$|V_{cb}|$ Discussion Items

Florian & Paolo with **your** Input

Inclusive Decays – Discussion Items

- Global $|V_{cb}|$ fits
  - **Correlations** of **theory** errors
  - **Correlations** of **experimental** values
- **QED corrections** → also see Marzia's talk tomorrow
- **Useful measurements** @ Belle II
  - Redo spectral moment measurements of q^2, M_X , n_X, and E_l moments in a single analysis
    - Very valuable to capture the full experimental correlations
  - A_{FB} & other differential measurements (q^2, M_X and E_l moments for forward and backward events)
  - Do measurements w/ and w/o QED FSR corrections
- Differential measurements could be useful to validate or use lattice information

Inclusive Decays – Discussion Items

- Useful Calculations:
  - Complete **O(α_s)** for q^2 and analytica formulae for M_X, E_l moments
  - q^2 moments at **O($1/m_b^4$, $1/m^5$)**
  - **QED**
- Lattice Input to constrain or test relevance of higher order corrections?
- What is the **ultimate** precision that can be reached? [arXiv:2310.20324]

The q^2 moments in inclusive semileptonic B decays

G. Finauri[a] P. Gambino[a,b,c]

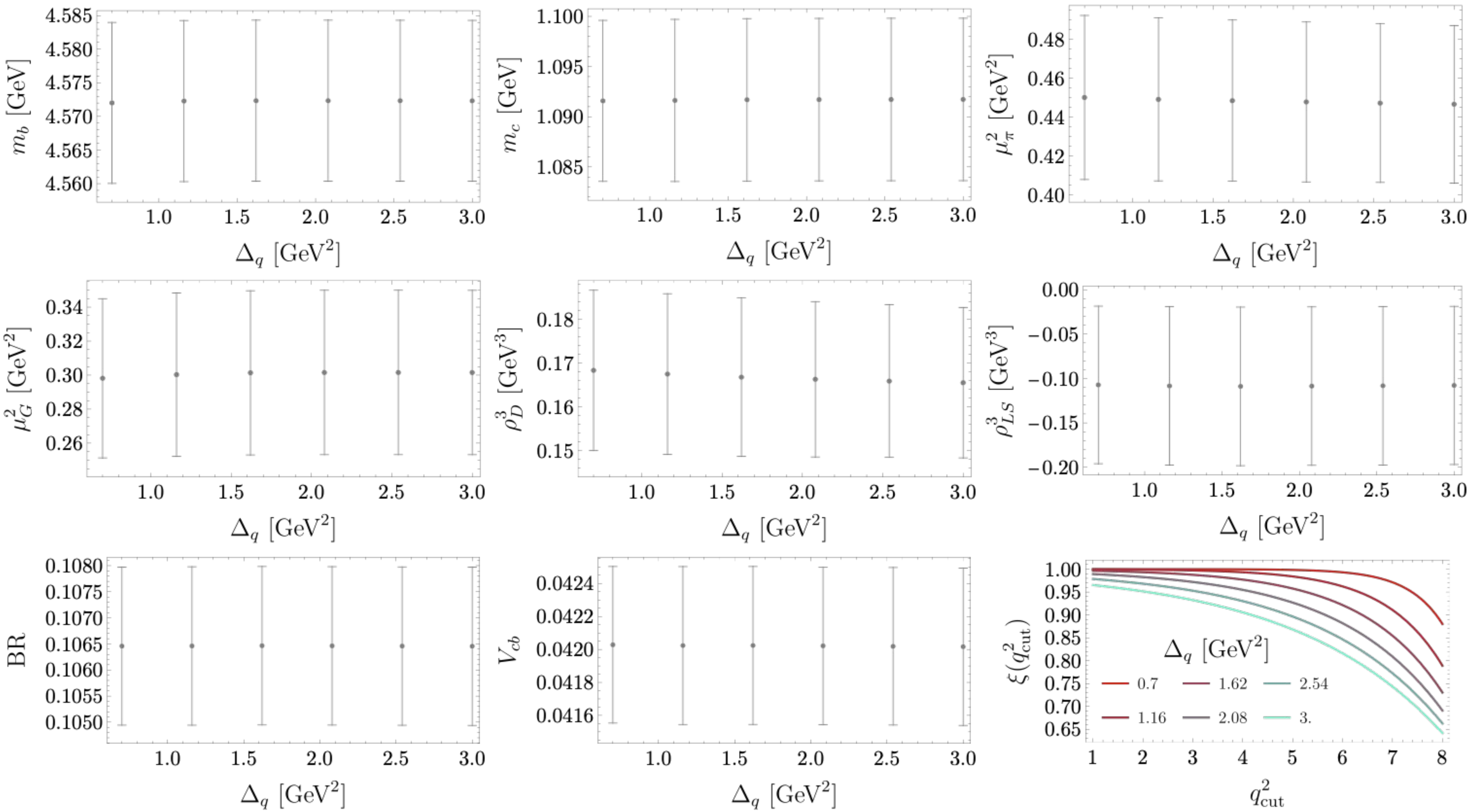
$$|V_{cb}| = (41.97 \pm 0.27_{exp} \pm 0.31_{th} \pm 0.25_{\Gamma}) \times 10^{-3} = (41.97 \pm 0.48) \times 10^{-3}\,.$$

Inclusive Decays – Discussion Items

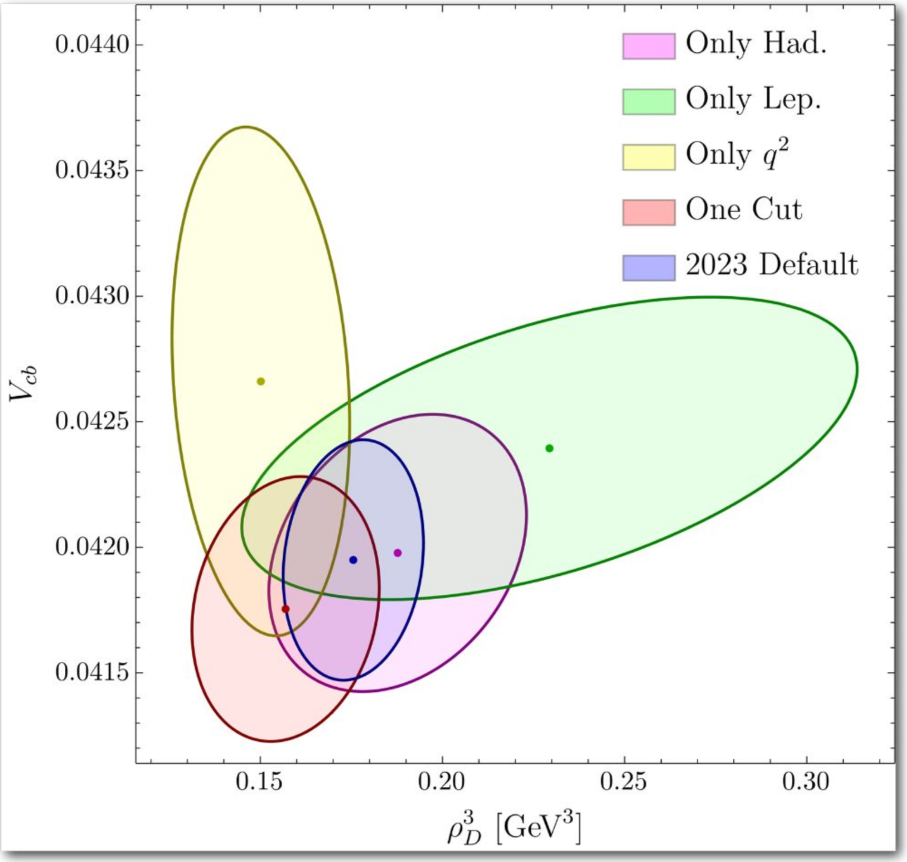
- Should provide measurements w/o FSR QED effects
  - Very interesting to test our understanding of QED !
- BF BF BFs – we need **new** measurements

