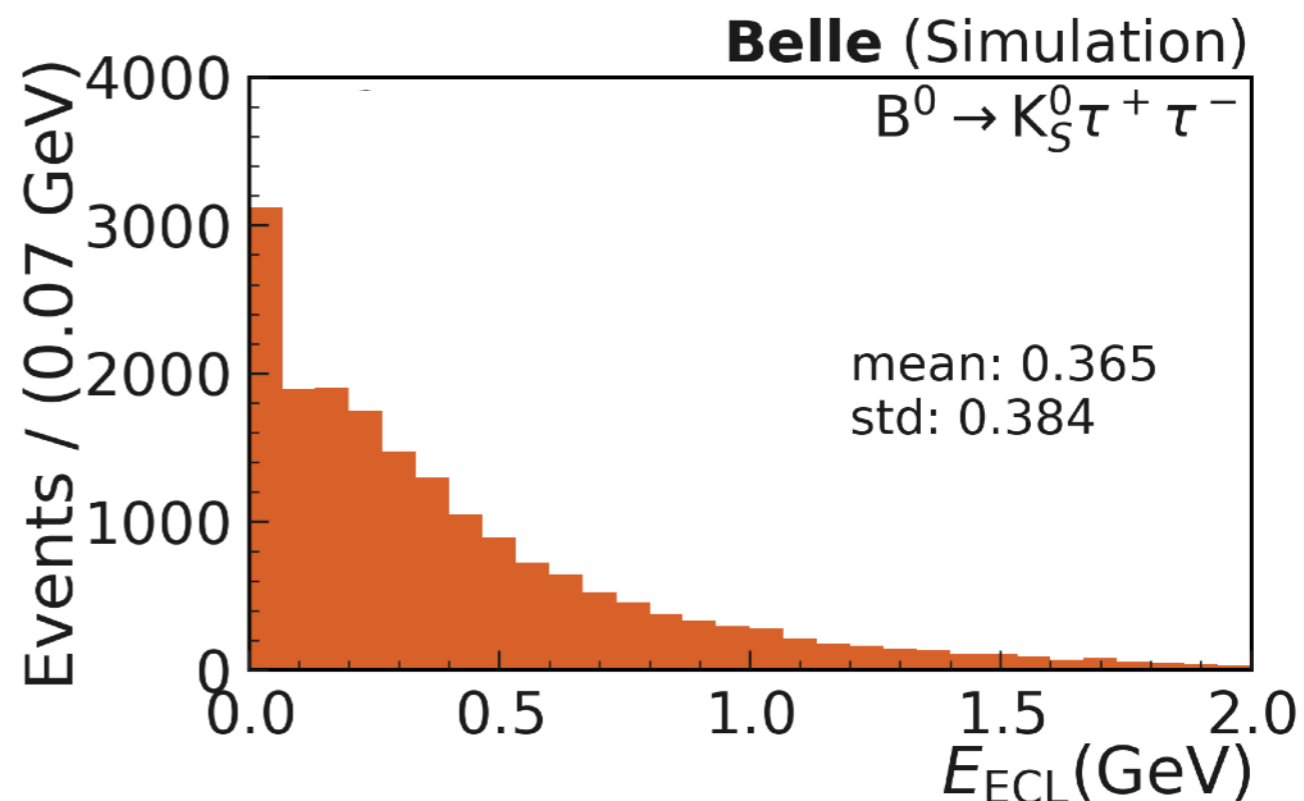
- Physics

- Fake photons

Track's deposits in the crystals can be wrongly divided into multiple clusters

Some of them are unmatched with respect to the source track when being extrapolated to the ECL → they are reconstructed as photons

Hadronic showers can also produce fake photons as their energy deposits are scattered and asymmetrically arranged



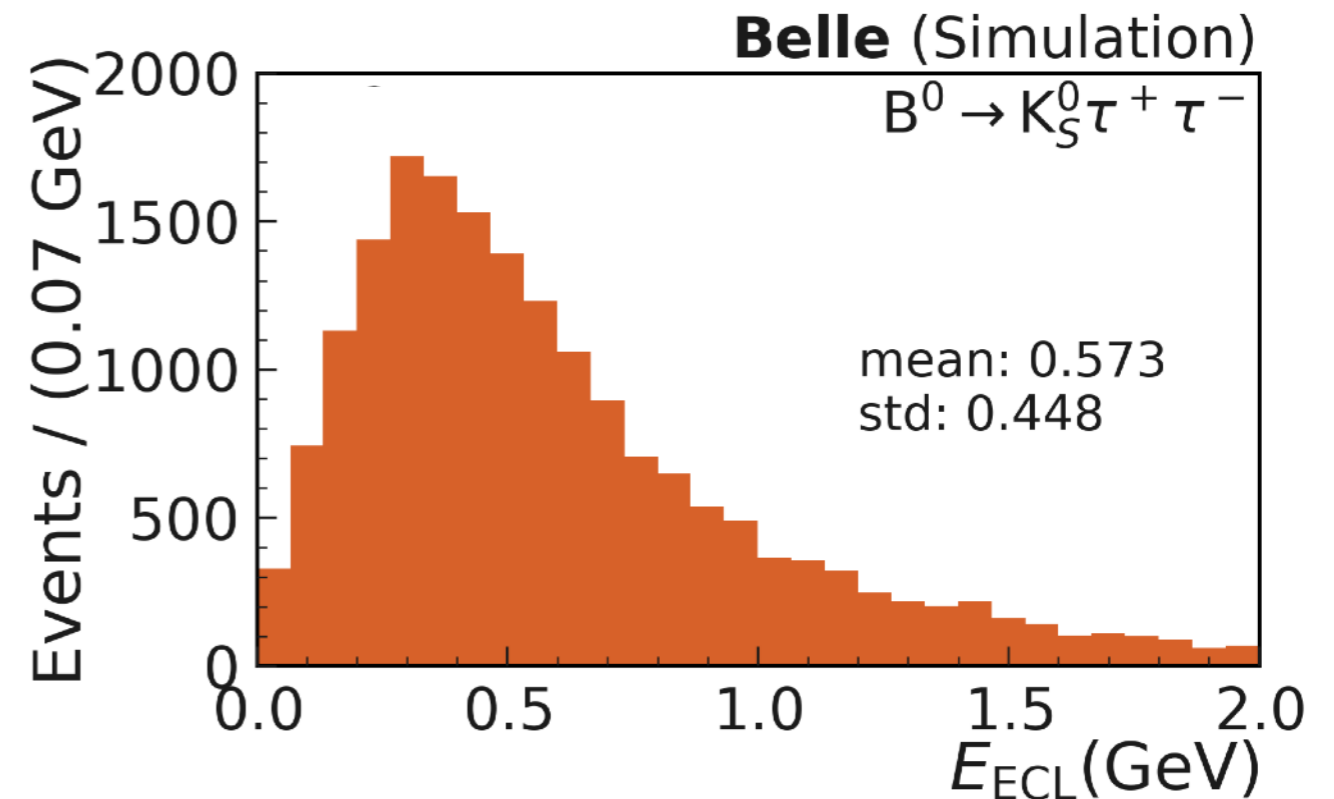
HOW DOES E_{ECL} LOOK LIKE?

Once signal and tag B's are reconstructed, the process producing the cluster in the calorimeter is

- Physics
- Fake photons
- Beam background

Energy from photons or neutrons that do not come from e^+e^- collision nor indicate a mis-reconstruction

In order to reproduce the effects of beam backgrounds, data recorded with random triggers are overlaid on simulated events



Dominant beam background sources are: Touschek effect, beam-gas scattering, radiative bhabha process...

E_{ECL} CLEANUP

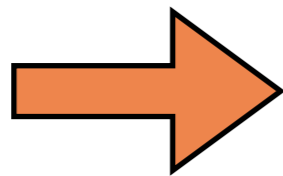
Summing over ALL ROE ECL clusters in the event is NOT OPTIMAL

Adding extra clusters due to

- wrongly reconstructed/matched clustering algorithm
 - beam background
 - secondary interactions of primary particles produced in the e⁺e⁻ collision
- leads to DEGRADED RESOLUTION

$$n_{\text{ECL}} = \sum_i (\text{clus}_{\text{ECL}})_i$$

$$E_{\text{ECL}} = \sum_i E(\text{clus}_{\text{ECL}})_i$$



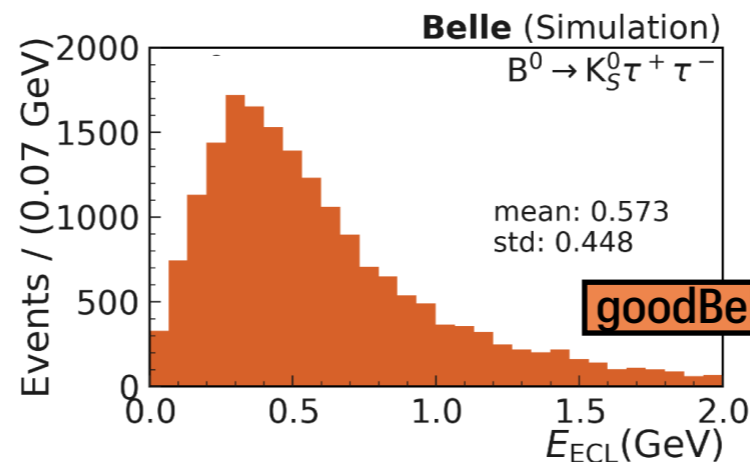
$$n_{\text{ECL}}^{\text{mask}} = \sum_{i \in \text{mask}} (\text{clus}_{\text{ECL}})_i$$

$$E_{\text{ECL}}^{\text{mask}} = \sum_{i \in \text{mask}} E(\text{clus}_{\text{ECL}})_i$$

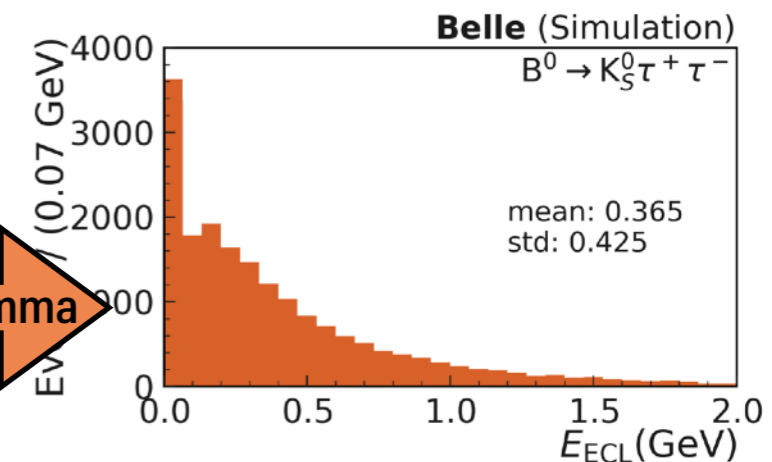
Mask: A set of selection criteria applied to the ROE objects

Mask examples:

- clusterE > 0.05 GeV
- clusterE > 0.10 GeV ($\theta \in \text{FWD}$) or
clusterE > 0.05 GeV ($\theta \in \text{BRR}$) or
clusterE > 0.15 GeV ($\theta \in \text{BWD}$)
- MVA-based cuts



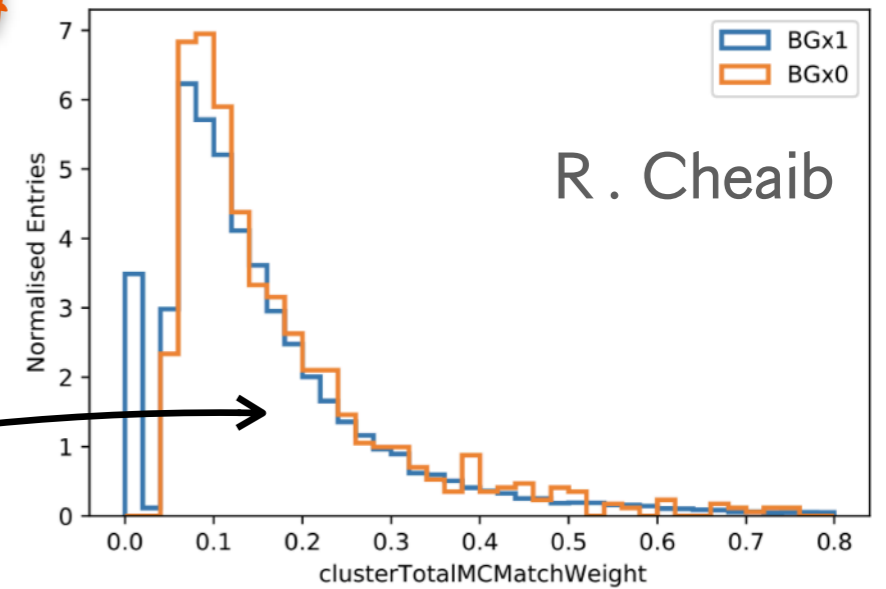
goodBelleGamma



ECL CLUSTERS CLASSIFICATION *for MVA*

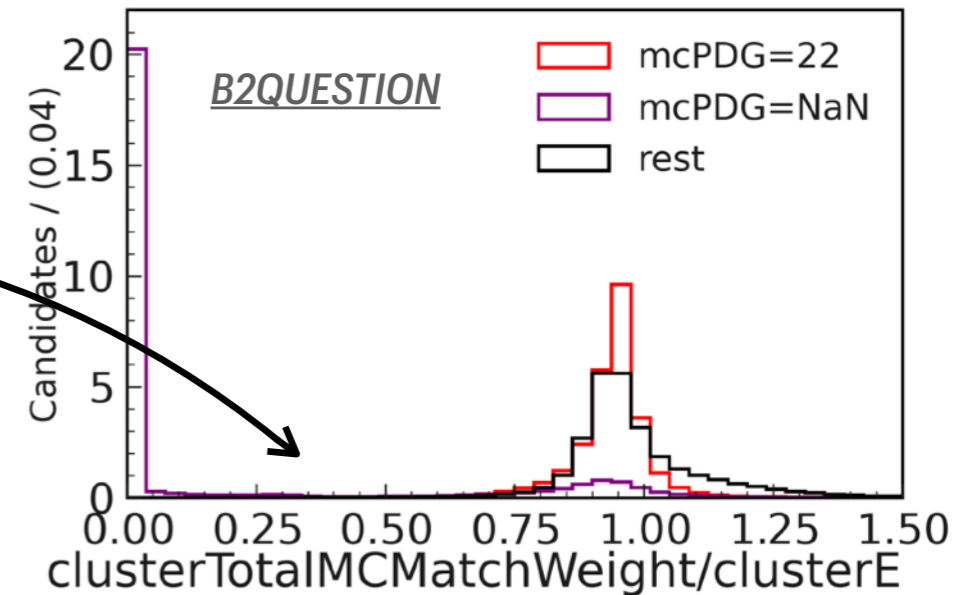
clusterMCMatchWeight

The energy fraction deposited by the MC-matched MC particle in the crystals of a cluster



clusterTotalMCMatchWeight

The energy deposited by all particles in the crystals of a cluster (sum of all weights)



clusterTotalMCMatchWeight/clusterE

Peaks ~0 for beam background and ~1 for physics interactions
For low energies, the resolution of this ratio is worse

	Photons	Beam	Fake
Belle	mcPDG==22	mcPDG==911	mcPDG!=22 and mcPDG!=911
Belle II	mcPDG==22	mcPDG==NaN and clusterTotalMCMatchWeight<0.053	[mcPDG==NaN or mcPDG!=22] and [clusterTotalMCMatchWeight>=0.053]

Various definitions have been proposed @ Belle II. Working on a common, agreed definition (Debjit Ghosh, Yo Sato)

MVA FOR PHOTON CLEANUP

Variable	Belle II	
	BB	FG
clusterE	✓	✓
minC2TDist		✓
clusterTiming	✓	✓
clusterPSDMVA	✓	✓
clusterZernikeMVA	✓	✓
clusterTheta	✓	✓

