Challenges in ϕ_3 **Combination** @ Belle + Belle II

Belle II Note: Note-PH-2023-024
Maillist:b2n-2023-024

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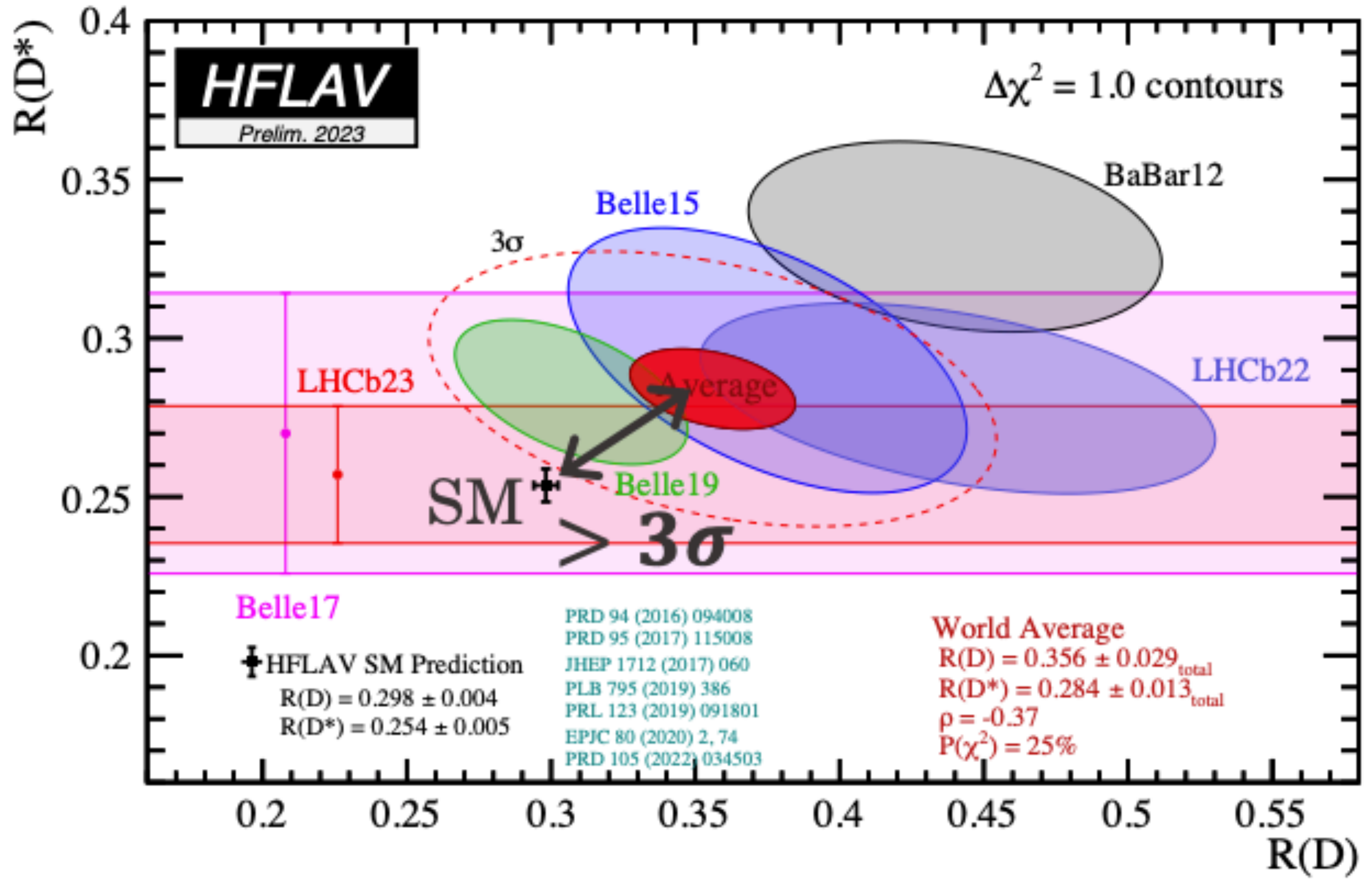
2023 Belle II Physics Week
10.30–11.03