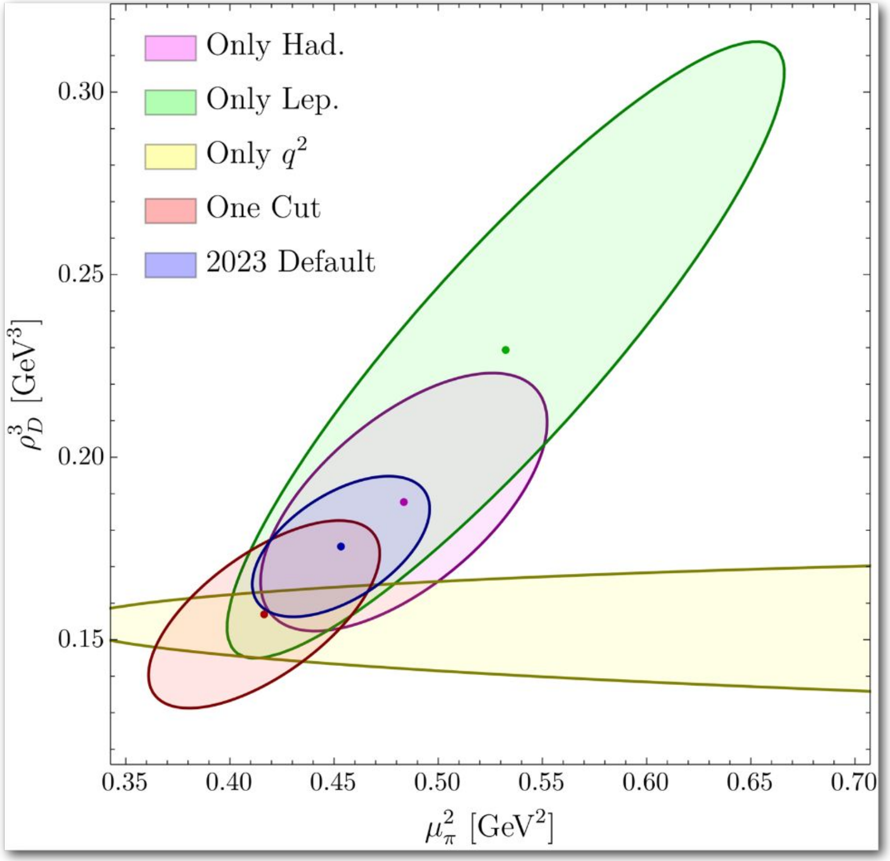
	$\mathcal{B}(B \to X \ell \bar{\nu}_\ell)$ (%)	$\mathcal{B}(B \to X_c \ell \bar{\nu}_\ell)$ (%)	In Average
Belle [63] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.54 ± 0.31	✓
Belle [63] $E_\ell > 0.4\,\mathrm{GeV}$	-	10.58 ± 0.32	
CLEO [65] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [65] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [62] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [64] $E_\ell > 0.6\,\mathrm{GeV}$	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [63] & BaBar [64] ($E_\ell > 0.6\,\mathrm{GeV}$)	-	10.63 ± 0.19	

Theory Correlations in $\mathbf{q^2}$ Moments

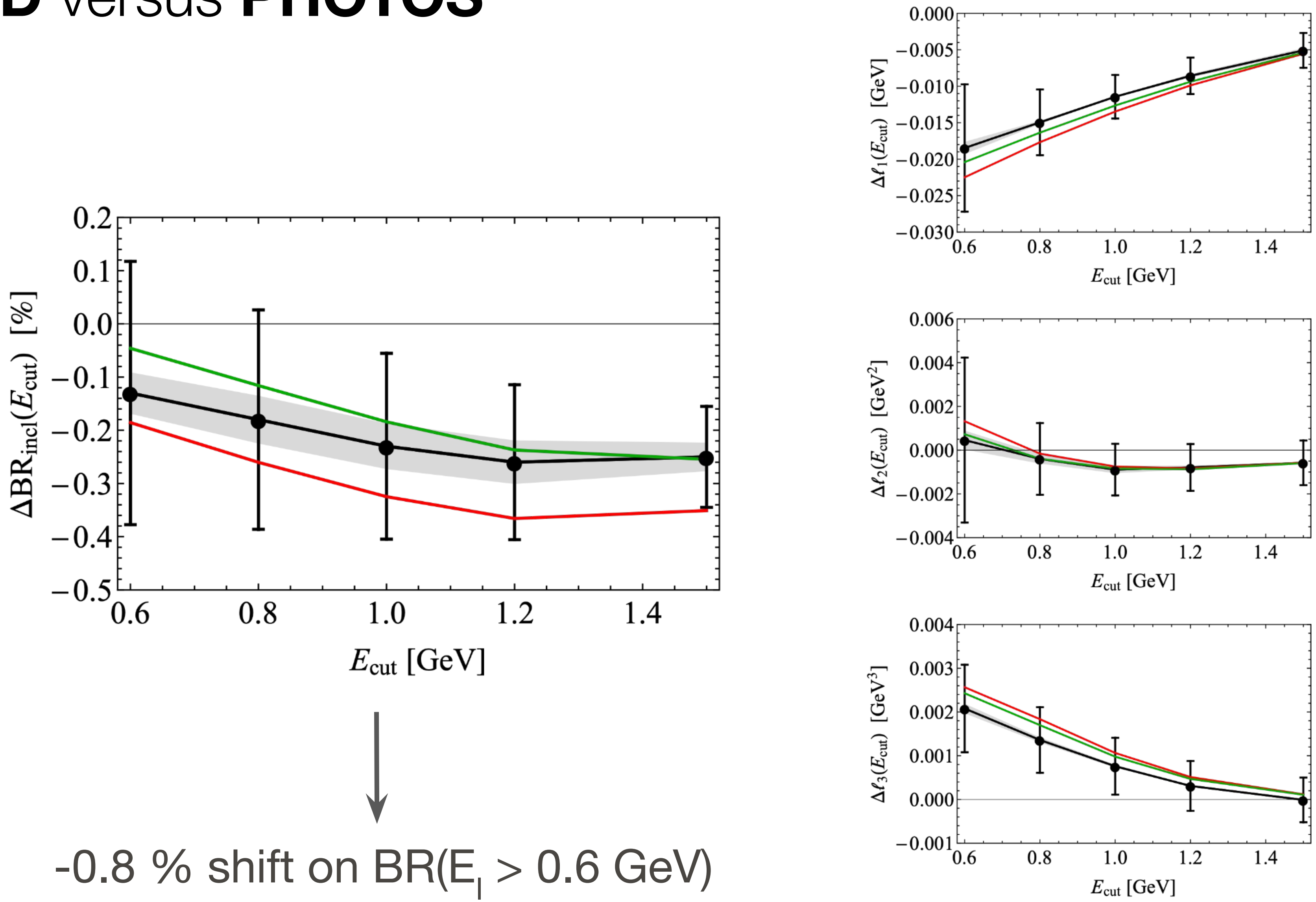


Details from Global Fit of [arXiv:2310.20324]





QED versus **PHOTOS**



-0.8 % shift on BR(E_l > 0.6 GeV)

Exclusive Decays

- How to truncate the **BGL expansion**? How to implement unitarity?
- QED correction and Coulomb factor
  - In inclusive decays, it dominates the correction to the total width (but not the w spectrum)
  - Structure Dependent contributions?
  - $1 + \alpha\pi$ from scalar QED (B^0 ->D, D*) versus ($1 + \mathbf{4/9}\,\alpha\pi$) inclusive (both B^0 and B^+)
- Lattice Calculations prospects in 3 years?
- Assumptions of isospin (+/- 1%)

Exclusive Decays

- What should be measured?
  - Angular coefficients are an excellent way to parametrize and share results.