MVA tools available from light-2302-genetta onwards

Contact expert Priyanka Cheema

Changes:

New training

Fake photons: ~~clusterE1E9, clusterLAT and clusterSecondMoment,~~
clusterTiming, clusterTheta

Beam bg: ~~clusterE1E9, clusterLAT and clusterSecondMoment~~

New usage (fillParticleList → stdList)

More info at the
[CONFLUENCE PAGE](#)

Useful modular Analysis functions ...

```
stdPhotons(listtype='cdc', beamBackgroundMVAWeight='MC15ri',
           fakePhotonMVAWeight='MC15ri', path=main)
buildRestOfEvent('Upsilon(4S):BB', path=main)
```

Training on MC15rd is also available

```
addAlias("bbScore", "extraInfo(beamBackgroundSuppression)")
addAlias("fgScore", "extraInfo(fakePhotonSuppression)")
```

```
mask_0 = ('mask0', track_cut, f'[E>0.05]')
mask_1 = ('mask1', track_cut, f'[E>0.05] and [bbScore>0.2] and [fgScore>0.5]')
appendROEMasks('Upsilon(4S):BB', [mask_0, mask_1], path=main)
```

```
cutAndCopyList('gamma:inroe', gamma, 'isInRestOfEvent == 1', writeOut=True, path=roe_path)
main.for_each('RestOfEvent', 'RestOfEvents', path=roe_path)
```

MVA FOR PHOTON CLEANUP

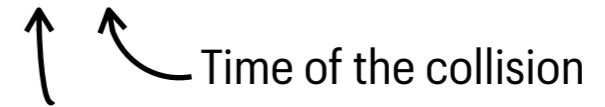
Variable	Belle II	
	BB	FG
clusterE	✓	✓
minC2TDist		✓
clusterTiming	✓	✓
clusterPSDMVA	✓	✓
clusterZernikeMVA	✓	✓
clusterTheta	✓	✓

Clusters from beam bg tend to have lower energies

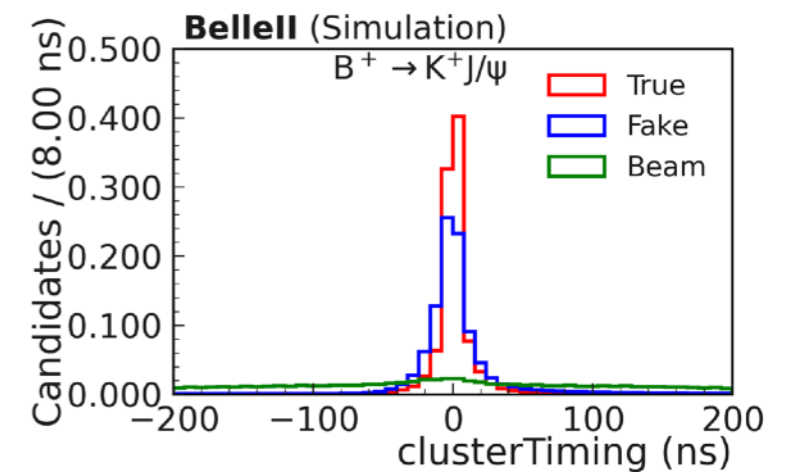
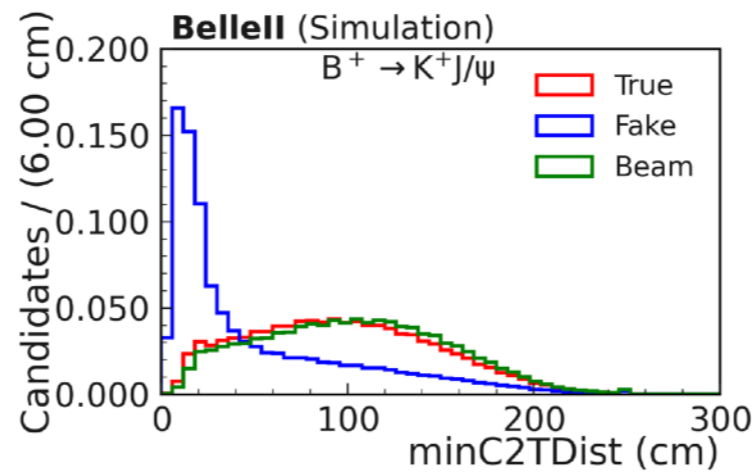
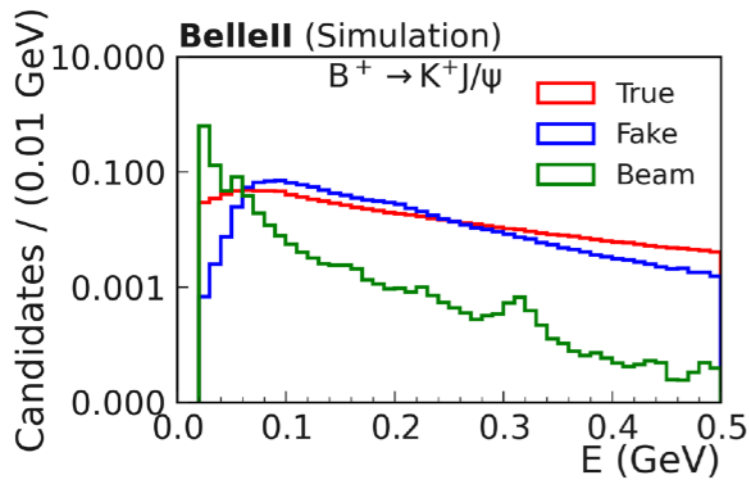
Fake photons have smaller minimum distance with tracks

Clusters from beam bg are off-time

cluster Time: $t - t_0$



Time of the cluster (highest energetic crystal)



Variable	Belle II	
	BB	FG
clusterE	✓	✓
minC2TDist		✓
clusterTiming	✓	✓
clusterPSDMVA	✓	✓
clusterZernikeMVA	✓	✓
clusterTheta	✓	✓

Clusters from beam bg tend to have lower energies

Fake photons have smaller minimum distance with tracks

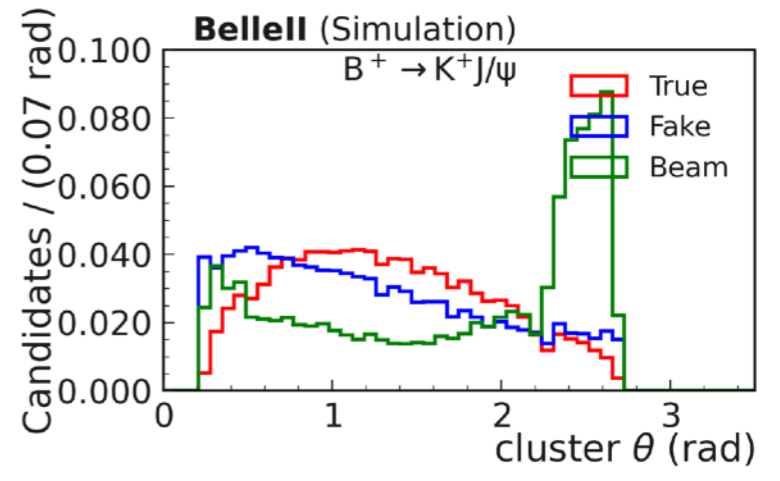
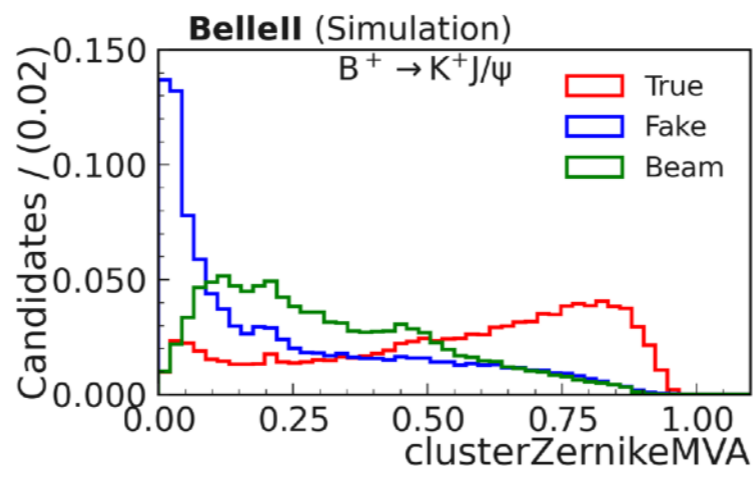
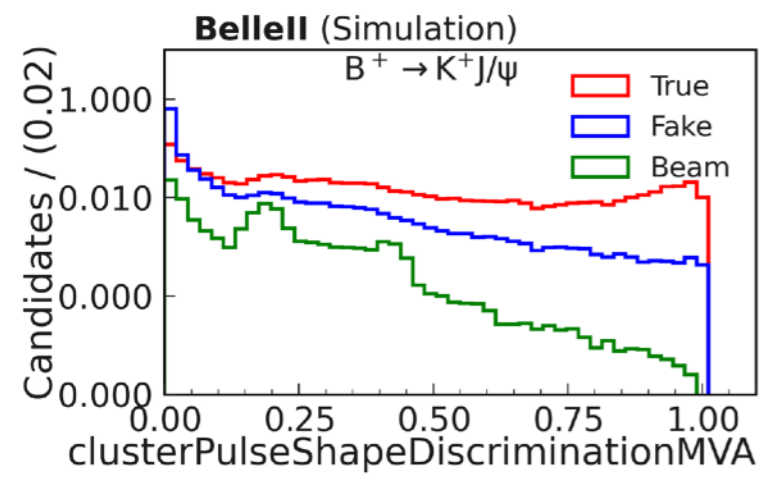
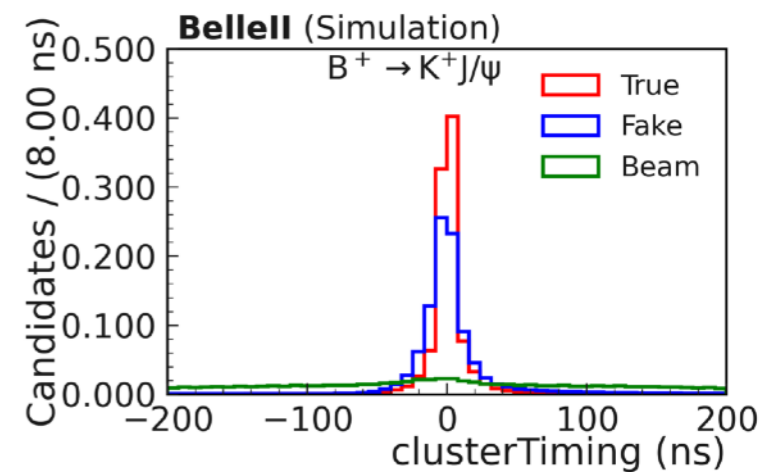
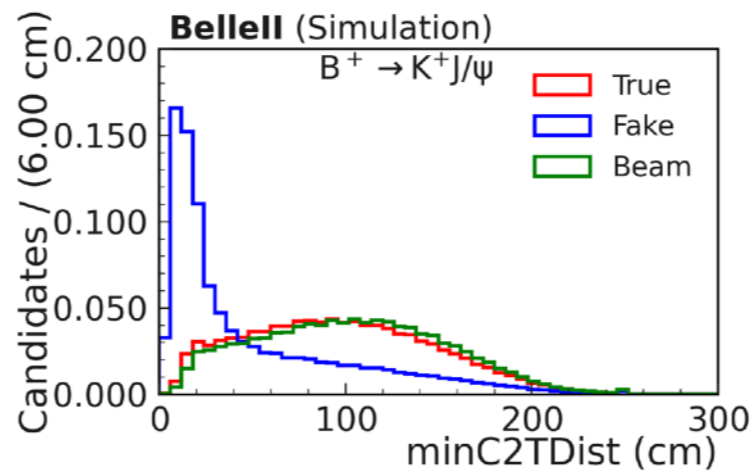
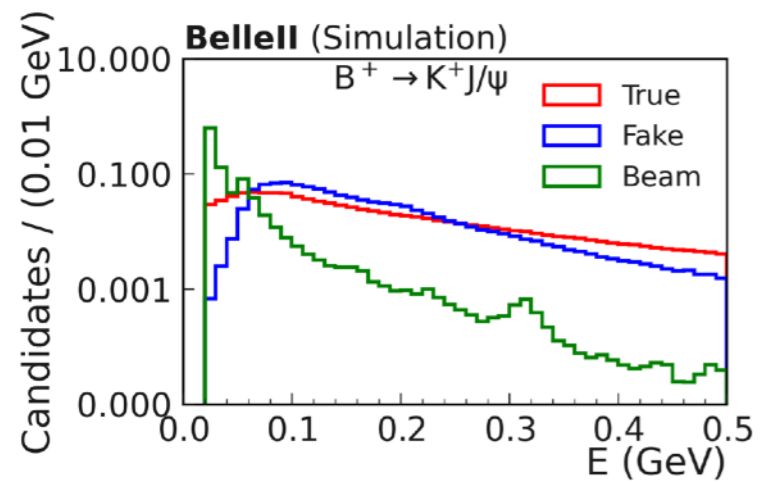
Clusters from beam bg are off-time

Crystals waveforms are recorded for $E > 50$ MeV

Energy distribution as a function of an angular rotation around the central crystal

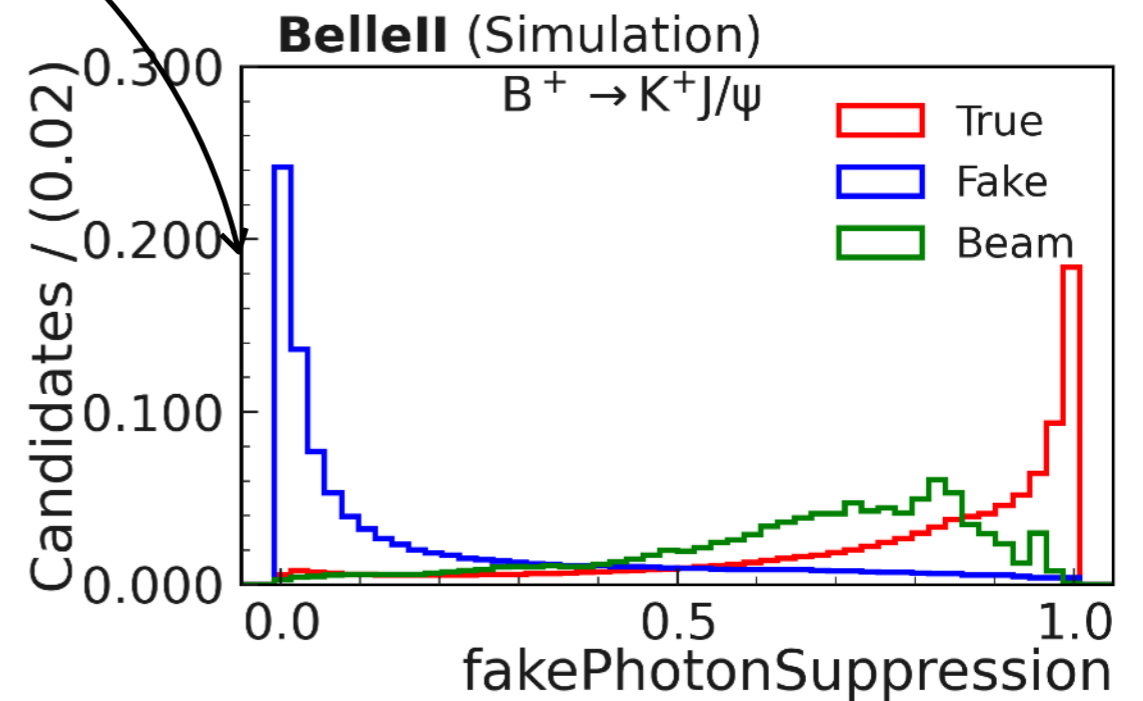
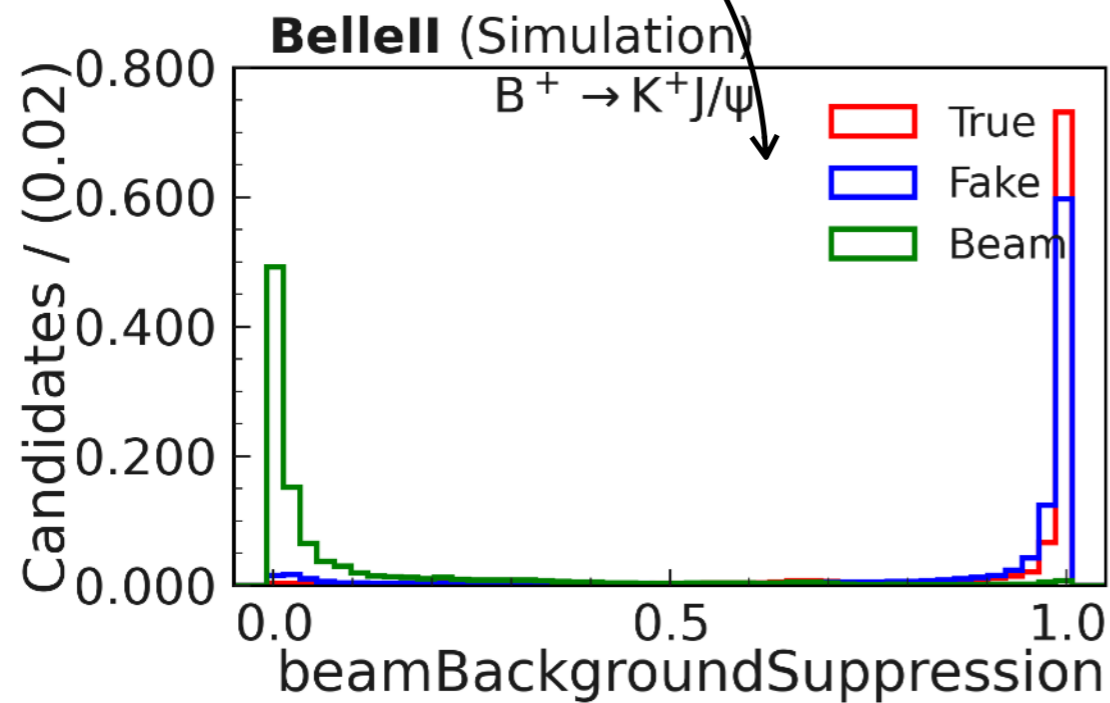
The effect of beam background is more severe in the endcaps