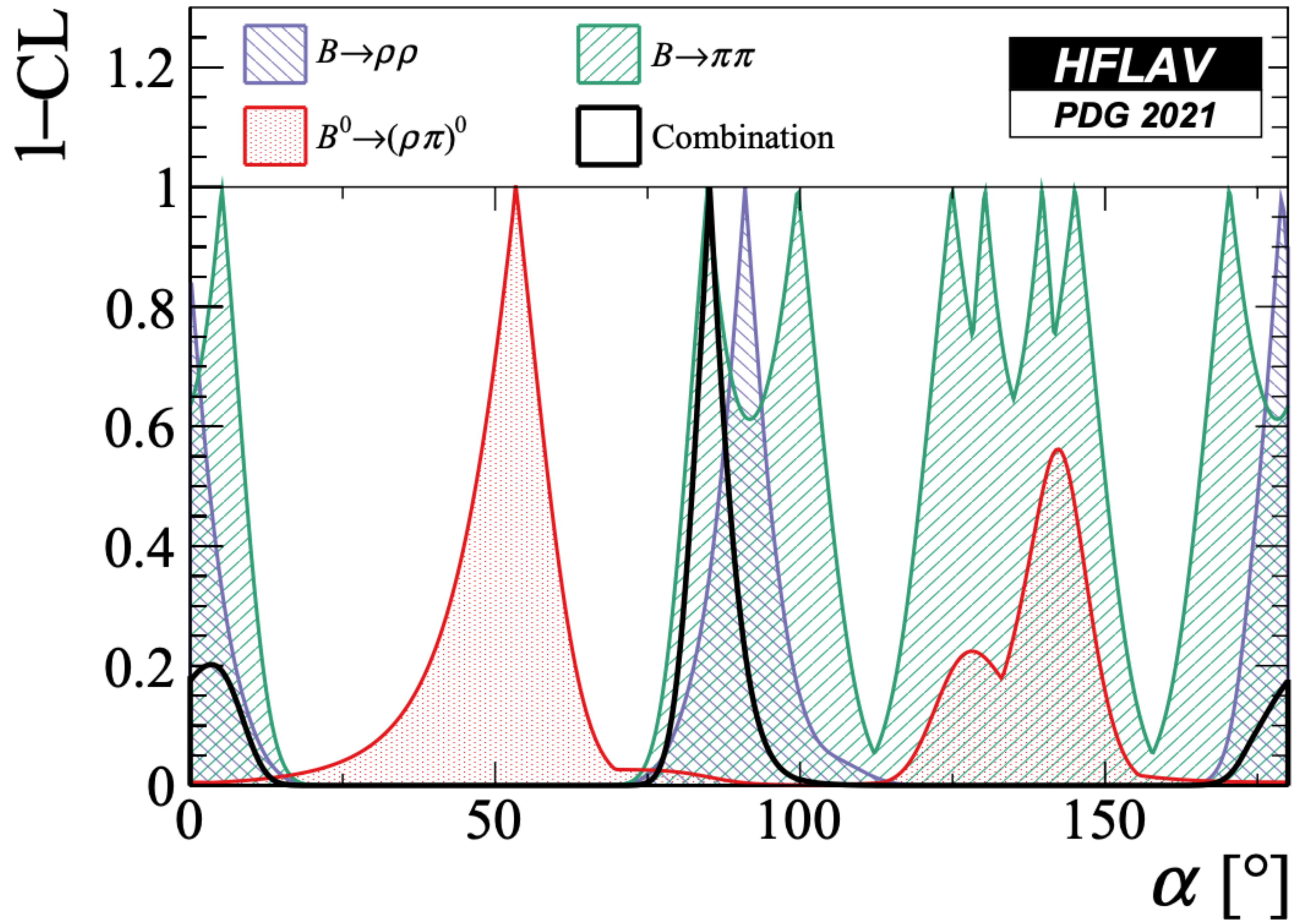
BELLE

Belle II

Motivation

- In particle physics, we may measure same parameter of interests through a few different channels, from different experiments.
- Sometimes, we need to do combination among few different channels.