New on arxiv today!
https://arxiv.org/pdf/2310.20286.pdf

Measurement of Angular Coefficients of $\bar{B} \to D^* \ell \bar{\nu}_\ell$: Implications for $|V_{cb}|$ and Tests of Lepton Flavor Universality

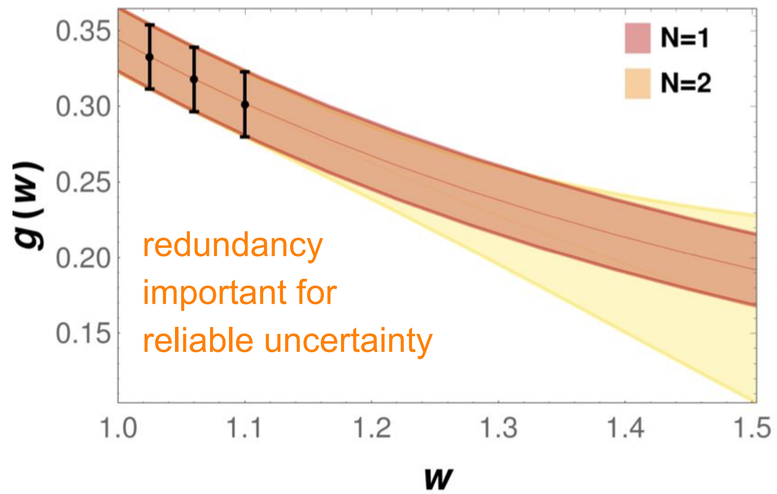
$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^* \ell \bar{\nu}_\ell)}{\mathrm{d}w\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\mathrm{V}\,\mathrm{d}\chi} = \frac{2G_\mathrm{F}^2 \eta_\mathrm{EW}^2 |V_\mathrm{cb}|^2 m_B^4 m_{\mathrm{D}^*}}{2\pi^4} \times \Big(J_{1s}\sin^2\theta_\mathrm{V} + J_{1c}\cos^2\theta_\mathrm{V} + (J_{2s}\sin^2\theta_\mathrm{V} + J_{2c}\cos^2\theta_\mathrm{V})\cos 2\theta_\ell + J_3 \sin^2\theta_\mathrm{V}\sin^2\theta_\ell\cos 2\chi + J_4\sin 2\theta_V \sin 2\theta_\ell \cos\chi + J_5 \sin 2\theta_V \sin\theta_\ell\cos\chi + (J_{6s}\sin^2\theta_\mathrm{V} + J_{6c}\cos^2\theta_\mathrm{V})\cos\theta_\ell + J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8 \sin 2\theta_V \sin 2\theta_\ell\sin\chi + J_9\sin^2\theta_\mathrm{V}\sin^2\theta_\ell\sin 2\chi \Big) .$$

Need to make sure we have matching bin boundaries in w

Model Independence versus Overfitting

Model independence vs overfitting

$$\phi(z) = \sum_i^{\infty} a_i z^i, \qquad \sum_i^{\infty} a_i^2 < 1$$



1. Where do we truncate the series?
2. How can we include unitarity constraints?
3. These questions are related.

Model Independence versus Overfitting

Models on the market :

Different options with various pro/cons:

1. Frequentist fits with strong χ^2 ***penalty*** outside unitarity; increase BGL order till χ^2_{min} is stable. Can compute CL intervals ***New: Feldman-Cousins***. Bigi, PG, 1606.08030, Jung,Schacht,PG 1905.08209
2. Frequentist fit with ***Nested Hypothesis Test or AIC*** to determine optimal truncation order: go to order $N+1$ if $\Delta\chi^2 = \chi^2_{min,N} - \chi^2_{min,N+1} \geq 1{,}2$ Check unitarity a posteriori Bernlochner et al, 1902.09553
3. ***Bayesian inference*** using unitarity constraints as prior with BGL Flynn, Jüttner, Tsang 2303.11285 or in the ***Dispersive Matrix approach (which avoids truncation!)***, Martinelli, Simula, Vittorio et al. 2105.02497

1D Example

$$-1 \le \mu \le 1$$

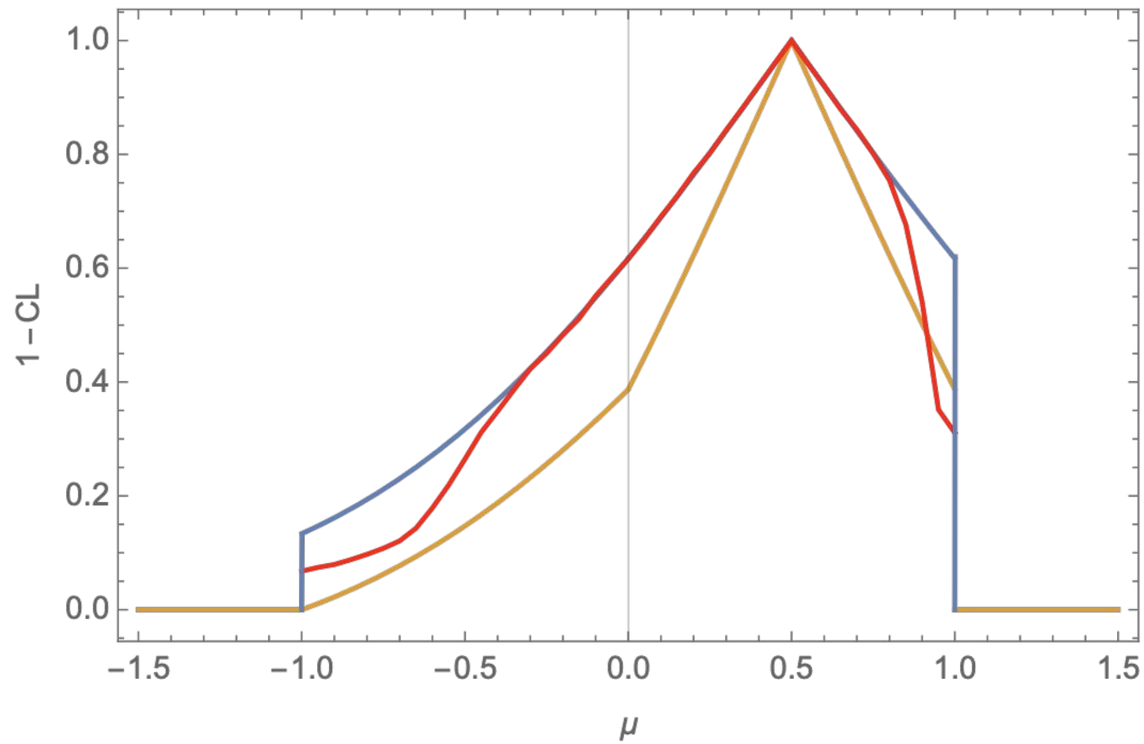
$$\text{exp} \quad \mu = 0.5 \pm 1$$

gaussian

A) Bayesian approach: cut gaussian PDF, compute CL from new PDF

B) Large χ^2 penalty outside physical region

C) Feldman Cousins toy MC coverage



$$1 - CL = \frac{N(\Delta\chi^2_{toy} > \Delta\chi^2_{data})}{N_{toys}}$$

$$\Delta\chi^2_{\rm data} \equiv \min(\chi^2_{{\rm data},c}) - \min(\chi^2_{\rm data})$$

$$\Delta\chi^2_{\rm toys} \equiv \min(\chi^2_{{\rm toy},c}) - \min(\chi^2_{\rm toy})\,.$$