Help distinguishing between photon and hadronic showers



MVA FOR PHOTON CLEANUP

Variable	Belle II	
	BB	FG
clusterE	✓	✓
minC2TDist		✓
clusterTiming	✓	✓
clusterPSDMVA	✓	✓
clusterZernikeMVA	✓	✓
clusterTheta	✓	✓



Looks great in MC but consistency with data must be checked... See later!

MVA FOR PHOTON CLEANUP

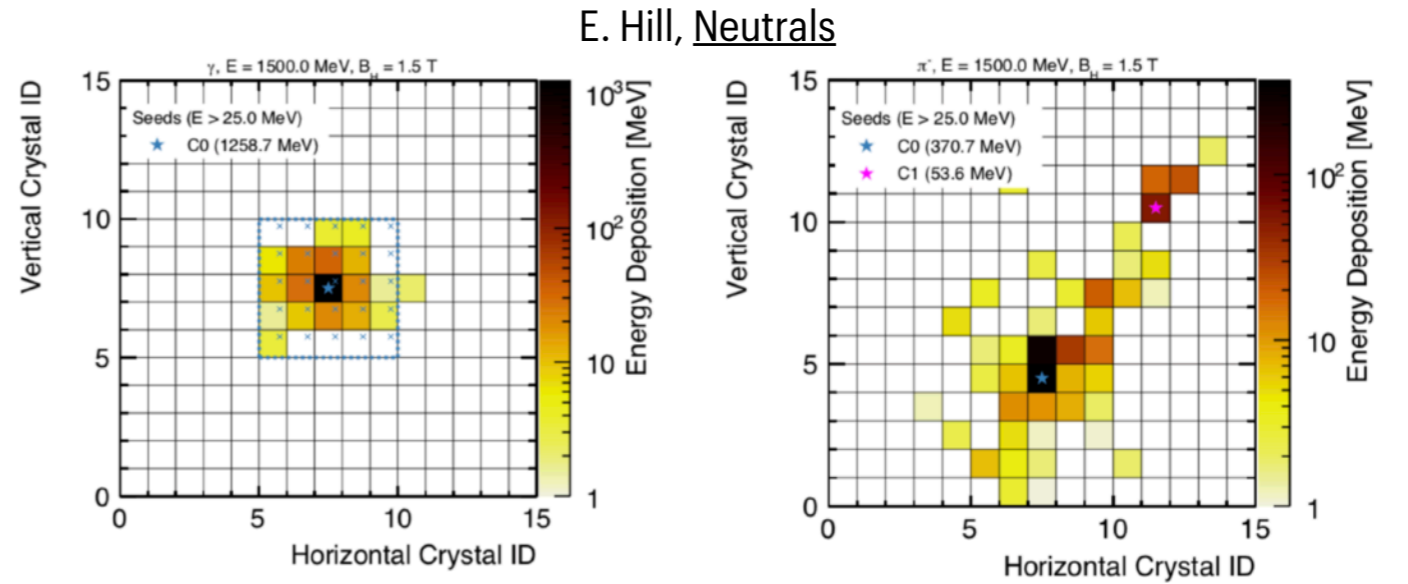
Variable	Belle II		Belle	
	BB	FG	BB	FG
clusterE	✓	✓	✓	✓
minC2TDist		✓	✓†	✓†
clusterTiming	✓	✓	*	*
clusterPSDMVA	✓	✓		
clusterZernikeMVA	✓	✓		
clusterTheta	✓	✓	✓	✓
clusterE9E25			✓	✓
clusterLat			✓	✓
clusterHighestE			✓	✓
clusterNHits			✓	✓

What about Belle? How can we suppress backgrounds?

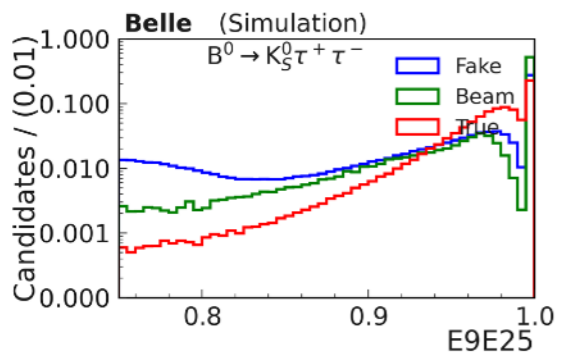
† An alternative definition of minC2TDist is available in b2bii (from light-2305-korat onwards) **NEXT SLIDE**

* cluster timing information is not available at Belle (neither in MC nor SVD1 data)

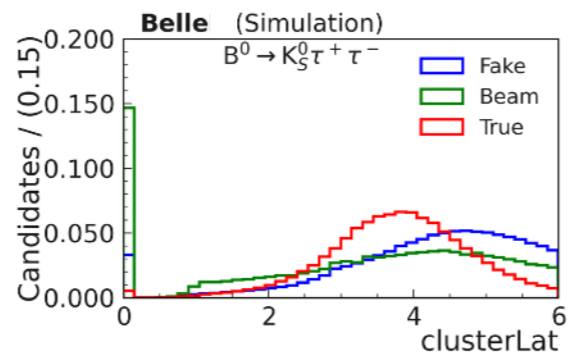
Variable	Belle II		Belle	
	BB	FG	BB	FG
clusterE	✓	✓	✓	✓
minC2TDist		✓	✓†	✓†
clusterTiming	✓	✓	*	*
clusterPSDMVA	✓	✓		
clusterZernikeMVA	✓	✓		
clusterTheta	✓	✓	✓	✓
clusterE9E25			✓	✓
clusterLat			✓	✓
clusterHighestE			✓	✓
clusterNHits			✓	✓



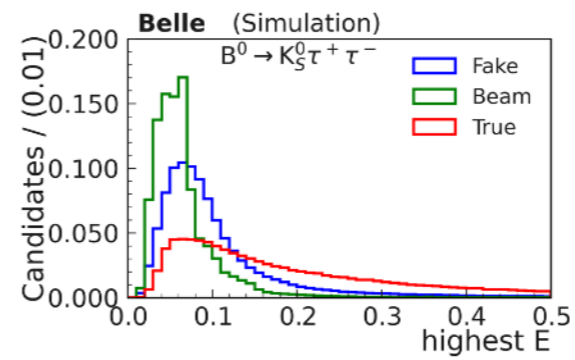
Ratio of energies in inner 3×3 crystals (E9) and 5×5 (E25)
 Energy distribution in the plane perpendicular to the shower axis
 The energy of the highest energetic crystal in the ECL cluster (⇔ E1 for true photons)
 Number of crystals in the cluster



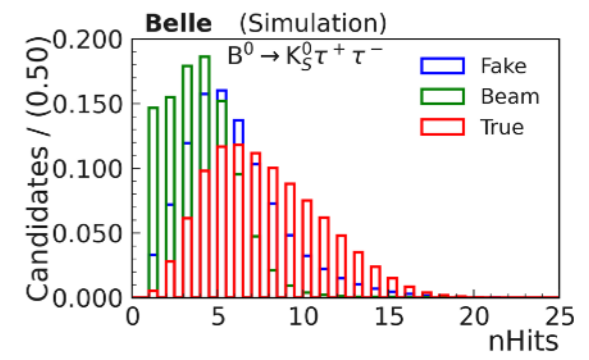
Gives info on how wide the shower is



True γ have more radially symmetric EM showers, hadronic events have larger clusterLat



Photons have most of their energy in the central crystal



Background photons tend to have lower nhits than the good ones

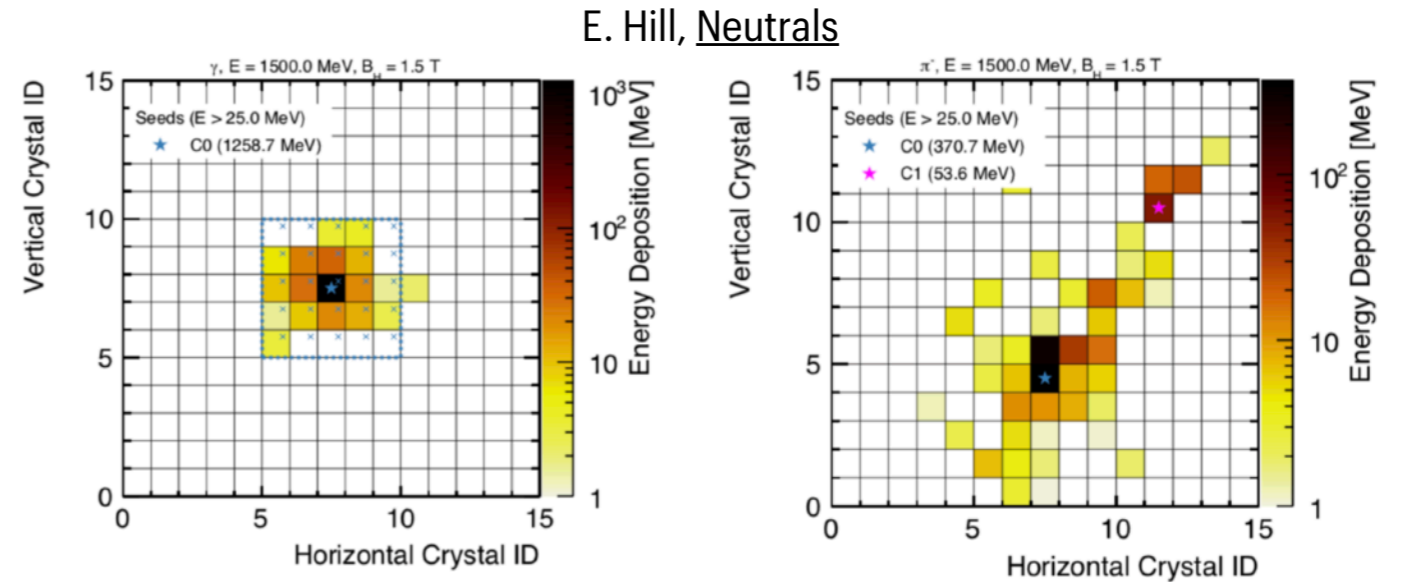
Variable	Belle II		Belle	
	BB	FG	BB	FG
clusterE	✓	✓	✓	✓
minC2TDist		✓	✓†	✓†
clusterTiming	✓	✓	*	*
clusterPSDMVA	✓	✓		
clusterZernikeMVA	✓	✓		
clusterTheta	✓	✓	✓	✓
clusterE9E25			✓	✓
clusterLat			✓	✓
clusterHighestE			✓	✓
clusterNHits			✓	✓

Ratio of energies in inner 3×3 crystals (E9) and 5×5 (E25)

Energy distribution in the plane perpendicular to the shower axis

The energy of the highest energetic crystal in the ECL cluster (⇔ E1 for true photons)

Number of crystals in the cluster



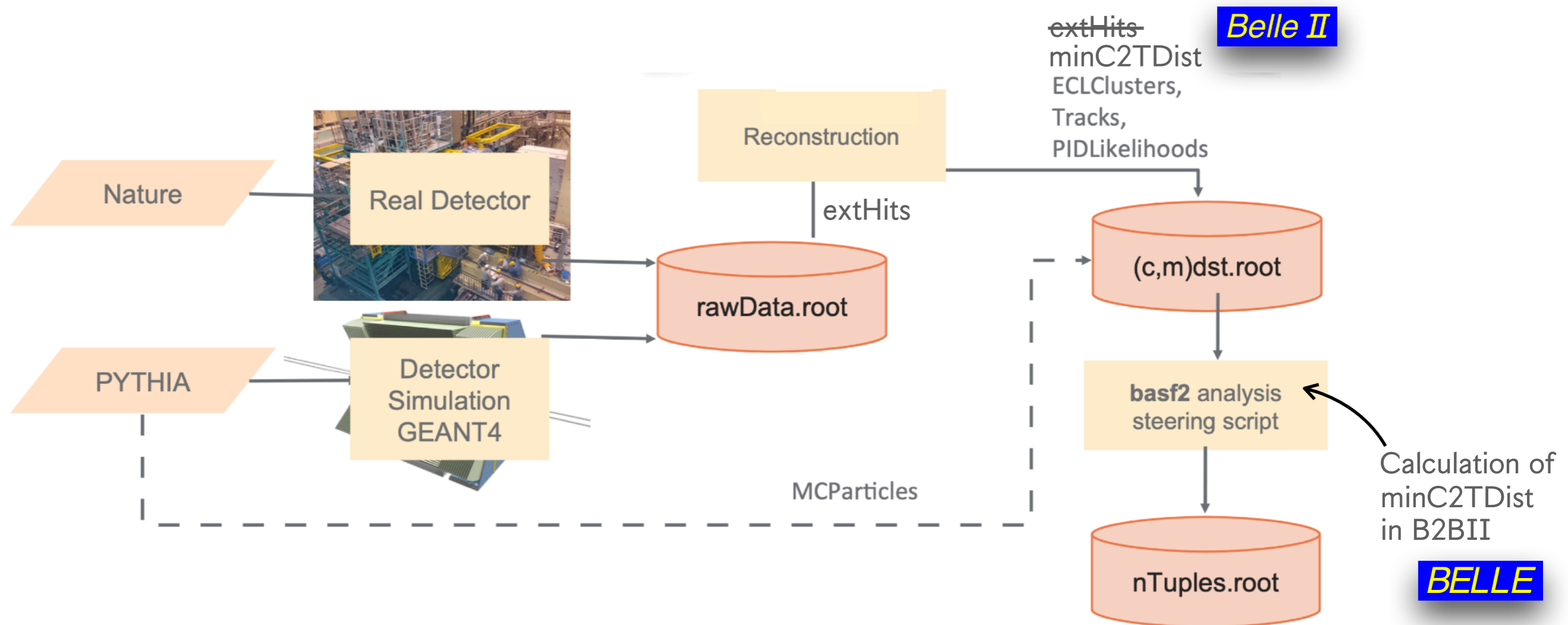
BeamBackgroundMVA and FakePhotonMVA SOON available for b2bii (MH Liu, C-L Hsu)

```
main.add_module('MVAExpert', listNames=['gamma:gbg'], extraInfoName='bbScore',
                identifier='./MyMVAMethod_BeamB.root')
```