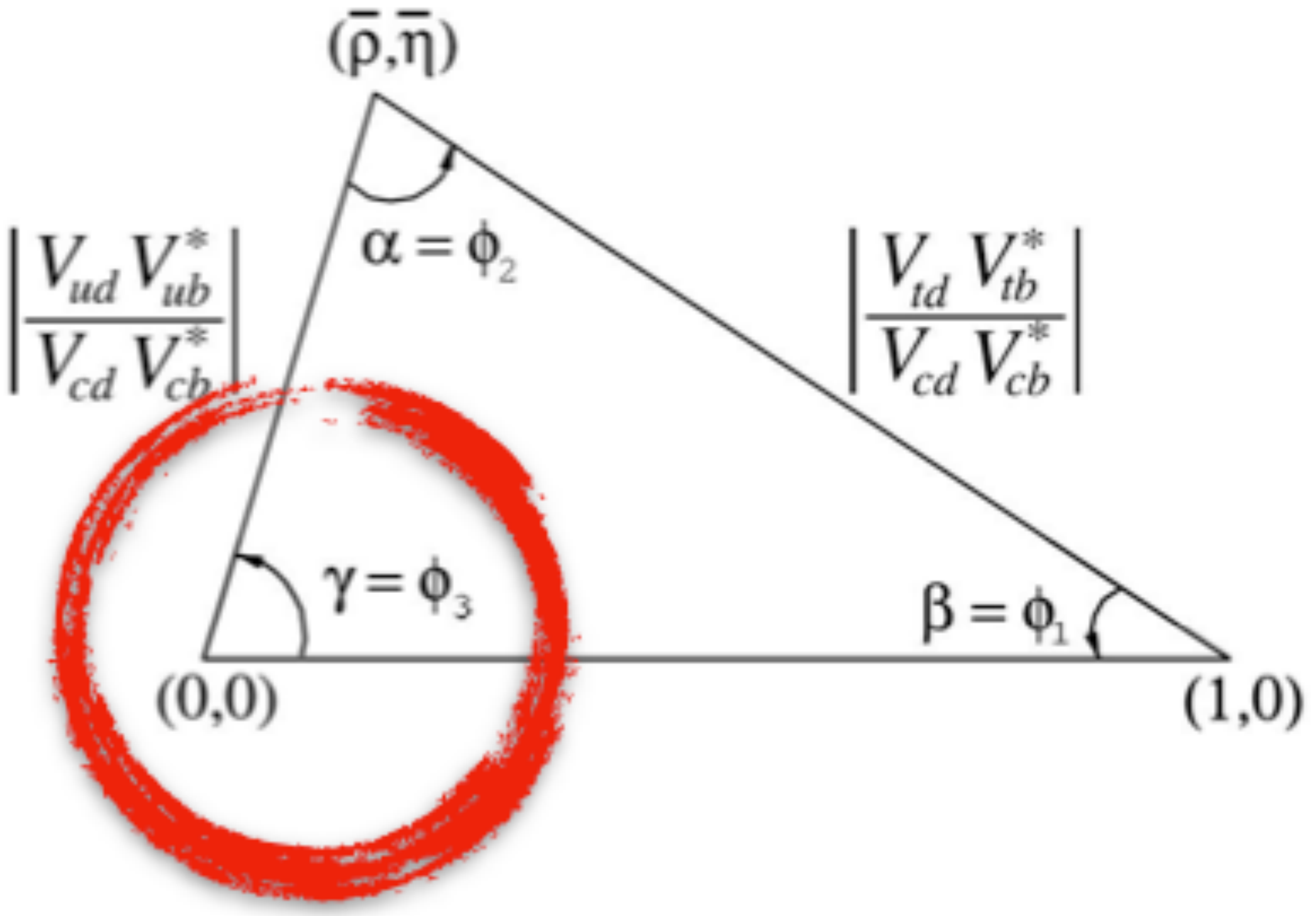




- If there is no combination, how to compare one SM point with several experimental results?
- For α/ϕ_2 from isospin study, multi-favored solution from one channel. Combination may tell us a single solution!

About ϕ_3 combination

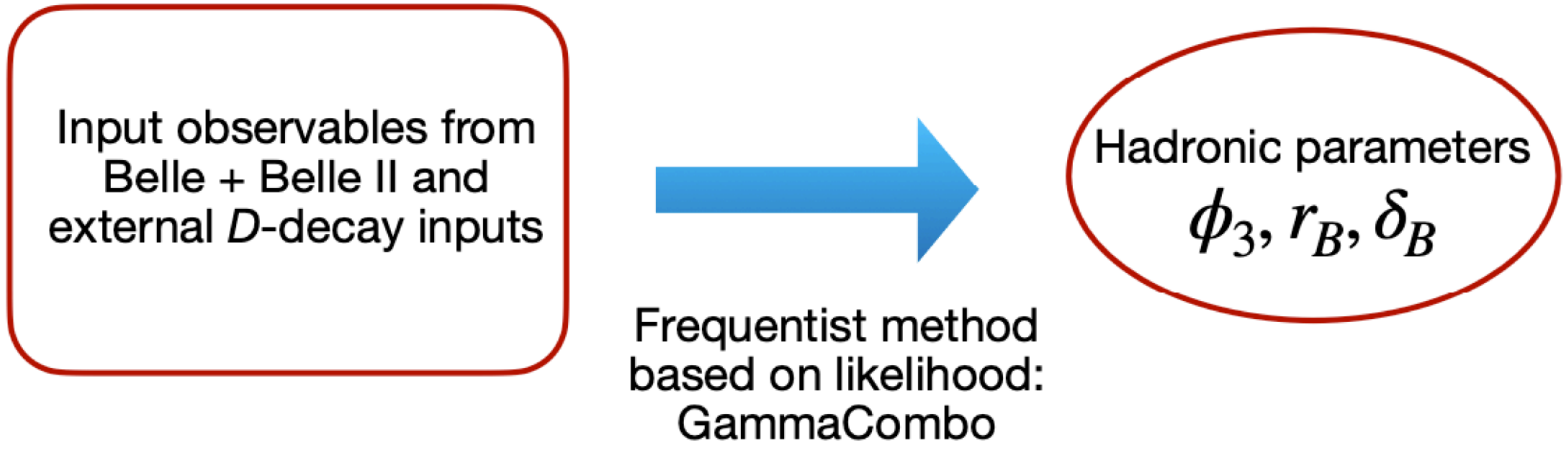
- $\phi_3 = \gamma = \arg(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*})$, one angle of the CKM triangle.
- Current W.A.: $\phi_3 = (66.2^{+3.4}_{-3.6})^\circ$[HFLAV], statistically uncertainty dominated.
- Theoretically clean, non-tree SM contribute ~10^{-7} [arXiv:1308.5663]
- Experimental precision will achieve < 1° by both LHCb and Belle II in next decades.



B decay	D decay	Method	Data set (Belle + Belle II) [fb^{-1}]	
$B^+ \to Dh^+$	$D \to K_S^0 h^- h^+$	BPGGSZ	$711 + 128$	[JHEP 02 063 (2022)]
$B^+ \to Dh^+$	$D \to K_S^0 \pi^- \pi^+ \pi^0$	BPGGSZ	$711 + 0$	[JHEP 10 178 (2019)]
$B^+ \to Dh^+$	$D \to K_S^0 \pi^0, K^- K^+$	GLW	$711 + 189$	[arxiv:2308.05048]
$B^+ \to Dh^+$	$D \to K^+ \pi^-, K^+ \pi^- \pi^0$	ADS	$711 + 0$	[PRL 106 231803 (2011)]
$B^+ \to Dh^+$	$D \to K_S^0 K^- \pi^+$	GLS	$711 + 362$	[arxiv:2306.02940]
$B^+ \to D^* K^+$	$D \to K_S^0 \pi^- \pi^+$	BPGGSZ	$605 + 0$	[PRD 81 112002 (2010)]
$B^+ \to D^* K^+$	$D \to K_S^0 \pi^0, K_S^0 \phi, K_S^0 \omega, K^- K^+, \pi^- \pi^+$	GLW	210+0	[PRD 73 051106 (2006)]

- Many results on ϕ_3 from Belle/Belle II, using different channels/methods.
- Why not combine them?
- A single ϕ_3 is more natural.