Feldman / Cousins

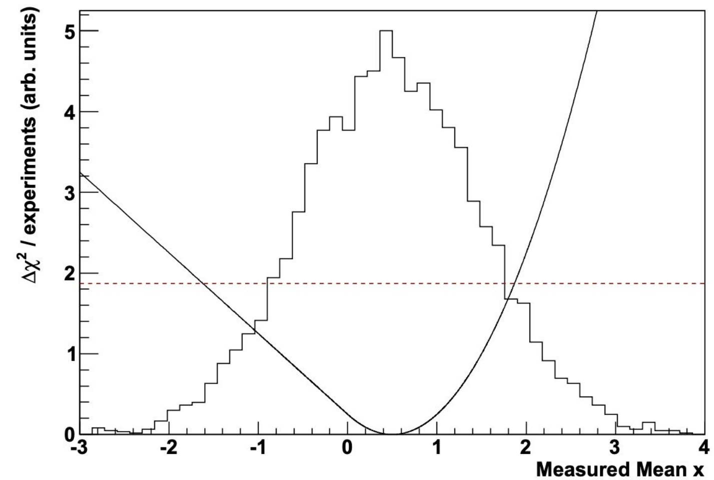
$$-2\ln R \equiv \Delta\chi^2 = \chi^2 - \chi^2_{\text{best}}$$
$$= \chi^2(x,\mu) - \chi^2(x,\mu_{\text{best}}) \ ,$$

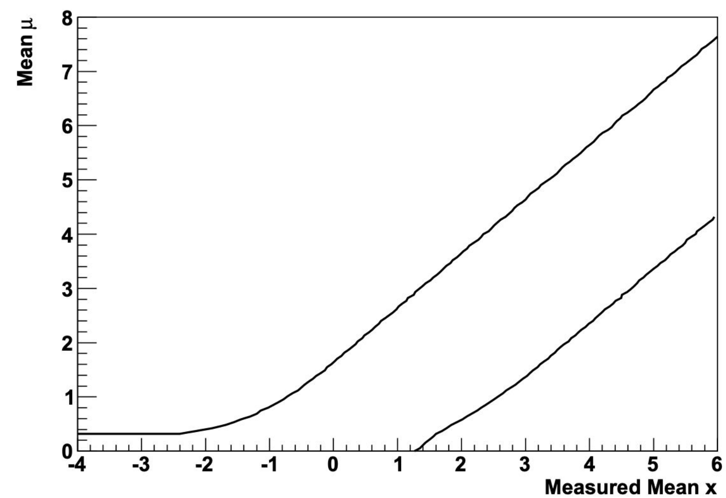
1. At the considered value of the true parameter μ_0, generate a toy experiment by drawing a value x_{toy} from the p.d.f. That is, draw from the unit Gaussian $G(\sigma = 1, \mu = \mu_0)$.

2. Compute $\Delta\chi^2_{\text{toy}}$ for the toy experiment, following Eq. 6. For $\chi^2(x,\mu)$, x is set to x_{toy} and μ is set to μ_0. For $\chi^2(x,\mu_{\text{best}})$, $x = x_{\text{toy}}$ and $\mu_{\text{best}} = \max(0, x_{\text{toy}})$ implementing the boundary at zero.

3. Find the value $\Delta\chi^2_c$, such that α of the toy experiments have $\Delta\chi^2_{\text{toy}} < \Delta\chi^2_c$.

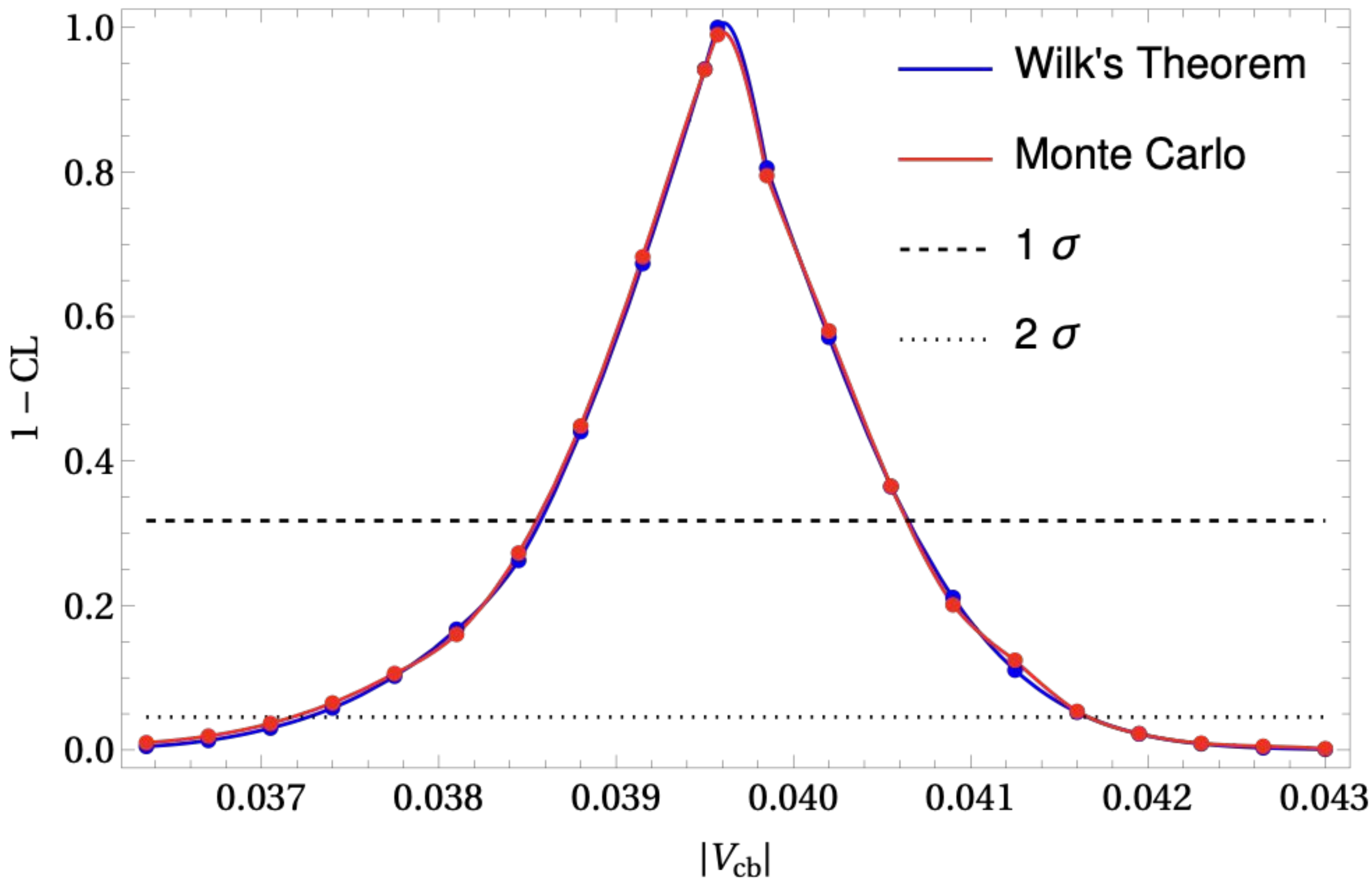
4. The interval $[x_1, x_2]$ is given by all values of x such that $\Delta\chi^2(x,\mu_0) < \Delta\chi^2_c$.