MINIMUM DISTANCE VARIABLE(S)

- minC2TDist is defined as the distance between the ECL cluster and its nearest track
It is computed using the hits (extHits) obtained extrapolating the tracks through the ECL
- The extHits are not available in Belle mdsts → obtained by computing the entrance point of the tracks into ECL surface based on helix extrapolation

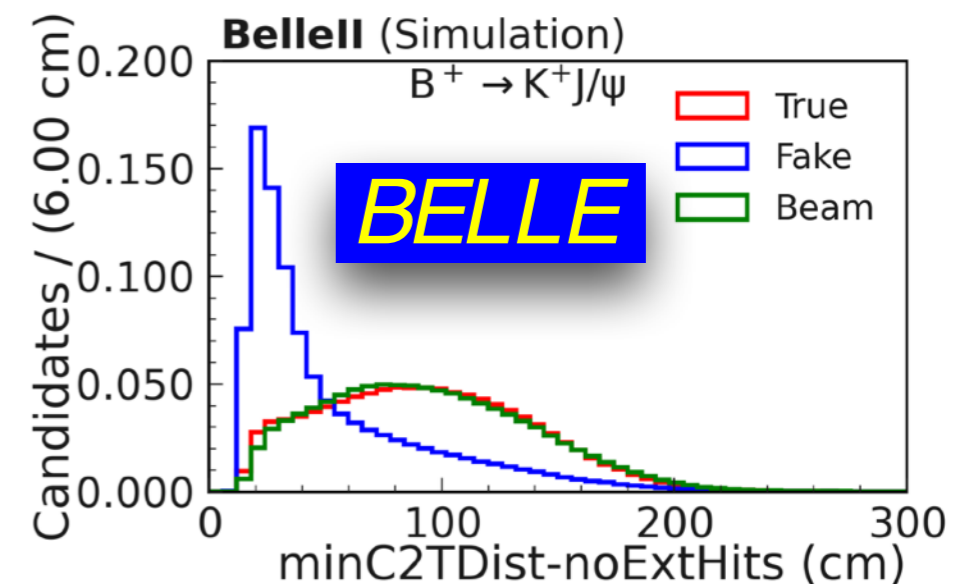
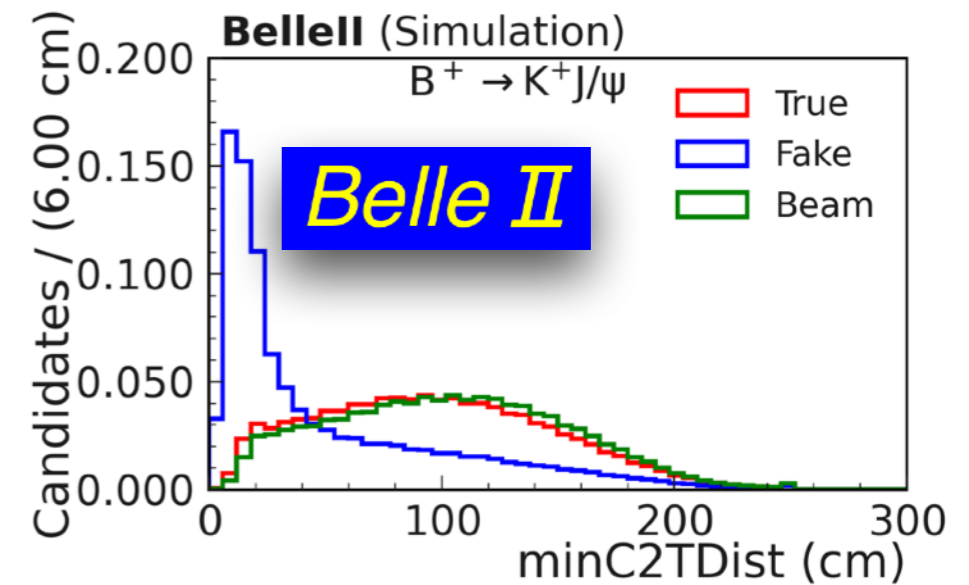
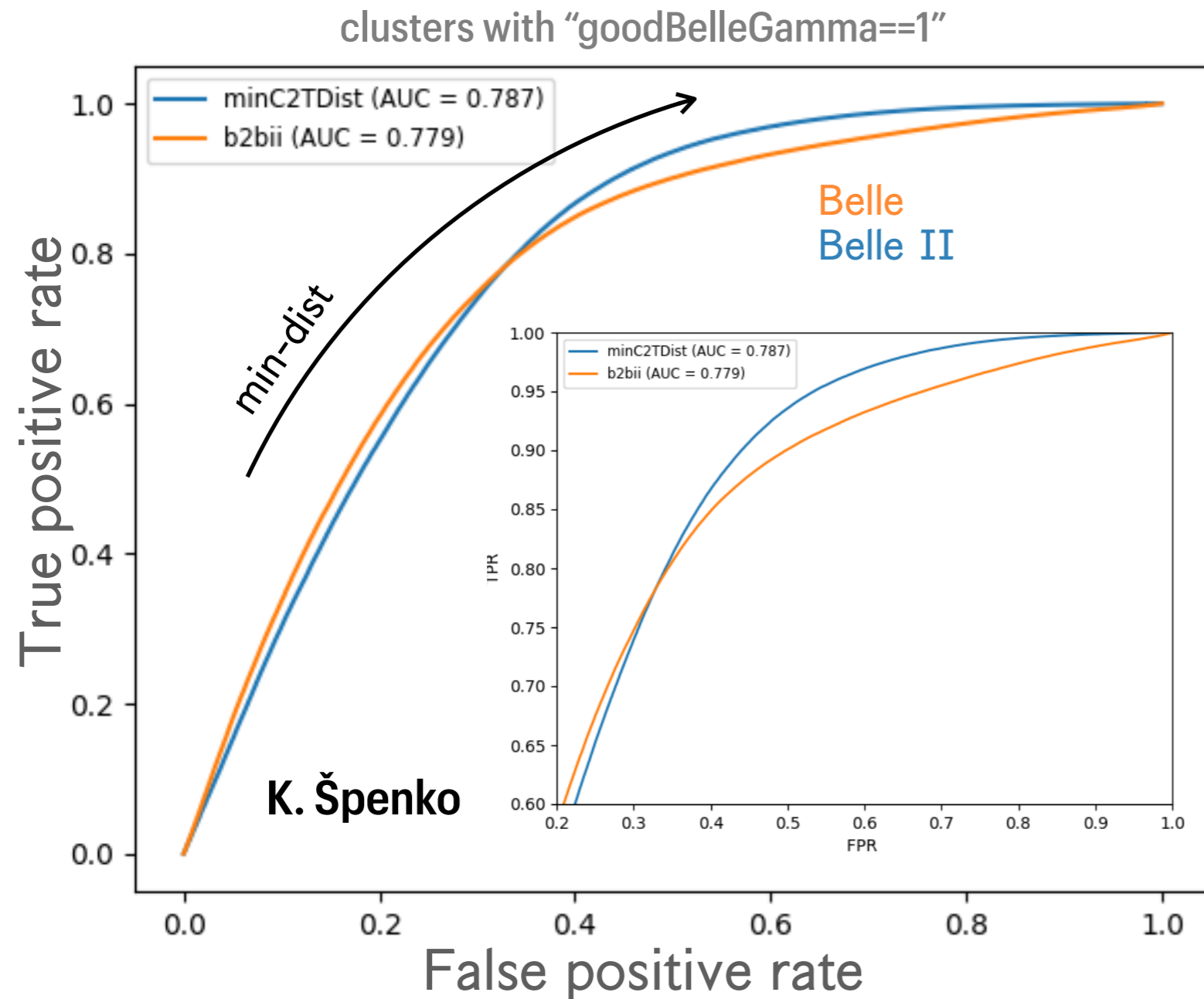
MH Liu



MINIMUM DISTANCE VARIABLE(S)

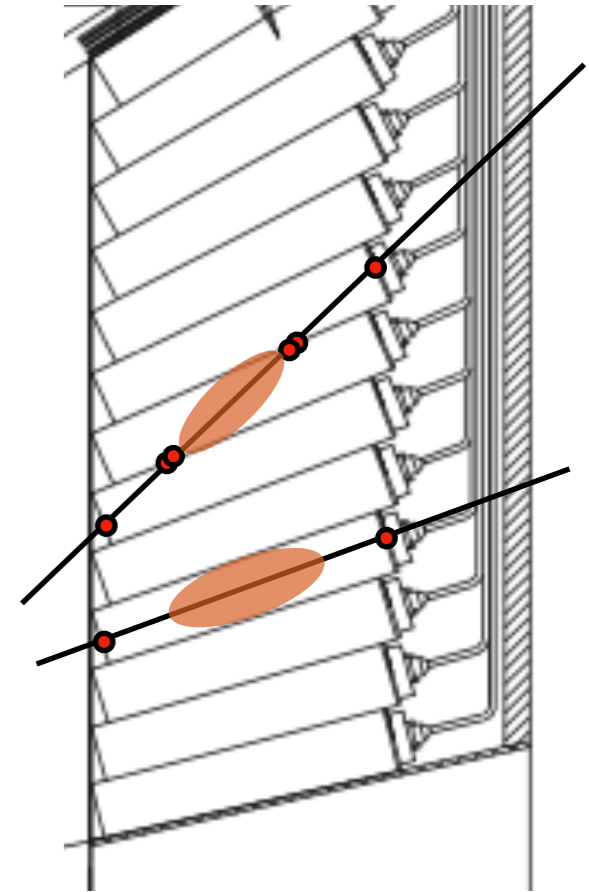
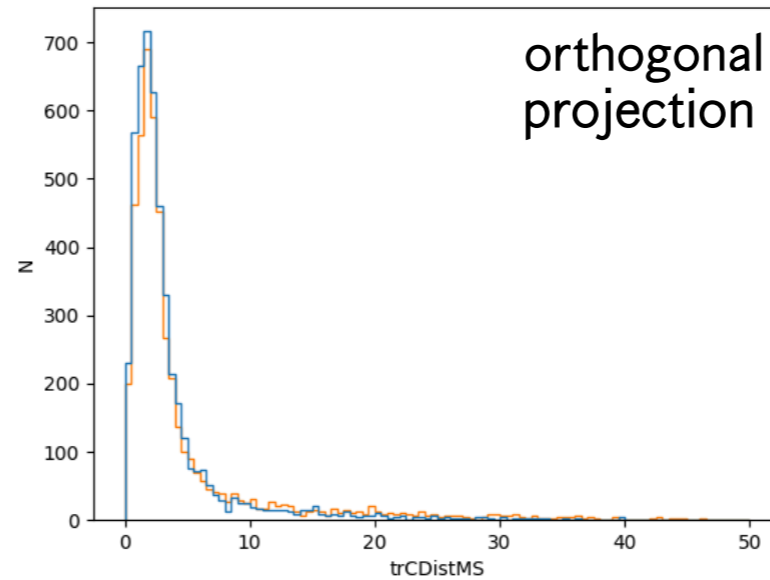
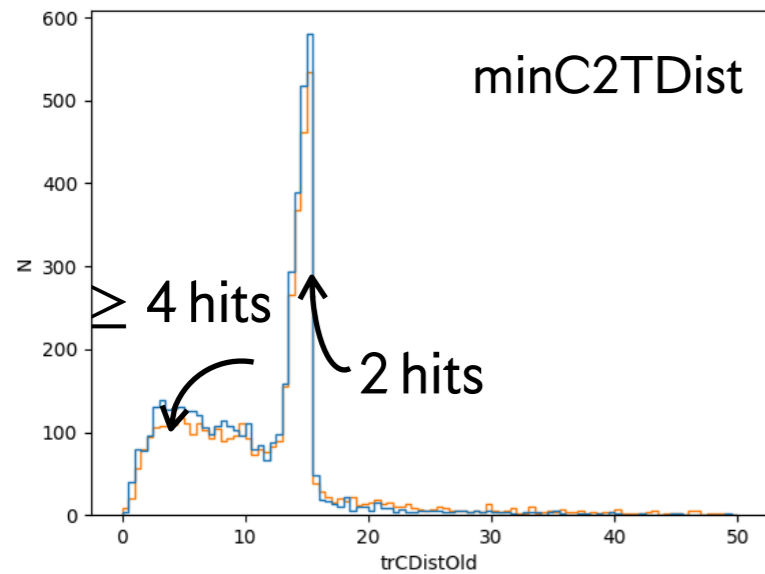
- minC2TDist is defined as the distance between the ECL cluster and its nearest track
It is computed using the hits (extHits) obtained extrapolating the tracks through the ECL
- The extHits are not available in Belle mdsts → obtained by computing the entrance point of the tracks into ECL surface based on helix extrapolation

MH Liu



MINIMUM DISTANCE VARIABLE(S)

- Studies with track particle gun have shown that for small track-cluster distances the current definition leads to unexpected feature
- A better resolution would be achieved using the projection

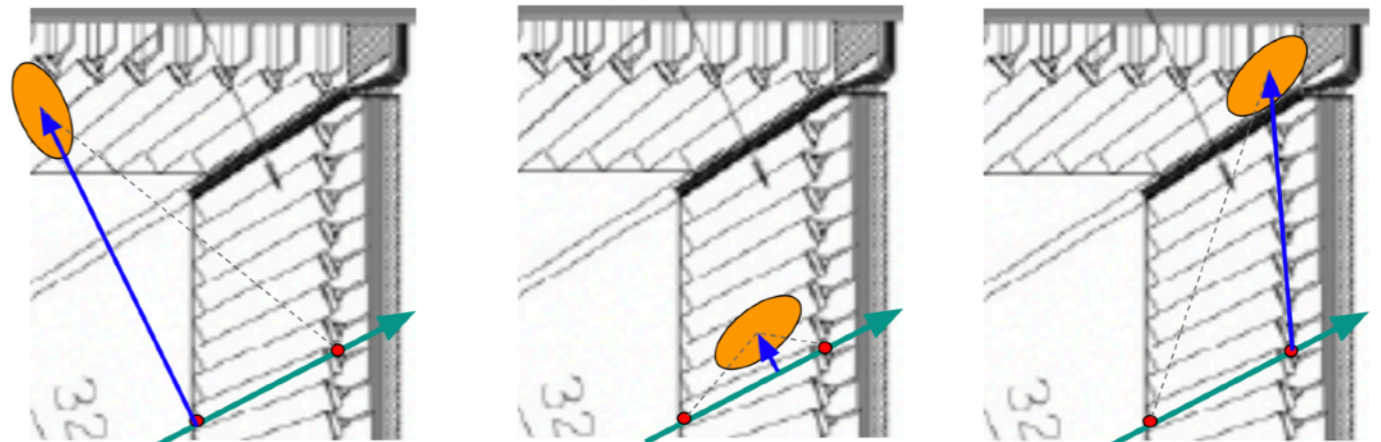


Possible improvements

- Use either the extHit-cluster distance (minC2TDist) or the orthogonal distance based on the relative position
- Use helix extrapolation whenever the extHits are not available
- Do not consider tracks for which $E(\text{track}, \pi \text{ mass}) < E(\text{cluster})$

From K. Špenko
#9856

No large improvement is expected for analyses using E_{ECL} !!