Workflow



① Read all Belle/Belle II ϕ_3 papers

② Construct a combined likelihood

③ Find the solution with minimum -LLH

$$\mathcal{L}(\vec{\alpha}|\vec{A}^{\rm obs}) = \prod_i f_i(\vec{A}_i^{\rm obs}|\vec{\alpha}),$$

$$f_i(\vec{A}_i^{\rm obs}|\vec{\alpha}) \propto \exp\left(-\frac{1}{2}(\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\rm obs})^T \underline{V_i^{-1}} (\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\rm obs})\right)$$

V covers uncertainties and correlations.

Sounds very easy, especially we have the available package from LHCb: GammaCombo.

In practice, still some **challenges** during the combination.

Challenge 1: asymmetric uncertainties

$$f_i(\vec{A}_i^{\mathrm{obs}}|\vec{\alpha}) \propto \exp\left(-\frac{1}{2}(\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\mathrm{obs}})^T V_i^{-1}(\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\mathrm{obs}})\right)$$

- In likelihood construction, use symmetric Gaussian.
- But sometimes, the measured observables are not.

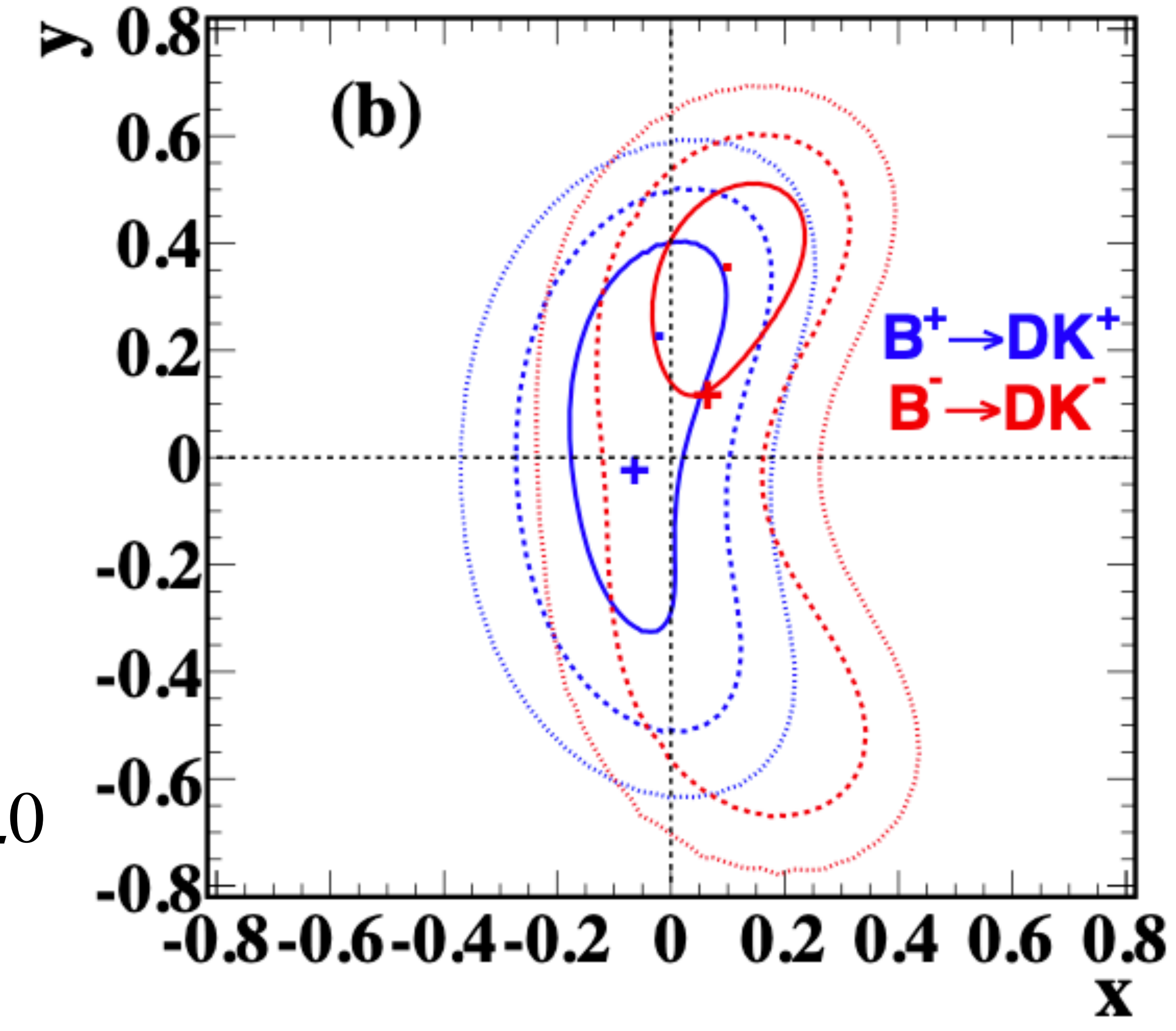
e.g.

$$\mathcal{A}_{DK} = -0.39^{+0.26}_{-0.28}(\mathrm{stat})^{+0.04}_{-0.03}(\mathrm{syst}),$$
$$\mathcal{A}_{D\pi} = -0.04 \pm 0.11(\mathrm{stat})^{+0.02}_{-0.01}(\mathrm{syst}),$$

$B^+ \to Dh^+, D \to K^+\pi^-$

Phys. Rev. Lett. 106 (2011) 231803

e.g.