From Moritz Karbach, **[arXiv:1109.0714]**

1 - CL from Feldman Cousins



Convergence order by order

	$\lvert V_{cb}\rvert$	χ^2	d.o.f.	N	$\lvert\rho_{max}\rvert$
BGL_{111}	40.4 ± 0.8	45.6	34	3	0.70
BGL_{112}	40.9 ± 0.9	43.4	33	4	0.98
$\mathbf{BGL_{121}}$	40.7 ± 0.9	45.2	33	4	0.60
BGL_{122}	41.5 ± 1.1	42.3	32	5	0.98
BGL_{131}	38.1 ± 1.7	41.7	32	5	0.98
BGL_{132}	39.0 ± 1.6	37.5	31	6	0.98
BGL_{211}	39.7 ± 1.0	42.7	33	4	0.99
BGL_{212}	40.4 ± 1.0	39.3	32	5	0.99
BGL_{221}	37.1 ± 1.2	37.7	32	5	0.99
BGL_{222}	37.9 ± 2.0	37.5	31	6	1.00
BGL_{231}	37.2 ± 1.8	37.7	31	6	0.99
BGL_{232}	38.8 ± 1.7	37.2	30	7	0.98
BGL_{311}	38.5 ± 0.9	40.1	32	5	0.95
BGL_{312}	39.9 ± 1.1	36.9	31	6	0.98
BGL_{321}	37.3 ± 1.2	37.3	31	6	0.97
BGL_{322}	38.9 ± 2.1	36.5	30	7	0.99
BGL_{331}	39.6 ± 2.3	36.3	30	7	0.99
BGL_{332}	40.1 ± 2.3	35.9	29	8	0.99

[2301.07529, Table XVI from PRD]

	$\lvert V_{cb}\rvert$	χ^2	dof	N	ρ_{max}
BGL_{111}	40.3 ± 0.8	45.7	32	3	0.71
BGL_{112}	40.8 ± 0.8	42.6	31	4	0.97
$\mathbf{BGL_{121}}$	$\mathbf{40.6 \pm 0.9}$	**45.3**	**31**	**4**	**0.62**
BGL_{122}	41.4 ± 1.0	41.5	30	5	0.97
BGL_{131}	39.9 ± 0.9	42.4	30	5	0.61
BGL_{132}	40.7 ± 1.0	39.3	29	6	0.98
BGL_{211}	39.8 ± 0.9	42.1	31	4	0.99
BGL_{212}	40.4 ± 0.9	37.5	30	5	0.99
BGL_{221}	40.9 ± 1.0	45.1	30	5	0.93
BGL_{222}	39.2 ± 1.0	36.5	29	6	0.96
BGL_{231}	40.3 ± 1.0	41.8	29	6	0.94
BGL_{232}	41.0 ± 1.0	39.0	28	7	0.97
BGL_{311}	39.8 ± 0.9	42.1	30	5	0.99
BGL_{312}	40.4 ± 0.9	37.4	29	6	0.99
BGL_{321}	38.5 ± 0.9	39.4	29	6	0.65
BGL_{322}	39.2 ± 1.0	36.4	28	7	0.96
BGL_{331}	38.3 ± 0.9	38.1	28	7	0.86
BGL_{332}	38.7 ± 1.5	36.0	27	8	0.99

Belle18+HPQCD

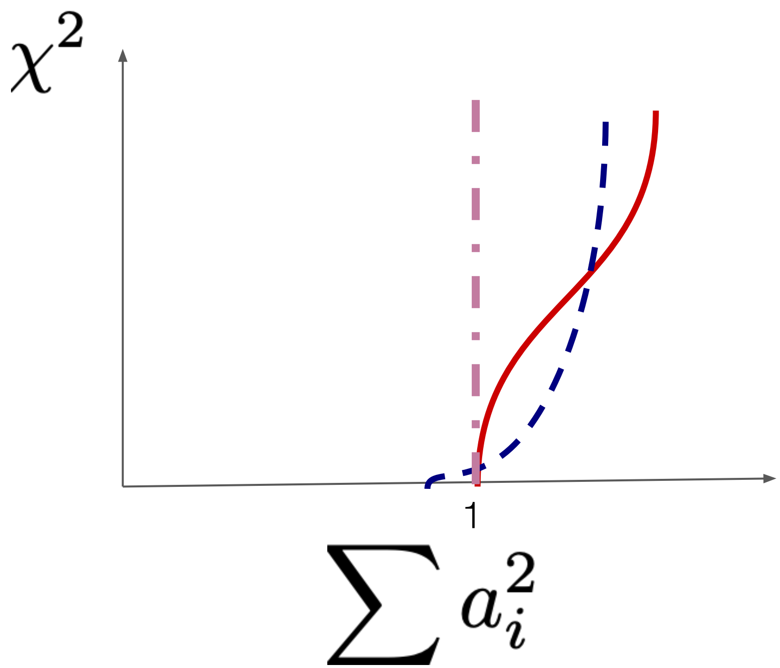
BGL exp	χ^2	$\lvert V_{cb}\rvert$
0001	78	41.0(8)
0101	68	41.2(8)
0111	57	40.8(8)
1111	57	40.8(8)
1121	54	40.6(8)
1222	52	40.6(8)
2222	50	40.4(8)
2232	50	40.4(8)
3333	50	40.4(8)

Lack of convergence?

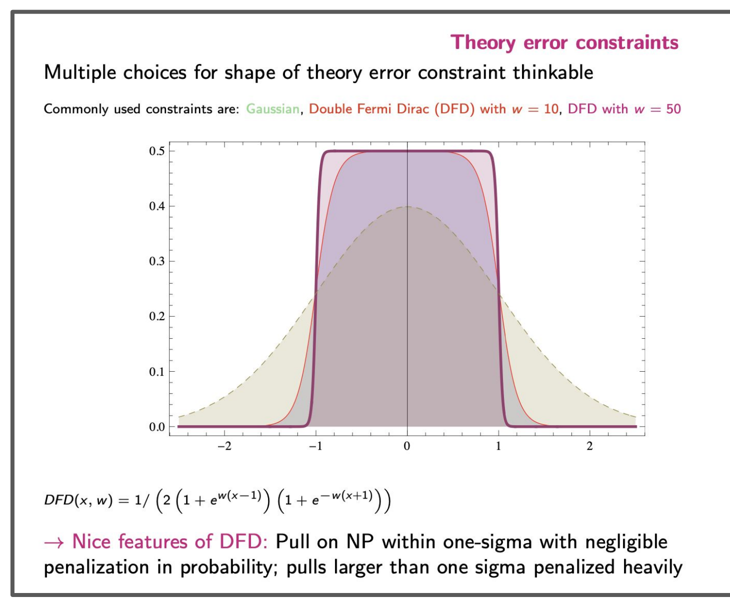
The problem with constraints

If we impose UT constraints directly into the fit, there are many shapes possible