Possible sources of discrepancies between data and MC

- `Split-offs`

Detailed modelling of hadronic , secondary showers (partial energy deposit) is difficult

- **Additional background**

in data, which can contain more soft photon candidates originating from beam-related background

**Should improve
with MCrd samples**

- **Imperfect modelling**

- of discriminative variables, e.g. photon shower shape variables (lateral moment, number of crystals in a shower) cluster timing \Leftrightarrow simulated detector response

- of generic BB/qq events

DATA/MC DISCREPANCY CHECK

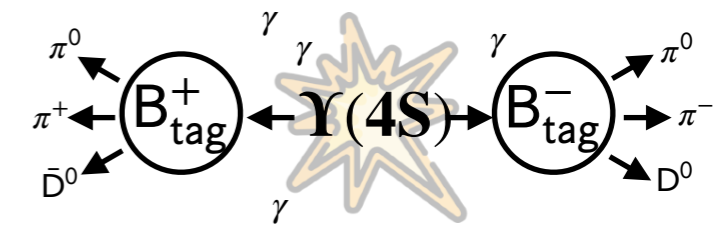
The two variables to check are ROE ECL cluster multiplicity
ROE ECL cluster total energy

How-to get an enriched sample of fake photons and beam background components in ROE

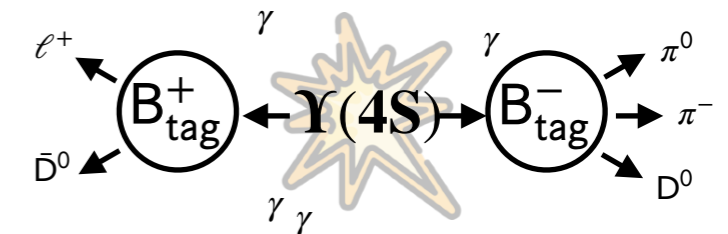
Double tag method

The B_{tag} is reconstructed, say via hadronic FEI, and the second B decay is also exclusively reconstructed

- **Hadronically** \Leftrightarrow Doubly-tagged events (Gaudino et al. $B^+ \rightarrow \tau^+ \nu_\tau$)



- **Semileptonically** \Leftrightarrow e.g. $B^+ \rightarrow \bar{D}^0 \ell^+ \nu_\ell$ (Ghosh et al. $B^+ \rightarrow K^+ \tau^+ \tau^-$)



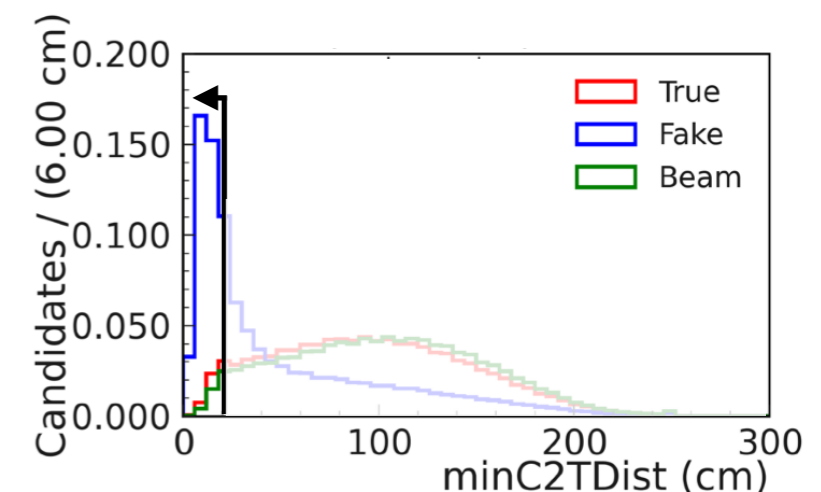
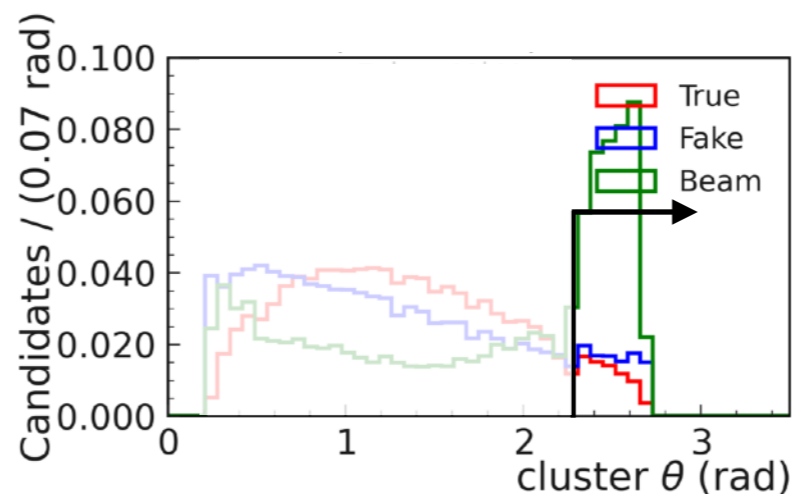
○ Fake photons/splitoffs

minC2TDist ✓

○ Beam background

clusterTiming, worse agreement with data ✗

clusterTheta ✓

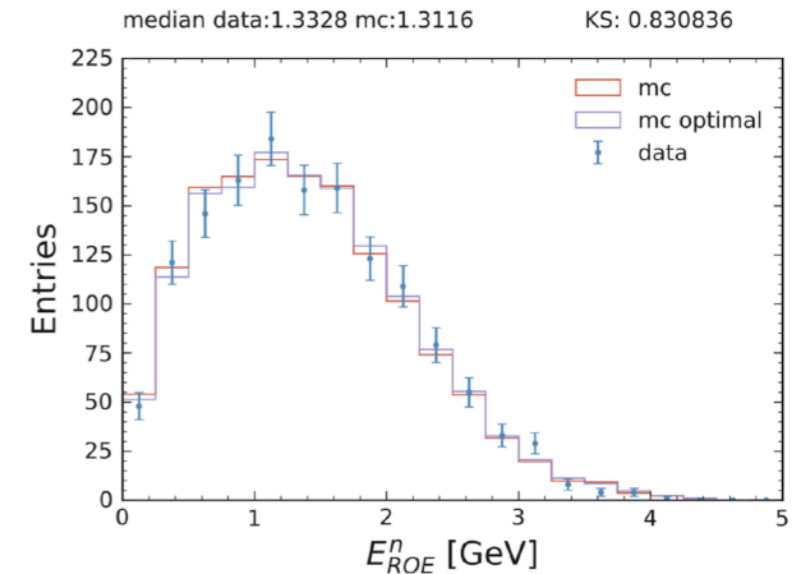
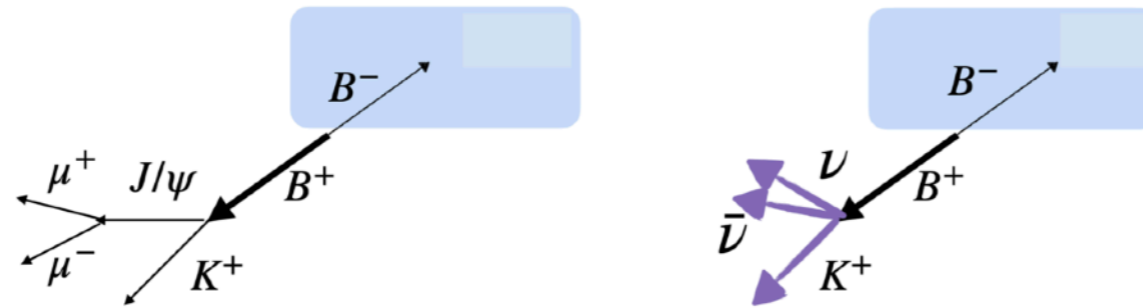


DATA/MC DISCREPANCY CHECK

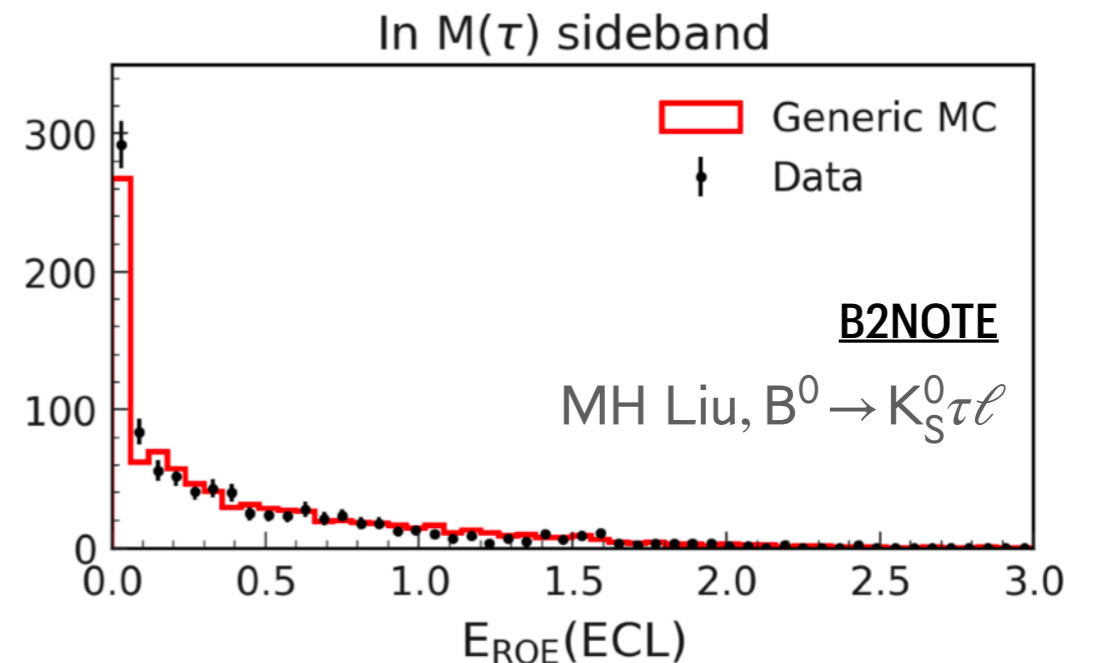
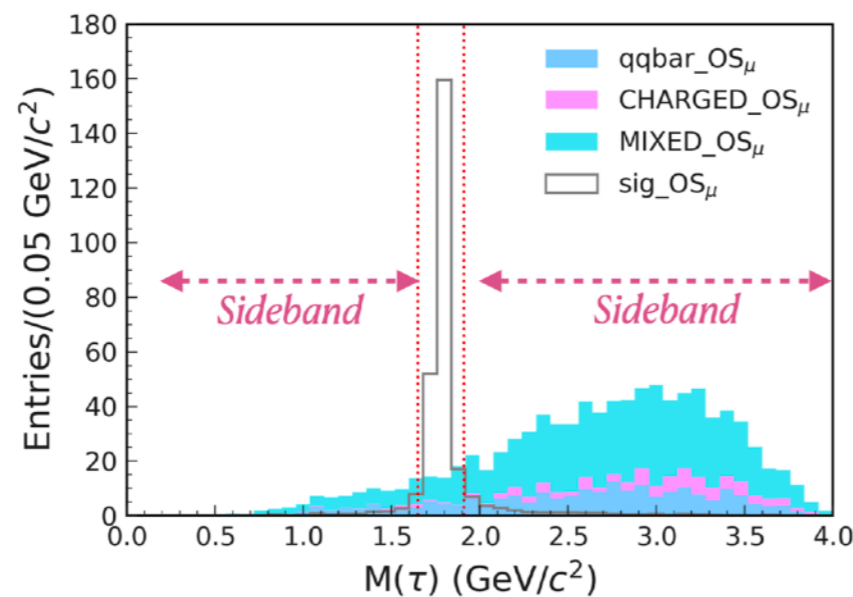
The two variables to check are
 ROE ECL cluster multiplicity
 ROE ECL cluster total energy

How-to validate the agreement?

- Embedded sample $B^+ \rightarrow J/\psi(\mu\mu)K^+$ ($B^+ \rightarrow K^+\nu\nu$)



- Analysis-specific sidebands (same flavor: S Moneta, tau sideband: MH Liu)



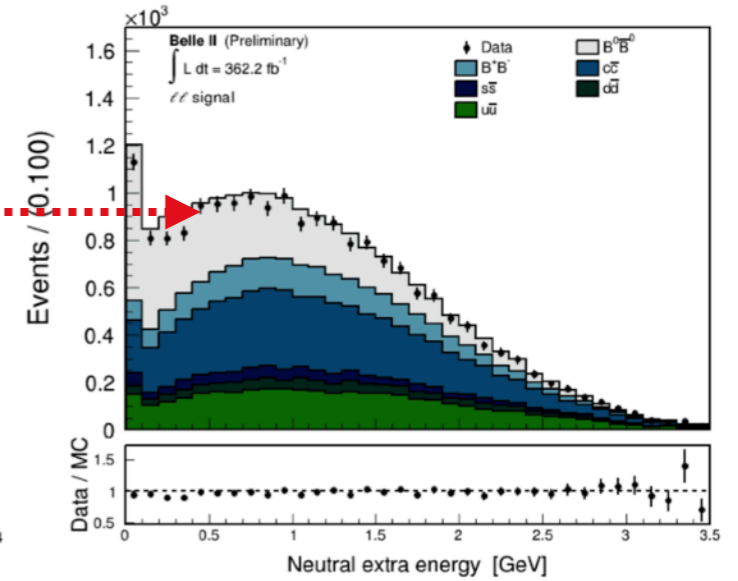
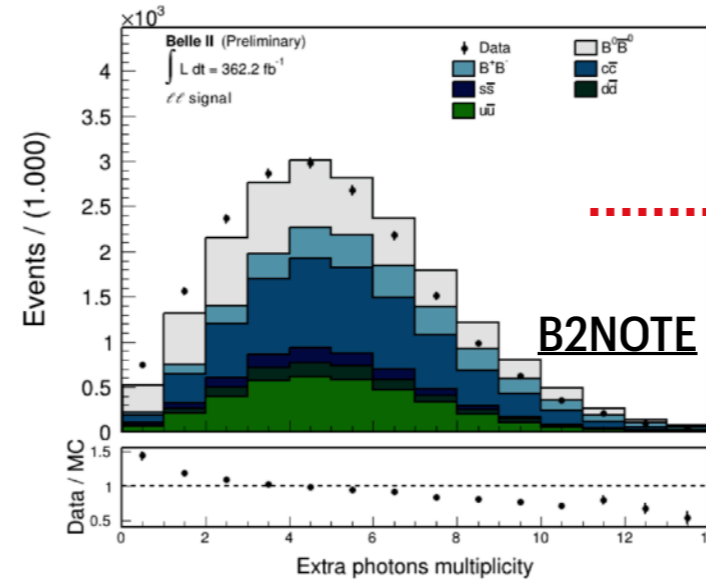
DATA/MC DISCREPANCY CORRECTIONS