$B^+ \to Dh^+, D \to K_S^0\pi^+\pi^-\pi^0$

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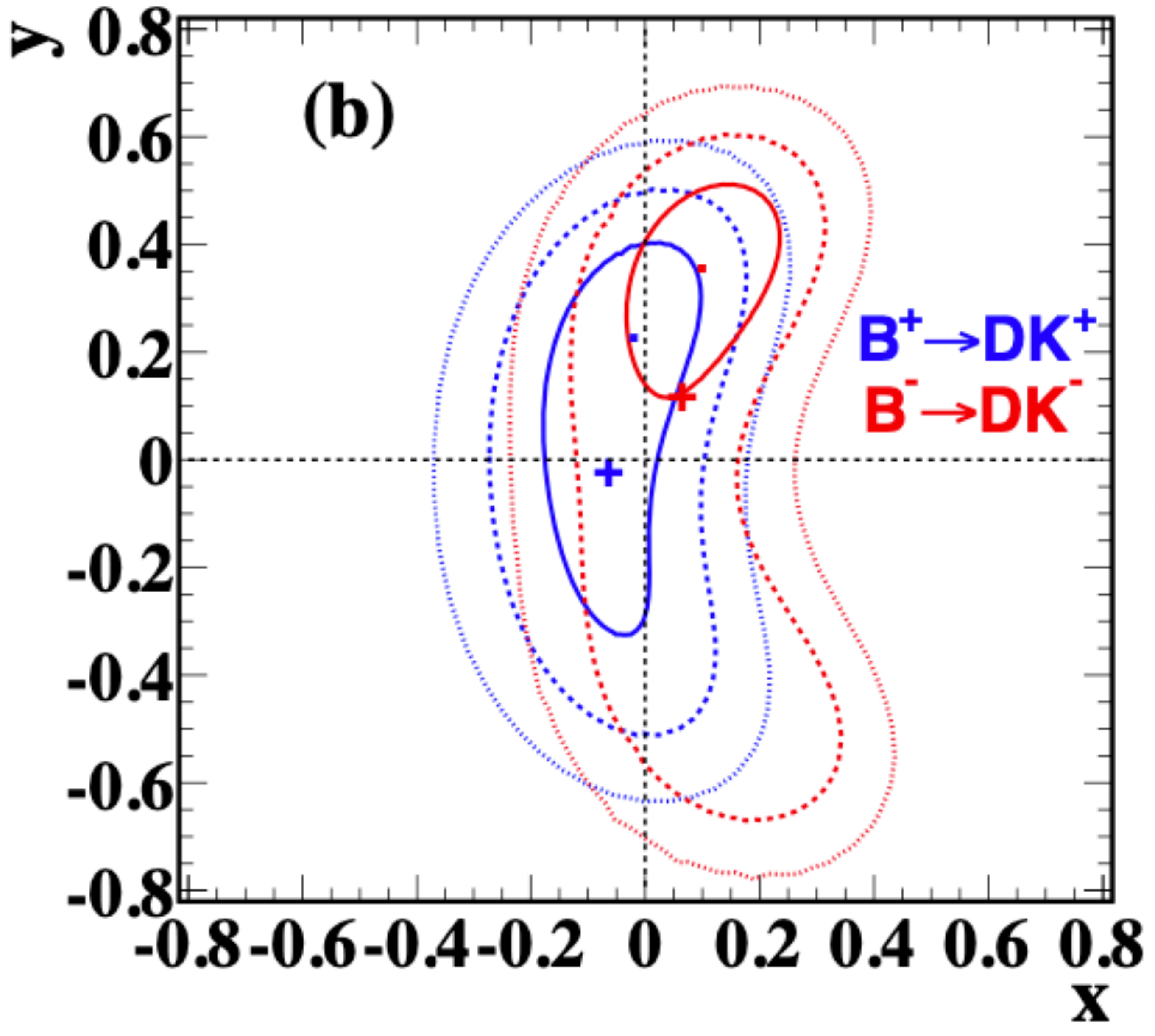
	$B^\pm \to D\pi^\pm$	$B^\pm \to DK^\pm$
x_+	$0.039 \pm 0.024\ ^{+0.018}_{-0.013}\ ^{+0.014}_{-0.012}$	$-0.030 \pm 0.121\ ^{+0.017}_{-0.018}\ ^{+0.019}_{-0.018}$
y_+	$-0.196\ ^{+0.080}_{-0.059}\ ^{+0.038}_{-0.034}\ ^{+0.032}_{-0.030}$	$0.220\ ^{+0.182}_{-0.541} \pm 0.032\ ^{+0.072}_{-0.071}$
x_-	$-0.014 \pm 0.021\ ^{+0.018}_{-0.010}\ ^{+0.019}_{-0.010}$	$0.095 \pm 0.121\ ^{+0.017}_{-0.016}\ ^{+0.023}_{-0.025}$
y_-	$-0.033 \pm 0.059^{+0.018}_{-0.019}\ ^{+0.019}_{-0.010}$	$0.354\ ^{+0.144}_{-0.197}\ ^{+0.015}_{-0.021}\ ^{+0.032}_{-0.049}$

Challenge 2: irregular correlation

$$f_i(\vec{A}_i^{\rm obs}|\vec{\alpha}) \propto \exp\left(-\frac{1}{2}(\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\rm obs})^T V_i^{-1}(\vec{A}_i(\vec{\alpha}) - \vec{A}_i^{\rm obs})\right)$$

- In likelihood construction, the correlation is described by the matrix.
- But sometimes, the correlation is more complex. Not just simple ellipse.

e.g.