Problem is that the ‘graduality’ on how the constraint is imposed, has an impact on the uncertainties of the higher coefficients. Posed differently, it may be that their uncertainties do not have a strict meaning in terms of a CL



These different penalties will give you different Uncertainties for higher order coefficients

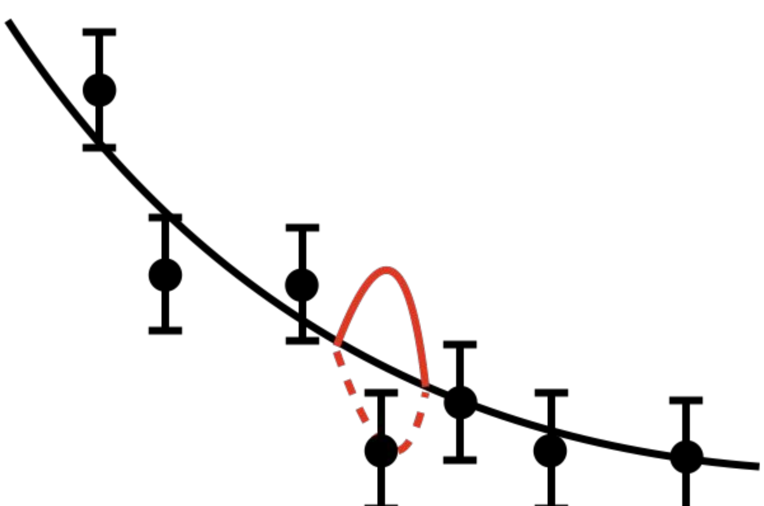


Watch out for biases

From talking with Patrick and Zoltan

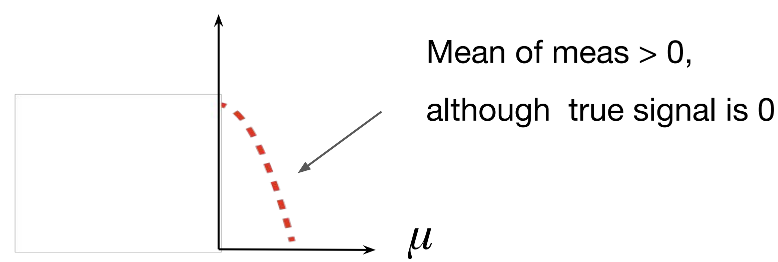
Patrick and Zoltan voiced **concerns** that imposing UT could lead to **biases**.

Zoltan gave a historical example from BaBar and Patrick the generic example of enforcing positive signal yields.



Textbook example:

- Search for small Signal over huge Bkg (true signal e.g. zero or negligible)
- If you impose $\mu >= 0$ and repeat your measurements you will always fit a value in [0, positive number]



Benchmarking different approaches

How can we make progress on this?

Let’s benchmark it!

Talked with Patrick, Paolo, Dean, Zoltan, Markus and others about this

I think we can actually solve this (and add a useful piece to the report)

Zoltan framed the challenge extremely well:

“I would like to have an approach that works for 5/ab now, that does not need adjustments.”

Patrick gave us the other critical ingredient: We can produce a **large number** of **possible BGL shapes** as true underlying distributions **that respect unitarity**

Benchmark procedure steps:

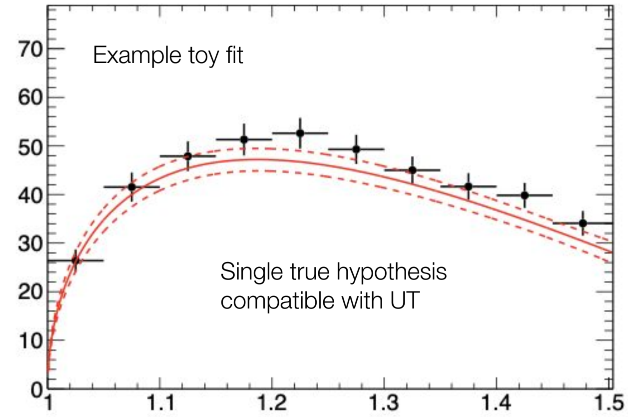
Step 1 : Produce a **large number** of **possible BGL shapes** as true underlying distributions **that respect unitarity** and a **given true $|V_{cb}|$** value

Step 2 : Use these shapes and **produce toys** / replica measurements **with** our **current** (or a future) experimental precision / covariance

Step 3 : Apply the different procedures (NHT, AIC, Feldman/Cousins, stability by eye, …) to determine FFs, $|V_{cb}|_{toy}$ and σ_{toy}

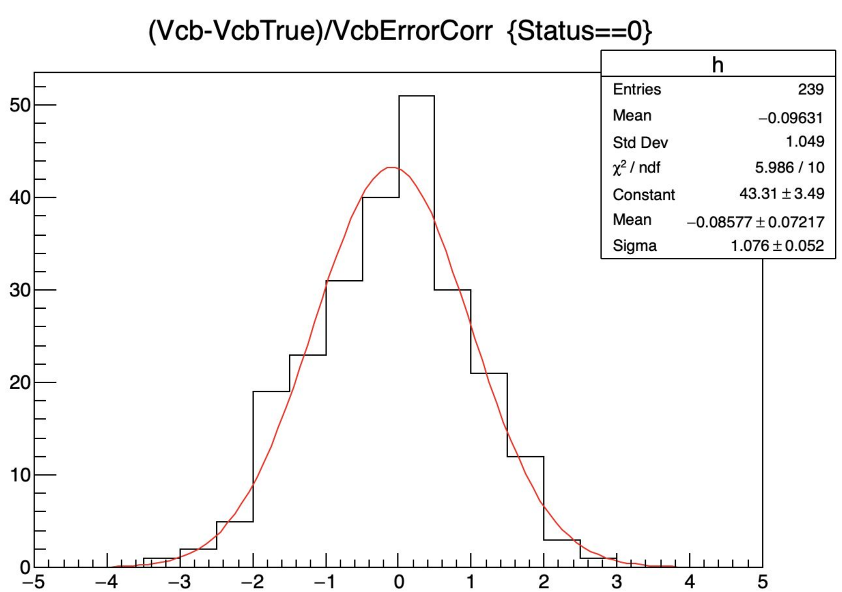
Step 4 : Study pulls of toys : $(|V_{cb}|_{toy} - |V_{cb}|_{true}) / \sigma_{toy}$

Toy example (very preliminary)