Two possible approaches to correct for data/MC disagreements

1. Correct the overall MC cluster multiplicity

This should improve the agreement in E_{ECL}

$w_i = n_i^{DATA} / n_i^{MC}$ applied to the events belonging to the i -th bin

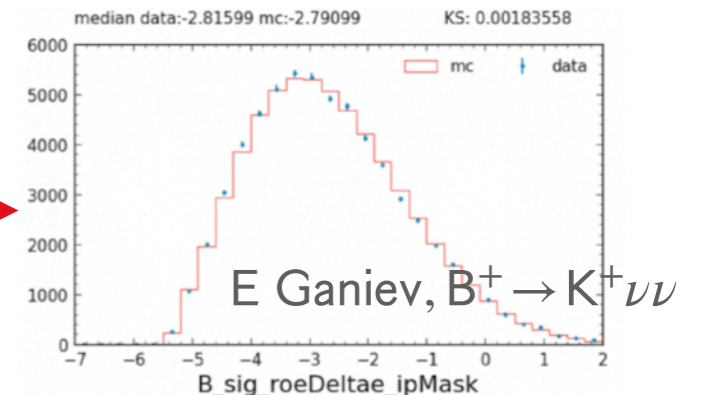
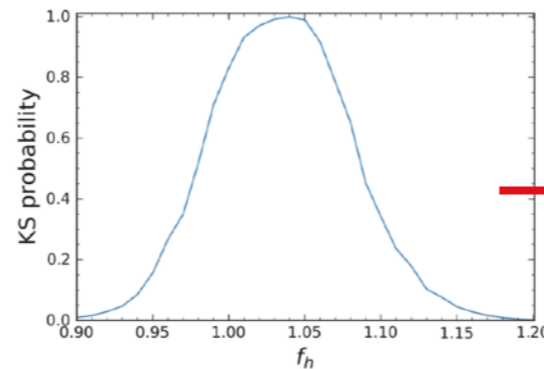
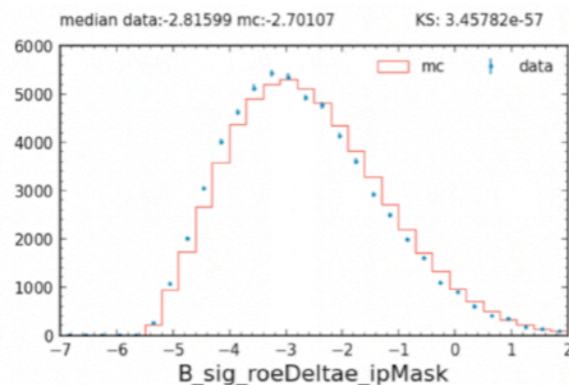


2. Correct E_{ECL}

Looks like it is enough to rescale the fake photon component for an overall better agreement

$$E_{ECL}(f_h) = \sum_{i \in \text{match}} E_i + f_h \sum_{j \in \text{unm}} E_j$$

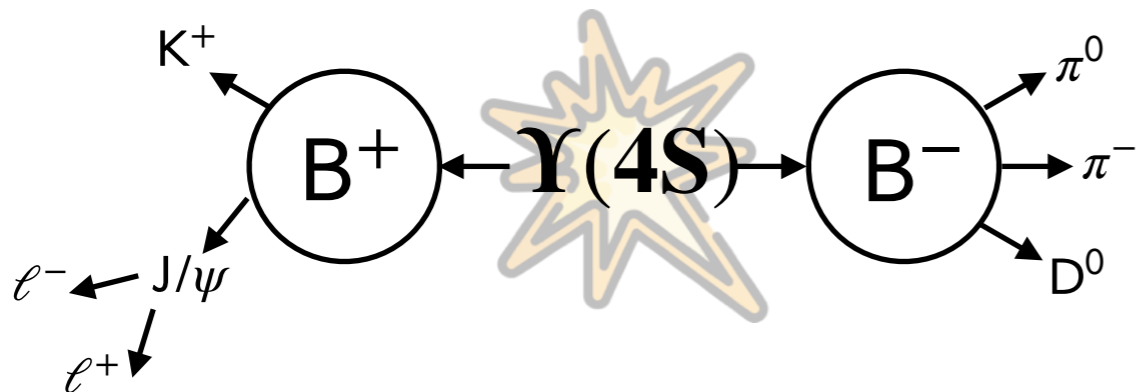
Scan in f_h to find best agreement



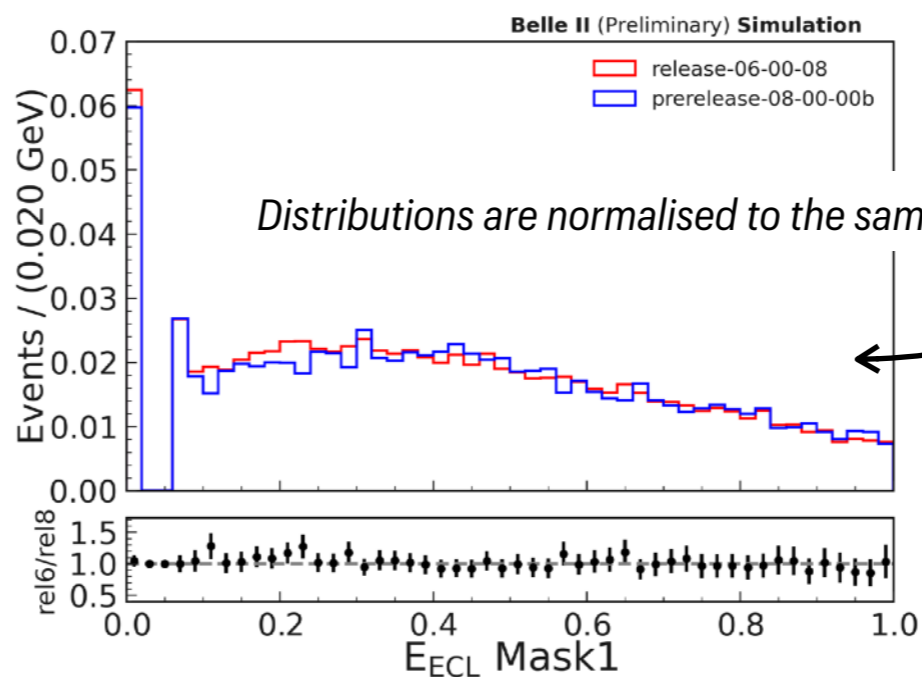
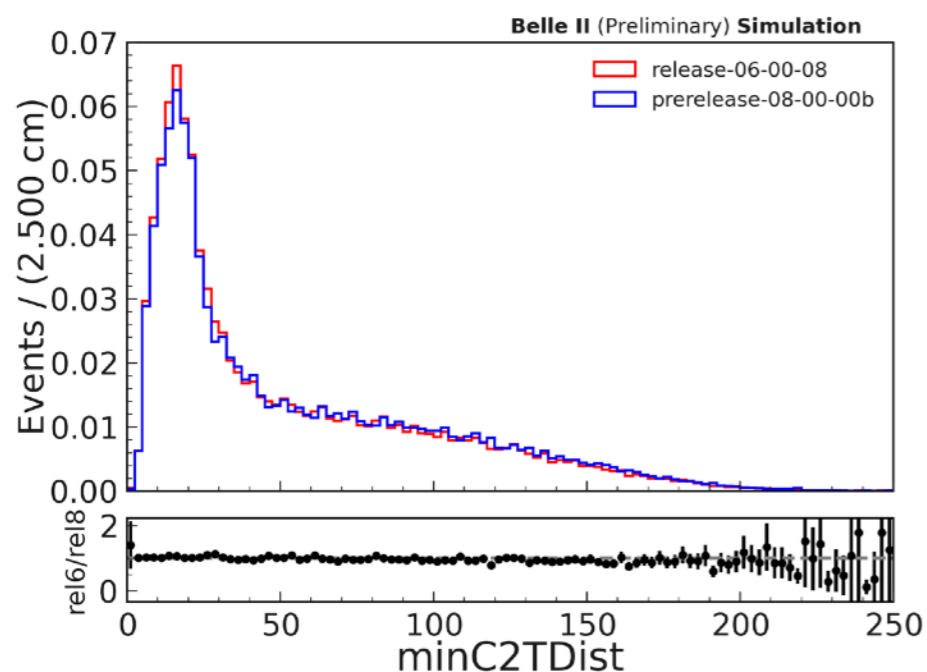
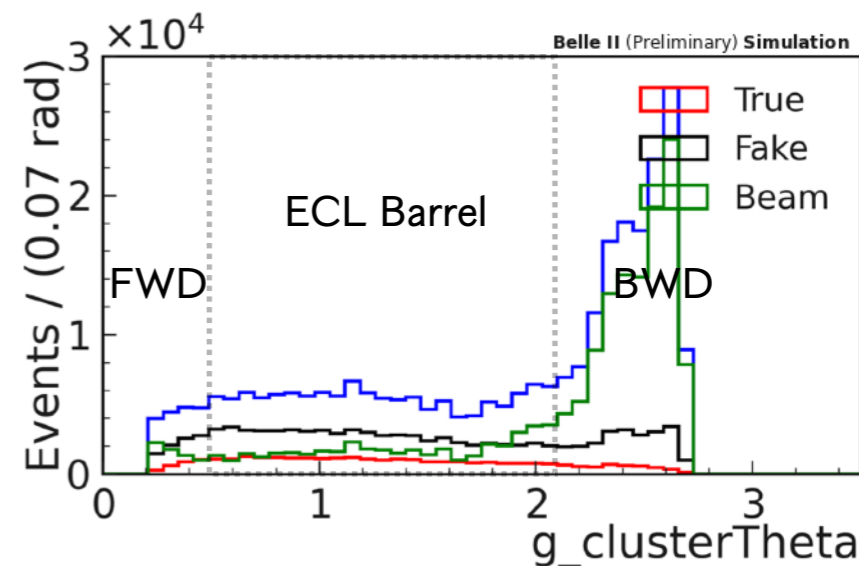
E Ganiev, $B^+ \rightarrow K^+ \nu \nu$

E_{ECL} IN VALIDATION

Additional mode for the validation framework to probe ECL reconstruction for missing energy analyses



Good for MC, but for data?



Distributions are normalised to the same area

goodGamma: $E > 0.1$ GeV (FWD)
 $E > 0.06$ GeV (BRR)
 $E > 0.15$ GeV (BWD)
 +TM B_{tag}

E_{ECL} FIT, PEAKING BACKGROUNDS

- When using E_{ECL} to extract the signal one must consider the presence of peaking backgrounds and derive the relative syst. uncertainties
- Modes with K_L⁰, n can be interpreted as missing energy, thus mimicking the signal

Lot of investigation from EWP/WG1 people on

- X_c → K_L⁰X

MC modelling (BF, decay models)

Upscaling of X_c → K_L⁰X components

- K_L⁰ reconstruction efficiency in ECL

Calibration

Inefficient MCMatching for K_L⁰ - induced energy deposits in ECL

Introduced a geometrical matching K_L⁰-ECL cluster

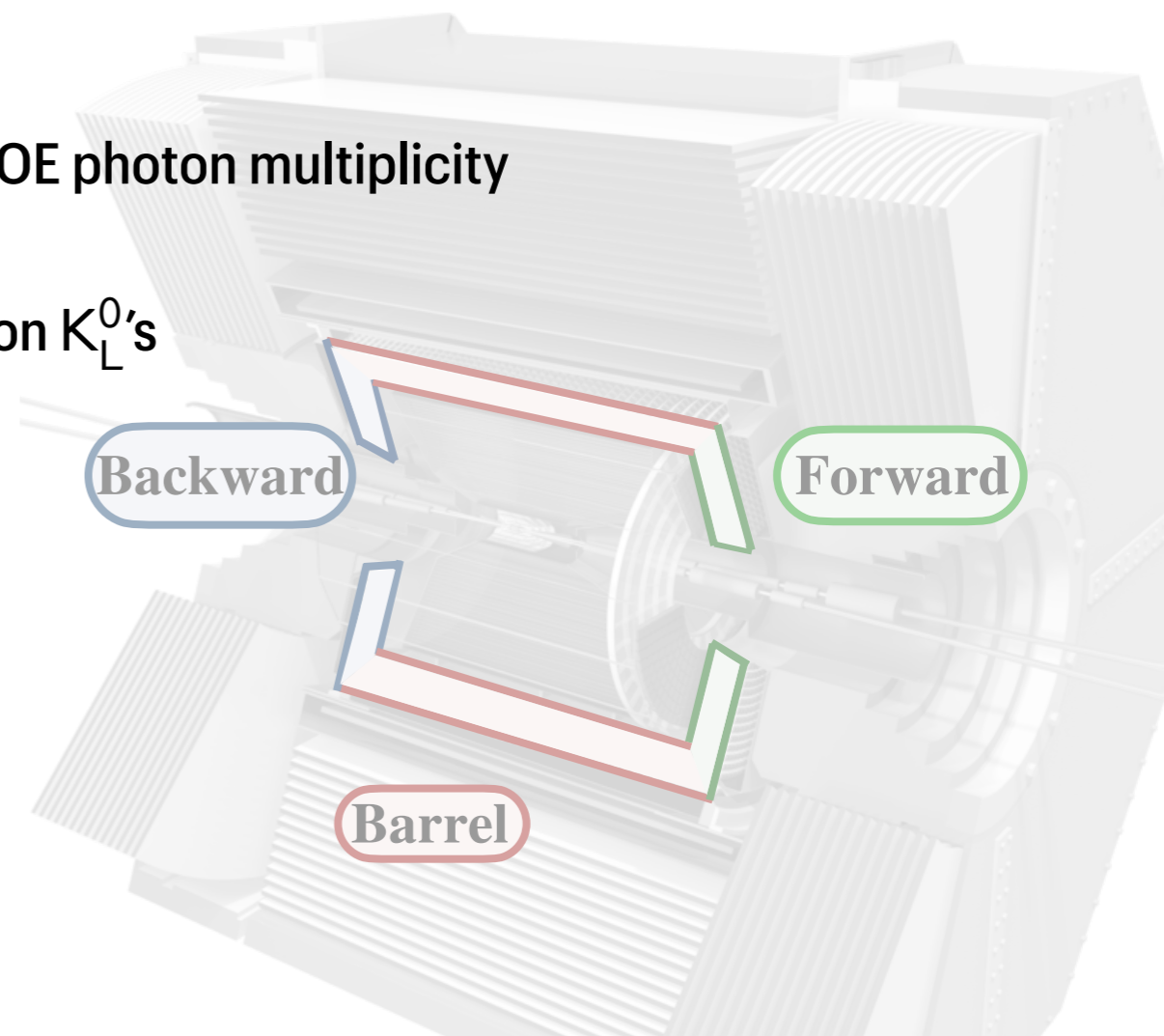
	PDG	MC
$\mathcal{B}(D^+ \rightarrow K^0/\bar{K}^0 X)$	$(61 \pm 5) \%$	57.5
$\mathcal{B}(D^0 \rightarrow K^0/\bar{K}^0 X)$	$(47 \pm 4) \%$	40

More details from the
[Joint \(S\)L/EWP mini-workshop](#)