$B^+ \to Dh^+, D \to K_S^0\pi^+\pi^-\pi^0$

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No same!

	x_+	y_+	x_-	y_-
x_+	1	0.486	0.172	−0.231
y_+		1	−0.127	0.179
x_-			1	0.365
y_-				1

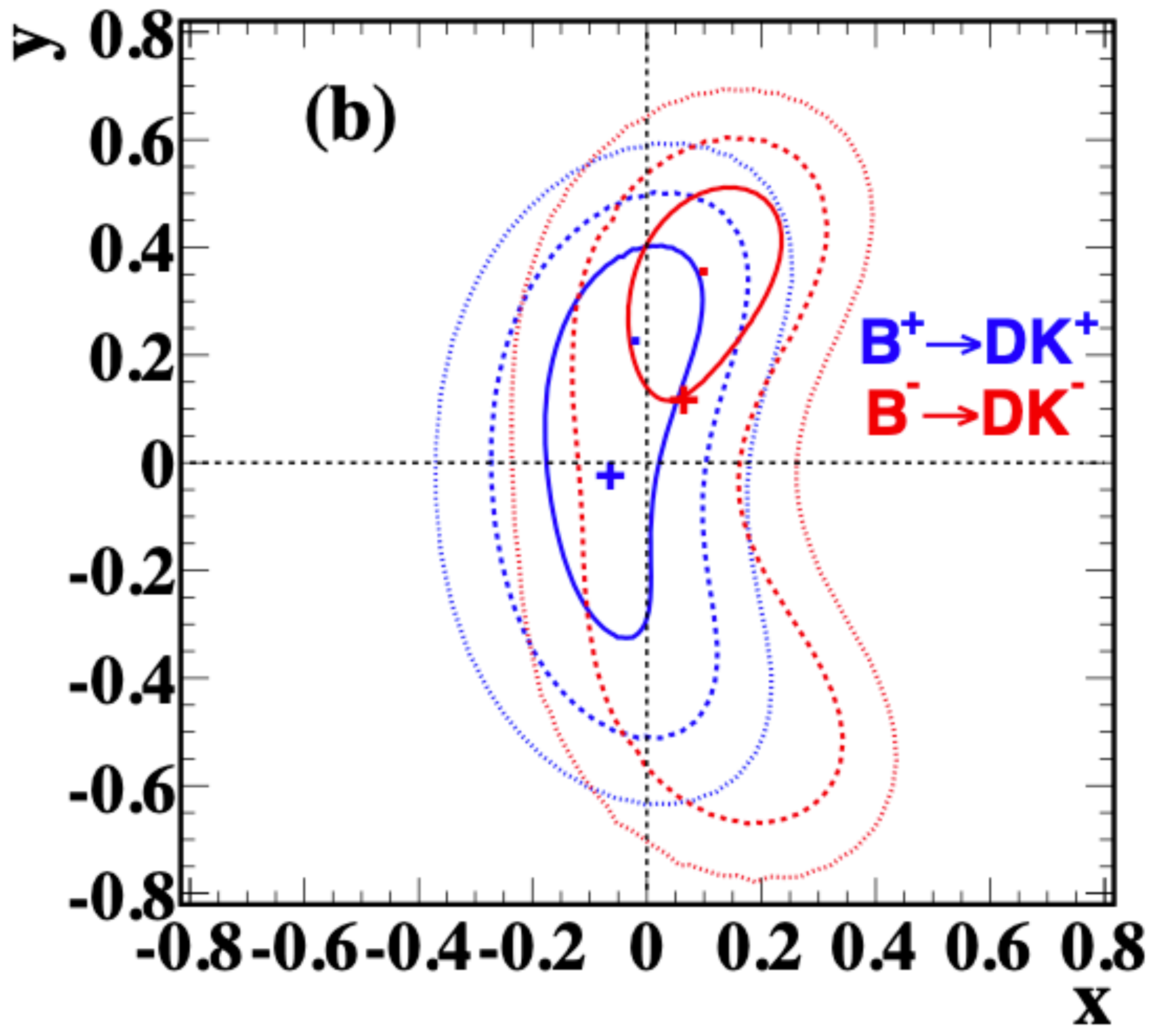
Matrix used in the combination.

Challenge 3: unknown correlation in some results

- Some results didn't provide the correlation matrix in their paper.
- How to include these results properly in the combination?

The ideal solution to challenge 1/2

$$f_i(\overrightarrow{A}_i^{\rm obs}|\overrightarrow{\alpha}) \propto \exp\left(-\frac{1}{2}(\overrightarrow{A}_i(\overrightarrow{\alpha}) - \overrightarrow{A}_i^{\rm obs})^T V_i^{-1}(\overrightarrow{A}_i(\overrightarrow{\alpha}) - \overrightarrow{A}_i^{\rm obs})\right)$$



- The most ideal solution is using the entire likelihood function, not just Gaussian function.
- So the irregular correlation, asymmetric uncertainties are included in the combination.