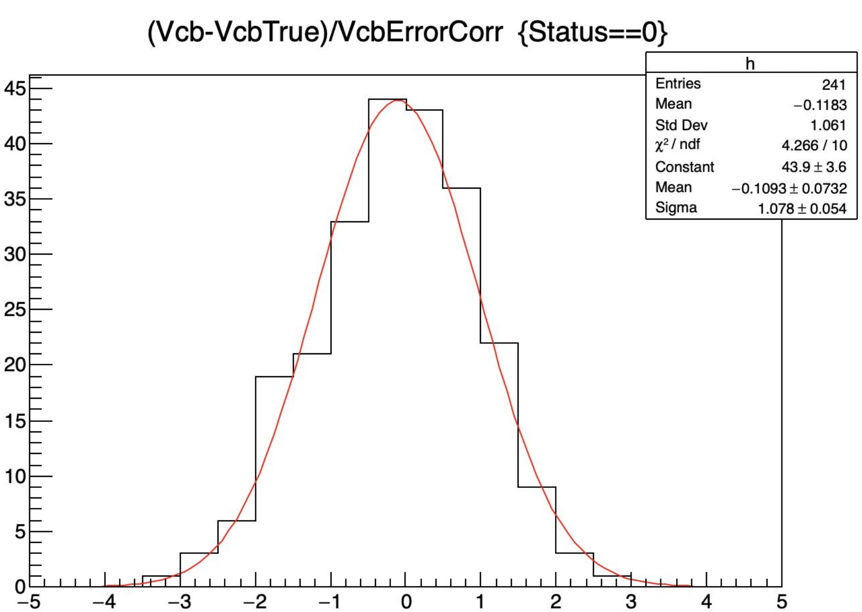


NHT

AIC



$(|V_{cb}|_{toy} - c) / \sigma_{toy}$



$(|V_{cb}|_{toy} - c) / \sigma_{toy}$

Remaining Truncation Uncertainty

Input from Bob

Bob had another intriguing thought : can we not just use the UT bound to assess how dependent we are on truncating the BGL series?

We certainly could !

(although this assumes that we do not exceed UT, i.e. somehow choose to impose it in the fit)

Idea: e.g. fit N coefficients

Chose **next coefficient** as shown below; check if $|V_{cb}|$ is affected

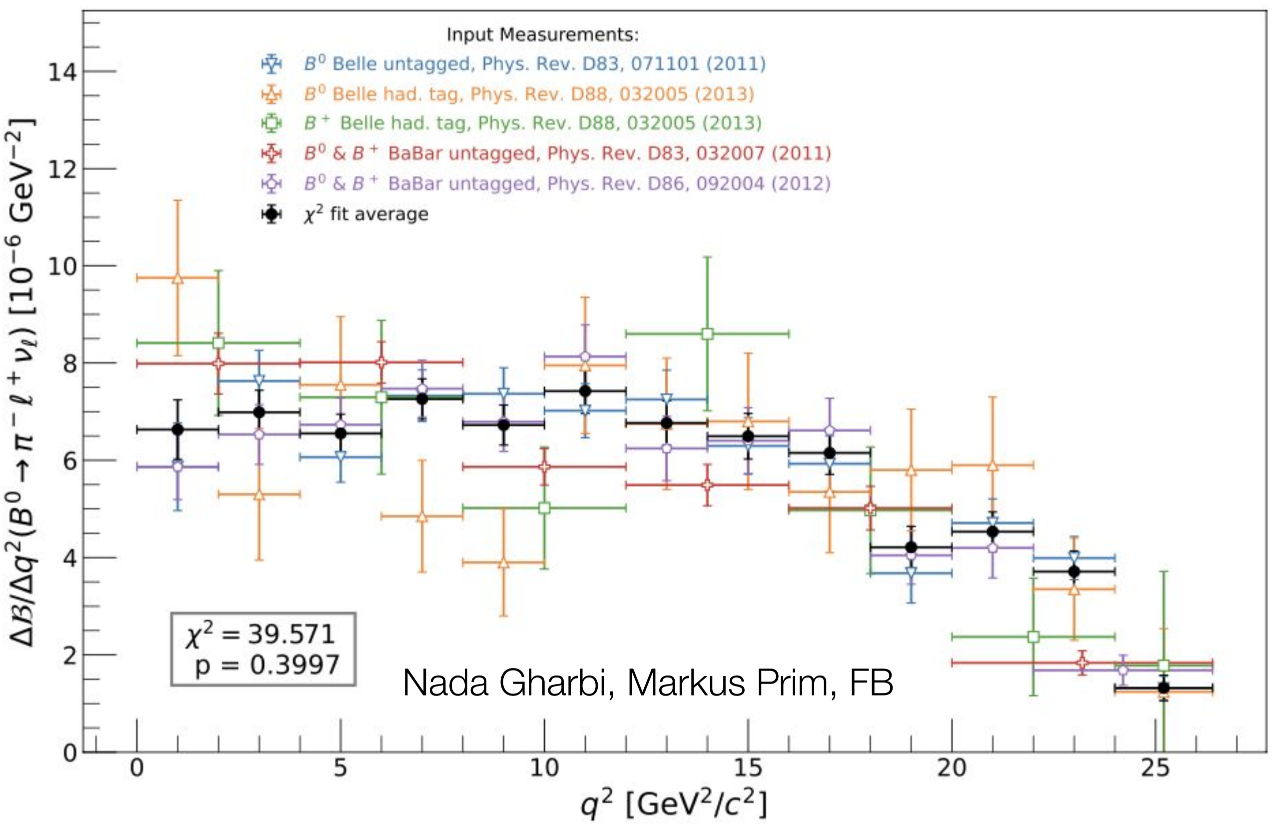
$$\sum_n^N a_n^2 < 1 \longrightarrow a_{n+1} = \sqrt{1 - \sum_n^N a_n^2}$$