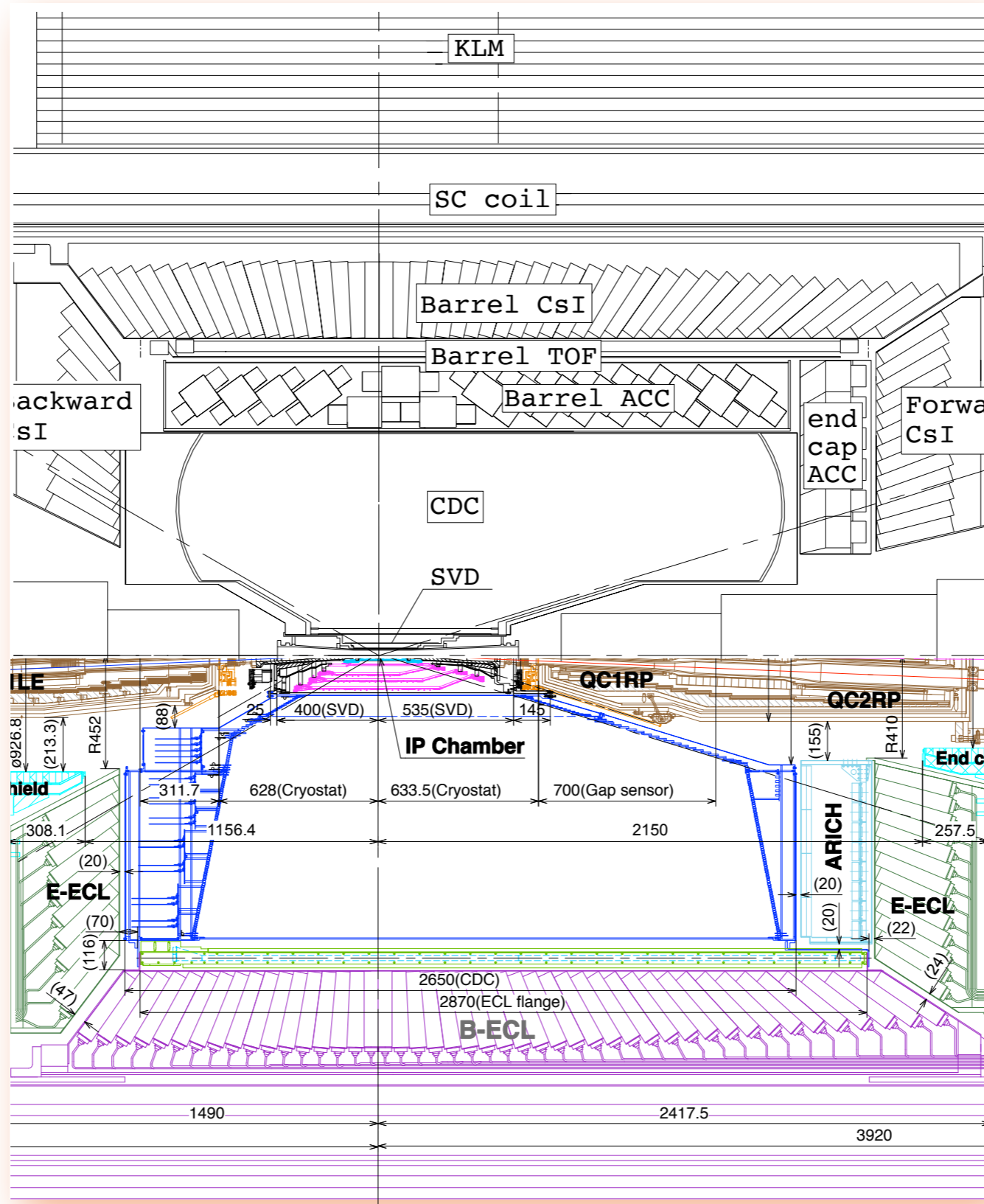
CONCLUSION

- Excellent control on E_{ECL} is a crucial element for missing energy analyses
- Belle/Belle II specificities for combined analyses
- MVA's pros&cons
- Correction approaches: fake ROE photons energy, ROE photon multiplicity
- Need for more: feedback on MCrd samples, studies on K_L^0 's

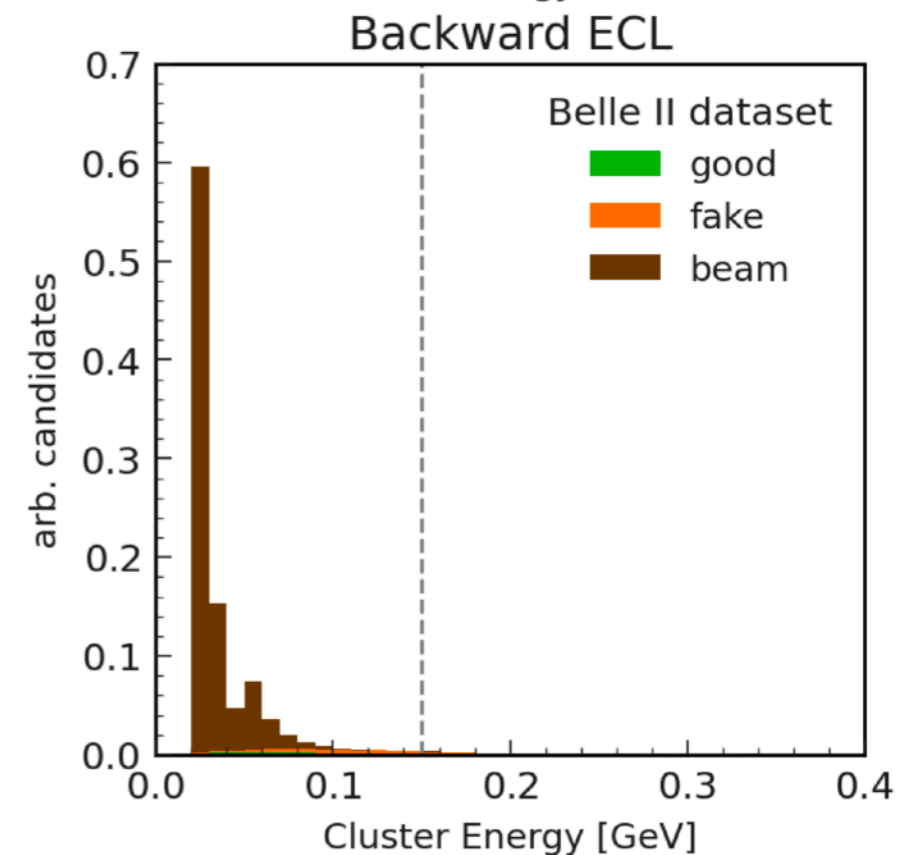
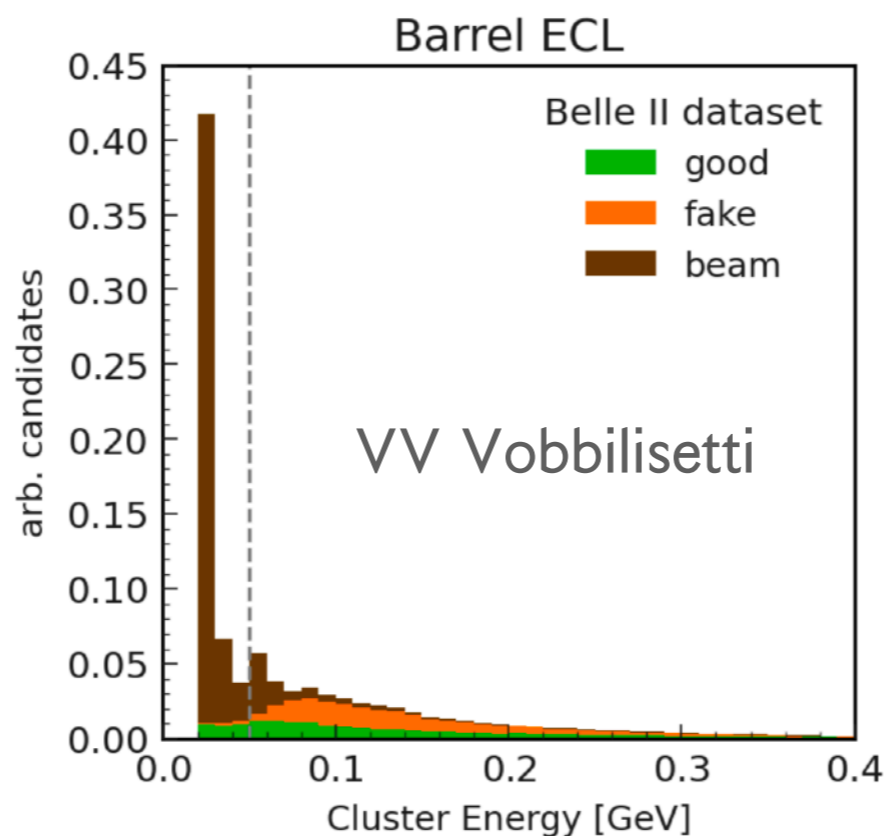
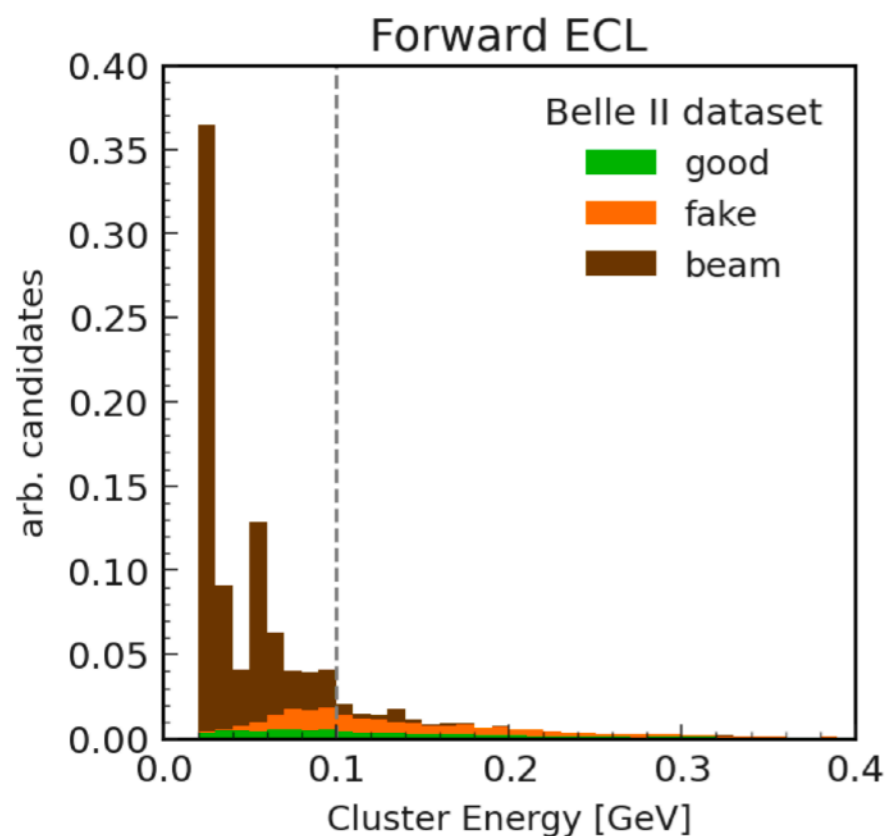
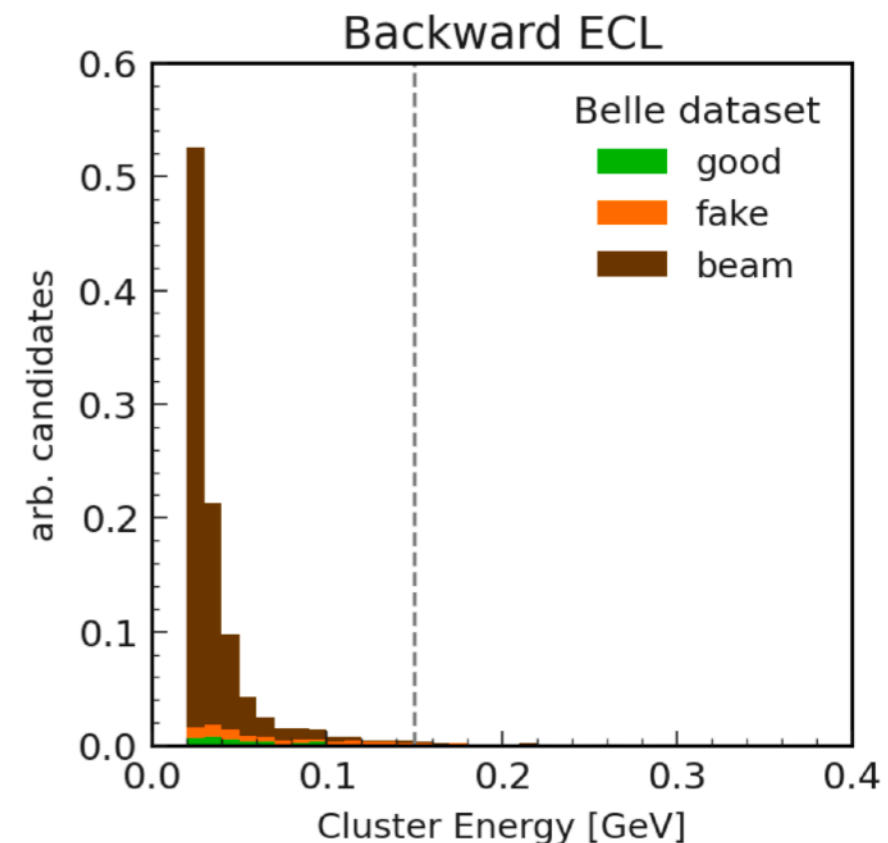
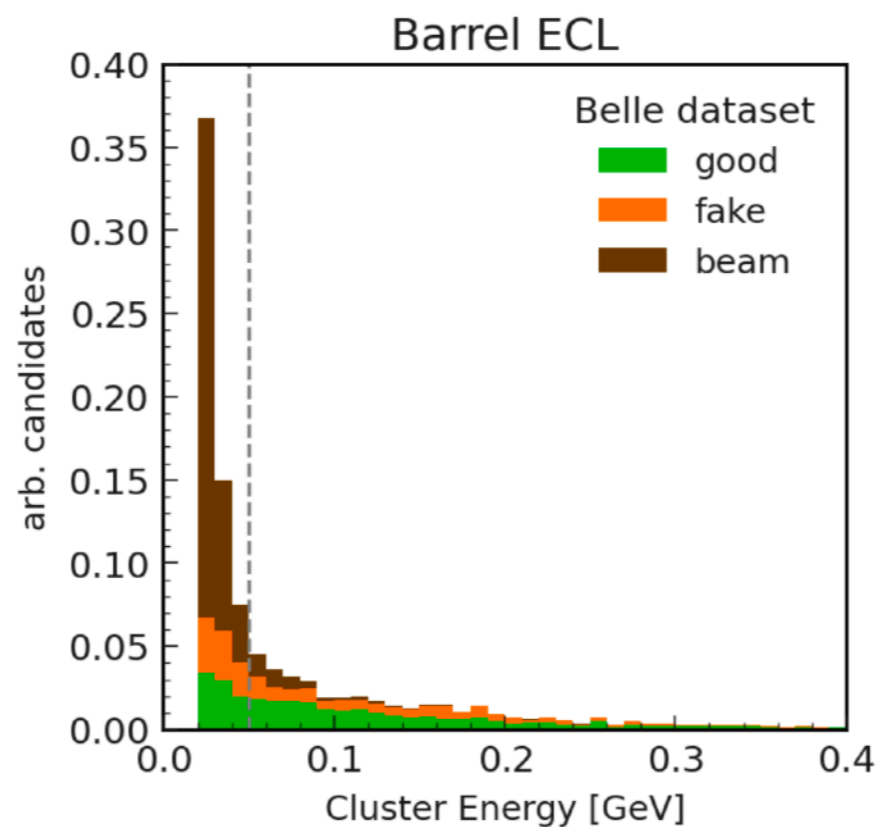
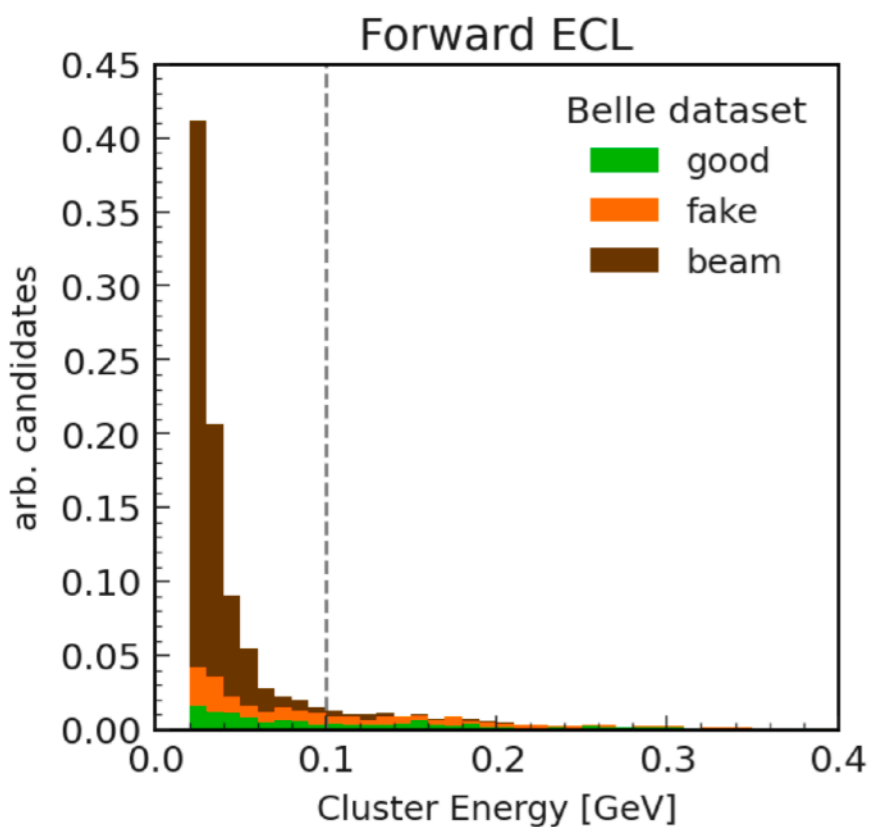