- In practice, such information are lost. “The student graduated.” “The result was 10 years ago. No files are left.” etc.
- Let’s see the compromise solution in ϕ_3 combination…

Solution to challenge 1: asymmetric uncertainties

- Symmetrize it!
  - Generate toy MC samples with asymmetric Gaussian.
  - Take the **standard width** of the sample as a new uncertainty.
  - Keep the origin mean value unchanged, as it's the point with maximum likelihood.

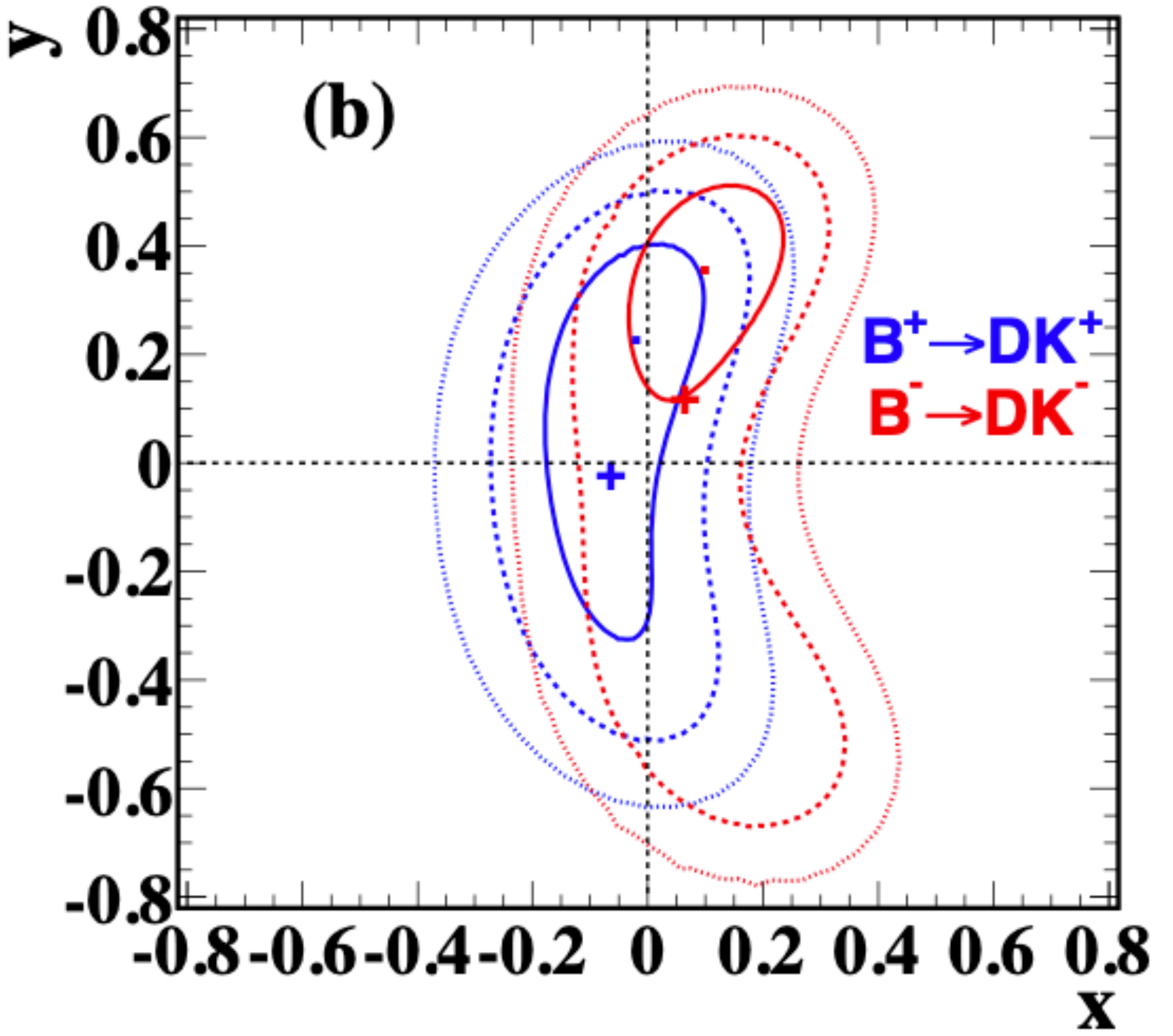
	$B^\pm \to D\pi^\pm$	$B^\pm \to DK^\pm$
x_+	$0.039 \pm 0.024\,^{+0.018}_{-0.013}\,^{+0.014}_{-0.012}$	$-0.030 \pm 0.121\,^{+0.017}_{-0.018}\,^{+0.019}_{-0.018}$
y_+	$-0.196\,^{+0.080}_{-0.059}\,^{+0.038}_{-0.034}\,^{+0.032}_{-0.030}$	$0.220\,^{+0.182}_{-0.541} \pm 0.032\,^{+0.072}_{-0.071}$
x_-	$-0.014 \pm 0.021\,^{+0.018}_{-0.010}\,^{+0.019}_{-0.010}$	$0.095 \pm 0.121\,^{+0.017}_{-0.016}\,^{+0.023}_{-0.025}$
y_-	$-0.033 \pm 0.059\,^{+0.018}_{-0.019}\,^{+0.019}_{-0.010}$	$0.354\,^{+0.144}_{-0.197}\,^{+0.015}_{-0.021}\,^{+0.032}_{-0.049}$

→ $0.22 \pm 0.376 \pm 0.032 \pm 0.072$

More discussion about asymmetric uncertainty [arXiv:physics/0401042] Thanks Lu Cao.