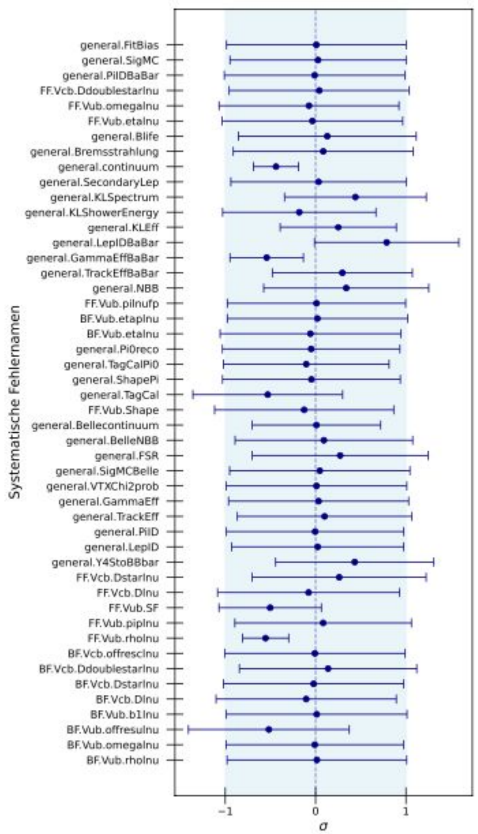
Averages of experimental information

Good example: B → $\pi \ell \nu$ average from **HFLAV**

1. **Average** all the **experimental information**

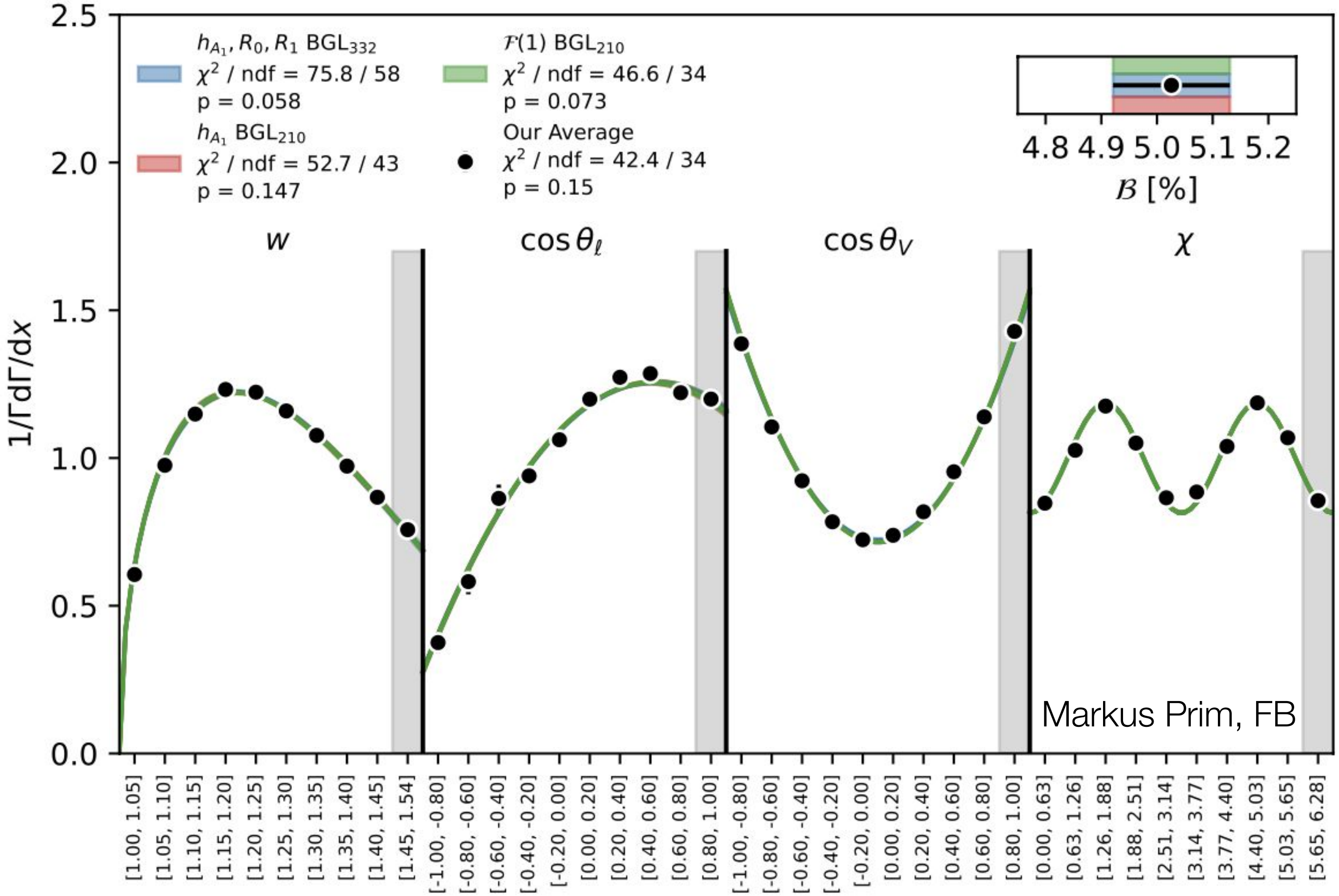
2. Then do a **V_{ub} fit**



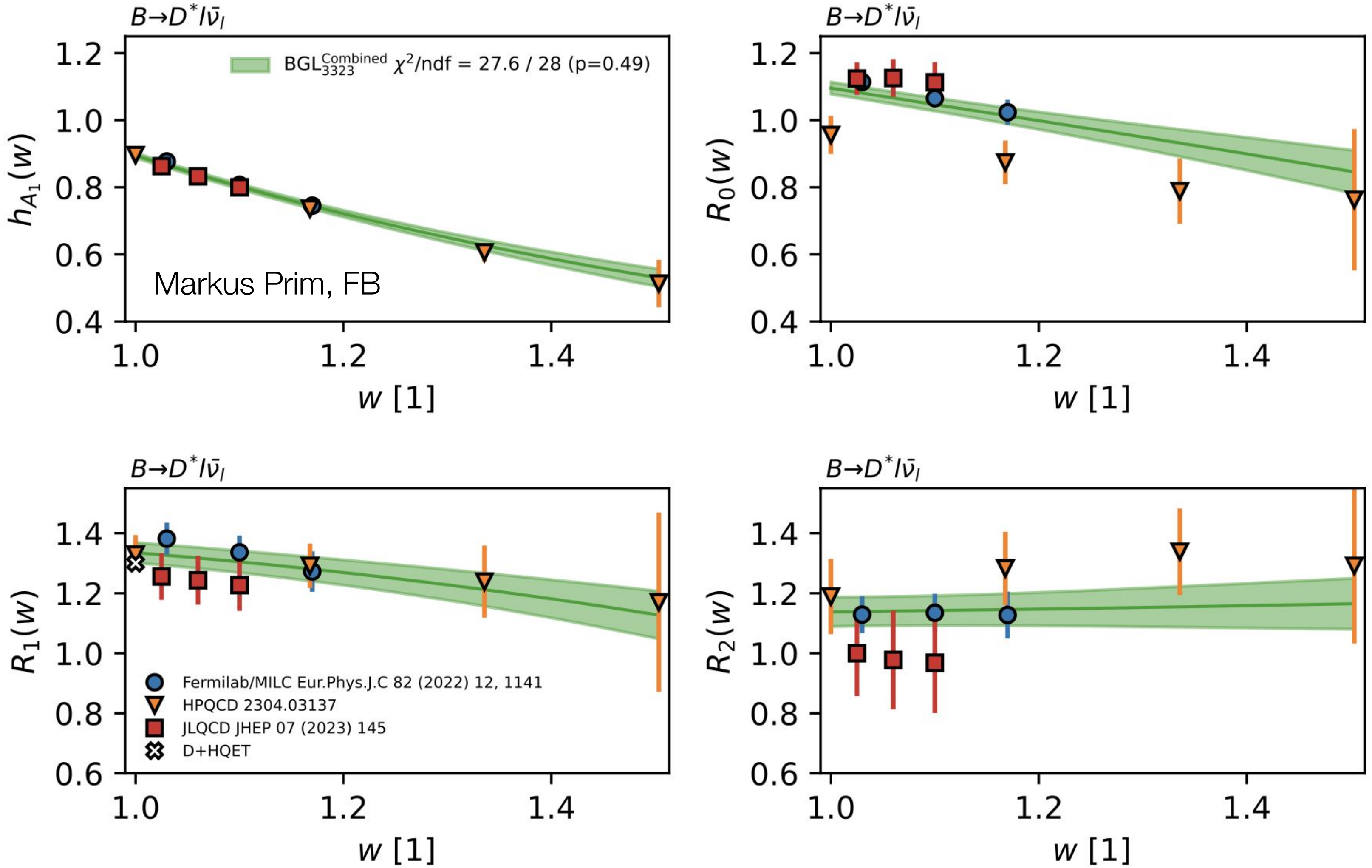


Averages of experimental information

Average of **tagged Belle** and **untagged Belle II** B → D* $\ell\nu$ in prep. for HFLAV report



Lattice (dis)agreements



- QED effects → should provide measurements w/ QED effects included
- Exclusive case (QED)
- More angular analysis
    - Moments in forward versus backward
    - Maybe more granular useful?
- BF measurements?
-

- Test from Bob
    - $\text{Sum}^n\ a^2 < 1 \rightarrow$ add $a_{n+1} = 1 -$ sum