ADDITIONAL MATERIAL

ECL, REMINDER

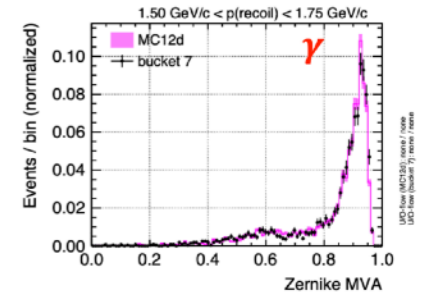
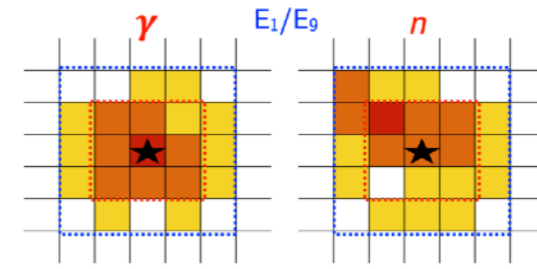


PHOTONS SPECTRA

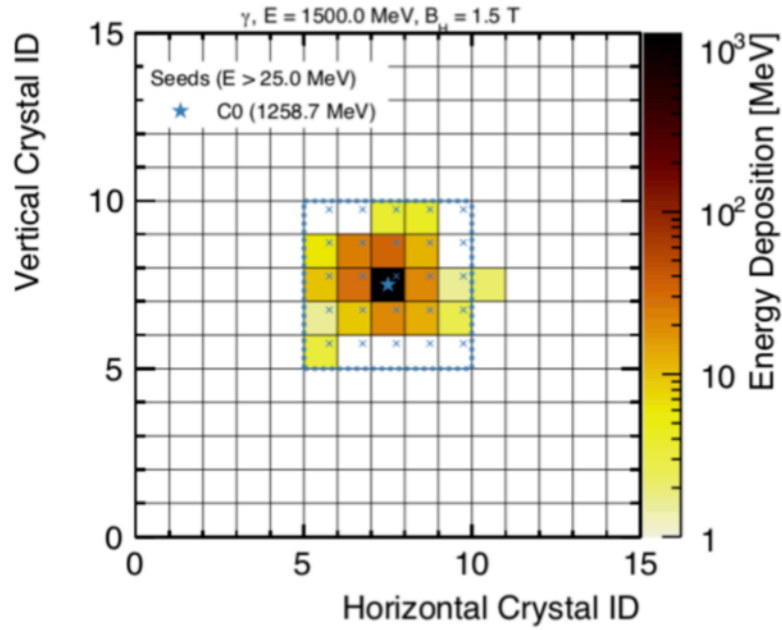


2023.11.03 - GdM - PHYSICS WEEK

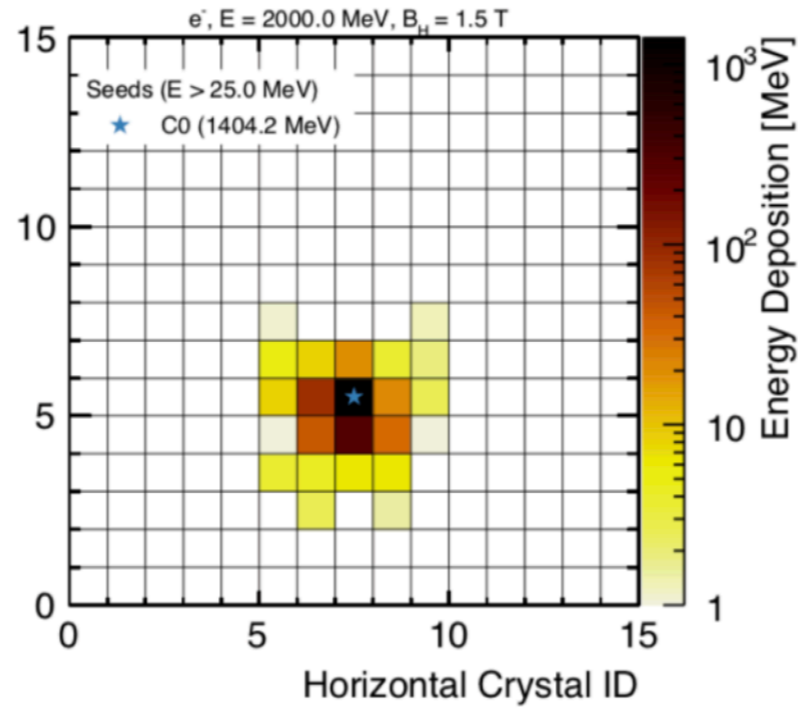
CLUSTER SHAPE



- Radially symmetric shape
- Usually contained in 5x5 cells

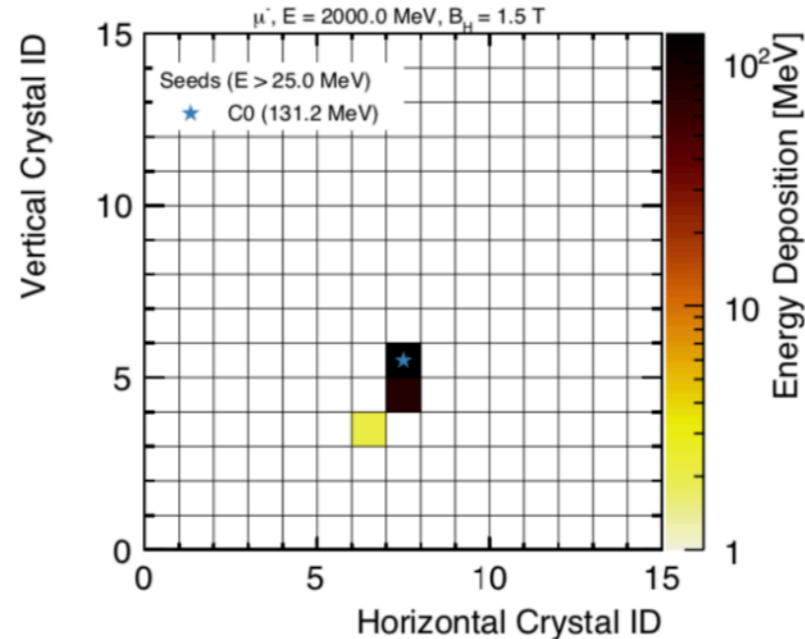
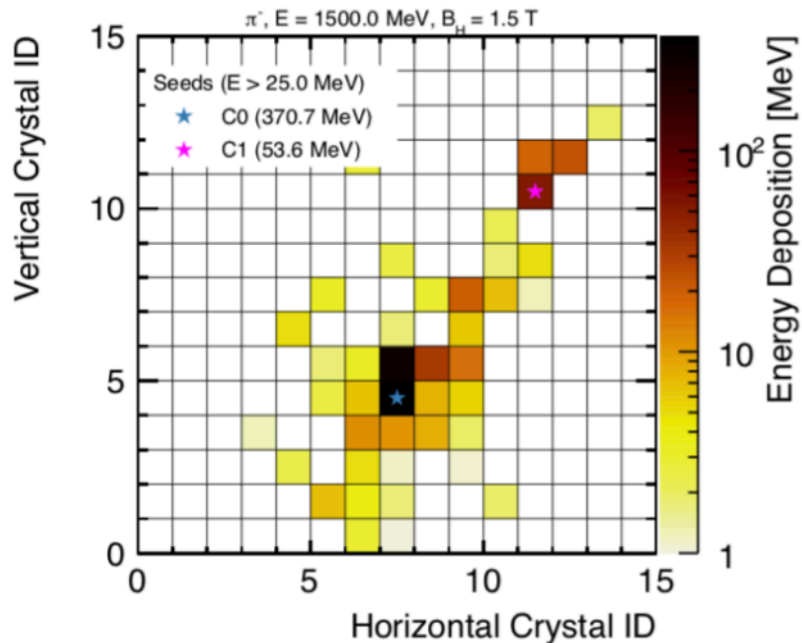


- Similar shape to gamma
- Less symmetric (B field bend and Brems gamma emitted before ECL)



- Ionisation loss contained in 1-2 cells
- Asymmetric lateral spread due to hadronic interactions

- Pure MIP behaviour
- $\langle E_{cluster} \rangle \sim 200$ MeV



clusterZernikeMVA

Returns output of a MVA using eleven Zernike moments of the cluster. Zernike moments are calculated per shower in a plane perpendicular to the shower direction via

$$|Z_{nm}| = \frac{n+1}{\pi} \frac{1}{\sum_i w_i E_i} \left| \sum_i R_{nm}(\rho_i) e^{-im\alpha_i} w_i E_i \right|$$

where n , m are the integers, i runs over the crystals in the shower, E_i is the energy of the i -th crystal in the shower, R_{nm} is a polynomial of degree n , ρ_i is the radial distance of the i -th crystal in the perpendicular plane, and α_i is the polar angle of the i -th crystal in the perpendicular plane. As a crystal can be related to more than one shower, w_i is the fraction of the energy of the i -th crystal associated with the shower.

More details about the implementation can be found in [BELLE2-NOTE-TE-2017-001](#) .

More details about Zernike polynomials can be found in [Wikipedia](#) .

For cluster with hypothesisId==N1: raw MVA output.

For cluster with hypothesisId==N2: $1 - \text{prod}\{\text{clusterZernikeMVA}\}$, where the product is on all N1 showers belonging to the same connected region (shower shape variable).

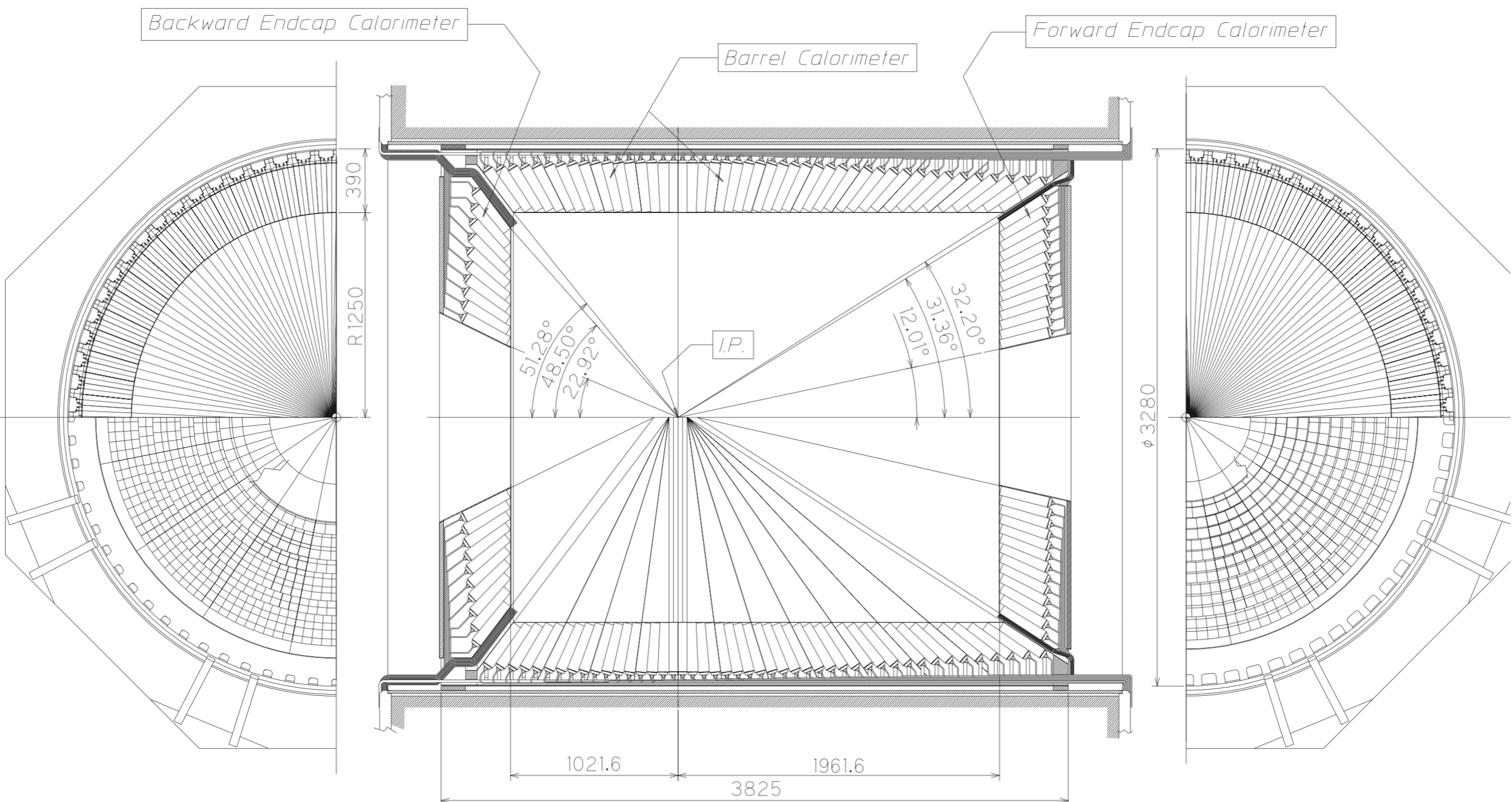
clusterPulseShapeDiscriminationMVA

Returns MVA classifier that uses pulse shape discrimination to identify electromagnetic vs hadronic showers.

- 1 for electromagnetic showers
- 0 for hadronic showers

Table 47: Summary of experimental measurements of semitauonic B decays.

Exp.	Tag method	τ^- decays	Observables	Fit variables
Belle [249]	Untagged	$e^- \nu_\tau \bar{\nu}_e, \pi \nu_\tau$	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau)$	M_{bc}^{tag}
Belle [266]	Untagged	$\ell^- \nu_\tau \bar{\nu}_\ell, \pi \nu_\tau$	$\mathcal{B}(B^- \rightarrow D^{(*)0} \tau^- \bar{\nu}_\tau)$	M_{bc}^{tag} and p_{D^0}
Belle [251]	Hadronic	$\ell^- \nu_\tau \bar{\nu}_\ell$	$R_D, R_{D^*}, q^2, p_\ell^* $	M_{miss}^2 and \mathcal{O}_{NB}
Belle [267]	Semileptonic	$\ell^- \nu_\tau \bar{\nu}_\ell$	$R_{D^*}, p_\ell^* , p_{D^*}^* $	E_{ECL} and \mathcal{O}_{NB}
Belle [268]	Hadronic	$h^- \nu_\tau$	$R_{D^*}, P_\tau(D^*)$	E_{ECL} and $\cos \theta_{\text{hel}}$
Belle [269]	Semileptonic	$\ell^- \nu_\tau \bar{\nu}_\ell$	R_D, R_{D^*}	E_{ECL} and \mathcal{O}_{BDT}
BaBar [250, 270]	Hadronic	$\ell^- \nu_\tau \bar{\nu}_\ell$	R_D, R_{D^*}, q^2	M_{miss}^2 and p_ℓ
LHCb [252]	—	$\ell^- \nu_\tau \bar{\nu}_\ell$	$E_\mu^*, m_{\text{miss}}^2, q^2$	
LHCb [271]	—	$h^- h^+ h^- \nu_\tau$	$q^2, t_\tau, \mathcal{O}_{BDT}$	



unit (mm)