Solution to challenge 2: irregular correlation

- We can't get the full information, not just these contours.
- Still use correlation matrix only. Nothing we can do.

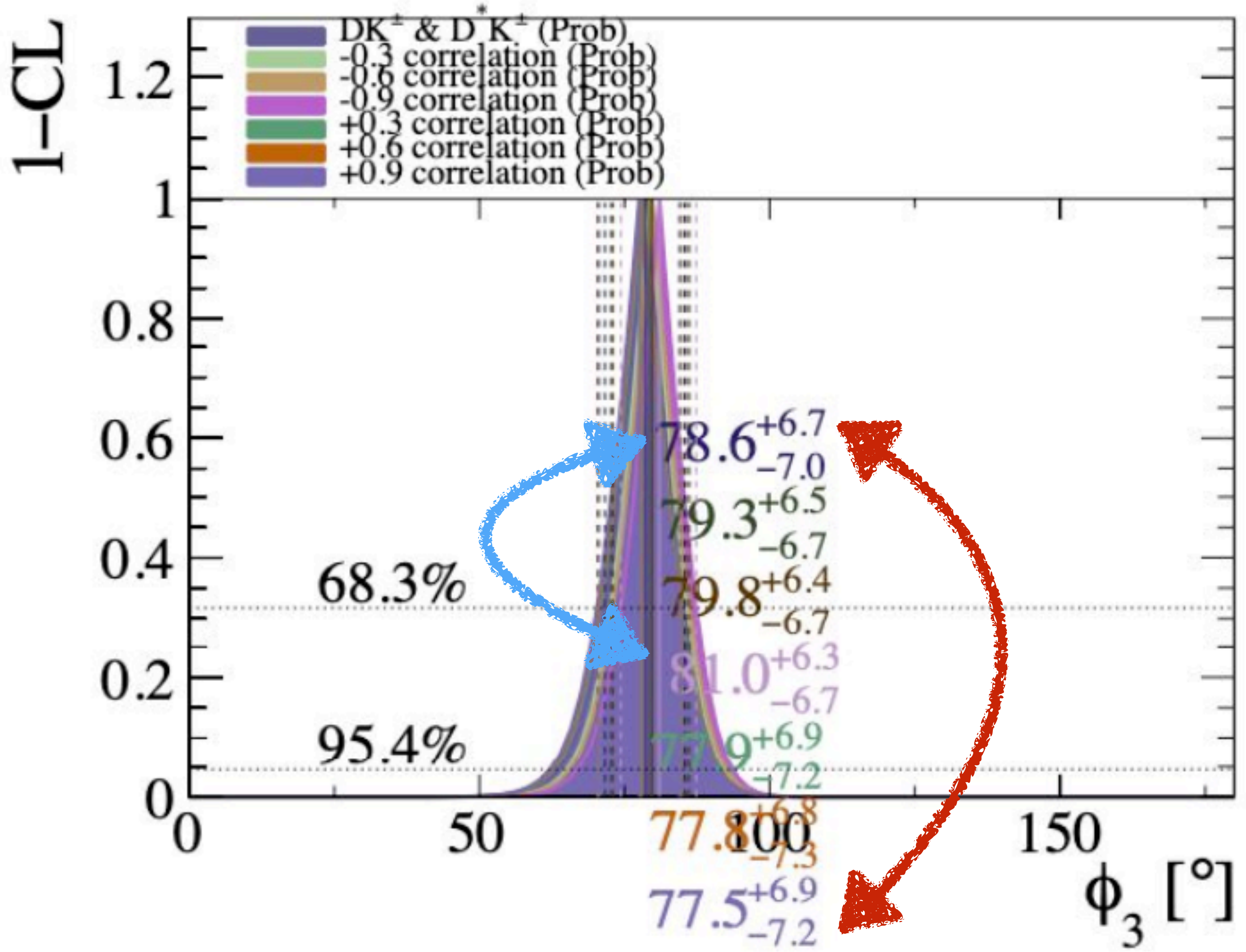


- Lucky thing: $B^+ \to Dh^+, D \to K_S^0\pi^+\pi^-\pi^0$ contribute little in this ϕ_3 combination.
- Lesson here: if the correlation is quite irregular. Better to save the full information, so your result will be used correctly by others.

Solution to challenge 3: unknown correlation in some results

- Try contact the author first!

- Solution, if the information is really lost:
  - assign 0 correlation for the nominal result.
  - Vary correlation up to ± 0.9 to check possible bias; take the maximum bias as additional systematic uncertainty.



- additional uncertainties to cover this missing correlation: $(78.6^{+6.8+2.4}_{-7.2-1.1})^{\circ}$
- Lucky, the bias are not much.
- Lessons: if you measure multi-observables, check the correlation and report it! Make your result more precise.

Summary

- Several issues and compromise solutions in combination ϕ_3 study.
- Reminder:
  - if you measure more than one observable, don't forget check the correlation and report the correlation matrix;
  - if the correlation is non-trivial, store the entire shape of likelihood scan for future precise combination.